$B\to\eta(\eta')K(\pi)$ in the standard model with flavor symmetry

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The observed branching ratios for $B\to K\eta'$ decays are much larger than factorization predictions in the standard model (SM). Many proposals have been made to reconcile the data and theoretical predictions. In this paper we study these decays within the SM using flavor SU(3) symmetry. Treating the singlet η_1 and octet $(\pi^{\pm,0},K^{\pm},\bar{K}^0,K^0,\eta_8)$ pseudoscalar mesons as a nonet multiplet, we find that if small annihilation amplitudes are neglected only 11 hadronic parameters are needed to describe $B\to PP$ decays where P can be one of the π, K, η and η' mesons. We find that existing data are consistent with the SM. We also predict several measurable branching ratios and CP asymmetries for $B\to K(\pi)\eta(\eta')$, $\eta(\eta')\eta(\eta')$ decays. Experiments in the near future can provide important tests for the standard model with flavor symmetry.

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Experimental data from CLEO, BaBar and Belle [1–4] have measured branching ratios of $B\to K\eta'$ around 6×10^{-5} which are substantially larger than theoretical calculations based on the naive factorization approximation in the standard model (SM) [5]. Although there are some improvements in calculating the branching ratios in the last few years by using the QCD improved factorization method [6], there are still large uncertainties in calculating the branching ratios for $B\to K\eta'$ because of issues related to η_1-η_8 mixing and the QCD anomaly associated with η_1. There are also many speculations about possible new physics beyond the SM in these decays [7]. Before any claim can be made about new physics, one must study the SM contributions in all possible ways to see if it is really inconsistent with experimental data.

In this paper we carry out a systematic study of $B\to K\eta'$ and, more generally, of processes of $B\to PP$ decays, with P being one of the π, K, η, η' in the final states, by using flavor symmetry in the SM. This way one can relate different decays to predict unmeasured branching ratios and CP asymmetries. Drastic deviations between the predicted relations and experimental data can provide information about the SM and models beyond. Similar considerations based on $SU(3)$ have been applied to $B\to PP$ decays, with P being one of the π or K in the pseudoscalar octet $(\pi^{\pm,0},K^{\pm},\bar{K}^0,K^0,\eta_8)$ [8], and have been shown to be consistent with data [9]. When considering $B\to PP$, with at least one of the P being an η or an η', one can introduce a singlet η_1 as an independent multiplet into the theory to form additional amplitudes to describe these decays [10]. One may also consider combining the singlet η_1 and the octet to form a nonet such that η_1 is automatically included in the theory.

Flavor $SU(3)$ symmetry with a nonet multiplet has been studied in kaon decays. There there are non-negligible symmetry breaking effects. For B decays one may also expect symmetry breaking effects to exist. There are also some studies of similar flavor symmetry for B decays with η' included [11,12]. Present data, however, are not able to make a clear statement about whether this symmetry is badly broken. In this paper we will take the flavor SU(3) symmetry with $(\pi^{\pm,0},K^{\pm},\bar{K}^0,K^0,\eta_{8,1})$ as a nonet as a working hypothesis and study whether experimental data can be explained by carrying out a systematic analysis. We find that the SM with such a flavor symmetry can explain all existing data, in particular large branching ratios for $B\to K\eta'$ decays can be obtained. We also predict some unmeasured branching ratios and CP asymmetries which can be used to further test the theory.

The quark level effective Hamiltonian can be written as [13]

$$H^q_{eff}=\frac{G_F}{\sqrt{2}}\left[V_{ub}V^*_{uq}(c_1O_1+c_2O_2)-\sum_{i=3}^{11}(V_{ub}V^*_{uq}c_i^{uc}+V_{tb}V^*_{tq}c_i^{tc})O_i\right]. \tag{1}$$

Here V_{ij} are KM matrix elements. The coefficients $c_{1,2}$ and c_i^{jk} are the Wilson coefficients which have been evaluated by several groups [13] with $|c_{1,2}|\gg|c_i^{jk}|$. O_i are operators consist of quarks and gluons.

The $B\to PP$ decay amplitudes can be parametrized as

$$A(B\to PP)=\langle PP|H^q_{eff}|B\rangle=\frac{G_F}{\sqrt{2}}[V_{ub}V^*_{uq}T+V_{tb}V^*_{tq}P], \tag{2}$$

where $B=(B_u, B_d, B_s)=(B^-, \overline{B}^0, \overline{B}^0_s)$, which form a fundamental representation of $SU(3)$. The amplitudes T and P are related to the hadronic matrix elements $\langle PP|O_i|B\rangle$ which are very difficult to calculate. For our purpose, however, we only need to know that under $SU(3)$ $O_{1,2}$, $O_{3-6,11}$, and O_{7-10} transform as $\overline{3}+\overline{3}'+6+\overline{15}$, $\overline{3}$, and $\overline{3}+\overline{3}'+6+\overline{15}$, respectively [8], and parameterize the amplitudes according to the flavor symmetry to be described below.

As mentioned earlier, there are two approaches to the problem from the flavor symmetry point of view. We first work with the approach of taking η_1 and $(\pi^{\pm,0}, K^\pm, \overline{K}^0, K^0, \eta_8)$ as a nonet. The nonet elements M^i_j are given as

$$(M^i_j)=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{3}}\eta_1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\eta_1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}.$$

One can write the T amplitude for $B\to PP$ in terms of the flavor symmetry invariant amplitudes as

$$\begin{aligned} T=&A^T_{\overline{3}}B_iH(\overline{3})^i(M^k_lM^l_k)+C^T_{\overline{3}}B_iM^i_kM^k_jH(\overline{3})^j \\ &+\tilde{A}^T_6B_iH(6)^{ij}_kM^l_jM^k_l+\tilde{C}^T_6B_iM^i_jH(6)^{jk}_lM^l_k \\ &+A^T_{\overline{15}}B_iH(\overline{15})^{ij}_kM^l_jM^k_l+C^T_{\overline{15}}B_iM^i_jH(\overline{15})^{jk}_lM^l_k \\ &+B^T_{\overline{3}}B_iH(\overline{3})^iM^j_jM^k_k+\tilde{B}^T_6B_iH(6)^{ij}_kM^k_jM^l_l \\ &+B^T_{\overline{15}}B_iH(\overline{15})^{ij}_kM^k_jM^l_l+D^T_{\overline{3}}B_iM^i_jH(\overline{3})^jM^l_l, \end{aligned} \tag{3}$$

where $H(\overline{3},6,\overline{15})$ are the operators that correspond to the irreducible $\overline{3},6,\overline{15}$ representations in the effective Hamiltonian.

In Table I we list all decay amplitudes involving $\eta_{1,8}$. The amplitudes containing only K and π in the final states can be found in Ref. [9]. There are a few new features for the amplitudes in Eq. (3) compared with the amplitudes for $B\to PP$ where P can only be one of the octet mesons [9]. The last four terms are new. In the octet case, because of the traceless condition of M^j_i (η_1 is removed from M), these terms are automatically zero. The amplitudes $\tilde{A}_6$ and $\tilde{C}_6$ always appear in the combination of $\tilde{C}_6-\tilde{A}_6$ [8]. This degeneracy is naively lifted in processes with η_1 in the final states. It seems that there is the need to have both $\tilde{C}_6$ and $\tilde{A}_6$ describe the decays, thus increasing the total number of hadronic parameters by one. However, this is not true since the $\tilde{A}^T_6$ and $\tilde{C}^T_6$ terms in decay modes with η_1 in the final state can be written as $C^T_6=\tilde{C}^T_6-\tilde{A}^T_6$, and the additional $\tilde{A}^T_6$ can be absorbed by redefining the amplitude $B^T_6=\tilde{B}^T_6+\tilde{A}^T_6$. In Table I we therefore have listed the decay amplitudes in terms of the independent amplitudes, $C^T_{\overline{3},6,\overline{15}}$, $A^T_{\overline{3},\overline{15}}$, $B^T_{\overline{3},6,\overline{15}}$ and $D^T_{\overline{3}}$.

We now describe the other approach that includes η_1 in $B\to PP$ decays. Here one treats η_1 as an independent singlet of $SU(3)$ and parametrizes the decay amplitudes according to SU(3) symmetry. In this case there are also an additional four new terms,

$$\begin{aligned} T_{new}=&a^TB_iH(\overline{3})^i\eta_1\eta_1+b^TB_iM^i_j(8)H(\overline{3})^j\eta_1 \\ &+c^TB_iH(6)^{ik}_lM^l_k(8)\eta_1+d^TB_iH(\overline{15})^{ik}_lM^l_k(8)\eta_1. \end{aligned} \tag{4}$$

Here $M(8)$ is the octet part of in the nonet M.

In the nonet limit, we have

$$a^T=A^T_{\overline{3}}+3B^T_{\overline{3}}+\frac{1}{3}C^T_{\overline{3}}+D^T_{\overline{3}},\quad b^T=\frac{2}{\sqrt{3}}C^T_{\overline{3}}+\sqrt{3}D^T_{\overline{3}},$$

$$c^T=\frac{2}{\sqrt{3}}\tilde{A}^T_6+\sqrt{3}\tilde{B}^T_6+\frac{1}{\sqrt{3}}\tilde{C}^T_6,$$

$$d^T=\frac{2}{\sqrt{3}}A^T_{\overline{15}}+\sqrt{3}B^T_{\overline{15}}+\frac{1}{\sqrt{3}}C^T_{\overline{15}}. \tag{5}$$

We also see from the above that one can use C^T_6 and B^T_6 to absorb $\tilde{A}^T_6$ by writing, $c^T=\sqrt{3}B^T_6+(1/\sqrt{3})C^T_6$. It is interesting to note that both approaches discussed above introduce the same number of new parameters, four of them, into the theory. In our analysis we will work with the nonet approach described in the above.

To obtain the amplitudes for B decays with at least one $\eta(\eta')$ in the final states, one also needs to consider η-η' mixing,

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_1\end{pmatrix}. \tag{6}$$

The averaged value of the mixing angle θ is $-15.5°\pm1.3°$ [14]. We will use $\theta=-15.5°$ in our fit.

There are similar invariant amplitudes for the penguin contributions. We indicate them as $C^P_{\overline{3},6,\overline{15}}$, $A^P_{\overline{3},\overline{15}}$, $B^P_{\overline{3},6,\overline{15}}$ and $D^P_{\overline{3}}$. The amplitudes A_i and B_i are referred to as annihilation amplitudes because the B mesons are first annihilated by the interaction Hamiltonian and two light mesons are then created. In total there are 18 complex hadronic parameters (36 real parameters with one of them being an overall unphysical

TABLE I. Decay amplitudes for $B\to PP$ with at least one of the P being a η_8 or η_1.

$\Delta S=0$	$\Delta S=-1$
$T^{B_u}_{\pi^-\eta_8}(d)=\frac{2}{\sqrt{6}}(C^T_{\bar 3}-C^T_6+3A^T_{\overline{15}}+3C^T_{\overline{15}})$,	$T^{B_u}_{\eta_8K^-}(s)=\frac{1}{\sqrt{6}}(-C^T_{\bar 3}+C^T_6-3A^T_{\overline{15}}+9C^T_{\overline{15}})$,
$T^{B_d}_{\pi^0\eta_8}(d)=\frac{1}{\sqrt{3}}(-C^T_{\bar 3}+C^T_6+5A^T_{\overline{15}}+C^T_{\overline{15}})$,	$T^{B_d}_{\eta_8\bar K^0}(s)=-\frac{1}{\sqrt{6}}(C^T_{\bar 3}+C^T_6-A^T_{\overline{15}}-5C^T_{\overline{15}})$,
$T^{B_d}_{\eta_8\eta_8}(d)=\frac{1}{\sqrt{2}}(2A^T_{\bar 3}+\frac{1}{3}C^T_{\bar 3}-C^T_6-A^T_{\overline{15}}+C^T_{\overline{15}})$,	$T^{B_s}_{\pi^0\eta_8}(s)=\frac{2}{\sqrt{3}}(C^T_6+2A^T_{\overline{15}}-2C^T_{\overline{15}})$,
$T^{B_s}_{K^0\eta_8}(d)=-\frac{1}{\sqrt{6}}(C^T_{\bar 3}+C^T_6-A^T_{\overline{15}}-5C^T_{\overline{15}})$,	$T^{B_s}_{\eta_8\eta_8}(s)=\sqrt{2}(A^T_{\bar 3}+\frac{2}{3}C^T_{\bar 3}-A^T_{\overline{15}}-2C^T_{\overline{15}})$,
$T^{B_u}_{\pi^-\eta_1}(d)=\frac{1}{\sqrt{3}}(2C^T_{\bar 3}+C^T_6+6A^T_{\overline{15}}+3C^T_{\overline{15}}$ $+3B^T_6+9B^T_{\overline{15}}+3D^T_{\bar 3}$,	$T^{B_d}_{K^-\eta_1}(s)=\frac{1}{\sqrt{3}}(2C^T_{\bar 3}+C^T_6+6A^T_{\overline{15}}+3C^T_{\overline{15}}$ $+3B^T_6+9B^T_{\overline{15}}+3D^T_{\bar 3}$,
$T^{B_d}_{\pi^0\eta_1}(d)=\frac{-1}{\sqrt{6}}(2C^T_{\bar 3}+C^T_6-10A^T_{\overline{15}}-5C^T_{\overline{15}}$ $+3B^T_6-15B^T_{\overline{15}}+3D^T_{\bar 3}$,	$T^{B_d}_{\bar K^0\eta_1}(s)=\frac{1}{\sqrt{3}}(2C^T_{\bar 3}-C^T_6-2A^T_{\overline{15}}-C^T_{\overline{15}}$ $-3B^T_6-3B^T_{\overline{15}}+3D^T_{\bar 3}$,
$T^{B_d}_{\eta_1\eta_8}(d)=\frac{1}{3\sqrt{2}}(2C^T_{\bar 3}-3C^T_6+6A^T_{\overline{15}}+3C^T_{\overline{15}}$ $-9B^T_6+9B^T_{\overline{15}}+3D^T_{\bar 3}$,	$T^{B_s}_{\pi^0\eta_1}(s)=\frac{-2}{\sqrt{6}}(C^T_6-4A^T_{\overline{15}}-2C^T_{\overline{15}}$ $+3B^T_6-6B^T_{\overline{15}}$,
$T^{B_d}_{\eta_1\eta_1}(d)=\frac{\sqrt{2}}{3}(3A^T_{\bar 3}+C^T_{\bar 3}+9B^T_3+3D^T_{\bar 3})$,	$T^{B_s}_{\eta_1\eta_8}(s)=\frac{-\sqrt{2}}{3}(2C^T_{\bar 3}-6A^T_{\overline{15}}-3C^T_{\overline{15}}$ $-9B^T_{\overline{15}}+3D^T_{\bar 3}$,
$T^{B_s}_{K^0\eta_1}(d)=\frac{1}{\sqrt{3}}(2C^T_{\bar 3}-C^T_6-2A^T_{\overline{15}}-C^T_{\overline{15}}$ $-3B^T_6-3B^T_{\overline{15}}+3D^T_{\bar 3}$,	$T^{B_s}_{\eta_1\eta_1}(s)=\frac{\sqrt{2}}{3}(3A^T_{\bar 3}+C^T_{\bar 3}+9B^T_3+3D^T_{\bar 3})$,

phase). However, simplification can be made because of the following relations in the SM:

$$
\begin{aligned}
C^P_6(B^P_6)&=-\frac{3}{2}\,\frac{c_9^{tc}-c_{10}^{tc}}{c_1-c_2-3(c_9^{uc}-c_{10}^{uc})/2}C^T_6(B^T_6)\\
&\approx-0.013C^T_6(B^T_6),
\end{aligned}
$$

$$
\begin{aligned}
C^P_{\overline{15}}(A^P_{\overline{15}},B^P_{\overline{15}})&=-\frac{3}{2}\,\frac{c_9^{tc}+c_{10}^{tc}}{c_1+c_2-3(c_9^{uc}+c_{10}^{uc})/2}C^T_{\overline{15}}(A^T_{\overline{15}},B^T_{\overline{15}})\\
&\approx+0.015C^T_{\overline{15}}(A^T_{\overline{15}},B^T_{\overline{15}}).
\end{aligned}
\tag{7}
$$

Here we have used the Wilson coefficients obtained in Ref. [13].

We comment that in finite order perturbative calculations the above relations are renormalization scheme and scale dependent. One should use a renormalization scheme consistently. We checked with different renormalization schemes and found that numerically the changes are less than 15% for different schemes. In obtaining the above relations, we also neglected small contributions from $c_{7,8}$ which cause less than 1% deviations.

Using the above relations the number of independent hadronic parameters are reduced, which we chose to be $C^{T,P}_{\bar 3}(A^{T,P}_{\bar 3})$, C^T_6, $C^T_{\overline{15}}(A^T_{\overline{15}})$, $B^{T,P}_3$, B^T_6, $B^T_{\overline{15}}$, $D^{T,P}_{\bar 3}$. An overall phase can be removed without loss of generality; we will set $C^P_{\bar 3}$ to be real. There are in fact only 25 real independent parameters for $B\to PP$ in the SM with pseudoscalar nonet,

$$
\begin{aligned}
&C^P_{\bar 3},\ C^T_{\bar 3}e^{i\delta_{\bar 3}},\ C^T_6e^{i\delta_6},\ C^T_{\overline{15}}e^{i\delta_{\overline{15}}},\ A^T_{\bar 3}e^{i\delta_{A^T_{\bar 3}}},\ A^P_{\bar 3}e^{i\delta_{A^P_{\bar 3}}},\\
&A^T_{\overline{15}}e^{i\delta_{A^T_{\overline{15}}}},B^T_3e^{i\delta_{B^T_3}},\ B^P_3e^{i\delta_{B^P_3}},\ B^T_6e^{i\delta_{B^T_6}},\ B^T_{\overline{15}}e^{i\delta_{B^T_{\overline{15}}}},\\
&D^T_{\bar 3}e^{i\delta_{D^T_{\bar 3}}},\ D^P_{\bar 3}e^{i\delta_{D^P_{\bar 3}}}.
\end{aligned}
$$

Further, the amplitudes A_i and B_i correspond to annihilation contributions and are expected to be small, which is also supported by data [9]. If the annihilation amplitudes are neglected, there are only 11 independent hadronic parameters

$$
C^P_{\bar 3},\ C^T_{\bar 3}e^{i\delta_{\bar 3}},\ C^T_6e^{i\delta_6},\ C^T_{\overline{15}}e^{i\delta_{\overline{15}}},\ D^T_{\bar 3}e^{i\delta_{D^T_{\bar 3}}},\ D^P_{\bar 3}e^{i\delta_{D^P_{\bar 3}}}.
\tag{8}
$$

TABLE II. The central values and 68% C.L. allowed ranges for branching ratios (in units of 10^{-6}) and CP asymmetries for processes with no η or η' in the final states.

	Branching ratios			CP asymmetries		
	Experiment	Fit value		Experiment	Fit value	
$B_u \to \pi^- \pi^0$	5.3 ± 0.8	$5.1^{+0.8}_{-0.8}$	$(5.1^{+0.8}_{-0.8})$	-0.07 ± 0.15	$0.00^{+0.00}_{-0.00}$	$(0.00^{+0.00}_{-0.00})$
$B_u \to K^- K^0$	0.1 ± 0.6	$0.9^{+0.4}_{-0.3}$	$(0.9^{+0.4}_{-0.3})$		$-0.66^{+0.61}_{-0.28}$	$(-0.69^{+0.59}_{-0.27})$
$B_d \to \pi^+ \pi^-$	4.6 ± 0.4	$4.6^{+0.5}_{-0.5}$	$(4.6^{+0.5}_{-0.5})$	0.51 ± 0.19	$0.34^{+0.10}_{-0.11}$	$(0.34^{+0.10}_{-0.11})$
$B_d \to \pi^0 \pi^0$	1.9 ± 0.5	$2.1^{+0.5}_{-0.5}$	$(2.1^{+0.5}_{-0.5})$		$0.51^{+0.11}_{-0.14}$	$(0.51^{+0.11}_{-0.14})$
$B_d \to \bar{K}^0 K^0$	0.8 ± 0.8	$0.8^{+0.4}_{-0.3}$	$(0.8^{+0.4}_{-0.3})$		$-0.66^{+0.61}_{-0.28}$	$(-0.69^{+0.59}_{-0.27})$
$B_u \to \pi^- \bar{K}^0$	19.7 ± 1.5	$20.1^{+1.0}_{-1.0}$	$(20.1^{+1.0}_{-1.0})$	-0.02 ± 0.07	$0.03^{+0.03}_{-0.03}$	$(0.03^{+0.03}_{-0.03})$
$B_u \to \pi^0 K^-$	12.8 ± 1.1	$11.2^{+0.5}_{-0.5}$	$(11.2^{+0.6}_{-0.6})$	0.00 ± 0.07	$0.08^{+0.04}_{-0.04}$	$(0.08^{+0.04}_{-0.04})$
$B_d \to \pi^+ K^-$	18.2 ± 0.7	$19.0^{+0.8}_{-0.7}$	$(19.0^{+0.8}_{-0.8})$	-0.09 ± 0.04	$-0.09^{+0.03}_{-0.03}$	$(-0.08^{+0.03}_{-0.03})$
$B_d \to \pi^0 \bar{K}^0$	11.2 ± 1.4	$8.6^{+0.4}_{-0.4}$	$(8.6^{+0.6}_{-0.6})$	0.03 ± 0.37	$-0.12^{+0.05}_{-0.04}$	$(-0.12^{+0.05}_{-0.05})$
$B_s \to K^+ \pi^-$		$4.3^{+0.5}_{-0.5}$	$(4.3^{+0.5}_{-0.5})$		$0.34^{+0.10}_{-0.11}$	$(0.34^{+0.10}_{-0.11})$
$B_s \to K^0 \pi^0$		$1.9^{+0.5}_{-0.5}$	$(1.9^{+0.5}_{-0.5})$		$0.51^{+0.11}_{-0.14}$	$(0.51^{+0.11}_{-0.14})$
$B_s \to K^+ K^-$		$17.9^{+0.7}_{-0.7}$	$(17.9^{+0.7}_{-0.7})$		$-0.09^{+0.03}_{-0.03}$	$(-0.08^{+0.03}_{-0.03})$
$B_s \to K^0 \bar{K}^0$		$17.7^{+0.9}_{-0.9}$	$(17.7^{+0.9}_{-0.9})$		$0.03^{+0.03}_{-0.03}$	$(0.03^{+0.03}_{-0.03})$

The phases in the above can be defined in such a way that all $C_i^{T,P}$ and $D_i^{T,P}$ are real positive numbers.

At present many $B \to PP$ decay modes have been measured at B factories [2–4]. It is tempting to use experimental data to fix all the hadronic parameters described earlier. It has been shown that if processes involving $\eta(\eta')$ are not included, it is indeed possible to determine all the invariant amplitudes, A_i and C_i [8,9]. When processes involving $\eta(\eta')$ are also included, a meaningful determination of all hadronic parameters (25 of them) is, however, not possible at present because of too many parameters. Therefore, in the following we neglect the annihilation amplitudes, which are anticipated to be small, to see if all data can be reasonably explained, in particular to see if large $B \to K\eta'$ branching ratios can be obtained, with only 11 parameters given in Eq. (8). This is a nontrivial task. Remarkably we find that all data can, indeed, be well explained.

We use the averaged CLEO, BaBar and Belle data [2–4] shown in Tables II and III to fix the unknown 11 hadronic parameters by carrying out a global χ^2 analysis. The results

TABLE III. The central values and their 68% C.L. allowed ranges for branching ratios (in units of 10^{-6}) and CP asymmetries with at least one of the final mesons to be an η or η'.

	Branching ratios			CP asymmetries		
	Experiment	Fit value		Experiment	Fit value	
$B_u \to \pi^- \eta$	4.4 ± 0.9	$3.6^{+0.9}_{-0.8}$	$(3.6^{+0.8}_{-0.8})$	-0.510 ± 0.200	$-0.24^{+0.16}_{-0.16}$	$(-0.24^{+0.16}_{-0.16})$
$B_u \to \pi^- \eta'$	2.8 ± 1.3	$3.6^{+1.2}_{-1.0}$	$(3.6^{+1.2}_{-1.1})$		$-0.28^{+0.52}_{-0.50}$	$(-0.27^{+0.52}_{-0.50})$
$B_d \to \pi^0 \eta$	(<2.9)	$0.7^{+0.4}_{-0.3}$	$(0.7^{+0.3}_{-0.3})$		$-0.85^{+0.36}_{-0.14}$	$(-0.86^{+0.35}_{-0.13})$
$B_d \to \pi^0 \eta'$	(<5.7)	$1.7^{+1.0}_{-0.7}$	$(1.7^{+1.0}_{-0.7})$		$-0.99^{+0.31}_{-0.01}$	$(-0.99^{+0.30}_{-0.01})$
$B_u \to K^- \eta$	3.2 ± 0.8	$3.2^{+0.7}_{-0.7}$	$(3.2^{+0.7}_{-0.7})$	-0.320 ± 0.220	$-0.32^{+0.07}_{-0.07}$	$(-0.32^{+0.07}_{-0.07})$
$B_u \to K^- \eta'$	77.5 ± 4.6	$73.5^{+3.8}_{-3.7}$	$(73.6^{+3.8}_{-3.7})$	0.022 ± 0.037	$0.03^{+0.03}_{-0.03}$	$(0.03^{+0.03}_{-0.03})$
$B_d \to \bar{K}^0 \eta$	2.6 ± 0.9	$2.3^{+0.6}_{-0.6}$	$(2.2^{+0.6}_{-0.6})$		$-0.10^{+0.09}_{-0.08}$	$(-0.12^{+0.07}_{-0.06})$
$B_d \to \bar{K}^0 \eta'$	60.3 ± 5.7	$67.0^{+3.6}_{-3.5}$	$(66.9^{+3.5}_{-3.5})$	-0.042 ± 0.132	$0.07^{+0.03}_{-0.03}$	$(0.07^{+0.03}_{-0.03})$
$B_d \to \eta\eta$		$1.7^{+0.7}_{-0.6}$	$(1.7^{+0.6}_{-0.6})$		$-0.56^{+0.15}_{-0.16}$	$(-0.54^{+0.15}_{-0.16})$
$B_d \to \eta\eta'$		$2.1^{+0.8}_{-0.7}$	$(2.1^{+0.8}_{-0.7})$		$-0.57^{+0.23}_{-0.23}$	$(-0.55^{+0.23}_{-0.24})$
$B_d \to \eta'\eta'$		$1.0^{+0.6}_{-0.5}$	$(1.0^{+0.6}_{-0.5})$		$-0.67^{+0.40}_{-0.29}$	$(-0.66^{+0.39}_{-0.29})$
$B_s \to K\eta$		$1.3^{+0.5}_{-0.5}$	$(1.3^{+0.5}_{-0.5})$		$0.16^{+0.15}_{-0.15}$	$(0.19^{+0.12}_{-0.12})$
$B_s \to K\eta'$		$5.1^{+1.8}_{-1.5}$	$(5.1^{+1.9}_{-1.6})$		$-0.89^{+0.20}_{-0.11}$	$(-0.88^{+0.18}_{-0.12})$
$B_s \to \pi^0 \eta$		$0.1^{+0.3}_{-0.1}$	$(0.1^{+0.3}_{-0.3})$		$0.93^{+0.07}_{-0.10}$	$(0.93^{+0.07}_{-0.12})$
$B_s \to \pi^0 \eta'$		$0.1^{+0.3}_{-0.1}$	$(0.1^{+0.3}_{-0.3})$		$0.89^{+0.09}_{-0.11}$	$(0.88^{+0.10}_{-0.12})$
$B_s \to \eta\eta$		$6.0^{+2.0}_{-2.0}$	$(6.3^{+1.6}_{-1.6})$		$-0.14^{+0.05}_{-0.05}$	$(-0.13^{+0.05}_{-0.05})$
$B_s \to \eta\eta'$		$24.0^{+2.0}_{-1.8}$	$(24.4^{+1.4}_{-1.4})$		$0.01^{+0.03}_{-0.04}$	$(0.01^{+0.03}_{-0.03})$
$B_s \to \eta'\eta'$		$68.5^{+4.7}_{-4.7}$	$(67.9^{+4.4}_{-4.3})$		$0.05^{+0.04}_{-0.04}$	$(0.05^{+0.04}_{-0.04})$

TABLE IV. The best fit values and their 68% C.L. ranges for the hadronic parameters. The values without and with parentheses are for the fits with A, ρ and η fixed to their central values and varied within the allowed regions, respectively (the same for all other tables).

	Central value		Error range	
$C^P_{\bar{3}}$	0.137	(0.137)	0.002	(0.008)
$C^T_{\bar{3}}$	0.178	(0.177)	0.069	(0.075)
C^T_6	0.278	(0.287)	0.059	(0.066)
$C^T_{\overline{15}}$	0.142	(0.145)	0.010	(0.017)
$\delta_{\bar{3}}$	42.4°	(43.3°)	20.1°	(21.4°)
δ_6	75.1°	(75.4°)	13.0°	(12.8°)
$\delta_{\overline{15}}$	−10.0°	(−10.2°)	10.1°	(10.1°)
$D^P_{\bar{3}}$	0.110	(0.112)	0.012	(0.012)
$D^T_{\bar{3}}$	0.395	(0.398)	0.113	(0.121)
$\delta_{D^P_{\bar{3}}}$	−81.3°	(−82.8°)	8.8°	(7.0°)
$\delta_{D^T_{\bar{3}}}$	−106.9°	(−107.6°)	20.3°	(20.4°)
A		(0.854)		(0.046)
ρ		(0.190)		(0.041)
η		(0.337)		(0.028)

are shown in Table IV. In our analysis, due to the lack of knowledge of the error correlations from experiments in obtaining the averaged error bars, we have, for simplicity, taken them to be uncorrelated and assumed they obey a Gaussian distribution, taking the larger one between σ_+ and σ_- to be on the conservative side. Experimental data on ϵ_K, $B-\bar{B}$ mixing, $|V_{cb}|$, $|V_{ub}/V_{cb}|$, and $\sin2\beta$ provide very stringent constraints on the *KM* matrix elements involved in our analysis [9,15,16]. The known parameters with the values $\lambda=0.2196$, $A=0.854\pm0.046$, $\rho=0.178\pm0.046$ and $\eta=0.341\pm0.028$ were determined from the most recent data [16]. We consider two scenarios in order to include parameters of the *KM* matrix in our fit. One is to fix them to their central values, and another is to vary them in the allowed ranges.

Using the above determined hadronic parameters, we study other unmeasured branching ratios and *CP* violating rate asymmetries A_{CP} for $B\to PP$, defined by

$$A_{CP}=\frac{\Gamma(B_i\to PP)-\Gamma(\bar{B}_i\to\overline{PP})}{\Gamma(B_i\to PP)+\Gamma(\bar{B}_i\to\overline{PP})}. \tag{9}$$

The results are shown in Tables II and III.

We now discuss some implications of the results obtained and draw conclusions. The minimal χ^2 in our fit are 1.25 χ^2 per degree of freedom and 1.26 per degree of freedom for the two scenarios we are considering. We have checked in detail that there are no other local minima with a χ^2 per degree of freedom near the one we found. The values for the χ^2 per degree of freedom indicate that the fits are reasonable ones. These facts are also reflected in the best fit values for the branching ratios. As can be seen from Tables II and III the best fit values for the known branching ratios are in good agreements with data. We note that large $B\to K\eta'$ can be obtained.

The relevant values for the two scenarios regarding the handling of the *KM* matrix elements are listed in Table IV. These results indicate that varying the *KM* parameters in the allowed ranges do not affect the final results very much. In our fit the η-η' mixing parameter θ is fixed at the averaged value determined from other data [14]. We checked the sensitivity of the final results on θ within the allowed region and find the changes are small. We also find that if one reduces the nonet to an octet, just fitting data on $B\to\pi\pi,\pi K,KK$, the values obtained for C_i are not very much different than what we obtained here. This indicates that the parameters C_i are stable when replacing the octet multiplet by a nonet multiplet.

Since we neglected annihilation contributions, the mode $B_d\to K^-K^+$ has a vanishing branching ratio which is consistent with data. When neglecting annihilation terms, $Br(B_u\to K^-K^0)$ and $Br(B_d\to\bar{K}^0K^0)$ have the same SU(3) structures. These two decays should have the same decay width. These can be used as a test for the assumption of small annihilation contributions. The consistence of our fit with experimental data also can be taken to support the assumption. When annihilation contributions are included, even small contributions can affect the results, but the general features will not be changed too much. This has been shown to be true in the case that does not include including η and η' in the final states [9]. At present a complete analysis with annihilation contributions included cannot be carried out because there are not enough data points. One needs to wait until more data become available to check the consistence in full.

Theoretical calculation for the hadronic parameters of C_i is a very difficult task although there is some progress in using QCD improved factorization [6]. We have calculated the hadronic parameters C_i using QCD improved factorization developed in Ref. [6] with $C^P_{\bar{3}}=0.09$, $C^T_{\bar{3}}=0.35$, $C^T_6=0.20$, $C^T_{\overline{15}}=0.15$, $\delta_{\bar{3}}=3^0$, $\delta_6=8^0$, $\delta_{\overline{15}}=-3^0$. The magnitudes for C_i are similar to the fitting results in Table IV. However, the strong phases are very different. In our analysis the large branching ratios for $B\to\eta' K$ are due to the parameters $D_{\bar{3}}$. It is well known that factorization calculations have difficulties obtaining the observed branching ratios for $B\to\eta' K$. Therefore the parameters $D_{\bar{3}}$ obtained from factorization calculations are expected to be unreliable. The central values of theoretical estimates for $D_{\bar{3}}$ are typically a factor of 2 smaller (or even smaller) than what we obtained in Table IV. More reliable methods to evaluate hadronic parameters are needed.

Using the hadronic parameters determined from existing data, we have predicted several unmeasured branching ratios. These predictions can be used to test the theory. Several of them involve at least one η (or a η') in the final states for B_d decays. The ranges predicted for $B_d\to\pi^0\eta,\pi^0\eta'$ are consistent with the existing bounds. There are also seven B_s decay modes with at least one η or η' in the final states. Several of the branching ratios are predicted to be large, in particular the predicted branching ratio for $B_s \to \eta' \eta'$ is about 7×10^{-5} which can be measured at future hadron colliders and can provide another crucial test for theory.

We have also obtained interesting predictions for *CP* asymmetries in $B \to PP$ decay modes. Many of the predicted central values for the *CP* asymmetries are larger than 10% which can be measured in the near future. These modes can provide important information about *CP* violation in the standard model.

A SU(3) analysis for $B \to PP$ with at least one of the P being an η or an η' has been carried out recently in Ref. [12]. In our analysis we tried to fix the invariant amplitudes by a global fit of the available data on $B \to PP$, while the authors in Ref. [12] determined some of the amplitudes or combinations by some decays, and determined or bounded the remaining ones by some theoretical considerations. Due to this difference, the predictions for some of the branching ratios are different, but they are still in consistent ranges.

In our analysis we have assumed nonet symmetry. There may be effects from violation of this assumption, breaking of the nonet symmetry and also breaking of SU(3) symmetry. These breaking effects will cause deviations from what was obtained here. These effects can in principle be included in the analysis. To include the nonet breaking effects into the analysis, one needs to use the parameters in Eq. (4) without assuming the relations in Eq. (5). This will introduce more parameters in the analysis and cannot be meaningfully done at present. With more data becoming available, this can be achieved. One can also take the leading SU(3) breaking effects into account by identifying that the breaking effects come from the fact that the strange quark mass is significantly larger than the *u* and *d* quark masses. One can parametrize the effects in a similar way as we did for the SU(3) amplitudes, by inserting the quark mass matrix $M_{breaking} = \mathrm{diag}(m_u, m_d, m_s)$ at all possible places in Eq. (3). However, this will introduce several new parameters into the decay amplitudes. Again it is impossible to carry out a comprehensive analysis with all SU(3) breaking effects taken into account. What we can conclude from our analysis regarding this is that the fit was carried out assuming SU(3) (nonet) symmetry was consistent with data with a reasonable χ^2 per degree of freedom. The present data do not indicate large SU(3) (nonet) breaking effects. The predicted branching ratios and *CP* asymmetries can serve as further tests.

In conclusion, we have carried out a systematic analysis for $B \to PP$ decays in the SM with P being one of the nonet mesons. We find that all existing data can be explained; in particular, large branching ratios for $B \to K\eta'$ are possible. There is no conflict between the standard model and present experimental data. We have also predicted several unmeasured branching ratios and *CP* asymmetries within the reach of near future *B* factories. Future experimental data will provide crucial information on flavor symmetries and also on the standard model.

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