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Experiment for measuring the post-Maxwellian parameters of nonlinear electrodynamics of vacuum with laser-interferometer techniques

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An experiment is proposed to measure the post-Maxwellian parameters of the nonlinear electrodynamics of a vacuum. It is shown that the system for recording small differences of optical paths used in the Laser Interferometer Gravitational-Wave Observatory (LIGO) and in other laser interferometers for gravitational wave detectors permits us to measure the nonlinear electrodynamics corrections predicted by quantum electrodynamics. However, the use of LIGO or other full-scale laser interferometers of the gravitational wave detectors for this purpose does not seem sensible. Therefore it is suggested to perform this experiment on one of the laboratory prototypes used to develop interferometry for gravitational wave detection. The sensitivity of these laboratory prototypes is about the same as the sensitivity of the full-scale instruments, and the other parameters are most similar to the optimal values allowing us to observe the considered effect: the arm length l_0 of these prototypes is about several meters and in the Fabry-Perot interferometer about $N \sim 2\times 10^5$ bounces can be realized. A further increase in the detector sensitivity is noted to offer also an opportunity of experimentally studying the nonlinearly electrodynamic corrections predicted by the Born-Infeld theory.

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I. INTRODUCTION

The recent experiments [1] with inelastic laser photon scattering by gamma rays have shown that the electrodynamics in vacuum is a nonlinear theory. Therefore, the various models for nonlinear electrodynamics of vacuum, as well as their predictions, deserve serious attention. The present-day field theory treats some models for nonlinear electrodynamics of vacuum, of which the Born-Infeld [2] is the earliest. The Born-Infeld electrodynamics Lagrangian is

$$L=-\frac{1}{4\pi a^2}\left[\sqrt{1+a^2(\mathbf{B}^2-\mathbf{E}^2)-a^4(\mathbf{B}\cdot\mathbf{E})^2}-1\right], \tag{1}$$

where a is a certain parameter evaluated in Ref. [2] as $a \approx 10^{-16}\ \mathrm{G}^{-1}$.

As shown in Refs. [2,3], the Born-Infeld electrodynamics is a theory singled out in many respects. In the case of weak electromagnetic fields ($a^2\mathbf{E}^2 \ll 1$, $a^2\mathbf{B}^2\ll 1$), the Lagrangian (1) takes the following post-Maxwellian form:

$$L=\frac{1}{8\pi}(\mathbf{E}^2-\mathbf{B}^2)+\frac{a^2}{32\pi}[(\mathbf{B}^2-\mathbf{E}^2)^2+4(\mathbf{B}\cdot\mathbf{E})^2]. \tag{2}$$

The first part of this expression is Lagrangian of Maxwell electrodynamics, while the remaining parts are the corrections, arising from the nonlinearity of electromagnetic field interactions in vacuum.

The Heisenberg-Euler electrodynamics [4] following from quantum electrodynamics is another well-known model for nonlinear electrodynamics of vacuum. In the case where the electromagnetic fields are below the characteristic value $B_q = m^2c^2/e\hbar \approx 4.41\times 10^{13}$ G, the respective Lagrangian is

$$L=\frac{1}{8\pi}[\mathbf{E}^2-\mathbf{B}^2]+\frac{\alpha\{(\mathbf{B}^2-\mathbf{E}^2)^2+7(\mathbf{B}\cdot\mathbf{E})^2\}}{360\pi^2B_q^2},$$

where $\alpha = e^2/\hbar c \approx 1/137$ is the fine structure constant.

Comparing this formula with Eq. (2), it is easy to observe that there is no way of choosing the constant a^2 in order that these Lagrangians coincide. In order to facilitate the comparison among the predictions of the various models for nonlinear electrodynamics of vacuum, the parametrized post-Maxwellian formalism was proposed [5,6], which is similar to the post-Newton formalism [7] used extensively in the gravitation theory. Accordingly, the weak-field approximation of the post-Maxwellian Lagrangian is presented as

$$L=\frac{[\mathbf{E}^2-\mathbf{B}^2]+\xi[\eta_1(\mathbf{E}^2-\mathbf{B}^2)^2+4\eta_2(\mathbf{B}\cdot\mathbf{E})^2]}{8\pi}, \tag{3}$$

where $\xi = 1/B_q^2$; the value of the dimensionless post-Maxwellian parameters η_1 and η_2 is defined by the choice of a particular theoretical model for nonlinear electrodynamics of vacuum, namely, $\eta_1=\alpha/(45\pi)=5.1\times 10^{-5}$, $\eta_2 = 7\alpha/(180\pi) = 9.0\times 10^{-5}$ in the Heisenberg-Euler electrodynamics and $\eta_1=\eta_2=a^2B_q^2/4 \approx 4.9\times 10^{-6}$ in the Born-Infeld electrodynamics.

The nonlinear electrodynamics equations of electromagnetic field derived from the Lagrangian (3) are similar to the macroscopic electrodynamics equations

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$$\text{curl}\ \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \quad \text{div}\ \mathbf{D} = 0,$$

$$\text{curl}\ \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \text{div}\ \mathbf{B} = 0,$$

the only difference being in the relations of vectors **D** and **H** to vectors **B** and **E**:

$$\mathbf{D} = 4\pi\frac{\partial L}{\partial \mathbf{E}} = \mathbf{E} + 2\xi[\eta_1(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{E} + 2\eta_2(\mathbf{B}\cdot\mathbf{E})\mathbf{B}],$$

$$\mathbf{H} = -4\pi\frac{\partial L}{\partial \mathbf{B}} = \mathbf{B} + 2\xi[\eta_1(\mathbf{E}^2 - \mathbf{B}^2)\mathbf{B} - 2\eta_2(\mathbf{B}\cdot\mathbf{E})\mathbf{E}].$$

Quite a number of different experiments were proposed recently to study the effects of nonlinear electrodynamics of vacuum. The calculations [8–13] have shown that the effects must be most pronounced in the strong magnetic fields of pulsars ($B \sim 10^{12}$ G) and magnetars ($B \sim 10^{15}$ G). However, these astrophysical objects are so far away from the Earth that, having been measured by the extra-atmospheric astronomy tools, their electromagnetic radiation is of low intensity, thus making it difficult to detect the nonlinear electrodynamics effects.

With the fields $B, E \sim 10^6$ G obtainable at ground-based laboratories, the nonlinear corrections to the Maxwell equations are so small that their effects in vacuum become very difficult to observe. Nevertheless, the present-day facilities permit some optical experiments aimed at getting a deeper insight into the nonlinear electrodynamics of vacuum. Primarily, these are the ring laser experiments described in detail in Refs. [14–17]. In addition, the laser-interferometer techniques can be used in some other optical experiments to observe the nonlinear electrodynamics effects in vacuum. Let us consider one of them, which uses laboratory prototypes of the laser interferometer for the gravitational wave detection.

II. CALCULATION OF THE EFFECT

Let us write in the pseudo-Euclidean space-time with the metric tensor γ_{nm} the Lagrangian (3) in the general covariant form

$$L = \frac{\sqrt{-\gamma}}{32\pi}\{2J_2 + \xi[(\eta_1 - 2\eta_2)J_2^2 + 4\eta_2 J_4]\}, \tag{4}$$

where γ is the determinant of the metric tensor γ_{nm}, $J_2 = F_{ik}F^{ki}$, $J_4 = F_{ik}F^{km}F_{ml}F^{li}$ are the electromagnetic field tensor invariants and the tensor F_{km} indexes can be raised by the metric tensor γ^{nm}. The nonlinear electrodynamics equations of electromagnetic field with Lagrangian (4) have the form

$$\frac{1}{\sqrt{-\gamma}}\frac{\partial}{\partial x^n}\{\sqrt{-\gamma}H^{mn}\} = 0,$$

$$\frac{\partial F_{mn}}{\partial x^k} + \frac{\partial F_{nk}}{\partial x^m} + \frac{\partial F_{km}}{\partial x^n} = 0,$$

where for the compact record we denote

$$H^{mn} = \{1 + \xi(\eta_1 - 2\eta_2)J_2\}F^{mn} + 4\xi\eta_2 F^{ml}F_{lk}F^{kn}.$$

It follows from these equations [9,18] that the eikonal equation for a weak electromagnetic wave f_{km}, propagating in the external electromagnetic field F_{km}, depends, in the general case $\eta_1 \neq \eta_2$, on the electromagnetic wave polarization. For the first normal mode of a weak electromagnetic wave the eikonal equation can be written in the form

$$[\gamma^{mn} + 4\xi\eta_1 F^{mp}F_{p\cdot}^{\cdot n}]\frac{\partial S}{\partial x^m}\frac{\partial S}{\partial x^n} = 0,$$

while in the case of the second normal mode, we have

$$[\gamma^{mn} + 4\xi\eta_2 F^{mp}F_{p\cdot}^{\cdot n}]\frac{\partial S}{\partial x^m}\frac{\partial S}{\partial x^n} = 0,$$

where the all tensor F_{pk} indexes can be raised by the tensor γ^{mk}.

The basic concept of these equations is as follows. In terms of the nonlinear electrodynamics of vacuum constructed against the pseudo-Riemannian space-time with metric tensor $g_{(0)}^{ik}$, a weak electromagnetic wave front is known [9] to propagate along the geodesics of a certain effective pseudo-Riemannian space-time, whose metric tensor g^{ik} is defined by the weak electromagnetic wave polarization (birefringence of vacuum), by metric tensor $g_{(0)}^{ik}$ of the initial space-time, and by the tensor F_{ik} of external electromagnetic field.

The relevant calculations have shown that, in the case of the first normal mode of a weak electromagnetic wave that propagates in external magnetic field, the following relation holds among the tensors:

$$g^{ik} = g_{(0)}^{ik} + 4\eta_1\xi F^{in}F_{n\cdot}^{\cdot k}.$$

For the second normal mode, we get

$$g^{ik} = g_{(0)}^{ik} + 4\eta_2\xi F^{in}F_{n\cdot}^{\cdot k}.$$

Thus, the vacuum in a region, wherein an external magnetic field is generated, must show [19] the properties of a birefringent crystal, a fact that can be verified experimentally. The most promising experiment is as follows.

Examine the Michelson *L*-shaped interferometer, either arm of which is a Fabry-Perot interferometer, wherein N reflections of laser emission occur between mirrors spaced distance l_0 apart. Also, let a magnetic field $\mathbf{B}_0$ be generated normally to the laser beam along a path of light in one of the Fabry-Perot interferometers in a region of length l. Let the reference frame axes be so orientated that in this Fabry-Perot interferometer the wave vector of laser emission have components $\mathbf{k} = \{\pm k, 0, 0\}$ and the vector $\mathbf{B}_0 = \{0, 0, B_0\}$.

In the case of an electromagnetic wave polarized along vector $\mathbf{B}_0$ (the first normal mode), the components of the metric tensor g^{ik} of the effective pseudo-Riemannian space-time in the region with magnetic field will be

$$g^{00}=1, \quad g^{11}=g^{22}=-1+4\eta_1\xi\mathbf{B}_0^2, \quad g^{33}=-1.$$

In the case of an electromagnetic wave polarized normally to vector $\mathbf{B}_0$ (the second normal mode), we get

$$g^{00}=1, \quad g^{11}=g^{22}=-1+4\eta_2\xi\mathbf{B}_0^2, \quad g^{33}=-1.$$

Therefore, in the case of a single passage of laser emission inside the Fabry-Perot interferometer, wherein a magnetic field is generated, the optical path length will be different for electromagnetic waves of the two polarizations, namely, $l_{\mathrm{opt}}=l_0+2\eta_1\xi\mathbf{B}_0^2 l$ for a wave polarized along vector $\mathbf{B}_0$ and $l_{\mathrm{opt}}=l_0+2\eta_2\xi\mathbf{B}_0^2 l$ for an orthogonally polarized wave.

In the second Fabry-Perot interferometer, the absence of a magnetic field makes the optical path length of the two waves identical: $l_{\mathrm{opt}}=l_0$.

After N reflections from each mirror in the Fabry-Perot interferometers, the difference between the optical paths in two arms of the Michelson interferometer is

$$\Delta L_1=4\eta_1\xi\mathbf{B}_0^2 Nl. \tag{5}$$

A similar expression is obtainable for the second normal mode:

$$\Delta L_2=4\eta_2\xi\mathbf{B}_0^2 Nl. \tag{6}$$

III. THE POSSIBILITIES OF EXPERIMENT

Let us evaluate the obtained relations from the point of the experiment installation. The present-day hybrid magnets [20] are capable of producing a constant magnetic field $B_0=4.5\times10^5$ G in a region of length $l=3.2$ cm. Given the present-day quality of the interferometer mirrors [21], an interferometer with $N=2\times10^5$ can be realized.

Expressions (5) and (6), then, give $\Delta L_1=26.6\eta_1\times10^{-11}$ cm for the electromagnetic wave of the first normal mode and $\Delta L_2=26.6\eta_2\times10^{-11}$ cm for the wave of the second normal mode.

In the given experiment, magnetic field may be generated using also the pulsed magnets. The total pulse duration of the present-day pulsed magnets [20] is $T=20$ ms. The magnetic field of the magnets is peaking to $B_{\max}=5\times10^5$ G in a cylindrical region of diameter $l=2,4$ cm. To satisfy the condition $Nl_0/c\ll T$, the spacing of the Fabry-Perot interferometer mirrors must satisfy the inequality $l_0\ll30$ m. Therefore, we shall assume henceforth that $l_0=3$ m. Using the abovementioned values of the parameters in expressions (5) and (6), we obtain for the proposed experiment that the difference in the optical paths ΔL for the first normal mode will reach $\Delta L_1=24.7\eta_1\times10^{-11}$ cm. For the second normal mode, we get $\Delta L_2=24.7\eta_2\times10^{-11}$ cm.

Thus, the usage of the present-day constant and pulsed magnets in the proposed experiment gives the same order of the $\Delta L_{1,2}$ value. Estimate now the feasibility of measuring the post-Maxwellian parameters η_1 and η_2 in the proposed experiment. The modern laser interferometry for gravitational wave detection [22] has shown the feasibility of such measurements.

For example, the Laser Interferometer Gravitational-Wave Observatory (LIGO) designed by a Caltech-MIT collaboration for recording gravitational waves is similar in its design as the experimental facility for observing the effect of nonlinear electrodynamics of vacuum. In the detector of gravitational waves, the Fabry-Perot interferometer mirrors are spaced 4 km apart, with 30 bounces of laser emission. Since the present-day LIGO detector can record [22] a gravitational wave of dimensionless amplitude $h=10^{-21}$, the minimum measurable difference between the optical paths in the interferometer arms is $\Delta L=hL/2=1.2\times10^{-14}$ cm.

However, to realize the considering nonlinear electrodynamics experiment, the maximum length of the arm should be 3 m and it is necessary to make cavity with an effective number of bounces $\sim10^5$. Thus, using LIGO or other full-scale laser interferometer of gravitational wave detectors for this purpose seems not to be sensible. Thus an interesting possibility is to perform this experiment on one of the laboratory prototypes used to develop interferometry for gravitational wave detection. There are a number of them—in the USA, Germany, Scotland, Japan, and Australia [22,23].

The sensitivity of the laboratory prototypes is about the sensitivity of the full-scale laser interferometer for gravitational wave detectors, and the other parameters are similar to the optimal values allowing one to observe the considering effect: the arm length l_0 of these prototypes is about several meters and in the Fabry-Perot interferometer about $N\sim2\times10^5$ bounces can be realized. Nowadays, therefore, we can observe the displays of nonlinear electrodynamics of vacuum in the present-day pulsed magnet experiments in case $\eta_{1,2}\geqslant4.8\times10^{-5}$ and in the constant magnet experiments in case $\eta_{1,2}\geqslant4.5\times10^{-5}$. This is sufficient for the quantum-electrodynamics-predicted nonlinearly electrodynamic corrections ($\eta_1=5.1\times10^{-5}$, $\eta_2=9.0\times10^{-5}$), but insufficient for the Born-Infeld–theory–predicted nonlinearly electrodynamic corrections ($\eta_1=\eta_2\approx4.9\times10^{-6}$). However according to Ref. [22], perfecting of the laser interferometers for gravitational wave detector systems will permit one to decrease the measurable ΔL value by one or two orders.

The point is that the above estimations of the sensitivity of the instruments which are used in the gravitational wave detectors, are far enough from the quantum limit of the accuracy of the measurements of the difference in the optical paths of two rays. As follows from the estimations made in Refs. [24–26], this limit is about 10^{-17}–10^{-18} cm. To approach this limit is a very difficult task, however, the current intensive research, in this direction, give the definite optimism (see, for example, paper [27] and the scientific literature initiating there).

In the near future, therefore, perfection of the laser interferometer system for recording small differences in the optical paths will open up ways of experimentally studying the nonlinear electrodynamic corrections predicted by the Born-Infeld theory. It is necessary to note that in the proposed experiments L-form configuration of the instrument is not necessary; a parallel-arm interferometer might be better, if the magnetic field is well confined.

IV. CONCLUSION

Thus, the use of the laboratory prototypes of the laser interferometer for the gravitational wave detectors at the achieved sensitivity level gives the possibility to discover the nonlinear electrodynamics corrections predicted by the quantum electrodynamics. By the following increase on the one order of sensitivity of these instrument system of detection of the difference in the optical ray paths, it becomes possible to reveal the influence of nonlinear electrodynamics corrections predicted by the Born-Infeld theory.

Although the experiment we propose is very complex from a technical point of view, it constitutes a solvable problem. The main difficulty to overcome in view of successful detection of the effect seems to be the production of the ''actual'' vacuum, i.e., the pump down of a chamber, wherein the Fabry-Perot interferometers are placed, to attain an extremely low pressure. Otherwise, the Faraday and Cotton-Mouton effects will camouflage the effect of vacuum birefringence in magnetic field. This experiment is of great significance for physics since it clarifies which nonlinear electrodynamics of vacuum describes nature properly.

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