PHYSICAL REVIEW D **69**, 065020 (2004)

QCD and dimensional deconstruction

D. T. Son
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

M. A. Stephanov
Department of Physics, University of Illinois, Chicago, Illinois 60607-7059, USA
and RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA
(Received 2 May 2003; published 29 March 2004)

Motivated by phenomenological models of hidden local symmetries and the ideas of dimensional deconstruction and gauge/gravity duality, we consider the model of an ''open moose.'' Such a model has a large number K of hidden gauge groups as well as a global chiral symmetry. In the continuum limit $K\to\infty$ the model becomes a 4+1 dimensional theory of a gauge field propagating in a dilaton background and an external space-time metric with two boundaries. We show that the model reproduces several well known phenomenological and theoretical aspects of low-energy hadron dynamics, such as vector meson dominance. We derive the general formulas for the mass spectrum, the decay constants of the pion and vector mesons, and the couplings between mesons. We then consider two simple realizations, one with a flat metric and another with a ''cosh'' metric interpolating between two anti–de Sitter (AdS) boundaries. For the pion form factor, the single pole ρ-meson dominance is exact in the latter case and approximate in the former case. We discover that an AdS/conformal field theory-like prescription emerges in the computation of current-current correlators. We speculate on the role of the model in the theory dual to QCD.

DOI: 10.1103/PhysRevD.69.065020 PACS number(s): 12.38.Aw, 11.10.Kk, 11.25.Tq, 12.40.Vv

I. INTRODUCTION

Vector mesons [$\rho(770)$, $\omega(782)$, etc.] play a significant role in hadronic physics. Their interactions, though not constrained by low-energy theorems, apparently follow the broad pattern of vector meson dominance (VMD) [1]. There have been numerous efforts to incorporate vector mesons into field-theoretical frameworks. Historically, the Yang-Mills theory was discovered in an early attempt to treat the ρ meson [2]. More recently, interesting schemes based on ''hidden local symmetries'' (HLS) were developed by Bando *et al.* [3–5]. In the original model [3], the ρ meson is the boson of a spontaneously broken gauge group. The model has been extended to two hidden gauge groups [4]; then it also incorporates the lowest axial vector meson $a_1(1260)$. With suitable parameters, these models can be quite successful phenomenologically, although they cannot be systematically derived from QCD (except in the limit of very light ρ, if such a limit could be reached [6]).

In this paper we explore theories with very large, and even infinite number K of hidden local symmetries. Our motivation is twofold. First and most straightforwardly, there are excited states in the vector and axial vector channels [$\rho(1450)$, $a_1(1640)$, $\rho(1700)$, etc. [7]], which must become narrow resonances in the limit of large number of colors N_c. It is tempting to treat them as gauge bosons of additional broken gauge groups.[1]

The second motivation comes from recent theoretical developments. Many strongly coupled gauge theories are found to have a dual description in terms of theories with gravity in higher dimensions [9–12]. It was suggested that the string theory dual to large-N_c QCD must have strings propagating in five dimensions, in which the fifth dimension has the physical meaning of the energy scale [13]. In the framework of field theory, the fifth dimension can be ''deconstructed'' in models with a large number of gauge fields [14,15].

We discovered that the continuum limit $K\to\infty$ can lead to results that qualitatively, and in many cases even quantitatively, agree with phenomenology. Most remarkably, the vector meson dominance, which in the HLS theories required a tuning of parameters, becomes a natural consequence of the $K\to\infty$ limit. Another advantage of the limit $K\to\infty$ is the possibility of matching to the asymptotic behavior of the current-current correlator known from perturbative QCD.

As anticipated, a natural interpretation of this limit is a discretization, or deconstruction, of a 5-dimensional gauge theory. Further, to our amusement, in the calculation of current-current correlators we found a relation very similar to the one employed in the anti–de Sitter (AdS)/conformal field theory correspondence: the current-current correlator in 4D theory is expressed in terms of the variations of the classical 5D action with respect to the boundary values of the bulk gauge fields on the 4D boundaries.

We limit our discussion to the isospin-1 sector of QCD. It is straightforward to extend the discussion to the isospin-0 sector (η, ω, and f_1 mesons). The detailed treatment of the $U(1)_A$ problem, chiral anomaly, Wess-Zumino-Witten term, and baryons is deferred to future work.

The paper is organized as follows. In Sec. II we describe the open moose model. In Sec. III we compute different physical observables: the vector meson mass spectrum, the decay constants of the pion and the vector mesons, the coupling between the vector mesons and the pions, and the pion electromagnetic form factor. We also check the validity of

[1] To our knowledge, the earliest attempt to interpret the tower of ρ, ρ', etc. as a ''chain structure'' was made in Ref. [8].

Weinberg's spectral sum rules, and discover that the limit $K\to\infty$ automatically leads to exact VMD for the pion form-factor.

In Sec. IV we take the limit of infinite number of the hidden groups $K\to\infty$. We show that the theory can be understood as a 5D Yang-Mills theory in an external metric and dilaton background. We establish an AdS/CFT-type prescription for calculating the current-current correlators. We consider two concrete realizations of the open moose in Sec. V. We find that a "cosh" background metric interpolating between two AdS boundaries leads to correct asymptotic behavior of the current-current correlator. This allows us to establish a relationship between hadron parameters such as f_π, m_ρ, and the QCD parameter N_c. In Sec. VI we show that the instanton, which is a quasiparticle in $4+1$ dimensions, becomes a Skyrmion upon reduction to 4D, and thus describes the baryon. Section VII contains concluding remarks.

II. THE OPEN MOOSE

The model under consideration is described by the following Lagrangian[2]

$$\mathcal{L}=\sum_{k=1}^{K+1} f_k^2 \mathrm{Tr}|D_\mu \Sigma^k|^2 - \sum_{k=1}^{K} \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}^k)^2. \tag{2.1}$$

The covariant derivatives are defined as

$$D_\mu\Sigma^1=\partial_\mu\Sigma^1+i\Sigma^1(gA_\mu)^1, \tag{2.2a}$$

$$D_\mu\Sigma^k=\partial_\mu\Sigma^k-i(gA_\mu)^{k-1}\Sigma^k+i\Sigma^k(gA_\mu)^k, \tag{2.2b}$$

$$D_\mu\Sigma^{K+1}=\partial_\mu\Sigma^{K+1}-i(gA_\mu)^K\Sigma^{K+1}. \tag{2.2c}$$

A shorthand notation is used for the product of the gauge field $A_\mu=A_\mu^a\tau^a/2$ and its coupling constant: $g_kA_\mu^k \equiv (gA_\mu)^k$. If we assume $A_\mu^0=A_\mu^{K+1}=0$, then Eqs. (2.2a) and (2.2c) become special cases of Eq. (2.2b) for $k=1$ and $k=K+1$.

The model contains $K+1$ nonlinear sigma model fields $\Sigma^k\in \mathrm{SU}(2)$ [or, in general, $\mathrm{SU}(N_f)$], interacting via K "hidden" gauge bosons A_μ^k. The model has a chiral $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry and an $\mathrm{SU}(2)^K$ local symmetry:

$$\Sigma^1\to L\Sigma^1U_1^\dagger(x),$$

$$\Sigma^k\to U_{k-1}(x)\Sigma^kU_k^\dagger(x),\quad k=2,3,\dots, \tag{2.3}$$

$$\Sigma^{K+1}\to U_K(x)\Sigma^KR^\dagger.$$

In particular, the product

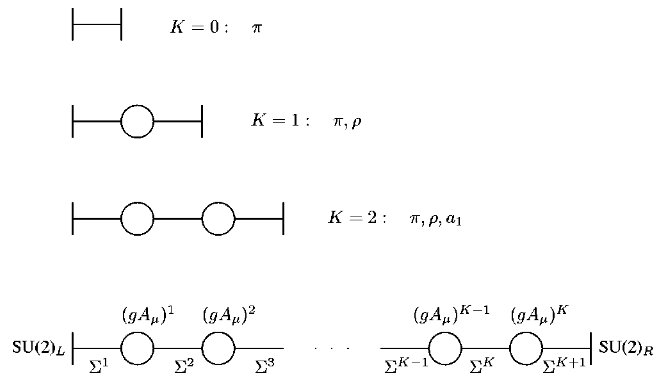

FIG. 1. Graphic representation of the Lagrangian (2.1). The examples of low K corresponding to previously considered theories are also shown. $K=0$ represents Weinberg's nonlinear sigma model for pions; $K=1$ represents a hidden symmetry Lagrangian describing π and ρ; and $K=2$ represents a description of π, ρ, and a_1 [3,4].

$$\Sigma=\Sigma^1\Sigma^2\cdots\Sigma^{K+1} \tag{2.4}$$

is the pion field, which can be seen from its transformation properties,

$$\Sigma\to L\Sigma R^\dagger. \tag{2.5}$$

The parameters entering (2.1) are $K+1$ decay constants f_k and K gauge couplings g_k. We shall assume they are invariant under a reflection with respect to the middle of the chain,

$$f_k=f_{K+2-k},\quad g_k=g_{K+1-k}, \tag{2.6}$$

which ensures parity is a symmetry in the theory (2.1).

In the case $K=0$ the model reduces to the chiral Lagrangian. For $K=1$ it is the version of the hidden local symmetry realized in the limit of very light ρ's [6]. The model with $K=2$ and a particular choice of parameters $f_1^2=f_3^2=2f_2^2$, $g_1=g_2$ has been considered in Ref. [4]. Graphically, the model can be represented by a "theory-space" diagram shown in Fig. 1. Since this diagram is the usual "moose diagram" cut open, we shall call the model (2.1) the "open moose" theory.

Note that (2.1) is not the most general Lagrangian satisfying all the symmetries and limited to lowest derivatives. In fact, terms of the following type are not forbidden:

$$|\partial_\mu(\Sigma^k\Sigma^{k+1})-i(gA_\mu)^{k-1}(\Sigma^k\Sigma^{k+1}) + i(\Sigma^k\Sigma^{k+1})(gA_\mu)^{k+1}|^2, \tag{2.7}$$

as well as analogous expressions containing products of more than two consecutive Σ's. In order to restrict the Lagrangian to the form (2.1) an additional condition of nearest neighbor locality in the k space should be imposed. It is this condition that enables us later to interpret this theory as a dimensionally deconstructed 5D gauge theory in the limit $K\to\infty$.

[2]We are using the usual 3+1 Minkowski metric $\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$, but write all indices as lower indices for simplicity, unless it could lead to a confusion.

III. PHYSICAL OBSERVABLES

In this section we derive expressions for physical observables, such as pion and vector meson decay constants, mass spectrum, and pion-vector couplings in terms of the parameters of the moose f_k and g_k.

A. Pion decay constant f_π

For computations, the following gauge is the most convenient,

$$\Sigma^k = \exp\left\{ i\Pi \frac{f}{2f_k^2} \right\}, \quad \text{where} \quad \Pi = \pi^a \frac{\tau^a}{2}, \tag{3.1}$$

where f is a function of all f_k, which we shall specify in a moment [in Eq. (3.4)]. The advantage of this gauge is that the pion field Π does not mix with other fields:

$$\mathcal{L}_{\text{mix}} = \sum_{k=1}^{K+1} f \,\text{Tr}\, \partial_\mu \Pi [(gA_\mu)^k - (gA_\mu)^{k-1}] = 0. \tag{3.2}$$

The value of f is fixed by requiring that the kinetic term for π^a is canonically normalized:

$$\mathcal{L}_{\pi^2} = \sum_{k=1}^{K+1} \frac{f^2}{4f_k^2} \text{Tr}(\partial_\mu \Pi)^2 = \sum_{k=1}^{K+1} \frac{f^2}{4f_k^2} \frac{1}{2} (\partial_\mu \pi^a)^2. \tag{3.3}$$

Therefore

$$\frac{4}{f^2} = \sum_{k=1}^{K+1} \frac{1}{f_k^2}. \tag{3.4}$$

To determine f_π we use Noether's theorem to construct the axial current $\mathcal{A}_\mu$. Let us consider an infinitesimal axial SU(2) transformation. It acts only on Σ^1 and Σ^{K+1} at the ends of the moose:

$$\Sigma^1 \to U\Sigma^1 \quad \text{and} \quad \Sigma^{K+1} \to \Sigma^{K+1} U,$$

where

$$U = \exp(i\alpha^a \tau^a/2). \tag{3.5}$$

If the parameter α depends on coordinates, the Lagrangian changes by $\delta\mathcal{L} = \mathcal{A}_\mu^a \partial_\mu \alpha^a$. On the other hand from (2.1) one finds

$$\delta\mathcal{L}_{\pi^2} = f\, \text{Tr}(\partial_\mu \Pi) \tau^a (\partial_\mu \alpha^a) = f \partial_\mu \pi^a (\partial_\mu \alpha^a), \tag{3.6}$$

which means $\mathcal{A}_\mu = f \partial_\mu \pi^a$, i.e., $f_\pi = f$. Equation (3.4) becomes

$$\frac{4}{f_\pi^2} = \sum_{k=1}^{K+1} \frac{1}{f_k^2}. \tag{3.7}$$

It is a simple exercise to verify that for $K=0,1,2$ this general formula is in agreement with corresponding results in these theories. It is also perhaps useful to observe that sending $f_k \to \infty$ on one of the links effectively sets gauge fields on the ends of this link equal to each other $[(gA)^k = (gA)^{k+1}]$, effectively eliminating this link and reducing K by one. The formula (3.7) obviously reflects this reduction—the corresponding term $1/f_k^2$ drops out.

B. Vector meson mass spectrum m_n

In our gauge, the vacuum is $\Sigma^k = 1$ for all k. Expanding to second order in A_μ^k, we find the terms that determine the masses of the vector mesons:

$$\mathcal{L}_{A^2} = \sum_{k=1}^{K+1} f_k^2 \text{Tr}[(gA_\mu)^{k-1} - (gA_\mu)^k]^2$$
$$\equiv \sum_{\substack{k=1 \\ k'=1}}^{K} (M^2)_{kk'} \text{Tr}\, A_\mu^k A_\mu^{k'}. \tag{3.8}$$

The mass matrix can be diagonalized by using an orthogonal matrix b_n^k satisfying

$$\sum_{\substack{k=1 \\ k'=1}}^{K} (M^2)_{kk'} b_m^k b_n^{k'} = m_n^2 \delta_{mn}; \quad b^T b = b b^T = 1. \tag{3.9}$$

In terms of new vector fields α_μ^n defined as

$$A_\mu^k = \sum_n b_n^k \alpha_\mu^n \quad \text{or} \quad \alpha_\mu^n = \sum_k b_n^k A_\mu^k, \tag{3.10}$$

the mass term (3.8) is diagonal.

Using Eq. (3.8) and the orthogonality of b_n^k, we can write the equation determining b_n^k and m_n as

$$f_{k+1}^2[(gb_n)^{k+1} - (gb_n)^k] - f_k^2[(gb_n)^k - (gb_n)^{k-1}]$$
$$= -\frac{m_n^2 b_n^k}{g_k}. \tag{3.11}$$

This is essentially a discretized version of a Sturm-Liouville problem. We shall write the corresponding differential equation in Sec. IV when we consider the continuum limit $K \to \infty$. We shall also use the discrete equation (3.11) in Sec. III D.

Without solving Eq. (3.11), we can conclude right away that there is a tower of eigenvalues m_n, $n=1,2,\ldots,K$, corresponding to the masses of vector and axial vector mesons. The lowest $n=1$ and $n=2$ states correspond to the ρ and a_1 mesons. Moreover, states with opposite parity alternate in the spectrum:

$$n=1,3,\ldots \quad (\rho,\rho',\ldots): \quad b_n^k = +b_n^{K+1-k},$$
$$n=2,4,\ldots \quad (a_1,a_1',\ldots): \quad b_n^k = -b_n^{K+1-k}, \tag{3.12}$$

Odd n states correspond to vector mesons and even n to axial vector mesons. In the real world, the trend of alternating parity can be seen in the hadronic spectrum for the few first vector and axial vector states.

C. Vector meson–pion-pion coupling $g_{n\pi\pi}$

Let us compute the coupling of nth vector meson to a pion pair, $g_{n\pi\pi}$. Expanding the Lagrangian in A and π, isolating $A\pi\pi$ terms, and using Eq. (3.10), we find

$$\mathcal{L}_{A\pi\pi}=i\sum_{k=1}^{K+1}\frac{f_\pi^2}{4f_k^2}\mathrm{Tr}[\partial_\mu\Pi,\Pi][(gA_\mu)^{k-1}+(gA_\mu)^k]$$
$$=i\sum_{n=1}^{K}\sum_{k=1}^{K+1}\frac{f_\pi^2}{4f_k^2}[(gb_n)^{k-1}+(gb_n)^k]\mathrm{Tr}[\partial_\mu\Pi,\Pi]\alpha_\mu^n. \tag{3.13}$$

Recall that we use the matrix notation where $\alpha=\alpha^a\tau^a/2$ and $\Pi=\pi^2\tau^a/2$. If we normalize $g_{n\pi\pi}$ so that the relevant coupling is

$$\mathcal{L}_{\alpha\pi\pi}=-g_{\alpha\pi\pi}\epsilon^{abc}\partial_\mu\pi^a\pi^b\alpha_\mu^c,\quad \alpha=\rho,\rho',\ldots \tag{3.14}$$

then

$$g_{n\pi\pi}=\sum_{k=1}^{K+1}\frac{f_\pi^2}{4f_k^2}\frac{1}{2}[(gb_n)^{k-1}+(gb_n)^k]. \tag{3.15}$$

Note that this $n\pi\pi$ coupling vanishes for axial mesons $n=2,4,\ldots$, as required by parity, because their "wave functions" b_k^n are odd under $k\to K+1-k$ [Eq. (3.12)].

D. Vector meson decay constants g_{nV} and g_{nA}

We define the decay constants for the vector and axial vector mesons via the matrix elements of the vector and axial vector currents between the vacuum and the one-meson states,

$$\langle 0|\mathcal{V}_\mu^a(0)|\alpha_n^b(p,\epsilon)\rangle=g_{nV}\delta^{ab}\epsilon_\mu, \tag{3.16a}$$

$$\langle 0|\mathcal{A}_\mu^a(0)|\alpha_n^b(p,\epsilon)\rangle=g_{nA}\delta^{ab}\epsilon_\mu. \tag{3.16b}$$

Here $|\alpha_n^b(p,\epsilon)\rangle$ is a single-particle state of the nth vector boson ($\alpha_1=\rho$, $\alpha_2=a_1$, etc.) with isospin b and polarization ϵ. Both g_{nV} and g_{nA} have the dimension of [mass^2]. $g_{nV}=0$ for axial vector mesons (n even) and $g_{nA}=0$ for vector mesons (n odd).

It is convenient to compute g_{nV} by looking at the vector current-current correlator $\langle\mathcal{V}_\mu(x)\mathcal{V}_\nu(0)\rangle$. The residues at the poles are easily related to g_{nV}. The correlator can be obtained by gauging the corresponding $SU(2)_V$ transformation and differentiating the action with respect to the gauge field B_μ:

$$\langle\mathcal{V}_\mu^a(x)\mathcal{V}_\nu^b(0)\rangle\equiv i\langle 0|T(\mathcal{V}_\mu^a(x)\mathcal{V}_\nu^b(0))|0\rangle$$
$$=-\frac{\delta^2W_{\mathrm{vac}}[B_\mu]}{\delta B_\mu^a(x)\,\delta B_\nu^b(y)}, \tag{3.17}$$

where $W_{\mathrm{vac}}[B_\mu]$ is the vacuum energy functional in the presence of the external field B. The $SU(2)_V$ transformation only affects the two Σ links at the ends of the chain according to Eq. (2.3):

$$\Sigma^1\to U\Sigma^1\quad\text{and}\quad\Sigma^{K+1}\to\Sigma^{K+1}U^\dagger, \tag{3.18}$$

and therefore the terms containing the gauge field B_μ are only from $k=1$ and $k=K+1$:

$$\mathcal{L}_B=f_1^2\mathrm{Tr}|\partial_\mu\Sigma^1-iB_\mu\Sigma^1+i\Sigma^1(gA_\mu)^1|^2$$
$$+f_{K+1}^2\mathrm{Tr}|\partial_\mu\Sigma^{K+1}-i(gA_\mu)^K\Sigma^{K+1}+i\Sigma^{K+1}B_\mu|^2. \tag{3.19}$$

Keeping only terms bilinear in the fields we find (the term ΠB is absent due to parity):

$$\mathcal{L}_{A^2,AB,B^2}=f_1^2(B_\mu^a)^2-B_\mu^af_1^2[(gA_\mu^a)^1+(gA_\mu^a)^K]+\mathcal{L}_{A^2}+\mathcal{L}_{F^2}. \tag{3.20}$$

Extremizing the action with respect to A at fixed B and then taking the second derivative with respect to B we find (this is also equivalent to taking the Gaussian integral over A and then differentiating the logarithm of this integral)

$$\langle\mathcal{V}_\mu^a(x)\mathcal{V}_\nu^b(y)\rangle$$
$$=2f_1^2\eta_{\mu\nu}\delta_{xy}\delta^{ab}$$
$$-f_1^4\langle[(gA_\mu^a)^1+(gA_\mu^a)^K](x)[(gA_\nu^b)^1+(gA_\nu^b)^K](y)\rangle, \tag{3.21}$$

where the $\langle AA\rangle$ is the propagator of A, i.e., the inverse of the quadratic form found in $\mathcal{L}_{A^2}+\mathcal{L}_{F^2}$. Diagonalizing this expression using Eq. (3.10) and performing a Fourier transformation with respect to the four-dimensional coordinate x, we find

$$\langle\mathcal{V}_\mu^a(q)\mathcal{V}_\nu^b(-q)\rangle$$
$$=2f_1^2\eta_{\mu\nu}\delta^{ab}-\sum_{n=1}^{K}\frac{g_{nV}^2}{-q^2+m_n^2}\delta^{ab}\left(\eta_{\mu\nu}-\frac{q_\mu q_\nu}{m_n^2}\right), \tag{3.22}$$

where the decay constants g_{nV} are determined to be

$$g_{nV}=f_1^2[(gb_n)^1+(gb_n)^K]. \tag{3.23}$$

Note that $g_{nV}=0$ for axial mesons for which $b_n^1=-b_n^K$.

One can also use Eq. (3.11) together with Eq. (3.23) to write a different representation for g_{nV}:

$$g_{nV}=f_1^2[(gb_n)^1+(gb_n)^K]=m_n^2\sum_{k=1}^{K}\frac{b_n^k}{g_k}. \tag{3.24}$$

It is perhaps easier to look at this equation as a discretized version of integration by parts as we shall do in Sec. IV.[3]

The calculation of the decay constants of the axial vector mesons g_{nA} is completely analogous. We introduce an auxiliary gauge field $\widetilde{B}$ coupled to the axial current $\mathcal{A}$:

$$\mathcal{L}_{\widetilde{B}}=f_1^2\mathrm{Tr}|\partial_\mu\Sigma^1-i\widetilde{B}_\mu\Sigma^1+i\Sigma^1(gA_\mu)^1|^2 +f_{K+1}^2\mathrm{Tr}|\partial_\mu\Sigma^{K+1}-i(gA_\mu)^K\Sigma^{K+1}-i\Sigma^{K+1}\widetilde{B}_\mu|^2. \tag{3.25}$$

In addition to the $A\widetilde{B}$ mixing and $\widetilde{B}^2$ contact terms there is now also the mixing with pion field $\Pi\widetilde{B}$. Differentiating the logarithm of the partition function twice, we obtain

$$\langle\mathcal{A}^a_\mu(q)\mathcal{A}^b_\nu(-q)\rangle=2f_1^2\eta_{\mu\nu}\delta^{ab}-\sum_{n=1}^K\frac{g_{nA}^2}{-q^2+m_n^2}\delta^{ab}\times\left(\eta_{\mu\nu}-\frac{q_\mu q_\nu}{m_n^2}\right)-f_\pi^2\frac{q_\mu q_\nu}{q^2}\delta^{ab}, \tag{3.26}$$

where

$$g_{nA}=f_1^2[(gb_n)^1-(gb_n)^K]. \tag{3.27}$$

Note that, as expected, $g_{nA}=0$ for vector mesons for which $b_n^1=b_n^K$.

E. Spectral sum rules

Weinberg [17] has derived two sum rules for the weighted integrals of the difference of spectral functions of the vector and axial vector current correlators, $\langle\mathcal{V}\mathcal{V}\rangle$ and $\langle\mathcal{A}\mathcal{A}\rangle$. We shall now verify that both sum rules hold by using transversality of the current-current correlators.

From Eq. (3.24) it is easy to show that the $\langle\mathcal{V}\mathcal{V}\rangle$ correlator is transverse. Transversality requires that the contact term in Eq. (3.22) is related to the pole terms through the following sum rule:

$$2f_1^2=\sum_{n=1}^K\frac{g_{nV}^2}{m_n^2}. \tag{3.28}$$

This can be checked by using Eq. (3.24) and orthogonality of b_n^k. We indeed find

$$\sum_{n=1}^K\frac{g_{nV}^2}{m_n^2}=\sum_{n=1}^K\sum_{k=1}^K f_1^2[(gb_n)^1+(gb_n)^K]\frac{b_n^k}{g_k}=2f_1^2. \tag{3.29}$$

By using Eq. (3.28) one can rewrite the correlator in a manifestly transverse form,

$$\langle\mathcal{V}^a_\mu(q)\mathcal{V}^b_\nu(-q)\rangle=\Pi_V(-q^2)\delta^{ab}(-q^2\eta_{\mu\nu}+q_\mu q_\nu);$$

where

$$\Pi_V(Q^2)=\sum_{n=1}^K\frac{g_{nV}^2}{m_n^2(Q^2+m_n^2)},\qquad Q^2=-q^2. \tag{3.30}$$

The transversality of the $\langle\mathcal{A}\mathcal{A}\rangle$ correlator (3.26) amounts to the following sum rule:

$$2f_1^2-f_\pi^2=\sum_{n=1}^K\frac{g_{nA}^2}{m_n^2}. \tag{3.31}$$

By comparing Eqs. (3.28) and (3.31) we conclude that

$$\sum_{n=1}^K\left(\frac{g_{nV}^2}{m_n^2}-\frac{g_{nA}^2}{m_n^2}\right)=f_\pi^2, \tag{3.32}$$

which is one of Weinberg's sum rules.[4] This sum rule holds for any K. Note that, for $K\to\infty$ both sums (3.28) and (3.31) must diverge, since f_k must become infinite at the ends of the moose [to ensure convergence in Eq. (3.7)]. However, their difference is finite.

The second Weinberg sum rule

$$\sum_{n=1}^K(g_{nV}^2-g_{nA}^2)=0 \tag{3.33}$$

also holds. It is easy to prove by using the definitions (3.23) and (3.27) and the orthogonality of b_n^k:

$$\sum_{n=1}^K(g_{nV}^2-g_{nA}^2)=4f_1^2\sum_{n=1}^K(gb_n)^1(gb_n)^K=(2f_1g_1)^2\delta_{1K}, \tag{3.34}$$

which vanishes for all $K>1$. In the case $K=1$, there is only one meson–ρ, and no axial mesons at all.

[3]Equations (4.10) or (3.24) can be also understood in the following way (the reader might recognize the discussion given in Refs. [16,1]). One should realize that because of the mixing of B with A the actual photon is not the field B, but a linear combination of B and all A that leaves vacuum $\Sigma^k=1$ invariant. This is similar to the mixing of the standard model hypercharge boson and weak isospin vector boson to produce the photon. The corresponding linear combination of the A fields has a "wave function" proportional to $1/g_k$ (each A_k enters with weight $1/g_k$). The mixing of the actual photon is now entirely through derivative terms in the expansion of $\mathcal{L}_{F^2}$. The corresponding coefficients are given by the overlap of the photon "wave function" $1/g$ and the nth vector meson "wave function" b_n. The factor m_n^2 is from the derivatives evaluated using the equation of motion for the nth meson.

[4]Weinberg's sum rules [17] involve the spectral functions $\rho_V(\mu^2)\equiv(\mu^2/\pi)\mathrm{Im}\,\Pi_V(-\mu^2-i0)$, where Π_V is defined via $\langle\mathcal{V}\mathcal{V}\rangle$ in Eq. (3.30). A similar equation defines ρ_A. The sum rules state that (i) $\int[\rho_V(\mu^2)-\rho_A(\mu^2)]\mu^{-2}d\mu^2=f_\pi^2$; (ii) $\int[\rho_V(\mu^2)-\rho_A(\mu^2)]d\mu^2=0$. In our theory, according to Eq. (3.30), $\rho_{V,A}=\Sigma_n g_{nV,A}^2\delta(-\mu^2+m_n^2)$.

F. Pion form factor and VMD

The pion form factor (defined to be the isovector part of the electromagnetic form factor),

$$\langle \pi^a(p')|\mathcal{V}^c_\mu(0)|\pi^b(p)\rangle = G_{V\pi\pi}(q)\epsilon^{abc}(p+p')_\mu \quad (3.35)$$

can be found isolating terms linear in B in the Lagrangian (3.19). There are two contributions to the form factor—the direct interaction, given by the term $B\pi\partial\pi$ in the Lagrangian and the interaction mediated by vector mesons given by the AB mixing terms and the couplings $A\pi\partial\pi$. One finds

$$G_{V\pi\pi}(q) = \frac{f_\pi^2}{4f_1^2} + \sum_n \frac{g_{nV}g_{n\pi\pi}}{Q^2+m_n^2}. \quad (3.36)$$

Using the expressions (3.24) and (3.15) for g_{nV} and $g_{n\pi\pi}$ and the orthogonality of the matrix b_n^k, the sum rule related to the total charge of pion can be verified,

$$G_{V\pi\pi}(0) = \frac{f_\pi^2}{4f_1^2} + \sum_n \frac{g_{nV}g_{n\pi\pi}}{m_n^2} = 1. \quad (3.37)$$

If we understand VMD as the statement that $G_{V\pi\pi}(q)$ is saturated by a sum over resonances (i.e., dominance by the whole tower of mesons), then in our model VMD is valid when the contribution of the direct interaction is negligible, $f_\pi^2/4f_1^2 \ll 1$. Thus VMD is a natural consequence of the $K\to\infty$ limit [due to Eq. (3.7)]. A stronger statement that $G_{V\pi\pi}(q)$ is saturated by a single ρ pole is not, in general, valid (see, however, Sec. V B).

IV. $K\to\infty$ AND CONTINUUM LIMIT

In the preceding section we derived formulas that are valid for an arbitrary K. Now we wish to consider the limit $K\to\infty$. In this limit the expressions that we found can be simplified, provided that f_k and g_k are sufficiently smooth functions of k. In this case we can consider replacing the discrete variable k by a continuum variable that we shall call u:

$$u = \left(k - \frac{K}{2}\right)a, \quad (4.1)$$

Here a plays the role of the "lattice spacing." If the limit $K\to\infty$ is performed in the following way,

$$K\to\infty \quad \text{and} \quad a\to 0, \quad Ka \equiv 2u_0 \quad \text{fixed}, \quad (4.2)$$

then u becomes a continuum replacement for k. If f_k and g_k are smooth functions of k, we can also replace them by functions of u,

$$af_k^2 = f^2\left(\left(k-\frac{K}{2}\right)a\right) = f^2(u), \quad (4.3a)$$

$$ag_k^2 = g^2\left(\left(k-\frac{K}{2}\right)a\right) = g^2(u). \quad (4.3b)$$

For the resonance "wave functions" b_n^k that vary smoothly we can write

$$\frac{1}{\sqrt{a}}b_n^k = b(u), \quad (4.4)$$

so that orthogonality of the matrix b_n^k translates into orthonormality of the functions $b_n(u)$,

$$\int_{-u_0}^{u_0} du\, b_n(u) b_{n'}(u) = \delta_{nn'}. \quad (4.5)$$

The wave functions of sufficiently high resonances with $n \sim K$ cannot be expected to be smooth, so they must be treated discretely. We shall always be interested in a finite number of lowest resonances, while $K\to\infty$.

A. Physical observables

Let us now write continuum limits for the main formulas we have derived in the preceding section. From Eq. (3.7),

$$\frac{4}{f_\pi^2} = \int_{-u_0}^{+u_0} \frac{du}{f^2(u)}. \quad (4.6)$$

From Eq. (3.11),

$$g(f^2(gb_n)')' = -m_n^2 b_n. \quad (4.7)$$

with Dirichlet boundary conditions $b_n(\pm u_0)=0$ (since we set $A_\mu^0 = A_\mu^{K+1} = 0$).

From Eq. (3.15),

$$g_{n\pi\pi} = \frac{f_\pi^2}{4}\int_{-u_0}^{+u_0} \frac{du}{f^2(u)} g(u) b_n(u). \quad (4.8)$$

From Eq. (3.23), using the fact that $(gb)^0 = (gb_n)^{K+1} = 0$,

$$g_{nV} = -[f^2(u)(g(u)b(u))']_{-u_0}^{+u_0}. \quad (4.9)$$

By using Eq. (4.7), we find the continuum limit of Eq. (3.24):

$$g_{nV} = -[f^2(u)(g(u)b(u))']_{-u_0}^{+u_0} = m_n^2 \int_{-u_0}^{+u_0} du \frac{b_n(u)}{g(u)}. \quad (4.10)$$

Analogously, Eq. (3.27) becomes

$$g_{nA} = [f^2(u)(g(u)b(u))']|_{+u_0} + [f^2(u)(g(u)b(u))']|_{-u_0}. \quad (4.11)$$

It is very interesting that the physical observables we calculated are all well behaved in the continuum limit $K\to\infty, a\to 0$ (provided the corresponding integrals over u converge). For reference, the equations for other vertex couplings are presented in Appendix A.

B. $d=4+1$ and dimensional deconstruction

Our long-moose theory with $K\gg 1$ can be also considered as a discretized (or deconstructed) five-dimensional continuum gauge theory in curved spacetime.[5] The variable u plays the role of the fifth, deconstructed, dimension. The smoothly varying fields Σ's can be interpreted as the link variables along the fifth dimension $u\equiv x^5$,

$$\Sigma^k \approx 1+iaA_5(u), \tag{4.12}$$

For this equation and for remainder of Sec. IV we shall make a temporary switch of notations, absorbing the gauge coupling constants g into the fields A: $gA\to A$. Then the action (2.1) can be written in the 5D notations as

$$S=-\mathrm{Tr}\int du d^4x\left(-f^2(u)F_{5\mu}^2+\frac{1}{2g^2(u)}F_{\mu\nu}^2\right). \tag{4.13}$$

We now compare this action to the action of a gauge field in a background of curved spacetime and a dilaton field. In the following, $|g|$ denotes the determinant of the metric tensor. The action is taken in the form:

$$S=-\frac{1}{2g_0^2}\mathrm{Tr}\int d^5x\sqrt{|g|}e^{-2\phi}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}, \tag{4.14}$$

where $\hat{\mu},\hat{\nu}$ are 5D Lorentz indices. The coupling to the dilaton field is written so that the effective gauge coupling is $g=g_0e^{\phi}$. In our simple model we consider the metric and the dilaton as classical background fields with no dynamics of their own. Taking the dilaton field to be dependent only on the fifth coordinate u, $\phi=\phi(u)$, and the metric to be of the warped form,

$$ds^2=-du^2+e^{2w(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \tag{4.15}$$

the action (4.14) can be expanded as

$$S=-\frac{1}{2g_0^2}\mathrm{Tr}\int d^5x(-2e^{2w-2\phi}F_{5\mu}^2+e^{-2\phi}F_{\mu\nu}^2). \tag{4.16}$$

Equation (4.16) coincides with Eq. (4.13) if one makes the following identification:

$$f^2(u)=\frac{1}{g_0^2}e^{2w-2\phi}, \tag{4.17a}$$

$$g^2(u)=g_0^2e^{2\phi}. \tag{4.17b}$$

Notice that the warp factor e^{2w} is equal to $f^2(u)g^2(u)$, i.e.,

$$ds^2=-du^2+f^2g^2\eta_{\mu\nu}dx^\mu dx^\nu. \tag{4.18}$$

It is also easy to see that the wave equation for the spin-1 mesons (4.7) is one of the Yang-Mills/Maxwell equations for the 5D (massive) gauge field in the curved background. Notice also that the pion field (2.4) is now the Wilson line stretching between the two boundaries,

$$\Sigma(x)=P\exp\left(i\int duA_5(u,x)\right). \tag{4.19}$$

C. AdS/CFT connection

We now show that correlators of conserved currents in our theory can be computed by using a prescription essentially identical to the AdS/CFT one. Namely, the generating functional for the correlation functions of the currents is equal to the action of a solution to the classical field equations, with the sources serving as the boundary values for the classical fields.

Recall our calculation of the current-current correlators in Sec. III D. Instead of the vector field B_μ and $\tilde{B}_\mu$ let us introduce two separate fields A_μ^L and A_μ^R, corresponding to gauging the $SU(2)_L$ and $SU(2)_R$ global symmetries of the theory. The appearance of these fields modify the first and last terms in the moose,

$$\mathcal{L}=f_1^2\mathrm{Tr}|\partial_\mu\Sigma^1-iA_\mu^L\Sigma^1+i\Sigma^1A_\mu^1|^2+\cdots +f_{K+1}^2\mathrm{Tr}|\partial_\mu\Sigma^{K+1}-iA_\mu^K\Sigma^{K+1}+i\Sigma^{K+1}A_\mu^R|^2. \tag{4.20}$$

In this section we also absorb the coupling g_k into the field A. Remember that there are no dynamical fields associated with the ends of the moose $k=0$ and $K+1$. We can treat the ends of the moose more equally with the other points by thinking that the values of the field A^k at the ends of the moose, at $k=0$ and $K+1$, are fixed at given values:

$$A_\mu^0=A_\mu^L \quad\text{and}\quad A_\mu^{K+1}=A_\mu^R. \tag{4.21}$$

If the field A^k is smooth, we can translate this into the continuum limit by setting boundary conditions on the continuous 5D field $A_\mu(u)$:

$$A_\mu(-u_0)=A_\mu^L \quad\text{and}\quad A_\mu(+u_0)=A_\mu^R. \tag{4.22}$$

At tree level, the generating functional is thus equal to

$$Z[A_\mu^L,A_\mu^R]=e^{iS_{\mathrm{cl}}[A_\mu^{\mathrm{cl}}]} \tag{4.23}$$

where A_μ^{cl} is the solution to the classical field equation that satisfies the boundary conditions (4.22). This formula is of the same form as the formula for AdS/CFT correspondence: the sources for the boundary theory (in our case $A_\mu^{L,R}$) serve as the boundary values for the bulk field. In particular, in order to compute the correlation functions for the conserved currents $\mathcal{L}_\mu=\frac{1}{2}(\mathcal{V}_\mu+\mathcal{A}_\mu)$ or $\mathcal{R}_\mu=\frac{1}{2}(\mathcal{V}_\mu-\mathcal{A}_\mu)$ one just needs to differentiate the classical action with respect to the corresponding boundary values, e.g.,

$$\langle\mathcal{L}_\mu(x)\mathcal{L}_\nu(y)\rangle=\frac{\delta^2S_{\mathrm{cl}}[A_\mu^{\mathrm{cl}}]}{\delta A_\mu^L(x)\,\delta A_\nu^L(y)}. \tag{4.24}$$

[5]Deconstruction of gauge theories in curved space was considered in Refs. [18–20].

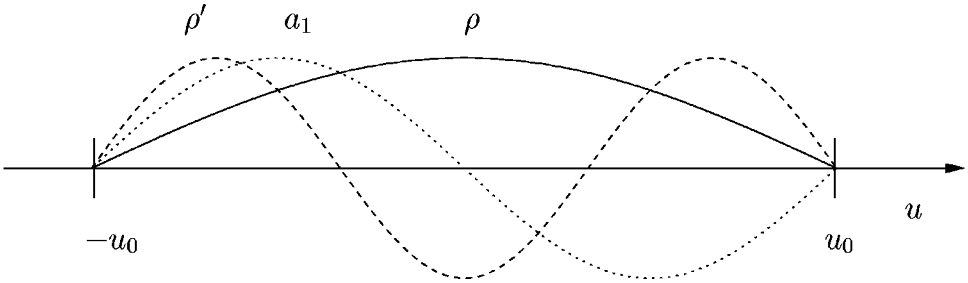

FIG. 2. A few first wave functions in the flat background.

The fact that we have arrived at an AdS/CFT-like formula (4.23) makes one wonder if the hidden local symmetry models for the ρ and a_1 vector mesons [3–5] are (very coarsely) discretized versions of a 5D theory dual to QCD. This could explain why these models enjoy certain phenomenological success, and why in the $K=2$ model [4] one is driven to choose the parameters so that it becomes a moose theory (i.e., nearest neighbor local).

A difference from the usual AdS/CFT correspondence is that there are *two* boundaries in the open moose theory. However, if so desired, one can reformulate the 5D theory (4.14) and (4.15) in the spatial region $0<u<u_0$ (which is one-half of the original $-u_0<u<u_0$) at the price of having two gauge fields obeying a matching condition at $u=0$. Then the spacetime will have only one boundary at $u=u_0$.

V. EXACTLY SOLVABLE EXAMPLES AND PHENOMENOLOGY

So far, our discussion has been general and valid for any choice of f_k and g_k. In this section we shall consider two concrete realizations of the open-moose theory. Our goal is to illustrate the general formulas, and to compare the results with the phenomenology of vector mesons. The two examples are chosen because they are exactly solvable: the spectrum of the vector mesons and the coupling constants can be found in the closed form. The first example is also the simplest possible model, but it has a significant physical drawback that we point out at the end. We think nevertheless that it is a useful reference point for comparison and for understanding the robustness/sensitivity of the results towards the change of the background parameters $f(u)$ and $g(u)$.

A. Example I: Flat background

Consider a moose with parameters f_k and g_k independent of k.[6] In the continuum limit $K\gg 1$ the corresponding functions are therefore constant,[7]

$$f(u)=f, \quad g(u)=g, \quad |u|<u_0=1. \tag{5.1}$$

Let us now apply general formulas from Sec. IV A to determine the properties of this theory in terms of the parameters f and g. From (4.6)

$$f_\pi^2=2f^2. \tag{5.2}$$

The spectrum and wave functions of the spin-1 mesons are given by Eq. (4.7), which becomes

$$b_n''+\frac{m_n^2}{f^2g^2}b_n=0, \quad b_n(\pm 1)=0. \tag{5.3}$$

This means

$$b_n(u)=\sin\left(\frac{\pi n}{2}(u+1)\right); \quad m_n=\frac{\pi f g}{2}n; \quad n=1,2,\ldots, \tag{5.4}$$

The few first wave functions are plotted in Fig. 2.

From Eq. (4.8), for $n=1,3,\ldots,$

$$g_{n\pi\pi}=\frac{f_\pi^2}{\pi}\frac{g}{f^2}\frac{1}{n}=\frac{2}{\pi}g\frac{1}{n}; \quad n=1,3,\ldots. \tag{5.5}$$

Consider the ρ meson, $n=1$. The ratio of m_ρ^2 to $g_{\rho\pi\pi}^2 f_\pi^2$ is dimensionless and is equal to

$$\frac{m_1^2}{g_{1\pi\pi}^2 f_\pi^2}=\frac{\pi^4}{32}\approx 3.04. \tag{5.6}$$

The coupling $g_{\rho\pi\pi}$ can be found from the width of the ρ, which decays predominantly to two pions: $\Gamma_\rho = g_{\rho\pi\pi}^2 m_\rho v_\pi^3/(48\pi)$, where v_π is the velocity of the final-state pions. Using $\Gamma_\rho\approx 150$ MeV, we find the ratio (5.6) to be around 1.9 in Nature. For comparison, the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [21,22] corresponds to this ratio being equal to 2, and the value in Georgi's vector limit (i.e., $K=1$ moose theory) is 4 [6]. Therefore, our model would underpredict Γ_ρ from experimental m_ρ and f_π.

The decay constants g_{nV} and g_{nA} are given by Eqs. (4.10) and (4.11) and are equal to

$$g_{nV,A}=m_n^2\frac{4}{\pi}\frac{1}{g}\frac{1}{n}=\pi f^2 g n. \tag{5.7}$$

In Eq. (5.7) $g_{nV,A}$ refers to g_{nV} for odd n's and g_{nA} for even n's. For $n=1$, we find $g_{\rho V}=\sqrt{2}f_\pi m_\rho$. We can now predict the rate of the electromagnetic decay $\rho^0\to e^+e^-$, using $\Gamma(\rho^0\to e^+e^-)=\frac{4}{3}\pi\alpha^2 g_{\rho V}^2 m_\rho^{-3}$ and the experimental values

[6]Such a theory can be easily solved even for finite K, but we shall only consider $K\gg 1$.

[7]The choice of u_0 does not affect the results; it is equivalent to rescaling f and g.

for f_π and m_ρ. We find $\Gamma(\rho^0\to e^+e^-)\approx 5.0$ keV, which is somewhat smaller than the measured value 6.85 ± 0.11 keV [7].

It is also interesting to consider the contribution of the ρ meson to pion form factor at $q=0$ [Eq. (3.37)],

$$\frac{g_{1V}g_{1\pi\pi}}{m_1^2}=\frac{8}{\pi^2}\approx 0.81. \tag{5.8}$$

Thus the single ρ-meson dominance holds to within 20%. The VMD is, however, exact if all vector mesons are included in the limit $K\to\infty$. Indeed the direct pion-photon interaction in Eq. (3.36) vanishes: $f_\pi^2/4f_1^2=af_\pi^2/4f^2=a/2\to 0$.[8]

The drawback of this model is that it fails to satisfy the asymptotic condition on $\Pi_V(Q^2)$ that follows from QCD: $\Pi_V(Q^2)\sim N_c\log(Q^2)$ when $Q^2\to\infty$. Instead, $\Pi_V(Q^2)$ in this model vanishes as $1/\sqrt{Q^2}$ when $Q^2\to\infty$. Indeed, according to Eq. (3.30), with values of g_{nV} and m_n found in Eqs. (5.7) and (5.4),

$$\Pi_V(Q^2)=\sum_{n=1,3,\ldots}\frac{g_{nV}^2}{m_n^2(Q^2+m_n^2)}=\frac{2f}{gQ}\tanh\left(\frac{Q}{fg}\right). \tag{5.9}$$

We shall now consider an exactly solvable model that will satisfy the condition $\Pi(Q^2)\sim\log Q^2$ at large Q^2.

B. Example II: "cosh" background

This model is given by

$$g(u)=g_5=\text{const}, \tag{5.10a}$$

$$f(u)=\frac{\Lambda}{g_5}\cosh u. \tag{5.10b}$$

According to Eqs. (4.17), this corresponds to a constant dilaton background and the following background metric,

$$ds^2=-du^2+\Lambda^2\cosh^2u\,\eta_{\mu\nu}dx^\mu dx^\nu. \tag{5.11}$$

The two boundaries are located at $u=\pm\infty$. Near the boundaries the metric becomes asymptotically AdS_5. According to the AdS/CFT philosophy, u has the physical meaning of the energy scale; large u's correspond to short distances. Therefore one can expect that the current correlators has the conformal form at short distance, i.e., as $Q^2\to\infty$,

$$\Pi_V(Q^2),\quad \Pi_A(Q^2)\sim\log(Q^2). \tag{5.12}$$

The main reason for choosing the background (5.10) is that $\cosh u$ is the simplest function interpolating between e^{-u} and e^u, and that the mass spectrum can be found exactly (see below). Otherwise, we have no reason to prefer this background over any other that has two AdS_5 boundaries.[9]

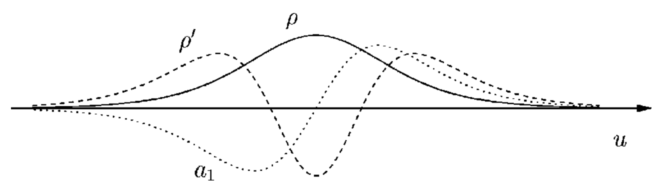

FIG. 3. A few first wave functions in the "cosh" background.

Applying Eq. (4.6), one finds

$$f_\pi^2=\frac{2\Lambda^2}{g_5^2}. \tag{5.13}$$

The wave equation for the vector mesons is

$$(\cosh^2u\,b_n')'=-\frac{m_n^2}{\Lambda^2}b_n, \tag{5.14}$$

which implies the following spectrum:

$$m_n^2=n(n+1)\Lambda^2,\quad n=1,2,\ldots \tag{5.15}$$

In particular, $m_\rho^2=2\Lambda^2$ and $m_{a_1}^2=6\Lambda^2=3m_\rho^2$. Taking m_ρ as an input, this predicts $m_{a_1}=1335$ MeV, which is not far from the observed 1230 ± 40 MeV. However, the masses of higher excitations grow faster with n than in the real world. The 5D eigenfunctions of the vector mesons are

$$b_n(u)=-c_n\frac{P_n^1(\tanh u)}{\cosh u},\quad c_n=\sqrt{\frac{2n+1}{2n(n+1)}}, \tag{5.16}$$

where P_n^1 are the associated Legendre functions. The first few wave functions are (see Fig. 3):

$$b_1(u)=\frac{\sqrt{3}}{2}\frac{1}{\cosh^2u}, \tag{5.17a}$$

$$b_2(u)=\frac{\sqrt{15}}{2}\frac{\sinh u}{\cosh^3u}, \tag{5.17b}$$

$$b_3(u)=-\frac{1}{2}\sqrt{\frac{21}{2}}\frac{1}{\cosh^2u}\left(\frac{5}{2\cosh^2u}-2\right). \tag{5.17c}$$

In order to establish Eq. (5.12), we compute the decay constants of vector mesons from Eqs. (4.10) and (4.11),

[8]This can be verified also by summing the contributions from all vector mesons in (3.37). Each contribution is proportional to $1/n^2$ and $\Sigma_{n=1,3,\ldots}1/n^2=\pi^2/8$.

[9]Curiously, (5.11) coincides with the 5D part of the induced metric on a probe D7 brane in $AdS_5\times S^5$ [23].

$$g_{nV,A}=\sqrt{2n(n+1)(2n+1)}\,\frac{\Lambda^2}{g_5}. \tag{5.18}$$

The correlation function for the vector current is found from Eq. (3.30),

$$\Pi_V(Q^2)=\frac{2\Lambda^2}{g_5^2}\sum_{n\text{odd}}\frac{2n+1}{Q^2+n(n+1)\Lambda^2}. \tag{5.19}$$

At large $Q^2\gg\Lambda^2$, the sum can be replaced by an integral, which is logarithmically divergent.[10] One thus finds for large Q^2

$$\Pi_V(Q^2)=-\frac{1}{g_5^2}\ln(Q^2),\quad Q^2\gg\Lambda^2. \tag{5.20}$$

The asymptotic behavior of $\Pi_A(Q^2)$ is the same. Thus the current correlators have the correct asymptotics at large Q^2. Moreover, they obey Weinberg's sum rules, as proven in Sec. III E. The constraints imposed by the $Q^2\to\infty$ behavior and Weinberg's sum rules on the masses m_n and decay constants $g_{nV,A}$ are quite nontrivial [24]. It is remarkable that the open-moose construction generates examples that automatically satisfy these constraints.

One can match the asymtotics (5.20) with the result found from QCD,

$$\Pi_V(Q^2)=-\frac{N_c}{24\pi^2}\ln(Q^2), \tag{5.21}$$

where N_c is the number of colors, to obtain

$$\frac{1}{g_5^2}=\frac{N_c}{24\pi^2}. \tag{5.22}$$

By using this relationship between g_5 and N_c together with $m_\rho=\sqrt{2}\Lambda$, we can now express all quantities in the model via a single mass m_ρ and the number of colors N_c. A short summary is given in Appendix B. For example, for f_π we find from Eq. (5.13)

$$f_\pi^2=\frac{N_c}{24\pi^2}m_\rho^2. \tag{5.23}$$

For $N_c=3$ Eq. (5.23) predicts $f_\pi=$ 87 MeV, rather close to the experimental value of 93 MeV. Interestingly, Eq. (5.23) coincides with the one obtained from QCD sum rules [25]. The large N_c scaling in (5.23) also matches: $m_\rho\sim 1, f_\pi\sim\sqrt{N_c}$.

Another distinct feature of the model is that the pion form factor is dominated by a single ρ pole. Indeed, the coupling $n\pi\pi$ is found by substituting Eq. (5.16) into Eq. (4.8),

$$g_{n\pi\pi}=-\frac{c_n}{2}g_5\int_{-1}^{1}d\xi\sqrt{1-\xi^2}P_n^1(\xi), \tag{5.24}$$

which vanishes for all $n\neq1$. This is due to the orthogonality of Legendre functions and the fact that $\sqrt{1-\xi^2}=P_1^1(\xi)$. Therefore ρ meson dominance is exact for the pion form factor: $G_{V\pi\pi}(q)=(Q^2+m_\rho^2)^{-1}$. For $n=1$ one finds

$$g_{1\pi\pi}^2\equiv g_{\rho\pi\pi}^2=\frac{g_5^2}{3}=\frac{8\pi^2}{N_c}=\frac{m_\rho^2}{3f_\pi^2}. \tag{5.25}$$

The KSRF ratio in this model is equal to 3. This means that the ρ width is underpredicted by a factor of about 2/3. However, it is still interesting to compute Γ_ρ for arbitrary N_c, in the chiral limit,

$$\Gamma_\rho=\frac{\pi}{6N_c}m_\rho. \tag{5.26}$$

The rate of the electromagnetic decay $\rho^0\to e^+e^-$ in this model,

$$\Gamma(\rho^0\to e^+e^-)=\frac{\alpha^2N_c}{6\pi}m_\rho, \tag{5.27}$$

is equal to 6.5 keV at $N_c=3$, which is rather close to the observed 6.85 ± 0.11 keV. Interestingly, the prediction from QCD sum rules [25] is close but different in this case: $\Gamma(\rho^0\to e^+e^-)_{\text{s.r.}}/\Gamma(\rho^0\to e^+e^-)_{\text{cosh}}=e/3$.

The phenomenology of the a_1 meson in this model is discussed in Appendix C. The excitations with $n>2$ have an unrealistic mass spectrum in our model, so we shall not discuss their phenomenology.

VI. INSTANTON ~ BARYON

The baryon appears in the framework of chiral Lagrangians as a solitonic object: a Skyrmion [26]. One wonders: what is the corresponding object in 5D that can describe the baryon? An obvious candidate is the instanton, which can be "lifted" to become a quasiparticle in 5D. Here we show that the instanton appears from the point of view of 4D as a Skyrmion. We are interested only in topological aspects, and defer the question of stability of such a solution to future work.[11]

On an intuitive level, to see the relation between the instanton and the Skyrmion, one can consider as an example the well-known instanton solution in the singular gauge (in the flat background metric):

[10]We perform a trivial regularization in Eq. (5.20), subtracting a constant equal to $(1/g_5^2)\log(K\Lambda)^2$ for $K\gg1$. Of course, the equation is only valid for $Q^2\ll(K\Lambda)^2$.

[11]It is interesting to note in this regard that the issue of stability of the Skyrmion in models with a ω, ρ, and a_1 mesons has been studied [27–30]. It was determined that vector mesons not only stabilize the Skyrmion, but also noticeably improve agreement with phenomenology.

$$A_\mu = \frac{\tau^a \bar{\eta}^a_{\mu\nu} x_\nu}{x^2(x^2+\rho^2)}. \tag{6.1}$$

In this solution we shall think of A_μ as a four-vector with coordinates μ running through 1, 2, 3, and 5 (i.e., x, y, z, and u) and $x^2 = \boldsymbol{x}^2 + u^2$. Note that the metric signature for these 4 coordinates is Euclidean $(-,-,-,-)$ [see (4.18)]. The solution we wish to use to describe a baryon is static, i.e., $A_0 = 0$, and there is no dependence on t. To see the behavior of the pion field we need to look at A_5 [see (4.19)]:

$$A_5 = \frac{\boldsymbol{\tau}\cdot \boldsymbol{x}}{x^2(x^2+\rho^2)}. \tag{6.2}$$

We see that at every fixed u the solution is a hedgehog, thus having the same topology as the Skyrmion made of the pion field.

We shall now show that for an arbitrary background metric the topological charge of the instanton is equal to the baryon charge of the pion-field Skyrmion. Our discussion is very similar to that of Refs. [31,32]. The 5D Yang-Mills theory possesses a conserved topological current,

$$\sqrt{|g|}\, j^{\hat\mu}_{5\mathrm{D}} = \frac{1}{32\pi^2} \epsilon^{\hat\mu\hat\nu\hat\lambda\hat\rho\hat\sigma} \mathrm{Tr}\, F_{\hat\nu\hat\lambda} F_{\hat\rho\hat\sigma}. \tag{6.3}$$

Here $\epsilon^{\hat\mu\hat\nu\hat\lambda\hat\rho\hat\sigma}$ is defined so that its elements are ± 1. For simplicity we assume that g_5 is a constant and absorb it into the gauge field, and drop the hat in the 5D Lorentz indices in subsequent formulas. The boundaries are assumed to be $u = \pm\infty$. That this current is conserved, $\partial_\mu(\sqrt{|g|} j^\mu) = 0$, can be shown by using the Bianchi identity $D_{[\mu} F_{\nu\lambda]} = 0$. The topological charge of a static solution is

$$Q = \int du\, d^3x \sqrt{|g|}\, j^0_{5\mathrm{D}} = \frac{1}{32\pi^2} \int du\, d^3x\, \epsilon^{0\mu\nu\lambda\rho} \mathrm{Tr}\, F_{\mu\nu} F_{\lambda\rho}. \tag{6.4}$$

The numerical coefficient in Eq. (6.3) was chosen so that the static instanton has unit total charge.

Now consider a field configuration where the pion field, given by the Wilson line along the u coordinate (4.19) has a nontrivial winding, and A_μ goes to 0 at the boundaries. To compute the topological charge of this configuration, it is convenient to perform a gauge transformation to set $A_5 = 0$. Explicitly,

$$A_\mu \to U A_\mu U^{-1} + i U \partial_\mu U^{-1},$$

$$U(u,\boldsymbol{x}) = P \exp\left(-i \int_{-u_0}^{u} du'\, A_5(u',\boldsymbol{x}) \right). \tag{6.5}$$

According to (4.19), $U(+u_0,\boldsymbol{x}) = \Sigma^{-1}(\boldsymbol{x})$. Thus while A_i remains 0 at the left boundary $u = -u_0$, it becomes nonzero at the right boundary:

$$A_i = i\Sigma^{-1}\partial_i \Sigma, \quad u \to +u_0, \quad i = 1,2,3. \tag{6.6}$$

By using the identity

$$\epsilon^{0\mu\nu\lambda\rho} \mathrm{Tr}\, F_{\mu\nu} F_{\lambda\rho} = \partial_\mu K^{0\mu},$$

$$K^{0\mu} = 4\epsilon^{0\mu\nu\lambda\rho} \mathrm{Tr}\left(A_\nu \partial_\lambda A_\rho - \frac{2i}{3} A_\nu A_\lambda A_\rho \right), \tag{6.7}$$

one can rewrite the topological charge (6.4) as

$$Q = \frac{1}{32\pi^2} \int d^3x K^{05} \Big|^{u=+u_0}_{u=-u_0} = \frac{1}{32\pi^2} \int d^3x K^{05} \Big|_{u=+u_0}, \tag{6.8}$$

since at $u = -u_0$ $A_i = 0$, and $K^{05} = 0$. By using Eq. (6.6) one transforms this expression to

$$Q = \frac{i}{24\pi^2} \int d^3x\, \epsilon^{ijk} \mathrm{Tr}[(\Sigma^{-1}\partial_i \Sigma)(\Sigma^{-1}\partial_j\Sigma)(\Sigma^{-1}\partial_k \Sigma)]. \tag{6.9}$$

It is now obvious that the topological charge becomes the winding number of the pion field. Therefore, the instanton becomes a Skyrmion, and corresponds to the physical baryon.

VII. CONCLUSIONS AND DISCUSSION

We considered a theory of an "open moose" given by Lagrangian (2.1) illustrated in Fig. 1. This model describes a multiplet of massless Goldstone bosons and a tower of vector and axial vector mesons. We developed a formalism for calculating the mass spectrum and the coupling constants in this theory for arbitrary parameters of the moose, f_k and g_k, and determine their values in the continuum limit, when the number of hidden symmetry groups K tends to infinity. We applied this formalism to two exactly solvable realizations of the model and found that the physics of the lowest modes match quite well with the phenomenology of the π, ρ, and a_1 mesons.

We also find that the open-moose theory naturally incorporates the phenomenon of vector meson dominance. For example, the pion form factor is saturated by poles from a tower of vector mesons. Moreover, since couplings between mesons are given by overlap integrals, the couplings of highly excited ρ's to the pion are suppressed by the oscillations of their wave functions in the fifth dimension. This means that the pion form factor should be well approximated by the sum of contributions from a few lowest ρ's. In the second example we considered (the "cosh" background) the situation is brought to an extreme: the pion form factor is saturated by a single pole ρ-meson dominance. We verified that both Weinberg's spectral sum rules are automatically obeyed, in a nontrivial way, in any open-moose theory.

One of our original motivations was to include the excited vector mesons beyond the lowest a_1. With respect to that goal, we achieved only limited success, at least within the two exactly solvable models we considered. On the one hand, we do find that vector and axial vector mesons alternate in the spectrum, as it seems to be the case in QCD, at least for a few excited states. On the other hand, in both our simple models, the mass of an n-th state m_n is $\mathcal{O}(n)$ at large n, which seems to be in contradiction with the real world, and with the theoretical prejudice that $m_n = \mathcal{O}(\sqrt{n})$. Further study of different backgrounds might provide a model that reproduces desired features of excited mesons and help understand constraints that phenomenology and QCD theorems impose on functions $f(u)$ and $g(u)$. Alternatively, it is also possible that the excited vector mesons have ‘‘stringy’’ nature and cannot, in principle, be incorporated into our field-theoretical scheme.[12]

The success that the model enjoys in describing the lowest states can be attributed to an apparent property of low-energy QCD: at intermediate distances correlation functions are reasonably well saturated by a single pole. In the ‘‘cosh’’ model the excited mesons ensure the correct behavior of the (averaged) spectral densities, thus playing the role of the continuum. This explains why some results of QCD sum rules are well reproduced.

We hope that the study of the open moose theories will deepen our understanding of QCD at the fundamental level. One intriguing fact discovered in these theories is the similarity to the AdS/CFT correspondence. The procedure of calculating current-current correlators is essentially equivalent to the well-known AdS/CFT prescription: the correlators are given by the variational derivatives of the classical 5D action of the dual theory with respect to the sources living on the 4D boundary. There is overwhelming evidence that the $\mathcal{N}=4$ supersymmetric Yang-Mills theory is described by a string theory. Perhaps, an open moose theory is a low-energy limit of the string theory dual to QCD.[13] In this regard, the result we found in the ‘‘cosh’’ model,

$$g_5^2 \sim \frac{1}{N_c},$$

is reassuring in view of a general expectation that such a dual theory should have a coupling proportional to $1/N_c$ in the ’t Hooft limit.

Among the questions left for further study is the detailed phenomenology of isoscalar mesons (η, ω, f_1, etc.). These mesons are described by an additional 5D Abelian gauge field, which should be introduced into the action (4.14). Most of our results should generalize straightforwardly to this case. However, there is an important new issue that the isoscalar sector brings into the theory. The global $\mathrm{U}(1)_A$ symmetry must be explicitly broken, e.g., η should not be massless. It is very encouraging that the 5D formulation of the theory provides a very natural mechanism for this. It is the topological 5D Chern-Simons term of the form

$$\int d^5x\, \epsilon^{\hat{\mu}\hat{\nu}\hat{\lambda}\hat{\rho}\hat{\sigma}} A_{\hat{\mu}} F^a_{\hat{\nu}\hat{\lambda}} F^a_{\hat{\rho}\hat{\sigma}}, \tag{7.1}$$

where $A_{\hat{\mu}}$ is the 5D vector field describing isoscalars. This term breaks the $\mathrm{U}(1)_A$ symmetry in the desired way. In particular, it is not invariant under $\mathrm{U}(1)_A$ transformations on the 4D boundary (although it is invariant under local transformations in the bulk of 5D). It is easy to see that it also provides $\pi^0 \to 2\gamma$ and other anomalous processes in QCD. The coefficient of the term (7.1) can be fixed by matching to QCD chiral anomaly, and is therefore proportional to N_c. The term (7.1) also couples the ω meson field to the baryon current, providing a hard-core repulsion between baryons, and preventing the baryon/instanton from shrinking to zero size (this effect is the origin of the stabilization of the Skyrmion observed in Ref. [27]). It would be also interesting to see how the open-moose theory realizes Di Vecchia–Veneziano–Witten Lagrangian [34] and the corresponding phenomenology. Other avenues for future study are the incorporation of finite quark masses, extension to three flavors and realization of the Wess-Zumino-Witten topological term (which does require a 5th dimension [35]).

ACKNOWLEDGMENTS

The authors thank S. R. Beane, G. Gabadadze, W-Y. Keung, and especially T. Imbo and M. Strassler for discussions. The authors also wish to acknowledge the review talk by R. L. Jaffe at the *QCD and String Theory* workshop at the Institute for Nuclear Theory in Seattle, which provided much of the motivation for this work. M.A.S. acknowledges the hospitality of the Institute for Nuclear Theory, University of Washington, where part of this work has been done, and thanks RIKEN BNL Center and U.S. Department of Energy (DE-AC02-98CH10886) for providing facilities essential for the completion of this work. D.T.S. is supported, in part, by DOE Grant No. DOE-ER-41132 and the Alfred P. Sloan Foundation. M.A.S. is supported, in part, by a DOE OJI grant and by the Alfred P. Sloan Foundation.

APPENDIX A: INTERACTION VERTICES

For reference, we provide here some additional formulas for interaction vertices in the continuum limit of an arbitrary open-moose model. Let us define $g_{\pi mn}$, $g_{\pi\pi mn}$, g_{mnp}, and g_{mnpq} so that the Lagrangian contains

$$\begin{aligned}\mathcal{L} = \cdots &- g_{\pi mn} \epsilon^{abc} \pi^a (\alpha^m_\mu)^b (\alpha^n_\mu)^c \\ &- g_{\pi\pi mn} \epsilon^{abc} \epsilon^{ade} \pi^b \pi^d (\alpha^m_\mu)^c (\alpha^n_\nu)^e \\ &- g_{mnp} \epsilon^{abc} (\alpha^m_\mu)^a (\alpha^n_\nu)^b \partial_\mu (\alpha^p_\nu)^c \\ &- \frac{1}{4} g_{mnpq} \epsilon^{abc} \epsilon^{ade} (\alpha^m_\mu)^b (\alpha^n_\nu)^c (\alpha^p_\mu)^d (\alpha^q_\nu)^e,\end{aligned} \tag{A1}$$

then

[12]It is possible to reproduce the behavior $m_n = \mathcal{O}(\sqrt{n})$ by a suitable choice of background, even an exactly solvable one. But we did not find such models viable in other respects.

[13]From this point of view, meson interactions in strongly coupled gauge theories with fundamental quarks [23,33] deserve further studies.

$$g_{\pi mn}=\frac{f_\pi}{4}\int du[(gb_m)(gb_n)'-(gb_m)'(gb_n)], \quad \text{(A2a)}$$

$$g_{\pi\pi mn}=-\frac{f_\pi^2}{8}\int \frac{du}{f^2(u)}b_m b_n, \quad \text{(A2b)}$$

$$g_{mnp}=\int dugb_m b_n b_p, \quad \text{(A2c)}$$

$$g_{mnpq}=\int dug^2 b_m b_n b_p b_q, \quad \text{(A2d)}$$

Direct couplings to external currents are suppressed in the continuum limit (this includes, in particular, vector-meson dominance by a whole tower of mesons).

A simple qualitative interpretation of these couplings exists in terms of the overlaps of the wave functions in the u space, which reflects the property of locality of the theory (2.1) or (4.14). It is straightforward for the last two, resonance-resonance, couplings (A2c) and (A2d). These terms come from the second, $F_{\mu\nu}^2$ term in (2.1). For the pion-resonance couplings (A2a), (A2b) and (4.8), one should bear in mind that the strength of the coupling is proportional to $f^2(u)$, and think of the pion wave function as being proportional to $1/f^2$ [looking at (3.1)]. The u derivatives in (A2a) are necessary to account for the fact that, although the pion wave function is even in $u\to -u$, the pion itself is a pseudoscalar.

APPENDIX B: SUMMARY OF RESULTS FOR THE "COSH" MODEL

Instead of expressing the results in terms of the parameters of the model Λ and g_5, we will use m_ρ and N_c. The relations are

$$m_\rho=\sqrt{2}\Lambda, \quad g_5^2=\frac{24\pi^2}{N_c}, \quad \text{(B1)}$$

$$m_n=m_\rho\sqrt{\frac{n(n+1)}{2}}, \quad \text{(B2)}$$

$$f_\pi=\frac{m_\rho}{2\pi}\sqrt{\frac{N_c}{6}}, \quad \text{(B3)}$$

$$g_{nV,A}=\frac{m_\rho^2}{4\pi}\sqrt{\frac{n(n+1)(2n+1)}{3}N_c}, \quad \text{(B4)}$$

The ρ-meson dominance of the pion form factor is described by

$$g_{n\pi\pi}=0, \quad n\neq 1, \quad \text{(B5)}$$

$$g_{1\pi\pi}\equiv g_{\rho\pi\pi}=\frac{2\sqrt{2}\pi}{\sqrt{N_c}} \quad \text{(B6)}$$

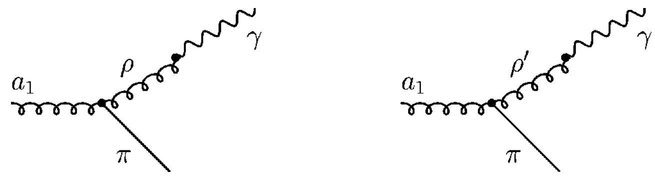

FIG. 4. Diagrams contributing to $a_1\to\pi\gamma$.

(also $g_{\rho\pi\pi}=m_\rho^2/g_{\rho V}$). There is a "$\Delta n=1$ rule" for pion emission:

$$g_{\pi mn}=0, \quad |m-n|\neq 1, \quad \text{(B7)}$$

$$g_{\pi n n+1}=m_\rho\pi(n+1)\sqrt{\frac{6n(n+2)}{(2n+1)(2n+3)N_c}}. \quad \text{(B8)}$$

There is also a "triangle rule" for triple resonance vertex:

$$g_{mnp}=0 \quad \text{if } m+1>n+p+2 \quad \text{or } n+1>p+m+2 \quad \text{or } p+1>m+n+2, \quad \text{(B9)}$$

i.e., the amplitude vanishes if a triangle (even a degenerate one) with sides $(m+1)$, $(n+1)$, and $(p+1)$ does not exist.

APPENDIX C: a_1 MESON IN THE "COSH" BACKGROUND

Let us discuss the phenomenology of the lowest axial vector meson (the $n=2$ excitation in the open moose). From Eq. (5.15), the mass of the a_1 meson in the "cosh" model is $m_{a_1}=\sqrt{3}m_\rho$. The a_1 decays into $\rho\pi$ with the coupling (B8),

$$g_{\pi\rho a_1}=2\pi m_\rho\sqrt{\frac{6}{5N_c}}. \quad \text{(C1)}$$

By using the formula [4]

$$\Gamma(a_1\to\rho\pi)=\frac{g_{\pi\rho a_1}^2}{4\pi}p_\rho\left(1+\frac{p_\rho^2}{3m_\rho^2}\right), \quad \text{(C2)}$$

we find

$$\Gamma(a_1\to\rho\pi)=\frac{4\pi}{9\sqrt{3}}N_c m_\rho\approx 210 \text{ MeV}. \quad \text{(C3)}$$

Experimentally, the total width of a_1 is 250 to 600 MeV, of which about 60% comes from $a_1\to\rho\pi$ [7].

The a_1 decay constant in our model is

$$g_{a_1 A}\equiv g_{2A}=\frac{m_\rho^2}{2\pi}\sqrt{\frac{5N_c}{2}}\approx 0.26 \text{ GeV}^2. \quad \text{(C4)}$$

A lattice measurement of this constant yields (in our normalization) 0.21 ± 0.02 GeV^2 [36], while an analysis of hadronic τ decays gives 0.177 ± 0.014 GeV^2 [37]. The agreement is fair, but not exceptionally good.

The decay $a_1 \to \pi\gamma$ occurs through two Feynman diagrams with intermediate ρ and ρ' (the $n=3$ excitation). Its amplitude is proportional to (see Fig. 4)

$$\frac{g_{\pi 12} g_{1V}}{m_1^2} + \frac{g_{\pi 32} g_{3V}}{m_3^2} \equiv \frac{g_{\pi\rho a_1} g_{\rho V}}{m_\rho^2} + \frac{g_{\pi\rho' a_1} g_{\rho' V}}{m_{\rho'}^2}. \quad \text{(C5)}$$

It can be checked that the two terms cancel each other exactly, so the amplitude vanishes. On the other hand, the partial width of this decay is quoted to be 640 ± 246 keV [7]. The simplest $K=2$ hidden local symmetry model also suffers from the same problem; in Ref. [4] this was cured by adding higher-derivative terms to the action. It would be interesting to see if this rate can be made nonzero by adding more terms to the action (4.14).

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