# Static post-Newtonian equivalence of general relativity and gravity with a dynamical preferred frame

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A generally covariant extension of general relativity (GR) in which a dynamical unit timelike vector field is coupled to the metric is studied in the asymptotic weak field limit of spherically symmetric static solutions. The two post-Newtonian parameters known as the Eddington-Robertson-Schiff parameters are found to be identical to those in the case of pure GR, except for some nongeneric values of the coefficients in the Lagrangian.

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## I. INTRODUCTION

Over the past several years quantum gravity considerations have led a number of researchers to contemplate violations of Lorentz invariance. Such violations are usually studied within the realm of particle physics, but they would also have implications for gravitation. Fixed background tensors breaking Lorentz symmetry also break the general covariance of general relativity (GR), generically resulting in overdetermined equations of motion. Together with a theoretical bias for preserving general covariance, this leads one to consider Lorentz breaking introduced by *dynamical* tensor fields.

The background value of a given dynamical tensor field "spontaneously" breaks Lorentz symmetry, either because of a potential that has a Lorentz violating (LV) minimum, or because the tensor is somehow constrained not to vanish. The simplest situations arise from scalar fields whose nonzero gradient is an LV vector, and vector fields. Vector fields without any potential or constraint were considered in the early 1970s by Will, Nordvedt, and Hellings in the spirit of alternate theories of gravity [1–4]. Vector fields with a potential leading to LV were studied by Kostelecký and Samuel [8] motivated by string theory considerations, and by Clayton and Moffat [5] motivated by the notion that a dynamically varying speed of light might solve some cosmological problems (see also [6,7]).

A vector field constrained to have a fixed timelike or spacelike length breaks local Lorentz symmetry in every configuration, much as a nonlinear sigma model spontaneously breaks gauge invariance. In the timelike case the residual symmetry is the 3D rotation group, while in the spacelike case it is the 2+1 Lorentz group. A particularly simple example of such a theory was considered by Kostelecký and Samuel [8] and also studied by Jacobson and Mattingly [9]. In this example, the action for the covariant vector field is just the square of the exterior derivative. Like the Maxwell action for the vector potential, this is independent of the connection components.

The most general action with up to two derivatives of a unit vector field has four terms. Three of these terms were included in the original study of Will and Nordvedt [1] (which did not include any constraint on the length of the vector) and all four were written down and studied using the tetrad formalism by Gasperini [11], who broke the local Lorentz symmetry by including in the action terms referring to a fixed "internal" unit timelike vector. The same theory in the metric formalism was written down in [9], and the linearized wave solutions in one special case (corresponding to the case focused on in [1]) were reported in [10]. The general constrained vector-tensor theory is quite complicated due to the derivative coupling to the vector field which includes connection components. Therefore the action for the vector in fact modifies the kinetic terms for the metric as well. Thanks to the unit timelike constraint, the vector has only three degrees of freedom, all corresponding to spacelike variations. Thus, unlike in other vector theories without gauge invariance, problems with negative energy modes need not arise.

We are interested in the observational signatures and constraints on the parameters in the general constrained vector-tensor theory. These can be obtained from the parametrized post-Newtonian (PPN) parameters, wave phenomena, and strong field effects. The PPN parameters for the general unconstrained vector-tensor theory were found by Will [4]. Those results do not directly apply to the constrained case, hence the analysis must be carried out anew. In this paper we begin that process, restricting at first to the static PPN parameters, i.e. the Eddington-Robertson-Schiff parameters $\beta$ and $\gamma$, which are the only ones that do not vanish when the isolated system is at rest with respect to the asymptotic rest frame defined by the vector field. We find that for generic values of the coefficients in the Lagrangian these parameters take the same values as in GR. This indicates that to observationally bound the coefficients in the Lagrangian one must consider higher order PPN contributions, preferred frame effects associated with the motion of the solar system relative to the asymptotic rest frame of $u^a$, and/or radiation or other effects.

We use metric signature $(+---)$ and units with $c=1$.

## II. ACTION AND FIELD EQUATIONS

Taking the viewpoint of effective field theory, we consider the action as a derivative expansion, keeping all terms con-

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TABLE I. Special cases of the action (1).

| Special case | parameter values in (2) | parameter values in [4] |
|---|---|---|
| General Relativity | $c_1=c_2=c_3=c_4=0$ | $\tau=\eta=\epsilon=0$ |
| equivalent to GR by field redefinition [12] | $c_1+c_4=0,\ c_2+c_3=0$ | $\tau=\epsilon=0$ |
| Will-Nordtvedt [1,4,10] | $c_2=c_3=c_4=0$ | $\eta=\epsilon=0$ |
| Hellings-Nordvedt [3] | $c_1+c_2+c_3=0,\ c_4=0$ | $\tau=0$ |
| Einstein-Maxwell-like [8,9] | $c_1+c_3=0,\ c_2=c_4=0$ | $\tau=\eta=0$ |

sistent with diffeomorphism symmetry. The most general Lagrangian scalar density involving the metric $g_{ab}$ and preferred frame unit vector $u^a$ with two or fewer derivatives is $\tilde{\mathcal{L}}=\sqrt{-g}\mathcal{L}$, with

$$\mathcal{L}=-R-K^{ab}{}_{mn}(\nabla_a u^m)(\nabla_b u^n)-\lambda(g_{ab}u^a u^b-1) \tag{1}$$

where

$$K^{ab}{}_{mn}=c_1 g^{ab}g_{mn}+c_2\delta^a_m\delta^b_n+c_3\delta^a_n\delta^b_m+c_4 u^a u^b g_{mn}. \tag{2}$$

Stationarity under variation of the Lagrange multiplier $\lambda$ constrains the preferred frame vector $u^a$ to be unit timelike. We have omitted in the Lagrangian terms that would vanish when the unit constraint is satisfied. The unit timelike vector $u^a$ is present everywhere in spacetime in every field configuration and specifies a locally preferred rest frame. It can be thought of as the four-velocity of a ubiquitous fluid and hence is naturally called the *aether* field. We sometimes use the term *aether theory* to refer to the theory described by the Lagrangian (1).

A general class of Lagrangians for vector-tensor theories with an unconstrained vector was parametrized by Will and Nordvedt [1,4]. Their term $\omega u^a u_a R$ does not appear here because of the unit constraint, and our $c_4$ term does not appear there. The relation between the parameters in Ref. [4] (neglecting $\omega$) and ours (neglecting $c_4$) is

$$\begin{aligned} \tau&=-(c_1+c_2+c_3) & c_1&=2\epsilon-\tau\\ \eta&=-c_2 & c_2&=-\eta\\ \epsilon&=-(c_2+c_3)/2 & c_3&=\eta-2\epsilon. \end{aligned} \tag{3}$$

In order to agree with observations the dimensionless coefficients $c_{1,2,3,4}$ must presumably be fairly small compared to unity. Special cases of this action are identified in Table I. Note that the "Einstein-Maxwell-like" case is a subcase of Hellings-Nordtvedt. This case has an extra gauge symmetry and was disfavored in [9] on account of the gradient singularities that generally develop in the vector field. Another notable subcase of Hellings-Nordtvedt is $c_1=c_4=0,\ c_2+c_3=0$, which is also one of the theories equivalent to GR via a field redefinition.

The metric equation with no matter source (other that the aether field) can be written in the form

$$G_{ab}=T_{ab} \tag{4}$$

where $G_{ab}$ is the Einstein tensor and $T_{ab}$ is the "aether stress" tensor obtained from varying the aether part of the action (1) with respect to the metric. For the metric variation we use the inverse metric, but we also have a choice whether to consider the independent aether field to be a covariant or a contravariant vector. We choose contravariant $u^a$ to simplify the stress tensor a bit, since the action in the contravariant form (1) has no metric dependence associated with the $c_2$ and $c_3$ terms in $K^{ab}{}_{mn}$. The field equations are thus obtained by requiring that the action (1) be stationary with respect to variations of $g^{ab}$, $u^a$, and $\lambda$.

The $\lambda$ variation imposes the unit constraint,

$$g_{ab}u^a u^b=1. \tag{5}$$

The $u^a$ variation gives

$$\nabla_a J^a{}_m-c_4\dot{u}_a\nabla_m u^a=\lambda u_m, \tag{6}$$

where to compactify the notation we have defined

$$J^a{}_m=K^{ab}{}_{mn}\nabla_b u^n \tag{7}$$

and

$$\dot{u}^m=u^a\nabla_a u^m. \tag{8}$$

Solving for $\lambda$ using Eq. (5) we find

$$\lambda=u^m\nabla_a J^a{}_m-c_4\dot{u}^2. \tag{9}$$

The $g^{ab}$ variation yields the aether stress tensor

$$\begin{aligned} T_{ab}=&\nabla_m(J_{(a}{}^m u_{b)}-J^m{}_{(a}u_{b)}+J_{(ab)}u^m)\\ &+c_1[(\nabla_m u_a)(\nabla^m u_b)-(\nabla_a u_m)(\nabla_b u^m)]\\ &+c_4\ \dot{u}_a\dot{u}_b\\ &+[u_n(\nabla_m J^{mn})-c_4\dot{u}^2]u_a u_b\\ &-\frac{1}{2}\mathcal{L}_u g_{ab}. \end{aligned} \tag{10}$$

In the above, expression (5) has been used to eliminate the term that arises from varying $\sqrt{-g}$ in the constraint term in Eq. (1), and in the fourth line $\lambda$ has been eliminated using Eq. (9). The first line contains all of the terms arising from varying the metric dependence of the connection. Note that it contains terms of second order in derivatives. In the last line the notation $\mathcal{L}_u$ refers to all of $\mathcal{L}$ in Eq. (1) except the Ricci scalar term.

## III. SPHERICALLY SYMMETRIC STATIC SOLUTIONS

Our objective in this paper is to consider the weak field limit of spherically symmetric static solutions to the aether-metric field equations. In spherical symmetry the $c_4$ term in the action can be absorbed by the change of coefficients

$$c_1 \rightarrow c_1 + c_4$$

$$c_3 \rightarrow c_3 - c_4. \tag{11}$$

To see why, note that any spherically symmetric vector field is hypersurface orthogonal, hence the twist

$$\omega_a = \epsilon_{abcd} u^b \nabla^c u^d \tag{12}$$

of the aether vanishes. The identity [12]

$$\dot{u}^2 = -\omega_a \omega^a + \nabla_a u_b \nabla^a u^b - \nabla_a u_b \nabla^b u^a, \tag{13}$$

valid for $u$ satisfying $u^2 = 1$, can be used to trade the $\dot{u}^2$ term in the action (1) for an $\omega^2$ term together with the substitution (11). Since the twist occurs quadratically and vanishes in spherical symmetry, that term will not contribute to the field equations, hence the $c_4 \dot{u}^2$ term simply modifies the coefficients as indicated in Eq. (11). Thus we henceforth set $c_4 = 0$ without loss of generality, as it can be reintroduced at the end via the replacements (11). [Although substitution of the identity (13) will not change the content of the field equations, it will change the value of the Lagrange multiplier $\lambda$ for a given solution.]

We have analyzed the asymptotic limit of such solutions and found that at first PPN order the two ERS parameters are exactly the same as in pure GR as long as $c_1 + c_2 + c_3 \neq 0$. The special case $c_1 + c_2 + c_3 = 0$ has no single characterization. We now describe how these results are obtained.

### A. Field equations

A common choice for a weak field analysis is isotropic coordinates $(t, r, \theta, \varphi)$, which we adopt here. In these coordinates the line element is

$$ds^2 = N(r) dt^2 - B(r)(dr^2 + r^2 d\Omega^2). \tag{14}$$

(Note that $r$ is not the usual Schwarzschild radial coordinate.) The aether field takes the form

$$u^t(r) \frac{\partial}{\partial t} + u^r(r) \frac{\partial}{\partial r} = a(r) \frac{\partial}{\partial t} + b(r) \frac{\partial}{\partial r} \tag{15}$$

and the unit constraint becomes

$$N(r) a(r)^2 - B(r) b(r)^2 = 1. \tag{16}$$

The aether field equation (6) has just $t$ and $r$ components, and the elimination of $\lambda$ reduces this pair to one independent equation.

Solving the field equations is obviously an enormous task given the form (10) of the stress tensor so we used the symbolic math program MAPLE and the Riemann tensor package [13]. With this package one can easily express the field equations in terms of the functions $N(r), B(r), a(r), b(r)$ and their derivatives. The end result is a set of coupled ordinary differential equations coming from both the Einstein equation and the aether field equation. Given that there are only three free functions left after applying the constraint (16), just three independent ODE's are needed. We used the aether field equation (6) and the $tt$ and $rr$ components of the metric equation. The equations are sufficiently complicated that it does not seem illuminating to display them here.

### B. Asymptotic weak field limit

Far from the source, the metric should approach flat Minkowski space. In order to examine what happens as $r$ approaches infinity we introduce the change of variables

$$x = \frac{1}{r}. \tag{17}$$

Around $x = 0$ the functions $N(x), B(x), b(x)$ will have power series behavior in the form of

$$N(x) = 1 + N_1 x + N_2 x^2 + N_3 x^3 + N_4 x^4 \tag{18}$$

$$B(x) = 1 + B_1 x + B_2 x^2 + B_3 x^3 + B_4 x^4 \tag{19}$$

$$b(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4. \tag{20}$$

At this stage it is convenient to use the constraint equation (16) to eliminate $a(r)$ in favor of the radial component $b(r)$. It turns out that asymptotic flatness and spherical symmetry generally require the aether to have no radial component at infinity ($a_0 = 1$, $b_0 = 0$) except in the Einstein-Maxwell-like case where the action takes a special form with an additional symmetry. The first order coefficient $N_1$ determines the Newtonian gravitational potential, so what we are really interested in are the post-Newtonian corrections to this associated with the $B_1$ and $N_2$ coefficients. The higher order coefficients are post-post Newtonian (and beyond). Substituting the above forms of the functions into the equations of motion and performing a series expansion in MAPLE around the point $x = 0$ ultimately gives a set of algebraic equations that can be solved to produce the local power series solutions for the fields.

### C. Series solutions

To illustrate our methods we first discuss the local power series solutions to pure Einstein gravity in isotropic coordinates. In this case, the $c_i$ parameters are all set to zero and we are left to consider the two coupled ODE's for the functions $N(x)$, $B(x)$ given by the vanishing $tt$ and $rr$ components of the Einstein tensor. Using the procedure described in Sec. III B we find

$$G_{tt} = \left[\frac{3}{4} B_1^2 - 2 B_2\right] x^4 + \left[N_1 \left(\frac{3}{4} B_1^2 - 2 B_2\right) - \frac{9}{4} B_1^3 + 7 B_1 B_2 - 6 B_3\right] x^5 + \cdots \tag{21}$$

$$G_{rr} = -(N_1+B_1)x^3 + \left[N_1^2 + \frac{1}{2}N_1B_1 + \frac{5}{4}B_1^2 - 2N_2 - 2B_2\right]x^4 + \cdots. \tag{22}$$

Thus, at third order in $x$, the $rr$ equation implies

$$B_1 = -N_1. \tag{23}$$

We can then substitute this result into the fourth order equations and solve simultaneously to determine that

$$N_2 = \frac{1}{2}N_1^2 \tag{24}$$

$$B_2 = \frac{3}{8}N_1^2. \tag{25}$$

For the additional coefficients in $B(x)$ and $N(x)$ we continue the process of examining higher order equations, substituting in lower order results, and solving simultaneously. These solutions can of course be verified by simply expanding the commonly known solution for the functions in isotropic coordinates in a power series.

Now we return to the case of interest and tune the $c_i$ parameters back up to nonzero values. At lowest (second) order in $x$ the aether field equation tells us that

$$-2(c_1+c_2+c_3)b_0(b_0^2+1) = 0, \tag{26}$$

which says that $b_0 = 0$ provided that $c_1+c_2+c_3 \neq 0$. This combination of parameters also appears in the aether field equation at third order in $x$,

$$-2(c_1+c_2+c_3)b_1 = 0. \tag{27}$$

From Eqs. (26) and (27) it is clear that we have two completely different cases depending on whether or not $c_1+c_2+c_3$ vanishes. These cases must be analyzed separately.

### D. Generic case: $c_1+c_2+c_3 \neq 0$

For this generic case Eq. (26) shows that asymptotic flatness of the metric implies that $b_0 = 0$, i.e. the aether has no radial velocity at infinity. Together with the constraint this implies that $a_1 = -N_1/2$. In addition, Eq. (27) tells us that $b_1 = 0$.

Now let us consider the metric equations. The $rr$ equation tells us that

$$B_1 = -N_1, \tag{28}$$

a result identical to pure GR. We have now determined all of the zeroth and first order coefficients in terms of $N_1$, but to examine the higher order ones we must consider the higher order terms in the expansions of the field equations. At fourth order the $u$ field equation is identically zero after substituting $b_1 = 0$ and $B_1 = -N_1$. Now all that remains at this order is to determine $B_2$ and $N_2$ using the two Einstein equations at fourth order in $x$. These have the form

$$\frac{5}{8}N_1^2c_1 - c_1N_2 - 2B_2 + \frac{3}{4}N_1^2 = 0 \tag{29}$$

$$\frac{7}{4}N_1^2 - 2N_2 - 2B_2 + \frac{1}{8}N_1^2c_1 = 0. \tag{30}$$

Solving these two equations simultaneously yields the final result

$$N_2 = \frac{1}{2}N_1^2 \tag{31}$$

$$B_2 = \frac{3}{8}N_1^2 + \frac{1}{16}N_1^2c_1. \tag{32}$$

To determine further coefficients of the power series expansion we move on to consider the field equations at fifth and sixth order in $x$. At fifth order in the $u$ field equation we recover the result

$$(c_1+c_2+c_3)(b_2N_1 - b_3) = 0 \tag{33}$$

indicating that $b_2$ is a new free parameter in addition to $N_1$, and $b_3 = N_1b_2$. The remaining metric equations at fifth order are quite complicated so we simply quote the final results

$$B_3 = -\frac{1}{16}N_1^3 - \frac{5}{96}N_1^3c_1 \tag{34}$$

$$N_3 = \frac{3}{16}N_1^3 - \frac{1}{96}N_1^3c_1. \tag{35}$$

We also examined the sixth order equations to find $N_4$ and $B_4$, but we will not give the results due to their complexity. However, we note that these coefficients depend on both $N_1$ and $b_2$.

As a final note, we also expanded the equation for lambda in Eq. (9) and used all of the above results for the expansion coefficients to determine at what order lambda contributes. This yields

$$\lambda = \frac{1}{2}N_1^2c_1x^4 + \cdots. \tag{36}$$

[As mentioned at the beginning of this section, the $c_4$ dependence of $\lambda$ cannot be obtained via the substitutions (11).]

### E. Special case: $c_1+c_2+c_3 = 0$

This special case corresponds to the Hellings-Nordtvedt theory [3] with a unit constraint on the vector field. Setting $c_3 = -c_2 - c_1$ from the beginning and repeating the procedure we find that the second order $tt$ and $rr$ metric equations imply

$$c_2b_0^2 = 0. \tag{37}$$

This special case thus further subdivides into the cases $c_2 \neq 0$ and $c_2 = 0$.

#### 1. $c_2 \neq 0$

If $c_2$ is nonzero we again find that $u^a$ has no radial component at infinity. There is no single characterization of this case. An exceptional subcase occurs if $c_1=0$, which falls into the class [12] that is equivalent to GR via a field redefinition (and the Lagrangian is just $R+c_2 R_{ab}u^a u^b$). In this class the aether field is completely unconstrained. If $c_1 \neq 0$ we again find $B_1=-N_1$ as in Eq. (28), while unlike Eq. (31) we find

$$N_2=\frac{1}{8}N_1^2(c_1+6c_2+4)/(c_2+1). \tag{38}$$

#### 2. $c_2=0$

The case $c_2=0$ yields the Einstein-Maxwell-like (plus $c_4$) sector of the theory, which was previously analyzed nonperturbatively in [9]. Working through the procedure for finding the local power series solutions we find the Reissner-Nordström solution (provided $\lambda=0$) with $b_0$, $b_1$, and $N_1$ as free parameters. The freedom appearing here in $b_0$ and $b_1$ is a result of the limited gauge symmetry

$$u_a \rightarrow u_a+\nabla_a f \tag{39}$$

preserving the unit constraint, as discussed in [9]. Specifically, $b_1$ is associated with an "aether charge" while $b_0$ corresponds to a scaling freedom. This is similar to the usual Reissner-Nordstrom case where the general solution for the co-vector potential $A_t$ is

$$A_t=\frac{Q+Dr}{r}, \tag{40}$$

where the $D$ constant is usually set to zero so that the field will be 0 at infinity.

The solutions with $\lambda \neq 0$ have the aether aligned with the Killing vector, i.e. $b(r)\equiv 0$. While there always exist such solutions in this special case, they are not asymptotically flat except in the even more special case $c_1=-c_3=2$, $c_2=0$. (In that case there is a full functional freedom in the solution, which corresponds in the charged dust interpretation of [9] to the case of extremally charged dust.) Thus the exterior solution for a star must have $\lambda=0$. On the other hand at the origin we must have $\lambda \neq 0$ to avoid a $1/r$ singularity in the $u$-field. It does not appear possible to match these, so it may be that there are no static spherically symmetric solutions that are regular at the origin. Since the Einstein-Maxwell case was already deemed unphysical [9] due to the generic appearance of aether shocks, we shall not belabor this point here.

### F. Eddington-Robertson-Schiff parameters

In the usual analysis of the post-Newtonian corrections to the gravitational field of a static spherical body the Schwarzschild line element is rewritten in terms of isotropic coordinates and those metric coefficients are then expanded to post-Newtonian accuracy. This takes the following form for a general gravitational theory [4]:

$$ds^2=\left(1-\frac{2M}{r}+2\beta\frac{M^2}{r^2}\right)dt^2-\left(1-2\gamma\frac{M}{r}\right)[dr^2+r^2 d\Omega^2] \tag{41}$$

where $M$ is the gravitating mass of the body in geometric units and $\gamma$ and $\beta$ are the Eddington-Robertson-Schiff (ERS) parameters of the theory. The parameter $\gamma$ measures the amount of space curvature produced by a unit rest mass and $\beta$ describes the amount of nonlinearity in the superposition law.

In the generic case $c_1+c_2+c_3 \neq 0$, we read off from Eqs. (28) and (31) of Sec. III D that

$$\gamma=1 \tag{42}$$

$$\beta=1, \tag{43}$$

in exact agreement with pure GR. The special case $c_1+c_2+c_3=0$ has no single characterization. If $c_1=0$ it is equivalent to GR via a field redefinition, and the aether field is arbitrary. If $c_2=0$ it is the Einstein-Maxwell-like sector, and the exterior is described by the Reissner-Nordström solution. Hence $\gamma=1$, but the value of $\beta$ depends upon the aether charge which is not determined by our method. If neither $c_1$ nor $c_2$ vanishes then, using Eq. (38) we find

$$\gamma=1 \tag{44}$$

$$\beta=\frac{1}{4}\frac{c_1+6c_2+4}{c_2+1}, \tag{45}$$

where Eq. (38) was used to obtain $\beta$. This special case corresponds to $\tau=\omega=0\neq\eta$ which, as shown by Will [4], is dynamically overdetermined in the linearized, unconstrained vector-tensor theory.

## IV. DISCUSSION

There are two important implications of this analysis. First, there appear to be only two free parameters in the local solution around infinity for the generic case $c_1+c_2+c_3 \neq 0$, namely $N_1$ and $b_2$. It is possible that analyzing the global behavior of the field equations may eliminate one of these or demonstrate the existence of even more parameters. Based on an analogy from pure GR, the metric parameter $N_1$ is determined by the mass of the presumed static, central object generating the field. The aether parameter $b_2$ cannot be associated with a "charge" as in the special case of Einstein-Maxwell due to the $1/r^2$ fall off.

The second implication is that in the generic case the aether model is quite close observationally to pure GR since the ERS parameters match. More precisely, the coefficients of the metric expansions are identical up to $B_2$, which differs by a term of relative size $c_1$ (or $c_1+c_4$). Other alternative theories of gravity with the same ERS parameters are the general vector-tensor theory without the unit constraint [14], and the bimetric theories with prior geometry of Rosen and of Rastall [4]. The fact that the ERS parameters are the same suggests that there may be a closer relation than might be expected between the PPN parameters of unconstrained and constrained vector tensor theories. It also is interesting to note the differences between this model and the Brans-Dicke scalar-tensor theory. The Brans-Dicke parameters are

$$\gamma=\frac{1+\omega}{2+\omega} \tag{46}$$

$$\beta=1 \tag{47}$$

where $\omega$ is the Dicke coupling constant, which must be greater than 500 in order to agree with observation.

In order to have a comprehensive check on the theory in the solar system we need to consider the full post-Newtonian approximation scheme. This allows for preferred frame effects due to the motion of the solar system with respect to the asymptotic preferred frame and is described by ten parameters (two of which are $\gamma$ and $\beta$). To determine these parameters one perturbatively integrates the field equations with a fluid source, imposing the condition of regularity at the origin. It seems likely that this would fix the value of $b_2$ in the generic case. Further tests of preferred frame effects will be found in gravitational wave phenomena (briefly mentioned in [10]) such as the orbital decay of binary pulsars, and in strong field settings such as black holes.

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