PHYSICAL REVIEW D **69**, 061701(R) (2004)

Possibility of a dynamical Higgs mechanism and of the respective phase transition induced by a boundary

A. N. Sissakian, O. Yu. Shevchenko, and V. N. Samoilov
Joint Institute for Nuclear Research, Dubna, Moscow Region 141980, Russia

(Received 11 August 2003; revised manuscript received 16 December 2003; published 18 March 2004)

The dynamical quantum effects arising due to the boundary presence with two types of boundary conditions (BC's) satisfied by scalar fields are studied. It is shown that while the Neumann BC's lead to the usual scalar field mass generation, the Dirichlet BC's give rise to the dynamical mechanism of spontaneous symmetry breaking. Because of the latter, there arises the possibility of the respective phase transition from the normal phase to the spontaneously broken one. In particular, at the critical value of the combined evolution parameter the usual massless scalar QED transforms to the Higgs model.

DOI: 10.1103/PhysRevD.69.061701 PACS number(s): 11.10.Wx, 11.30.Qc

The investigation of quantum field theory (QFT) systems with respect to their response to different external influences, such as different external fields, nonzero temperature, and density of the medium, etc., allows one to discover some new properties of these systems. For example, it is of interest to study the phase transitions in QFT systems with spontaneous symmetry breaking (such as the Higgs model [1]) at nonzero temperature [2,3]. It is of importance that the temperature (just as well as the finite medium density) always restores the initially broken symmetry and the phase transition from the broken to the normal (unbroken) phase occurs with a temperature increase [2,3].

On the other hand, it is possible to arrive at the very interesting class of external influences if one considers the QFT system quantized not in infinite space, as usual, but in space restricted by some boundary surfaces with the respective boundary condition (BC) satisfied by the fields. Such situations arise in physics very often. These are, for example, potential barriers for scalar mesons modeled by the Dirichlet and (or) Neumann BC in nuclear physics, the Casimir BC satisfied by the electromagnetic field on metal surfaces in QED, and the nucleon surface impenetrable for quarks and gluons modeled by the bag BC in QCD. It is well known that the Casimir effect occurs in all these cases (see [4] for an excellent review). However, the Casimir effect is the effect of zero order in the coupling constant and deals with free fields [5]. So, it is of interest to consider the possibilities of some purely dynamical phenomena, caused by interaction, in the boundary presence. In particular, we will be especially interested in the possibility of dynamical (and depending on the characteristic region size) particle mass generation in the initially massless theories. Namely such a situation occurs in QFT at finite temperature, for example in scalar field theory [3], where the initially massless particle becomes massive due to the temperature inclusion while the nontrivial part of dynamical mass depending on the temperature disappears in the zero temperature limit.

Let us consider the *massless scalar field theory* with $\mathcal{L}_{\rm int}=\lambda\varphi^4/4!$ quantized in the flat gap pictured in Fig. 1. We will consider two possible types of BC's satisfied by the field φ on the plates. These are the Dirichlet BC's,

$$\varphi_D|_{x_3=\pm L/2}=0, \tag{1}$$

and the Neumann BC's,

$$\partial\varphi_N/\partial x_3|_{x_3=\pm L/2}=0. \tag{2}$$

To study the dynamical effects arising in the translationally noninvariant case we deal with, it is convenient to start with the unrenormalized Schwinger-Dyson equation written in a coordinate representation for the full propagator $G(x,y)\equiv T\langle\phi(x)\phi(y)\rangle$,

$$-\partial_x^2 G(x,y)=\frac{\lambda}{3!}T\langle\varphi^3(x)\varphi(y)\rangle+i\delta(x-y),$$

which in the leading order in λ, by virtue of the Wick theorem, is rewritten as

$$-\partial_x^2 G_{D,N}=\frac{\lambda}{2}\mathcal{D}_{D,N}(x,x)G_{D,N}(x,y)+i\delta(x-y), \tag{3}$$

where $\mathcal{D}_{D,N}(x,y)$ are the propagators satisfying the free equation $-\partial_x^2\mathcal{D}_{D,N}(x,y)=i\delta(x-y)$ and the Dirichlet $\mathcal{D}_D|_{x_3=\pm L/2}=0$ or Neumann $\partial\mathcal{D}_N/\partial x_3|_{x_3=\pm L/2}=0$ BC's, respectively. These propagators are found by the method of mirror images, and in the nontrivial region inside the gap in which we are interested, the result reads [7]

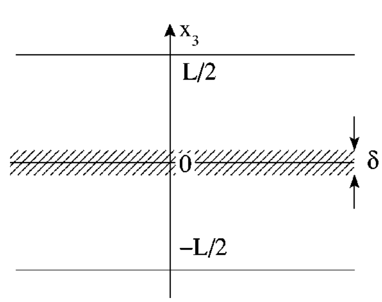

FIG. 1. The flat gap with the shadowed central δ region.

$$\mathcal{D}_{D,N}(x,y)=-(4\pi^2)^{-1}\sum_n (\mp 1)^n\{(\hat{x}-\hat{y})^2-[x_3-(-1)^n y_3-nL]^2\}^{-1} \tag{4}$$

$$=-(4\pi^2)^{-1}\sum_n ([(\hat{x}-\hat{y})^2-(x_3-y_3-2nL)^2]^{-1}\mp\{(\hat{x}-\hat{y})^2-[x_3+y_3-(2n-1)L]^2\}^{-1}), \tag{5}$$

where $\hat{x}^2\equiv x_0^2-x_1^2-x_2^2$. Introducing the quantity

$$\mu^2_{D,N}(\vec{x})=\frac{\lambda}{2}\lim_{x\to y}\tilde{\mathcal{D}}(x,y)+O(\lambda^2), \tag{6}$$

where

$$\tilde{\mathcal{D}}(x,y)\equiv\mathcal{D}(x,y)-\mathcal{D}_0(x-y), \tag{7}$$

and $\mathcal{D}_0(x)\equiv-(4\pi^2)^{-1}[x_0^2-\vec{x}^2]^{-1}$, one gets from Eq. (3) the equation

$$[-\partial_x^2-\mu^2_{D,N}(x_3)]G_{D,N}(x,y)=i\delta(x-y), \tag{8}$$

where now all quantities are renormalized [8] in leading order [9].

From Eq. (8) one can see that the x-dependent quantity μ can be considered an external "mass field" which acquires the sense of mass in its traditional (so that $E=\sqrt{\vec{p}^2+\mu^2}$) understanding only when it weakly (adiabatically) depends on the coordinate. So, we will call this quantity the "mass gap" in analogy with condensed matter physics [10].

We first consider the *zero-temperature* case. Using Eqs. (5)–(7), the formulas $\Sigma_1^\infty n^{-2}=\zeta(2)=\pi^2/6$, and

$$\sum_{-\infty}^{\infty}(n+a)^{-2}=-\pi d\cot(\pi a)/da=\pi^2/\sin^2(\pi a),$$

one easily gets

$$\mu^2{}_{D,N}=\frac{\lambda}{32L^2}\left\{\frac{1}{3}\mp\csc^2\left[\frac{\pi}{L}\left(x_3+\frac{L}{2}\right)\right]\right\}. \tag{9}$$

Let us analyze Eq. (9). First of all, one can see that there are two contributions to μ^2—translationally invariant and x_3-dependent, respectively. The translationally invariant contribution $\lambda/96L^2$ is the same for Dirichlet and Neumann BC's and comes from the translationally invariant part of the propagators $\mathcal{D}_{D,N}$—the first term in Eq. (5). Notice that this contribution can also be obtained from the well known result of QFT at finite temperature [3],

$$m_T^2=\lambda T^2/24 \tag{10}$$

with the substitution $T\to 1/2L$ if one, similarly to the case of a periodic BC [11], uses the analogy [12] of the eigenfrequency spectrum $w_n=\pi n/L$ for the Dirichlet and Neumann BC with the finite-temperature spectrum $w_n=2\pi nT$. However, let us stress that in this way one reproduces only a part [13] of the full μ^2 value, losing the x-dependent contributions. Moreover, it is seen from Eq. (9) that these contributions always dominate, i.e., they are the biggest at any x_3 values. The latter point leads to the crucial difference between Dirichlet and Neumann BC's, and this is of great importance for what follows: while in the case of a Neumann BC the mass gap square is always positive, $\mu_N^2>0$, in the case of a Dirichlet BC the mass gap square is always negative, $\mu_D^2<0$, and we will discuss this possibility later.

Looking at Eq. (9), one can notice that the expressions for $\mu^2_{D,N}$ are divergent on the gap boundaries $\pm L/2$. These divergences are not surprising since, in addition to the usual, ultraviolet singularity $(x-y)^{-2}|_{x\to y}$ subtracted by Eq. (7), the propagator (4) also contains contributions corresponding to $n=\pm 1$ in the sum that are divergent as $x\to y$ on the boundaries. Such so-called "surface divergences" are well known [4] from the calculation of the Casimir energy density with boundary conditions like Eqs. (1) and (2). It is known that these singularities arise because boundary conditions like Eqs. (1) and (2) are too idealized approximations to the real ones, and to avoid surface divergences one should deal with more realistic smooth boundary conditions. However, the transition from idealized sharp boundary conditions of total impenetrability to the realistic smooth ones causes enormous complications in all calculations. Fortunately, there is a possibility of getting reliable results even with the sharp boundary conditions like Eqs. (1) and (2). Indeed, the experience of the Casimir energy density calculations shows that the application of the smooth boundary conditions instead of the sharp ones does not influence the result in the region maximally distanced from the boundaries. So, we will ascribe to result (9) a physical sense only in such a region—in a strip with a small $(\delta/L\ll 1)$ width δ surrounding the central plane $x_3=0$ (see Fig. 1).

On the other hand, this central region has a remarkable property: since $\partial\mu^2_{D,N}/\partial x_3|_{x_3=0}=0$, the mass gap there is almost independent of x_3 $[\mu_{D,N}(\delta)\simeq\mu_{D,N}(0)]$ and can be considered a scalar field mass (but not yet as a real mass of scalar meson).

So, in the central δ region one gets instead of Eq. (9) the following expressions [14] for the scalar field masses:

$$\mu_N^2=\lambda/24L^2, \quad \mu_D^2=-\lambda/48L^2. \tag{11}$$

Thus, the Neumann and Dirichlet BC's differ drastically. While the Neumann BC leads to the usual dynamical mass generation of a scalar meson, $m_N=\mu_N=\lambda/24L^2$, the Dirichlet BC leads to the imaginary mass of the scalar field. This, as is well known, is the signal that the spontaneous violation of the ground-state symmetry $\langle\varphi\rangle\rightarrow\langle\varphi\rangle$ ought to happen, so that after the latter, the scalar meson acquires the real dynamical mass: $m_D^2=2|\mu_D^2|=\lambda/24L^2$. It is of interest that the real meson masses m_D and m_N happen to be equal to each other.

Let us show that the result $\mu_D^2<0$ is valid also in the case where *all three space dimensions are compactified* with the Dirichlet BC satisfied by the scalar field. Consider the parallelepiped with the edges L_1,L_2,L_3 centered around the coordinate origin. The respective scalar field propagator submitted to the Dirichlet BC on the plates, $\mathcal{D}(x,y)=0$ on $x_i=\pm L_i/2$, has a form [15]

$$\mathcal{D}(x,y)=\sum_N(-1)^{(n_1+n_2+n_3)}\mathcal{D}_0(x-y^{(N)}), \qquad (12)$$

where $N=(n_1,n_2,n_3)$, $y_i^{(N)}=(-1)^{n_i}y_i+n_iL_i$, and $\mathcal{D}_0$ is the free propagator in infinite space. Thus, for the cube ($L_1=L_2=L_3\equiv L$) in the small δ region of the coordinate origin, Eqs. (6) and (7) give

$$\mu_D^2|_{\rm cube}=\frac{\lambda}{4\pi^2L^2}\sum_{n_1,n_2,n_3}{}'\frac{(-1)^{n_1+n_2+n_3}}{n_1^2+n_2^2+n_3^2}, \qquad (13)$$

where the prime denotes that indexes n_1,n_2,n_3 in the sum do not equal zero simultaneously. Fortunately, the sum entering Eq. (13) is known from the crystal physics Madelung constant and can be found in Table 4 of Ref. [16]: $\Sigma'(-1)^{n_1+n_2+n_3}(n_1^2+n_2^2+n_3^2)^{-1}=d(2s)_{s=1}=-2.519\,35$. Thus, one again gets a negative result for μ_D^2,

$$\mu_D^2|_{\rm cube}=-0.063\,82\lambda/L^2. \qquad (14)$$

Returning now to the flat gap geometry, let us consider *massless scalar electrodynamics* with the Lagrangian density

$$-\frac{1}{4}F_{\mu\nu}^2+\frac{1}{2}\partial^\mu\varphi_a\partial_\mu\varphi_a-\frac{\lambda}{4!}\varphi^4$$

$$-e\epsilon_{ab}\partial^\mu\varphi_a\varphi_bA_\mu+\frac{1}{2}e^2\varphi^2A^2,$$

where $\varphi^2\equiv\varphi_a\varphi_a$, $\varphi^4\equiv(\varphi^2)^2$, $a=1,2$, and with the Dirichlet BC $\varphi_a|_{x_3=\pm L/2}=0$ satisfied by the scalar field on the gap boundaries. Here we will be interested in the dynamical effects caused by the *minimal* modification of the standard (infinite space) QFT. Thus, within the present paper, we do not impose [17] any BC's on the electromagnetic field, so that again one has only the tadpole diagram for both fields φ_1 and φ_2, contributing to the L-dependent scalar field mass gap in the one-loop approximation. Operating just as before, one gets in the central region (see Fig. 1)

$$\mu_D^2=-\lambda/36L^2. \qquad (15)$$

Thus, instead of the scalar QED, one arrives at the Higgs model with the "wrong" sign at the mass term, and this occurs only due to the dynamical corrections in the boundary presence.

So, after realization of the standard Higgs mechanism, one leaves with only the massive scalar meson with a mass $m_\varphi^2=2|\mu_D^2|=\lambda/18L^2$ interacting with the massive vector field with a mass $m_A^2=e^2|\mu_D^2|/\lambda=e^2/36L^2$.

Let us now *include the temperature* $T\equiv1/\beta$ and consider first $\lambda\varphi^4/4!$ theory. Following the well known imaginary-time Matsubara approach, one easily gets, instead of Eqs. (4) and (6), the equation $\mu_{D,N}^2=(\lambda/2)\tilde{\mathcal{D}}(x,x)+\mathcal{O}(\lambda^2)$ with

$$\tilde{\mathcal{D}}_{D,N}(x,y)=(4\pi^2)^{-2}\sum_{n,m}{}'(\mp1)^n\{(x_4-y_4+m\beta)^2+(\tilde{x}-\tilde{y})^2$$

$$+[x_3-(-1)^ny_3+nL]^2\}^{-1}, \qquad (16)$$

where $\tilde{x}\equiv(x_1,x_2)$ and the prime denotes that indexes n and m in the sum do not equal zero simultaneously. Then, in the case of Dirichlet BC's, near the central plane $x_3=0$, one gets

$$\mu_D^2(L,\beta)=(\lambda/8\pi^2)\sum_{n,m}{}'(-1)^n[\beta^2m^2+L^2n^2]^{-1}. \qquad (17)$$

Using $\Sigma_n{}'(-1)^nn^{-2}=-\pi^2/6$ and $\Sigma_n{}'n^{-2}=2\zeta(2)=\pi^2/3$, one can easily get the asymptotes $\mu_D^2(L,\infty)$ and $\mu_D^2(\infty,\beta)$ corresponding to the zero-temperature and infinite-space cases, respectively. In the first case, one arrives at the result (11) for μ_D^2; in the second case, one obtains the result (10): $m_T^2=\mu_D^2(\infty,\beta)=\lambda T^2/24$.

Let us now introduce two new evolution variables,

$$\chi_1\equiv\beta^2/L^2,\quad \chi_2\equiv\chi_1^{-1}=L^2T^2. \qquad (18)$$

Calculating the sums $\Sigma'_{n,m}(-1)^n[\chi_1m^2+n^2]^{-1}$ and $\Sigma'_{n,m}(-1)^n[m^2+\chi_2n^2]^{-1}$, one obtains the respective evolution pictures presented by Fig. 2 (top and bottom pictures, respectively). One can see the critical point $\chi_1^{\rm cr}=0.5698$ at which μ_D^2 changes the sign, i.e., there occurs the phase transition from the normal to the spontaneously broken phase. It is of importance that the phase transition can occur either because of changing the temperature at fixed L (bottom picture) or because of the gap size L changing at fixed temperature (top picture). The asymptotes of μ_D^2 as $\chi_1\rightarrow0$ ($L\rightarrow\infty$ while β is fixed) and as $\chi_2\rightarrow0$ ($\beta\rightarrow\infty$ while L is fixed) are presented by the left edges of the top and bottom pictures, respectively, and are in agreement with Eqs. (10) and (11). The results for the symmetric point $\beta=L$, i.e., $\chi_1=\chi_2=1$, are in agreement with the exactly calculated [18] sum $\Sigma'_{n,m}(-1)^n[n^2+m^2]^{-1}=(-1/2)\pi\ln2$.

Let us also stress the *essential advantage* of the just considered dynamical mechanism of the spontaneous symmetry breaking (restoration). In our case, there are no problems with the perturbative calculation of the critical point as it occurs at investigation of the spontaneously broken symmetry restoration at critical temperature [3], since we do not introduce into the Lagrangian the imaginary mass term by hand from the very beginning, that leads [19] to the complex value for the critical point.

It is obvious that in the case of *the massless scalar electrodynamics* with the Dirichlet BC on the gap plates, the

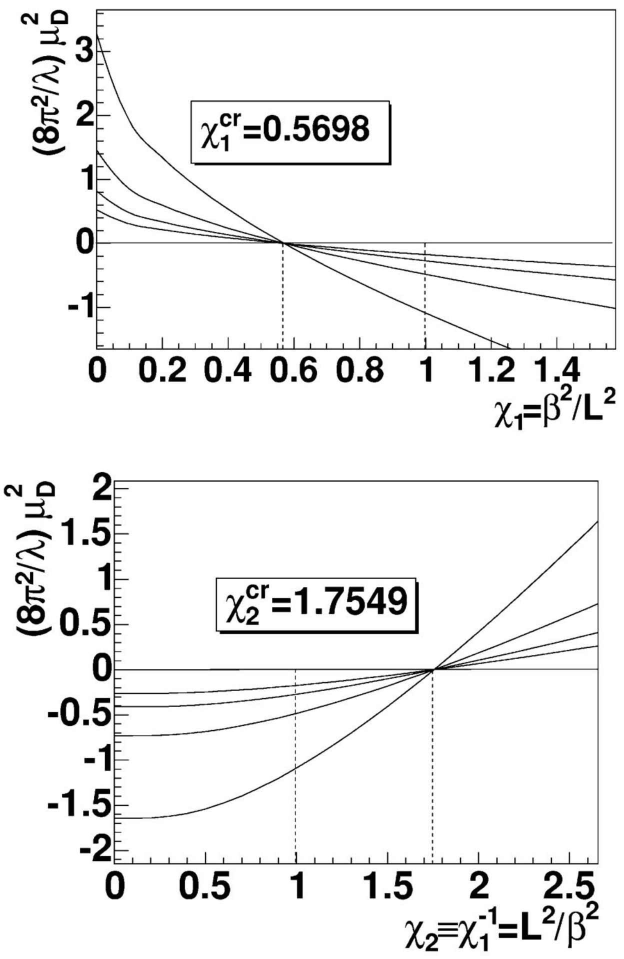

FIG. 2. $(8\pi^2/\lambda)\mu_D^2$ versus χ_1 (top) and $(8\pi^2/\lambda)\mu_D^2$ versus $\chi_2 \equiv \chi_1^{-1}$ (bottom). The natural system $\hbar=c=k=1$ is used. For each curve on the top picture, β is fixed while L evolves; for the bottom picture the reverse is true.

evolution pictures are analogous to those presented in Fig. 2 with the same critical point. The only difference is in the asymptotes. Namely, $\mu_D^2 \to \lambda T^2/18$ as $\chi_1 \to 0$ ($L\to\infty$ while T is fixed) and μ_D^2 tends to the zero-temperature result (15) as $\chi_2 \to 0$ ($\beta\to\infty$ while L is fixed). So, because of the gap size decreasing at fixed temperature at $\chi_1^{cr}=0.5698$, there occurs the following phase transition: the massless scalar electrodynamics with the Dirichlet BC satisfied by the scalar fields on the gap boundaries transforms to the Higgs model with the spontaneous symmetry violation. As a result, at $\chi_1 > \chi_1^{cr}$, after the realization of the standard Higgs mechanism, one leaves with the only massive scalar meson interacting with the massive vector boson.

Thus one can say that the boundary with the respective BC (Dirichlet here) and the temperature compete with each other: while the temperature always aspires to restore the broken symmetry, the boundary tends to violate it. This competition gives rise to a new type of phase transition: the decreasing in the characteristic size of the quantization region (the gap size here) and the increasing in the temperature tend to transport the system into the spontaneously broken or into the normal phase. The system evolves with a combined parameter reflecting the change in the temperature and in the size simultaneously. As a result, at the critical value of this parameter there occurs the phase transition from the normal to the spontaneously broken phase. In particular, the usual massless scalar electrodynamics transforms to the Higgs model. The latter, as is well known, is the key model supporting the foundations of the main directions in modern physics based on spontaneous symmetry breaking and the Higgs mechanism. In particular, this is superconductivity theory [20], which is just the nonrelativistic variant of the Abelian-Higgs model (see, for example, [21] and references therein) in condensed matter physics, and the Weinberg-Salam theory of electroweak interactions in high-energy physics. So, one can hope that the dynamical phenomena presented here caused by the boundary influence can lead to some new physical predictions in these important branches of modern physics.

In conclusion, let us stress that the present paper is, certainly, only one of the first steps in the investigation of the boundary-induced dynamical phenomena. To be sure that the mere *possibility* of the dynamical Higgs mechanism and of the respective phase transition discussed herein are indeed realizable in reality, one should answer the still-open questions. These include such problems as the calculation of the mass term with the softened BC and the subsequent investigation of the mass term behavior away from the central domain, where one ought to study the nontrivial momentum dependence of the mass term and to perform higher-order analysis of the vertex functions; investigation of the respective dynamical phenomena within the strong-coupling limit (lattice calculations); research on the influence of the nontrivial BC's (such as the Casimir ones) imposed on the gauge field; etc. This is, certainly, only a sketch of the main problems which will require detailed investigation in the future.

We are grateful to the specialists from the Scientific Center for Applied Research at JINR, G. Emelyanenko and O. Ivanov, for help in performing the numerical calculations. We are also grateful to N. Kochelev, E. Kuraev, S. Nedelko, and G. Piragino for fruitful discussions.

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[8] The subtraction recipe (7) means that only the nontrivial L-dependent part of μ^2 survives after renormalization, just as in the case of unrestricted space at finite temperature, where only the T-dependent part of the dynamical mass survives after renormalization [3]. It is obvious that another subtraction scheme can produce in addition only some trivial L-independent constants which can always be absorbed into the infinite mass renormalization constant.

[9] Notice also that in both the finite quantization region (compactified space dimensions) and finite temperature (compactified temporal dimension) cases, the renormalization procedure should be standard (the same as in the infinite space and/or at zero temperature). Being clear qualitatively, it was also shown for a nonzero temperature [3] by the explicit calculations beyond the one-loop approximation.

[10] Indeed, the same situation occurs, in particular, in the second kind of superconductors with chaotic distributed additions in condensed matter physics. There, the energetic gap Δ generally depends on coordinates. However, in the translationally invariant limiting case of a homogeneous superconductor, Δ determines the dispersion law of the elementary excitation and acquires the sense of the quasiparticle mass.

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[13] Contrary to the periodic BC over x_3, which does not violate translational invariance. The respective propagator is just the first term in Eq. (5) (with the replacement $2L \rightarrow L$), and, being rewritten to Euclidean space, it coincides with the finite temperature propagator with the replacement $T \rightarrow 1/L$. As a consequence, the trick [11] with the substitution $T \rightarrow 1/L$ in Eq. (10) just gives the correct μ^2 value for the periodic BC.

[14] Certainly, one can reproduce the results (11) at once, putting $x_3 = 0$ in Eqs. (4) and (6) and using that $\Sigma_{n\neq 0} n^{-2} = \pi^2/3$ and $\Sigma_{n\neq 0}(-1)^n n^{-2} = -\pi^2/6$.

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