Effects of color superconductivity on the structure and formation of compact stars

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(Received 5 November 2003; published 26 March 2004)

We show that if color superconducting quark matter forms in hybrid or quark stars it is possible to satisfy most of the recent observational boundaries on the masses and radii of compact stellar objects. An energy of the order of 10^{53} erg is released in the conversion from a (metastable) hadronic star into a (stable) hybrid or quark star in the presence of a color superconducting phase. If the conversion occurs immediately after the deleptonization of the protoneutron star, the released energy can help supernovae to explode. If the conversion is delayed the energy released can power a gamma-ray burst. A delay between the supernova and the subsequent gamma-ray burst is possible, in agreement with the delay proposed in the recent analysis of astrophysical data.

DOI: 10.1103/PhysRevD.69.057505 PACS number(s): 25.75.Nq, 26.50.+x, 26.60.+c,

The new accumulating data from x-ray satellites provide important information on the structure and formation of compact stellar objects. Concerning the structure, these data are at first sight difficult to interpret in a unique and self-consistent theoretical scenario, since some of the observations indicate rather small radii and other observations indicate large values for the mass of the star. Concerning the formation scenario, crucial information is provided by the very recent observations of gamma-ray bursts (GRBs), indicating the possibility that some of the GRBs are associated with a supernova (SN) explosion. It has not yet been clarified if the two explosions are always simultaneous or if, at least in a few cases, a time delay can exist, with the SN preceding the GRB [1–5].

The effect of the transition to deconfined quark matter (QM) on explosive processes such as SNs and GRBs has been discussed by many authors. In particular, the possibility that deconfinement takes place during the core collapse of massive stars at the moment of the bounce has been discussed, e.g., in Refs. [6,7], and this mechanism could help the SN to explode by increasing the mechanical energy associated with the bounce. However, it seems more plausible that deconfinement takes place only when the protoneutron star (PNS) has deleptonized and cooled down to a temperature of a few MeV [8,9]. The energy released in the conversion to QM produces a refreshed neutrino flux which can help the supernova to explode in a neutrino-driven scheme. Finally, another scenario is possible in which a neutron star having a small enough mass can exist as a metastable hadronic star (HS) if a nonvanishing surface tension is present at the interface between hadronic matter (HM) and QM. The process of quark deconfinement can then be a powerful source for GRBs and it can also explain the possible delay between a SN explosion and the subsequent GRB [10].

In recent years, much theoretical work has investigated the possible formation of a diquark condensate in QM, at densities reachable in the core of a compact star [11–13]. The formation of this condensate can greatly modify the structure of the star [14–17]. We present here an extension of the previous work, showing that it is possible to satisfy the existing boundaries on the mass and radius of a compact stellar object if a diquark condensate forms in a hybrid star (HyS) or a quark star (QS). Moreover, we show that the formation of a diquark condensate can significantly increase the energy released in the conversion from a purely HS into a more stable star containing deconfined QM.

To describe the high density equation of state (EOS) of matter we adopt standard models in the various density ranges. Concerning the hadronic phase we use relativistic nonlinear models [18,19]. At very low density we use the standard EOSs of Refs. [20,21]. For the QM phase we adopt an MIT bag-like model in which the formation of a diquark condensate is taken into account. To connect the two phases of our EOS, we impose Gibbs equilibrium conditions.

It is widely accepted that the color-flavor locking (CFL) phase is the real ground state of QCD at asymptotically large densities. We are interested in the bulk properties of a compact star and we adopt the simple scheme proposed in Refs. [14,17] where the thermodynamic potential is given by the sum of two contributions. The first term corresponds to a ''fictional'' state of unpaired QM in which all quarks have a common Fermi momentum chosen to minimize the thermodynamic potential. The other term is the binding energy Δ of the diquark condensate expanded up to order $(\Delta/\mu)^2$. In Ref. [14] the gap is assumed to be independent of the chemical potential μ. In the present calculation we consider a μ dependent gap resulting from the solution of the gap equation [11]. The resulting QM EOS reads

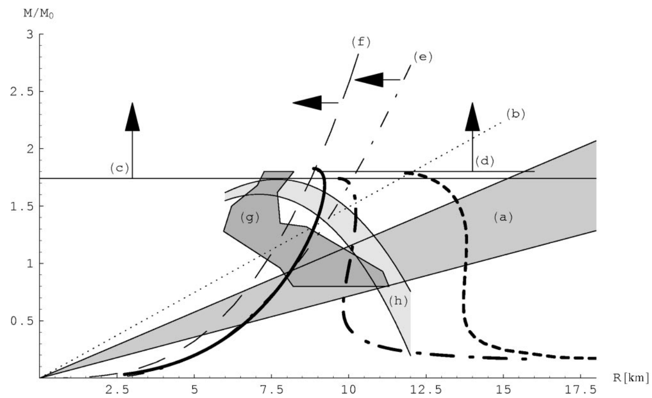

FIG. 1. Mass-radius plane with observational limits and representative theoretical curves: thick solid line indicates CFL quark stars; thick dot-dashed line, CFL hybrid stars; thick dashed line, hadronic stars (see text). Observational limits from (a) Sanwal *et al.* [22], (b) Cottam *et al.* [23], (c) Quaintrell *et al.* [24], (d) Heinke *et al.* [25], (e), (g) Dey *et al.* [26], (f) Li *et al.* [27], and (h) Burwitz *et al.* [28].

$$\Omega_{CFL}=\frac{6}{\pi^2}\int_0^{\nu}k^2(k-\mu)dk+\frac{3}{\pi^2}\int_0^{\nu}k^2(\sqrt{k^2+m_s^2}-\mu)dk-\frac{3\Delta^2\mu^2}{\pi^2}$$

with $\nu=2\mu-\sqrt{\mu^2+m_s^2/3}$, and the quark density ρ is calculated numerically by deriving the thermodynamic potential with respect to μ. The pressure and energy density read $P=-\Omega_{CFL}(\mu)-B-\Omega^e(\mu_e)$, and $E/V=\Omega_{CFL}(\mu)+\mu\rho+B+\Omega^e(\mu_e)+\mu_e\rho_e$.

In Fig. 1 we have collected most of the analysis of data from x-ray satellites concerning the masses and radii of compact stellar objects [22–28]. Observing Fig. 1, we notice that the constraints coming from a few data sets (labeled "e," "f,"[1] "g," and maybe also constraint "h"[2]) indicate rather unambiguously the existence of very compact stellar objects, having a radius smaller than $\sim$10 km. In contrast, at least in one case ("a" in the figure), the analysis of the data suggests the existence of stellar objects having radii of the order of 12 km or larger, if their mass is of the order of $1.4M_\odot$. We recall that it is difficult from an astrophysical viewpoint to generate compact stellar objects having a mass smaller than $1M_\odot$. Therefore the most likely interpretation of constraint "a" is that the corresponding stellar object does not belong to the same class of objects which have a radius smaller than $\sim$10 km. Concerning constraint "b," it can be satisfied either with a very compact star or with a star having a larger radius. The apparent contradiction between the constraints "e," "f," "g," and "a" can be easily accommodated in our scheme, since it can be the signal of the existence of a metastable purely HS which can collapse into a stable configuration when deconfined QM forms inside the star.

[1]A very recent reanalysis of the data of the pulsar SAX J1808.4-3658, discussed in Ref. [27], seems to indicate slightly larger radii, of the order of 9–10 km for a star having a mass of $(1.4–1.5)M_\odot$ [29].

[2]In Ref. [30] an indication for an even more compact stellar object can be found. In any case, the so-called thermal radius obtained in these analyses could be significantly smaller than the total radius of the star.

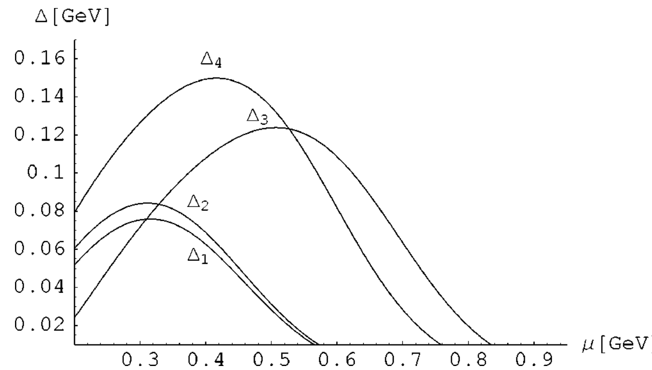

FIG. 2. Gap as function of the chemical potential, for four different parameter sets.

Finally, constraints "c"[3] and "d"[4] do not provide stringent limits on the radius of the star, but they put strong constraints on the lower value of its mass. It is in general not easy to obtain stellar configurations having both large masses and very small radii. As we will see, the existence of an energy gap associated with the diquark condensate helps in circumventing this difficulty, since the effect of the gap is to increase the maximum mass of stars having a huge content of pure QM.

In Fig. 1 we show a few theoretical M-R relations which correspond to the scenario we are proposing. More precisely, we show a thick dashed line corresponding to HSs (GM1), a thick dot-dashed line corresponding to HySs (GM1, $B^{1/4}=170$ MeV, Δ_2), and a thick solid line corresponding to QSs ($B^{1/4}=170$ MeV, Δ_4). Similar shapes can be obtained using the EOS of Ref. [19]. Both the HyS and the QS lines can satisfy essentially all the constraints derived from observations. The shapes of the gaps Δ_i are shown in Fig. 2. In conclusion, in our scheme most of the compact stars are either HySs or QSs having a mass in the range $(1.2–1.8)M_\odot$ and a radius $\sim$8.5–10 km. Stars having a significantly larger radius (like the one suggested by constraint "a") correspond in our scheme to metastable HSs which can exist if their mass is not too large, as we show in the following.

Let us now discuss ΔE, the energy released in the conversion from HS to HyS or QS. ΔE is the difference between the gravitational mass of the HS and that of the final HyS or QS having the same baryonic mass. As mentioned in the introduction, a possibility is that deconfinement takes place a few seconds after the bounce, when the PNS has deleptonized and its temperature has dropped down [8,9]. In particular, for stars having a small mass the formation of QM takes

[3]The result of Ref. [24] is $M/M_\odot=1.88\pm0.13$. In Fig. 1 only the lower limit is displayed.

[4]If the observed x-ray emission is due to continuing accretion, a smaller mass is allowed, $M/M_\odot=1.4$.

TABLE I. Energy released ΔE (measured in foe$=10^{51}$ erg) in the conversion from a $1.4M_{\odot}$ hadronic star into a hybrid or quark star having the same baryonic mass (labeled with an asterisk), for various sets of model parameters. BH indicates that the hadronic star collapses to a black hole. A dash indicates situations in which the Gibbs construction does not provide a mechanically stable EOS.

Hadronic model	$B^{1/4}$ (MeV)	ΔE				
		$\Delta=0$	Δ_1	Δ_2	Δ_3	Δ_4
GM3	160	95	172*	178*	204*	327*
GM3	170	40	83	89	133	236*
GM3	180	10	29	31	79	—
GM1	160	101	178*	184*	210*	333*
GM1	170	42	89	95	138	242*
GM1	180	6	28	31	BH	—

place only at $T \lesssim$ few MeV. Notice that for a star having a mass of order $1.4M_{\odot}$ and using the relativistic EOSs discussed in this paper, hyperons are present in the initial configuration, since the typical mass at which hyperons start forming is $\sim 1M_{\odot}$. The energy released during quark deconfinement powers a new neutrino flux which can be useful in making the supernova explode. ΔE is shown in Table I, for a PNS having a mass of $1.4M_{\odot}$. As can be seen, ΔE can be as large as 10^{53} erg, if the final configuration corresponds to a HyS and three times as large if a QS is obtained. The effect of the gap is to increase the energy released and to allow QS configurations in cases where a HyS would be obtained in the absence of quark pairing. Let us now remark that the deconfinement transition can be delayed if a nonvanishing surface tension at the interface between HM and QM exists and if the mass of the HS is not too large. This possibility was not discussed in Ref. [9] and it is the main ingredient of our model. To compute the time needed to form QM we use the technique of quantum tunneling nucleation. We can assume that the temperature has no effect in our scheme because, as discussed above, when QM forms the temperature is so low that only quantum tunneling is a practicable mechanism.

In Ref. [10] it was proposed that the central density of a pure HS (containing hyperons) can increase, due to spin down or mass accretion, until its value approaches the deconfinement critical density. At this point a spherical virtual drop of QM can form. The potential energy for fluctuations of the drop radius R has the following form [31]: $U(R) = \frac{4}{3}\pi R^3 n_q(\mu_q-\mu_h)+4\pi\sigma R^2+8\pi\gamma R$, where n_q is the quark baryon density, μ_h and μ_q are the hadronic and quark chemical potentials, all computed at a fixed pressure P, and σ is the surface tension for the interface separating quarks from hadrons. Finally, the term containing γ is the so called curvature energy. For σ we use standard values from 10 to 40 MeV/fm^2 and we assume that it also takes into account, in an effective way, the curvature energy. The value of σ was estimated in Ref. [32] to be ~ 10 MeV/fm^2. Values for σ larger than ~ 30 MeV/fm^2 are probably not useful in light of the results of Refs. [33,34].

The calculation proceeds in the usual way: after the computation (in the WKB approximation) of the ground state energy E_0 and of the oscillation frequency ν_0 of the virtual QM drop in the potential well $U(R)$, it is possible to calculate in a relativistic frame the probability of tunneling as [35] $p_0=\exp(-2A(E_0)/\hbar)$, where

$$A(E)=\int_{R_-}^{R_+} dR\sqrt{[2M(R)+E-U(R)][U(R)-E]}.$$

Here $M(R)=4\pi\rho_h(1-n_q/n_h)^2R^3$, ρ_h is the hadronic energy density, and n_h, n_q are the baryonic densities at the same and given pressure in the hadronic and quark phases, respectively. Finally, $R_{\pm}$ are the classical turning points. The nucleation time is then equal to $\tau=(\nu_0 p_0 N_c)^{-1}$, where N_c is the number of centers of droplet formation in the star, and it is of the order of 10^{48} [35]. τ can be extremely long if the mass of the metastable star is small enough but, via mass accretion, it can be reduced from values of the order of the age of the universe down to a value of the order of days or years. We can therefore determine the critical mass M_{cr} of the metastable HS for which the nucleation time corresponds to a fixed small value (1 yr in Table I).

In Table II we show the value of M_{cr} for various sets of model parameters. In the conversion process from a metastable HS into a HyS or a QS a huge amount of energy ΔE is released. We see in Table II that the formation of a CFL phase allows one to obtain values for ΔE which are one order of magnitude larger than the corresponding ΔE of the unpaired QM case $(\Delta=0)$. Moreover, we can observe that ΔE depends both on the magnitude and the position of the gap.

In the model we are presenting, the GRB is due to the cooling of the just formed HyS or QS via neutrino-antineutrino emission. The subsequent neutrino-antineutrino annihilation generates the GRB. In our scenario the duration of the prompt emission of the GRB is therefore regulated by two mechanisms: (1) the time needed for the conversion of the HS into a HyS or QS, once a critical-size droplet is formed and (2) the cooling time of the just formed HyS or QS. Concerning the time needed for the conversion into QM of at least a fraction of the star, the seminal work by [36] has been reconsidered in [37], where it has been shown that the stellar conversion is a very fast process, having a duration much shorter than 1 s. On the other hand, the neutrino trapping time, which provides the cooling time of a compact object, is of the order of a few tens of seconds [38], and it gives the typical duration of the GRB in our model.

In conclusion, comparing the theoretical mass-radius curves with recent observational data, we find that color superconductivity is a crucial ingredient in order to satisfy all the constraints coming from observations. The difficult problem posed by astrophysical data indicating the existence of stars which are both very compact and rather massive can be solved with either hybrid or quark stars. Concerning hybrid stars, the gap increases the maximum mass of the stable configuration, while keeping the corresponding radius $\lesssim 10$ km.

The superconducting gap also greatly affects the energy released in the conversion from a hadronic star into a hybrid or quark star. We assume that the deconfinement transition

TABLE II. Energy released ΔE in the conversion to a hybrid or quark star, for various sets of model parameters, assuming the hadronic star mean lifetime $\tau = 1$ yr (see text). M_{cr} is the gravitational mass of the hadronic star at which the transition takes place, for fixed values of the surface tension σ and of the mean lifetime τ. Notation as in Table I.

Hadronic model	$B^{1/4}$ (MeV)	σ (MeV/fm^2)	$M_{cr}/M_{\odot}$	ΔE				
				$\Delta=0$	Δ_1	Δ_2	Δ_3	Δ_4
GM3	170	10	1.12	18	52	57	86	178*
GM3	170	20	1.25	30	66	72	106	205*
GM3	170	30	1.33	34	75	81	120	221*
GM3	170	40	1.39	38	82	88	131	234*
GM3	180	10	1.47	BH	35	38	BH	—
GM3	180	20	1.50	BH	38	40	BH	—
GM3	180	30	1.52	BH	40	42	BH	—
GM1	170	10	1.16	18	58	64	94	189*
GM1	170	20	1.30	30	75	81	119	219*
GM1	170	30	1.41	43	90	96	141	244*
GM1	170	40	1.51	BH	105	111	163	267*
GM1	180	10	1.56	BH	52	54	BH	—
GM1	180	20	1.61	BH	65	65	BH	—
GM1	180	30	1.65	BH	BH	BH	BH	—

takes place only when the star has deleptonized and cooled down, in agreement with the results of Refs. [8,9]. If deconfinement occurs immediately after deleptonization, the energy released can help the SN to explode. If, in contrast, the transition is delayed, a metastable hadronic star can form. Its subsequent transition to a stable configuration, containing deconfined quark matter, can power a GRB via the annihilation of neutrinos and antineutrinos emitted during the cooling of the newly formed compact star. The energy released is significantly increased by the effect of the chemical-potential dependent superconducting gap and it can reach a value of the order of 10^{53} erg. The proposed mechanism could explain recent observations indicating a possible delay between a supernova and the subsequent gamma-ray burst [1–3].

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