Possible large phase in $\psi(2S)\to 1^-0^-$ decays

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The strong and electromagnetic amplitudes are analyzed on the basis of the measurements of $J/\psi, \psi(2S) \to 1^-0^-$ in e^+e^- experiments. The currently available experimental information is revised with the inclusion of the contribution from $e^+e^-\to\gamma^*\to 1^-0^-$. The study shows that a large phase around $-90°$ between the strong and electromagnetic amplitudes could not be ruled out by the experimental data for $\psi(2S)$.

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In recent years, increased information on J/ψ and $\psi(2S)$ decays from experiments has led to the analysis of strong and electromagnetic decay amplitudes in charmonium decay processes [1–6]. Such an analysis on J/ψ revealed that there exists a relative orthogonal phase between these two amplitudes for the two-body decay modes: 1^+0^- [1], 1^-0^- [2,3], 0^-0^- [4,5], 1^-1^- [5], and $N\overline{N}$ [6].

As for the $\psi(2S)$ decay, for which information from experiments is less abundant than J/ψ, it is questionable whether it decays in the same pattern. It has been argued [1] that the only large energy scale involved in the three-gluon decay of charmonia is the charm quark mass; one expects that the corresponding phase should not be much different between J/ψ and $\psi(2S)$ decays. There is also another theoretical argument which favors the $\pm 90°$ phase [7]. This large phase follows from the orthogonality of three-gluon and one-photon virtual processes. But an extensively quoted work [1] found that a fit to $\psi(2S)\to 1^-0^-$ with a large phase $\pm 90°$ is virtually impossible and concluded that the relative phase between strong and electromagnetic amplitudes should be around 180° [8]. So it is a matter of great concern as to whether the large phase is consistent with the $\psi(2S)$ experimental data.

Up to now, the most accurate data on $\psi(2S)\to 1^-0^-$ are from e^+e^- colliding experiments. However, in previous analyses, the contribution from the continuum one-photon annihilation

$$e^+e^-\to\gamma^*\to 1^-0^-$$

has been neglected [9]. In this analysis, such a contribution will be taken into account for both J/ψ and $\psi(2S)$. First the available data from $e^+e^-\to J/\psi$ are reanalyzed. To avoid the complexity and uncertainty of the mixing between SU(3) singlets and octets, only four processes are used, to wit

$$e^+e^-\to\omega\pi^0,$$

$$e^+e^-\to\rho\pi,$$

$$e^+e^-\to K^{*+}K^- + \text{c.c.},$$

$$e^+e^-\to K^{*0}\overline{K^0} + \text{c.c.} \tag{1}$$

It is found that the phase between strong and electromagnetic decay amplitudes is either $-72.0°$ or $+76.8°$. Then with the same scheme, the data on $e^+e^-\to\psi(2S)$ are reexamined. It is found that the currently available data from BES [10] accommodate the phases of both 180° and $-90°$.

In $e^+e^-\to 1^-0^-$ at J/ψ or $\psi(2S)$ resonance, the Born order cross section for the final state f is

$$\sigma_{Born} = \frac{4\pi\alpha^2}{s^{3/2}}|A_f|^2\mathcal{P}_f(s), \tag{2}$$

where $\mathcal{P}_f(s) = q_f^3/3$, with q_f being the momentum of either the 1^- or 0^- final state particle.

In e^+e^- annihilation experiments, there are three amplitudes [9,11]: the continuum one-photon annihilation amplitude a_c, the electromagnetic decay amplitude of the resonance a_γ, and the strong decay amplitude of the resonance a_{3g}. For the SU(3) breaking processes, a SU(3) breaking term ϵ is added to a_{3g}, so the strong decay amplitude is $a_{3g}+\epsilon$. With inclusion of a_c, the amplitudes of the four $e^+e^-\to 1^-0^-$ processes are expressed as

$$A_{\omega\pi^0} = 3(a_\gamma + a_c),$$

$$A_{\rho\pi} = a_{3g} + a_\gamma + a_c,$$

$$A_{K^{*+}K^-} = a_{3g} + \epsilon + a_\gamma + a_c,$$

$$A_{K^{*0}\overline{K^0}} = a_{3g} + \epsilon - 2(a_\gamma + a_c). \tag{3}$$

For $\omega\pi^0$, which goes only through the electromagnetic process, a_c and a_γ are related to the $\omega\pi^0$ form factor $\mathcal{F}_{\omega\pi^0}(s)$:

$$a_c = \frac{1}{3}\mathcal{F}_{\omega\pi^0}(s) \tag{4}$$

and

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$$a_\gamma = \frac{\sqrt{s}\Gamma_{ee}/\alpha}{s-M^2+iM\Gamma_t}\mathcal{F}_{\omega\pi^0}(s), \tag{5}$$

where α is the QED fine structure constant, M and Γ_t are the mass and total width of J/ψ or $\psi(2S)$, and Γ_{ee} is the partial width of e^+e^-. It has been assumed that there is no extra phase between a_c and a_γ, as in $e^+e^-\to\mu^+\mu^-$. Formally, the amplitude of the $\omega\pi^0$ process could be written as

$$A_{\omega\pi^0} = (1+B(s))\mathcal{F}_{\omega\pi^0}(s), \tag{6}$$

with the definition

$$B(s) \equiv \frac{3\sqrt{s}\Gamma_{ee}/\alpha}{s-M^2+iM\Gamma_t}.$$

If there is no a_c in Eq. (6), only the second term is left which describes the resonance decaying through electromagnetic process. Substituting it into Eq. (2), the commonly known Breit-Wigner form is then reproduced:

$$\sigma_{BW}(e^+e^-\to\text{Res.}\to\omega\pi^0) = \frac{12\pi\Gamma_{ee}\Gamma_{\omega\pi^0}}{(s-M^2)^2+\Gamma_t^2M^2},$$

with

$$\Gamma_{\omega\pi^0} = \frac{\Gamma_{ee}q^3_{\omega\pi^0}}{M}|\mathcal{F}_{\omega\pi^0}(M^2)|^2.$$

For the strong decay amplitude, the most interesting point lies in its phase and strength relative to the electromagnetic decay amplitude, so it is parametrized as

$$a_{3g} = \mathcal{C}e^{i\phi}a_\gamma, \tag{7}$$

where ϕ is the phase between the two amplitudes and $\mathcal{C}$ is taken to be real. For the SU(3) breaking strong decay amplitude, it is parametrized as its strength relative to the SU(3) conserved one:

$$\mathcal{R} = \frac{a_{3g}+\epsilon}{a_{3g}}. \tag{8}$$

As in Refs. [1,12,13], it is assumed that the SU(3) breaking amplitude $a_{3g}+\epsilon$ has the same phase as a_{3g} [14], so $\mathcal{R}$ is real. According to Eqs. (7) and (8), together with Eq. (6), the amplitudes of Eq. (3) could be expressed as:

$$\begin{aligned}
A_{\omega\pi^0} &= [1+B(s)]\mathcal{F}_{\omega\pi^0}(s),\\
A_{\rho\pi} &= [(\mathcal{C}e^{i\phi}+1)B(s)+1]\mathcal{F}_{\omega\pi^0}(s)/3,\\
A_{K^{*+}K^-} &= [(\mathcal{C}\mathcal{R}e^{i\phi}+1)B(s)+1]\mathcal{F}_{\omega\pi^0}(s)/3,\\
A_{K^{*0}\overline{K^0}} &= [(\mathcal{C}\mathcal{R}e^{i\phi}-2)B(s)-2]\mathcal{F}_{\omega\pi^0}(s)/3.
\end{aligned} \tag{9}$$

In this analysis, the branching ratios are converted into measured cross sections by multiplying the total resonance cross section. Special attention should be paid in calculating the cross sections where the experimental conditions must be taken into account properly [15,16]. The most important ones are the radiative correction and the energy spread of the collider, both of which reduce the height of the resonance and shift the position of the maximum cross section. Also experiments naturally tend to collect resonance data at the energy which yields the maximum inclusive hadron cross sections. This energy is higher than the nominal resonance mass, and it does not necessarily coincide with the maximum cross section of each exclusive mode. All these must be considered accordingly.

The experimental results for J/ψ decays relevant to the forementioned four channels are listed in Table I. The values of energy spread are obtained from Ref. [5]. The positions which yield the maximum inclusive hadronic cross section on each e^+e^- collider are calculated and listed in Table I as well.

The chi-square method is employed to fit the experiment data. The estimator is defined as

$$\begin{aligned}
\chi^2 = &\sum_i \frac{[R_i-\hat{R}_i(\vec{\eta})]^2}{\sigma_i^2} + \sum_j \frac{[\mathcal{B}_j/f_{mk3}-\hat{\mathcal{B}}_j(\vec{\eta})]^2}{(\sigma_j/f_{mk3})^2}\\
&+\sum_k \frac{[\mathcal{B}_k-\hat{\mathcal{B}}_k(\vec{\eta})]^2}{\sigma_k^2} + \sum_m \frac{[\mathcal{B}_m-\hat{\mathcal{B}}_m(\vec{\eta})]^2}{\sigma_m^2}\\
&+\sum_n \frac{[\mathcal{B}_n-\hat{\mathcal{B}}_n(\vec{\eta})]^2}{\sigma_n^2}.
\end{aligned} \tag{10}$$

In the above equation, the summation index i indicates the results from DM II [2]; j from Mark III [3]; k from Mark II [18] and Mark I [19]; m from CNTR [20], PLUTO [21], and DASP [22]; and n from BES [23]. R_i indicates the relative branching ratio—i.e., $R_i = \mathcal{B}_i/\mathcal{B}_{\rho\pi}$, where i denotes $\omega\pi^0$, $K^{*+}K^-+$c.c., and $K^{*0}\overline{K^0}+$c.c. The careted symbol in Eq. (10) indicates the theoretical expectation, and $\vec{\eta}$ denotes the parameter vector with five elements, four of which have been described in Eq. (9), and the fifth f_{mk3} is introduced to describe the correlation of data from Mark III, and correspondingly the 8.5% common error (the second term of error) for Mark III measurements in Table I is subtracted from the systematic uncertainty σ_j in Eq. (10).

The fitting gives a χ^2 of 4.1 with the number of degrees of freedom being 7. There are two minima with ϕ of opposite sign, while all other parameters have the same values up to the significant digits listed below:

$$\phi = -72.0°\pm3.6° \quad\text{or}\quad +76.8°\pm3.6°,$$

$$\mathcal{C} = 10.3\pm0.3,$$

$$\mathcal{R} = 0.775\pm0.013,$$

$$|\mathcal{F}_{\omega\pi^0}(M^2_{J/\psi})| = (0.075\pm0.004)\ \text{GeV}^{-1}.$$

f_{mk3} is 1.26 ± 0.11 from the fit, which means a global deviation of the Mark III values from other experiments. The above fitting results of $J/\psi\to1^-0^-$ decay deviate little from

TABLE I. Experimental results for $e^+e^-\to 1^-0^-$ processes at the J/ψ energy region.

Experiment	Accelerator	c.m. energy spread (MeV)	Data taking position [a] (GeV)	Final state	Branching ratio
DMII [2]	DCI	1.98	3.09707	$\omega\pi^0$	$(0.0272\pm 0.0021)\mathcal{B}_{\rho\pi}$
				$K^{*+}K^- +$c.c.	$(0.364\pm 0.013)\mathcal{B}_{\rho\pi}$
				$K^{*0}\overline{K^0}+$c.c.	$(0.300\pm 0.011)\mathcal{B}_{\rho\pi}$
Mark III [3]	SPEAR	2.40	3.09711	$\omega\pi^0$	$(4.82\pm 0.52\pm 0.41)\times 10^{-4}$
				$K^{*+}K^- +$c.c.	$(5.26\pm 0.32\pm 0.45)\times 10^{-3}$
				$K^{*0}\overline{K^0}+$c.c.	$(4.33\pm 0.29\pm 0.37)\times 10^{-3}$
				$\rho\pi$	$(1.42\pm 0.15\pm 0.12)\%$
MARK II [18]	SPEAR			$\rho\pi$	$(1.3\pm 0.3)\%$
MARK I [19]	SPEAR			$\rho\pi$	$(1.3\pm 0.3)\%$
CNTR [20]	DORIS	1.41	3.09701	$\rho\pi$	$(1.0\pm 0.2)\%$
PLUTO [21]	DORIS			$\rho\pi$	$(1.6\pm 0.4)\%$
DASP [22]	DORIS			$\rho\pi$	$(1.16\pm 0.16)\%$ [b]
BES [23]	BEPC	0.85	3.09696	$\rho\pi$	$(1.21\pm 0.20)\%$

[a]The data taking position is the energy which yields the maximum inclusive hadronic cross section.
[b]The latest PDG value of $\mathcal{B}_{\mu\mu}$ [17] is used to renormalize the branching ratio

$$\mathcal{B}_{\rho\pi}=(1.36\pm 0.28)\%\cdot\frac{\mathcal{B}_{\mu\mu}=(5.88\pm 0.10)\%(\text{PDG2002})}{\mathcal{B}_{\mu\mu}=(6.9\pm 0.9)\%(\text{used by DASP})}.$$

previous analyses without a_c [2,3], but the precision is improved. They support the following theoretical postulates [1,7].

(i) The relative phase between the strong and the electromagnetic amplitudes is large for $J/\psi\to 1^-0^-$ decays.
(ii) In the strong amplitude, the SU(3) breaking term ϵ is negative.

For $\psi(2S)$ decays, only two decay modes have been observed with finite branching ratios [10]:

$$\mathcal{B}_{K^{*0}\overline{K^0}+c.c.}=(0.81\pm 0.24\pm 0.16)\times 10^{-4},$$

$$\mathcal{B}_{\omega\pi^0}=(0.38\pm 0.17\pm 0.11)\times 10^{-4}.$$

These are given without subtracting the contribution from a_c. In the following calculations, they are converted into measured cross sections by multiplying the total $\psi(2S)$ cross section at 3.6861 GeV for an energy spread of 1.3 MeV [16]. The $\omega\pi^0$ mode gives the $\omega\pi^0$ form factor at $\psi(2S)$:

$$|\mathcal{F}_{\omega\pi^0}(M^2_{\psi(2S)})|=(0.039^{+0.009}_{-0.012})\ \text{GeV}^{-1}, \tag{11}$$

which is related to a_c and a_γ by Eqs. (4) and (5). Then it requires the input of ϕ together with the branching ratio $\mathcal{B}_{K^{*0}\overline{K^0}+c.c.}$ to obtain $|a_{3g}+\epsilon|=|a_\gamma|\mathcal{R}\mathcal{C}$. Here four different phases are assumed: $+76.8°$ and $-72.0°$ from J/ψ fitting, 180° from Ref. [1], and $-90°$ which is one of the phases favored by theory [7]. Then the cross sections and the branching ratios of $K^{*+}K^-$ are calculated and listed in Table II. Finally with the input of the SU(3) breaking magnitude $\mathcal{R}=0.775\pm 0.013$ from the J/ψ fitting, the cross sections of $\rho\pi$ are presented in Table II. For comparison, the upper experimental limits of $K^{*+}K^-$ and $\rho\pi$ cross sections are also listed.

From Table II, it can be seen that one of the phases $\phi=76.8°$ gives poorer agreement than the other phase $\phi=-72.0°$ does. The former predicts cross sections for $K^{*+}K^-$ and $\rho\pi$ more than 1.2 and 1.4 standard deviations above the available upper limits, while the latter yields 0.7 and 1.1 standard deviations from the experimental upper limits. This means that, although theory and experiment at J/ψ could not tell whether the large phase is positive or negative, the measurements at $\psi(2S)$ favor the negative one,

TABLE II. Calculated results for $\psi(2S)\to K^{*+}K^-$ and $\rho^0\pi^0$ with different ϕ.

ϕ	$\mathcal{C}=\left\|\frac{a_{3g}}{a_\gamma}\right\|$	$\sigma_{pre}(K^{*+}K^-)$(pb)	$\mathcal{B}^0_{K^{*+}K^-}(\times 10^{-5})$ [a]	$\sigma_{pre}(\rho^0\pi^0)$(pb)	$\mathcal{B}^0_{\rho^0\pi^0}(\times 10^{-5})$
$+76.8°$	$7.0^{+3.1}_{-2.2}$	37^{+24}_{-23}	$5.0^{+3.2}_{-3.1}$	64^{+43}_{-41}	$9.0^{+6.1}_{-6.0}$
$-72.0°$	$5.3^{+3.1}_{-2.6}$	19^{+14}_{-14}	$3.1^{+2.3}_{-2.3}$	33^{+25}_{-24}	$5.5^{+4.1}_{-4.0}$
$-90°$	$4.5^{+3.1}_{-2.6}$	12^{+9}_{-9}	$2.0^{+1.5}_{-1.5}$	22^{+17}_{-17}	$3.7^{+2.9}_{-2.9}$
180°	$3.4^{+3.0}_{-2.2}$	$4.0^{+4.3}_{-3.2}$	$0.39^{+0.42}_{-0.31}$	$7.8^{+8.6}_{-6.7}$	$1.0^{+1.1}_{-0.8}$
BES observed		<9.6		<5.8	

[a]The supscript 0 indicates that the continuum contribution in the cross section has been subtracted.

which indicates destructive interference between the a_{3g} and a_c for $\rho\pi$ and $K^{*+}K^-$ and constructive interference for $K^{*0}\overline{K^0}$ at the resonance.

Compared with $\phi=-72.0°$, the calculated cross sections with $\phi=-90°$ are closer to the measured upper limits for $K^{*+}K^-$ and $\rho\pi$, both of which are within one standard deviation. With $\phi=180°$, the evaluations of both cross sections also cover the experimental upper limits within one standard deviation.

From the above analysis, it shows that the large negative phase $-90°$ suggested by theory and $-72.0°$ from J/ψ could fit the $\psi(2S)$ data, after the one-photon annihilation amplitude being considered properly. It should be noted that the $\rho\pi$ cross sections in Table II are calculated under the assumption that the SU(3) breaking effect $\mathcal{R}$ has the same magnitude in J/ψ and $\psi(2S)$ decays. If this assumption is removed, the $\rho\pi$ cross section, for all values of ϕ, can be lower than those listed in Table II. In the extreme situation in which a_{3g} is very small, only a_c and a_γ contribute, and the $\rho\pi$ cross section is 1/9 of the $\omega\pi^0$ cross section, which is below the current upper limit by the experiment.

In conclusion, this work shows that the current available $\psi(2S)$ data accommodate both a large negative phase and $\phi=180°$ with the contribution from the continuum one-photon annihilation amplitude a_c taken into account. The theoretical favored phase $-90°$ could not be ruled out as analyzed in Ref. [1]. The data are also consistent with the assumption that the SU(3) breaking effect is of the same magnitude for J/ψ and $\psi(2S)$ decays. It requires more accurate $\psi(2S)\rightarrow 1^-0^-$ data to determine the phase between a_{3g} and a_γ. The most important information is whether the upper limits of $K^{*+}K^-$ and $\rho\pi$ will be further pushed down or finite cross sections will be observed.

In the end, it is also interesting to notice that if $\Gamma(\psi(2S)\rightarrow\gamma^*\rightarrow 1^-0^-)/\Gamma(\psi(2S)\rightarrow ggg\rightarrow 1^-0^-)$ is roughly equal to $\Gamma(\psi(2S)\rightarrow\gamma^*\rightarrow X)/\Gamma(\psi(2S)\rightarrow ggg\rightarrow X)$ [1], then it is expected that $\mathcal{C}\approx 4.5$, which is in good agreement with the fittings with $\phi=-72.0°$, $-90°$, and 180° in Table II; on the contrary, a similar relation for J/ψ implies $\mathcal{C}\approx 4$, which is far less than the fitted value $\mathcal{C}=10.3\pm 0.3$. So far as this point is concerned, the so-called "$\rho\pi$ puzzle" seems to be in J/ψ decays rather than in $\psi(2S)$.

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