

Charmonium options for the $X(3872)$

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(Received 14 November 2003; published 12 March 2004)*

In this paper we consider all possible $1D$ and $2P$ $c\bar{c}$ assignments for the recently discovered $X(3872)$. Taking the experimental mass as input, we give numerical results for the $E1$ radiative widths as well as the three principal types of strong decays; open-charm, $c\bar{c}$ annihilation and closed-charm hadronic transitions. We find that many assignments may be immediately eliminated due to the small observed total width. The remaining viable $c\bar{c}$ assignments are 1^3D_3 , 1^3D_2 , 1^1D_2 , 2^3P_1 and 2^1P_1 . A search for the mode $J/\psi\pi^0\pi^0$ can establish the C parity of the $X(3872)$, which will eliminate many of these possibilities. Radiative transitions can then be used to test the remaining assignments, as they populate characteristic final states. The 1^3D_2 and 1^1D_2 states are predicted to have large (ca. 50%) radiative branching fractions to $\chi_{c1}\gamma$ and $h_c\gamma$, respectively. We predict that the 1^3D_3 will also be relatively narrow and will have a significant (ca. 10%) branching fraction to $\chi_{c2}\gamma$, and should also be observable in B decay. Tests for non- $c\bar{c}$ $X(3872)$ assignments are also discussed.

DOI: 10.1103/PhysRevD.69.054008

PACS number(s): 12.39.-x, 13.20.Gd, 13.25.Gv, 14.40.Gx

I. INTRODUCTION

Several new meson states have recently been reported [1–5] whose properties are in disagreement with the predictions of quark potential models. Assuming experimental confirmation, this indicates the necessity of refinements in the models or the inclusion of additional dynamical effects.

The most recent of these discoveries is the $X(3872)$, which was reported by the Belle Collaboration [1] in the $J/\psi\pi^+\pi^-$ invariant mass distribution in the process $B^\pm \rightarrow K^\pm J/\psi\pi^+\pi^-$. The mass and width upper limit reported by Belle are

$$M = 3872.0 \pm 0.6(\text{stat}) \pm 0.5(\text{syst}) \text{ MeV}, \quad (1)$$

$$\Gamma_{X(3872)}^{\text{tot}} < 2.3 \text{ MeV}, \quad 95\% \text{ C.L.} \quad (2)$$

Note that the mass is very near the D^0D^{*0} threshold of 3871.5 ± 0.5 MeV. The width is consistent with experimental resolution. This observation has since been confirmed by the Collider Detector at Fermilab Collaboration (CDF) [2], who report a very similar mass of

$$M = 3871.4 \pm 0.7(\text{stat}) \pm 0.4(\text{syst}) \text{ MeV} \quad (3)$$

for a fixed experimental resolution of 4.3 MeV. A limit on the relative radiative branching fraction has also been reported [1]:

$$\frac{B(X(3872) \rightarrow \chi_{c1}\gamma)}{B(X(3872) \rightarrow J/\psi\pi^+\pi^-)} < 0.89, \quad 90\% \text{ C.L.} \quad (4)$$

There was a prior, unconfirmed, observation of a 2^{--} state, $\psi(3836 \pm 13)$ in $\pi^\pm N \rightarrow J/\psi\pi^+\pi^- + \text{anything}$ by the Fermilab E705 Collaboration [6].

An obvious assignment for the $X(3872)$ would be an $L=2$ $c\bar{c}$ level, since the 1^3D_2 and 1^1D_2 states are both expected to be narrow due to the absence of DD decay modes [7] and are expected to have sizeable production rates in B decays [8–10]. These assignments however have the problem that the mass of the $X(3872)$ is somewhat higher than most potential models predict for $1D$ $c\bar{c}$ states (see Table I). Another difficulty is that the Belle limit on the relative branching fraction $B_{\chi_{c1}\gamma}/B_{J/\psi\pi^+\pi^-}$ is much smaller than the ratio predicted by Eichten *et al.* [11] for the $C=(-)1^3D_2$ state, although this may simply be due to an inaccurate estimate of the problematic rate to $J/\psi\pi^+\pi^-$.

These difficulties have led to speculations that the $X(3872)$ may not be a conventional $1D$ $c\bar{c}$ state. The proximity to the DD^* threshold in particular has suggested that the $X(3872)$ might be a weakly bound DD^* molecule [12–18]. Other possibilities that have been discussed are a $2P$ $c\bar{c}$ state [13,15] or a charmonium hybrid [13,19].

In this article we compare the properties of the $X(3872)$ to theoretical predictions for the radiative transitions and strong decay rates of all $1D$ and $2P$ charmonium states. We begin by summarizing quark model predictions for the masses of the $1D$ and $2P$ $c\bar{c}$ states, followed by our predictions for radiative transitions and strong decay partial widths. From these results we determine which $c\bar{c}$ assignments appear consistent with the experimental data at present, following which we suggest measurements that can differentiate between these $c\bar{c}$ assignments as well as non- $c\bar{c}$ possibilities.

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TABLE I. Predicted and observed masses of $1D$ and $2P$ $c\bar{c}$ states.

State	Expt.	Theory					
		GI [23]	EF [24]	FU [25]	GRR [26]	EFG [27]	ZVR [28]
1^3D_3		3849	3840	3884	3830	3815	3830
1^3D_2		3838	3797	3871	3822	3813	3820
1^3D_1	3770	3819	3762	3840	3801	3798	3800
1^1D_2		3837	3765	3872	3822	3811	3820
2^3P_2		3979				3972	4020
2^3P_1		3953				3929	3990
2^3P_0		3916				3854	3940
2^1P_1		3956				3945	3990

II. SPECTROSCOPY

The spectrum of charmonium states has long provided important tests of our understanding of the forces between quarks. The mean multiplet positions are consistent with the “funnel-shaped potential” that follows from one gluon exchange and linear confinement. One gluon exchange implies additional spin-dependent forces, specifically the contact spin-spin interaction (evident in the $J/\psi - \eta_c$ splitting) and spin-orbit and tensor forces that affect the fine structure of $L > 0$ multiplets. The agreement of the predicted splittings of the χ_{cJ} states with experiment (including the negative spin-orbit contribution of scalar confinement) has until recently been considered a clear success of this model, and is the strongest experimental evidence in favor of Lorentz scalar confinement.

The discovery of the $X(3872)$, like the earlier reports of the $D_{sJ}^*(2317)^+$ and $D_{sJ}(2457)^+$, has called the accuracy of these models into question. In both cases, narrow states have been reported at masses that are rather far from the predictions of quark potential models. Either these new states are not dominantly quarkonia, or we are seeing evidence of important additional forces that were not previously incorporated in the models.

The most detailed predictions of the charmonium spectrum have come from quark potential models. These models typically assume a color Coulomb plus linear confining interaction, which is augmented by the spin-dependent forces that follow from one gluon exchange (OGE) and the confining interaction. These OGE terms are noncontroversial, and are the Breit-Fermi Hamiltonian times a color factor; they consist of a contact spin-spin term, a spin-orbit term and a smaller tensor interaction. The spin-dependent force that arises from confinement is rather controversial, as it depends on the assumed Lorentz structure of the confining interaction. The usual choice is scalar confinement, which gives an inverted spin-orbit term that partially cancels the OGE term for small L . The alternative choice of vector confinement (which was assumed in the Cornell model [20–22]) has a noninverted spin-orbit term, and unlike scalar confinement does not give a good description of the splittings of the χ_{cJ} states.

The numerical mass predictions for the $1D$ and $2P$ $c\bar{c}$ states given in Table I are taken from several of these

potential models [23–28]; note that most predict $1D$ states about 50–100 MeV below the $X(3872)$ mass, and the $2P$ states are predicted to lie above the $X(3872)$ by a similar amount. The results are rather similar numerically because they differ on relatively fine points such as relativizing quark motion, regularizing singular interactions, and the choice of experimental input. Clearly they all predict that the $1D$ $c\bar{c}$ multiplet has a much smaller multiplet splitting than is implied by the $X(3872)$ and the $\psi(3770)$. In this paper we tacitly assume that the potential model wave functions are approximately correct for $c\bar{c}$ states, and that the discrepancy in the spectrum is due to additional effects such as confinement spin-orbit terms or coupled-channel effects, which shift the various $c\bar{c}$ states by different amounts. The importance of these coupled-channel effects will be considered in future work.

Although the spectroscopy of charmonium states has been considered by many lattice gauge theory collaborations (for recent reviews see Refs. [29,30]), relatively few results have been reported for the orbitally and radially excited $1D$ and $2P$ multiplets, and these references quote rather large systematic and (for $2P$) statistical uncertainties, which at present imply an overall uncertainty of roughly ± 100 MeV [31]. The mean positions reported for the $1D$ [32,33] and $2P$ [32–36] multiplets are about 3.8 GeV and 4.0 GeV respectively, which are consistent with potential model estimates and with the experimental 1^3D_1 state $\psi(3770)$. Within the $1D$ multiplet there is some evidence from lattice gauge theory (LGT) that the 3^{--} state lies above the 2^{--} and 2^{-+} states [33]. Lattice gauge theory predictions for these higher excitations are clearly very important, and hopefully results with much smaller errors will become available in future. Studies of the mass differences of states within each multiplet would be especially interesting, and may be less sensitive to the large overall mass scale uncertainty.

III. RADIATIVE TRANSITIONS

Radiative transitions can provide sensitive tests of the spectroscopic assignments (angular quantum numbers) of heavy-quark mesons. As an example, radiative transitions have been proposed [37–39] as a means of determining the quantum numbers of the recently discovered $D_{sJ}^*(2317)^+$

TABLE II. Radiative transitions in scenario 1: Predictions for the $E1$ transitions $1D \rightarrow 1P$, $2P \rightarrow 2S$, $2P \rightarrow 1S$ and $2P \rightarrow 1D$, assuming in all cases that the initial $c\bar{c}$ state has a mass of 3872 MeV. The matrix elements were obtained using the wave functions of the Godfrey-Isgur model, Ref. [23]. Unless otherwise stated, the widths are given in keV and the final $c\bar{c}$ masses are PDG values [44].

Initial state $X(3872)$	Final state	M_f (MeV)	ω (MeV)	$\langle f r i\rangle$ (GeV^{-1})	C_{fi}	Width (keV)
1^3D_3	$\chi_{c2}(1^3P_2)\gamma$	3556.2	303	2.762	2/5	367
1^3D_2	$\chi_{c2}(1^3P_2)\gamma$	3556.2	303	2.769	1/10	92
	$\chi_{c1}(1^3P_1)\gamma$	3510.5	345	2.588	3/10	356
1^3D_1	$\chi_{c2}(1^3P_2)\gamma$	3556.2	303	2.769	1/90	10.2
	$\chi_{c1}(1^3P_1)\gamma$	3510.5	345	2.598	1/6	199
	$\chi_{c0}(1^3P_0)\gamma$	3415	430	2.390	2/9	437
1^1D_2	$h_c(1^1P_1)\gamma$	3517 ^a	339	2.627	2/5	464
2^3P_2	$\psi'(2^3S_1)\gamma$	3686	182	2.530	1/3	55.2
	$J/\psi(1^3S_1)\gamma$	3097	697	0.276	1/3	37.2 ^c
	$\psi''(1^3D_1)\gamma$	3770	101	-2.031	1/150	0.12
	$\psi_2(1^3D_2)\gamma$	3838 ^a	34	-2.208	1/10	0.08
	$\psi_3(1^3D_3)\gamma$	3849	23	-2.375	14/25	0.16
2^3P_1	$\psi'(2^3S_1)\gamma$	3686	182	2.723	1/3	63.9
	$J/\psi(1^3S_1)\gamma$	3097	697	0.150	1/3	11.0 ^c
	$\psi''(1^3D_1)\gamma$	3770	101	-2.244	1/6	3.7
	$\psi_2(1^3D_2)\gamma$	3838 ^a	34	-2.413	1/2	0.49
2^3P_0	$\psi'(2^3S_1)\gamma$	3686	182	2.899	1/3	72.4
	$J/\psi(1^3S_1)\gamma$	3097	697	-0.002	1/3	1.5 eV ^c
	$\psi''(1^3D_1)\gamma$	3770	101	-2.457	2/3	17.8
2^1P_1	$\eta_{c2}(1^1D_2)\gamma$	3837 ^a	35	-2.395	2/3	0.7
	$\eta'_c(2^1S_0)\gamma$	3638 ^b	227	2.303	1/3	89
	$\eta_c(1^1S_0)\gamma$	2980	789	0.304	1/3	65.4

^aMass predicted by the Godfrey-Isgur model, Ref. [23]. The masses given in Ref. [23] were rounded to 10 MeV; here we quote them to 1 MeV.

^bCurrent world average, from Ref. [45].

^cThese transitions are rather sensitive to the details of the model due to the node in the $2P$ wave functions, which lead to approximate cancellation in the $2P$ - $1S$ overlap integrals.

and $D_{sJ}(2457)^+$ [3–5]. In this section we calculate the $E1$ radiative widths that follow from various $c\bar{c}X(3872)$ assignments.

The partial width for an $E1$ radiative transition between $c\bar{c}$ states in the nonrelativistic quark model is given by

$$\Gamma(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} e_c^2 \alpha \omega^3 C_{fi} \delta_{SS'} |\langle n'^{2S'+1}L'_{J'} | r | n^{2S+1}L_J \rangle|^2 \quad (5)$$

(see for example Ref. [40]), where $e_c=2/3$ is the c -quark charge in units of $|e|$, α is the fine-structure constant, ω is the photon's energy, and the angular matrix element C_{fi} is given by

$$C_{fi} = \max(L, L') (2J'+1) \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2. \quad (6)$$

For convenience the coefficients $\{C_{fi}\}$ are listed in Tables II and III. The matrix elements $\langle n'^{2S'+1}L'_{J'} | r | n^{2S+1}L_J \rangle$ are given in Tables II and III, and were evaluated using the wave functions of Ref. [23]. Relativistic corrections are implicitly included in these $E1$ transitions through Siegert's theorem [41–43], by including spin-dependent interactions in the Hamiltonian used to calculate the meson masses and wave functions.

We give two sets of predictions for these radiative widths. In the first set (Table II) we assume in all cases that the initial meson has the mass of the $X(3872)$. While we appreciate that in some cases this is clearly an unlikely assignment, such as the 1^3D_1 [normally identified with the $\psi(3770)$], we wish to consider decays of all conceivable $X(3872)$

TABLE III. Radiative transitions in scenario 2: As in scenario 1, except that unknown masses are taken from the Godfrey-Isgur model.

Initial state	Final state	M_f (MeV)	ω (MeV)	$\langle f r i\rangle$ (GeV ⁻¹)	C_{fi}	Width (keV)
1 $^3D_3(3849)$	$\chi_{c2}(1\ ^3P_2)\gamma$	3556.2 ^a	282	2.762	2/5	295
1 $^3D_2(3838)$	$\chi_{c2}(1\ ^3P_2)\gamma$	3556.2 ^a	271	2.769	1/10	66
	$\chi_{c1}(1\ ^3P_1)\gamma$	3510.5 ^a	314	2.588	3/10	268
1 $^3D_1(3770)$ ^a	$\chi_{c2}(1\ ^3P_2)\gamma$	3556.2 ^a	208	2.769	1/90	3.3
	$\chi_{c1}(1\ ^3P_1)\gamma$	3510.5 ^a	251	2.598	1/6	77
	$\chi_{c0}(1\ ^3P_0)\gamma$	3415 ^a	338	2.390	2/9	213
1 $^1D_2(3837)$	$h_c(1\ ^1P_1)\gamma$	3517	307	2.627	2/5	344
2 $^3P_2(3979)$	$\psi'(2\ ^3S_1)\gamma$	3686 ^a	282	2.530	1/3	207
	$J/\psi(1\ ^3S_1)\gamma$	3097 ^a	784	0.276	1/3	53 ^c
	$\psi_3(1\ ^3D_3)\gamma$	3849	128	-2.375	14/25	29
	$\psi_2(1\ ^3D_2)\gamma$	3838	139	-2.208	1/10	5.6
	$\psi''(1\ ^3D_1)\gamma$	3770 ^a	204	-2.031	1/150	1.0
2 $^3P_1(3953)$	$\psi'(2\ ^3S_1)\gamma$	3686 ^a	258	2.723	1/3	184
	$J/\psi(1\ ^3S_1)\gamma$	3097 ^a	763	0.150	1/3	14.4 ^c
	$\psi_2(1\ ^3D_2)\gamma$	3838	113	-2.413	1/2	18.3
	$\psi''(1\ ^3D_1)\gamma$	3770 ^a	179	-2.244	1/6	20.7
2 $^3P_0(3916)$	$\psi'(2\ ^3S_1)\gamma$	3686 ^a	223	2.899	1/3	135
	$J/\psi(1\ ^3S_1)\gamma$	3097 ^a	733	-0.002	1/3	1.6 eV ^c
	$\psi''(1\ ^3D_1)\gamma$	3770 ^a	143	-2.457	2/3	51.2
2 $^1P_1(3956)$	$\eta_{c2}(1\ ^1D_2)\gamma$	3837	117	-2.395	2/3	26.6
	$\eta'_c(2\ ^1S_0)\gamma$	3638 ^b	305	2.303	1/3	217
	$\eta_c(1\ ^1S_0)\gamma$	2980 ^a	856	0.304	1/3	83

^aExperimental PDG mass [44].

^bCurrent world average, from Ref. [45].

^cThese transitions are rather sensitive to the details of the model due to the node in the $2P$ wave functions, which lead to approximate cancellation in the $2P$ - $1S$ overlap integrals.

$c\bar{c}$ assignments systematically, and will demonstrate that only a few possibilities are consistent with the existing $X(3872)$ data.

In the second set of radiative width predictions (Table III) we assume the $c\bar{c}$ masses predicted by the Godfrey-Isgur model [23] where no obvious experimental candidate exists. This should generally give more reliable predictions for the radiative widths of as yet unidentified $c\bar{c}$ states, and will hopefully provide useful guidance for experimental searches for these states.

IV. STRONG DECAYS

Strong decays provide crucial tests of the nature of the $X(3872)$, through the total width and relative branching fractions. We consider three types of strong decay:

- (1) Zweig-allowed open-charm decays, $(c\bar{c}) \rightarrow (c\bar{q}) + (q\bar{c})$ ($q = u, d, s$),
- (2) $c\bar{c}$ annihilation, $c\bar{c} \rightarrow gg, ggg, q\bar{q}g, \dots$, and

- (3) closed-flavor hadronic transitions, such as $(c\bar{c}) \rightarrow J/\psi\pi\pi, \eta_c\pi\pi, J/\psi\eta, \eta_c\eta, \dots$

We estimate the Zweig allowed decays using the 3P_0 decay model. The history of this model and related strong decay models has been reviewed recently by Barnes [46]; details of the approach may be found in the extensive literature (see for example Ackleh *et al.* [47] and Blundell and Godfrey [48]). The 3P_0 strong decay amplitudes are given by a dimensionless pair production amplitude γ times a convolution integral of the three meson wave functions. Based on our experience with light meson decays we set $\gamma = 0.4$. We assume simple harmonic oscillator (SHO) wave functions for the three mesons, with a universal Gaussian width parameter of $\beta = 0.5$ GeV; this is a rough average of β values that give maximum overlap with nonrelativistic Coulomb plus linear wave functions as well as Godfrey-Isgur wave functions. We also generalized the 3P_0 decay overlap integrals of Ackleh *et al.* [47] to accommodate different quark and antiquark masses in the final mesons. The single new parameter required here is the heavy-light quark mass ratio $r = m_c/m_q$,

TABLE IV. Partial widths and branching fractions (BF) for strong and electromagnetic transitions in scenario 1: We assume in all cases that the initial $c\bar{c}$ state has a mass of 3872 MeV. Details of the calculations are given in the text.

Initial state	Final state	Width (MeV)	BF (%)
1^3D_3	DD	4.04	84.2
	ggg	0.18	3.8
	$J/\psi\pi\pi$	0.21 ± 0.11	4.4
	$\chi_{c2}(1^3P_2)\gamma$	0.37	7.7
	Total	4.80	100
1^3D_2	ggg	0.08	10.8
	$J/\psi\pi\pi$	0.21 ± 0.11	28.4
	$\chi_{c2}(1^3P_2)\gamma$	0.09	12.2
	$\chi_{c1}(1^3P_1)\gamma$	0.36	48.6
	Total	0.74	100
1^3D_1	DD	184	98.9
	ggg	1.15	0.6
	$J/\psi\pi\pi$	0.21 ± 0.11	0.1
	$\chi_{c1}(1^3P_1)\gamma$	0.20	0.1
	$\chi_{c0}(1^3P_0)\gamma$	0.44	0.2
Total	186	100	
1^1D_2	gg	0.19	22.1
	$\eta_c\pi\pi$	0.21 ± 0.11	24.4
	$h_c(1^1P_1)\gamma$	0.46	53.5
	Total	0.86	100
2^3P_2	DD	21.1	82.4
	gg	4.4	17.2
	$\psi'(2^3S_1)\gamma$	0.06	0.2
	$J/\psi(1^3S_1)\gamma$	0.04	0.2
	Total	25.6	100
2^3P_1	$q\bar{q}g$	1.65	95.9
	$\psi'(2^3S_1)\gamma$	0.06	3.5
	$J/\psi(1^3S_1)\gamma$	0.01	0.6
	Total	1.72	100
2^3P_0	DD	13.7 (see text)	24.6
	gg	42.	75.3
	$\psi'(2^3S_1)\gamma$	0.07	0.1
	$\psi''(1^3D_1)\gamma$	0.02	4×10^{-2}
	Total	55.8	100
2^1P_1	ggg	1.29	81.6
	$gg\gamma$	0.13	8.2
	$\eta'_c(2^1S_0)\gamma$	0.09	5.7
	$\eta_c(1^1S_0)\gamma$	0.07	4.4
	Total	1.58	100

which we take to be 1.5/ 0.33 for u, d and 1.5/0.55 for s . Our results for the partial widths of these open-flavor modes are given in Table IV (with all initial masses set to 3872 MeV) and Table V (with all unknown masses set to the Godfrey-Isgur values).

TABLE V. As in Table IV, except that unknown masses are taken from the Godfrey-Isgur model.

Initial state	Final state	Width (MeV)	BF (%)
$1^3D_3(3849)$	DD	2.27	76.7
	ggg	0.18	6.1
	$J/\psi\pi\pi$	0.21 ± 0.11	7.1
	$\chi_{c2}(1^3P_2)\gamma$	0.30	10.1
	Total	2.96	100
$1^3D_2(3838)$	ggg	0.08	12.7
	$J/\psi\pi\pi$	0.21 ± 0.11	33.3
	$\chi_{c2}(1^3P_2)\gamma$	0.07	11.1
	$\chi_{c1}(1^3P_1)\gamma$	0.27	42.9
Total	0.63	100	
$1^3D_1(3770)$	DD	42.8	96.4
	ggg	1.15	2.6
	$J/\psi\pi\pi$	0.21 ± 0.11	0.5
	$\chi_{c1}(1^3P_1)\gamma$	0.08	0.2
	$\chi_{c0}(1^3P_0)\gamma$	0.21	0.5
Total	44.4	100	
$1^1D_2(3837)$	gg	0.19	25.7
	$\eta_c\pi\pi$	0.21 ± 0.11	28.4
	$h_c(1^1P_1)\gamma$	0.34	45.9
	Total	0.74	100
$2^3P_2(3979)$	DD	42.4	46.8
	DD^*	42.5	46.9
	D_sD_s	1.03	1.1
	gg	4.4	4.9
	$\psi'(2^3S_1)\gamma$	0.21	0.2
	$J/\psi(1^3S_1)\gamma$	0.05	6×10^{-2}
	$\psi_3(1^3D_3)\gamma$	0.03	3×10^{-2}
Total	90.6	100	
$2^3P_1(3953)$	DD^*	118.	98.4
	$q\bar{q}g$	1.65	1.4
	$\psi'(2^3S_1)\gamma$	0.18	0.2
	$J/\psi(1^3S_1)\gamma$	0.01	8×10^{-3}
	$\psi_2(1^3D_2)\gamma$	0.02	2×10^{-2}
	$\psi_1(1^3D_1)\gamma$	0.02	2×10^{-2}
	Total	120	100
$2^3P_0(3916)$	DD	0.0 (see text)	0
	gg	42.	99.5
	$\psi'(2^3S_1)\gamma$	0.14	0.3
	$\psi_1(1^3D_1)\gamma$	0.05	0.1
	Total	42	100
$2^1P_1(3956)$	DD^*	78.4	97.9
	ggg	1.29	1.6
	$gg\gamma$	0.13	0.2
	$\eta'_c(2^1S_0)\gamma$	0.22	0.3
	$\eta_c(1^1S_0)\gamma$	0.08	0.1
	Total	80	100

Note that the $1^3D_2 \psi_2$ and the $1^1D_2 \eta_{2c}$ cannot decay to $D\bar{D}$ due to parity conservation, and since they are below the next open flavor threshold (DD^*) they are expected to be narrow. The narrowness of the $1^3D_3 \psi_3$ in contrast is due to suppression by the $D\bar{D}$ F -wave angular momentum barrier.

Experience with light and strange meson strong decays suggests that these partial widths should be accurate to perhaps a factor of two (given the correct masses); the predicted width of the $\psi(3770)$ (in Table V), for example, is $\Gamma(1^3D_1(3770) \rightarrow D\bar{D}) = 42.8$ MeV, whereas the PDG experimental average is $\Gamma_{\psi(3770)}^{\text{tot}} = 25.3 \pm 2.9$ MeV [44].

Annihilation decays into gluons and light quarks make significant contributions to the total widths of some $c\bar{c}$ resonances. These decay rates been studied extensively using perturbative QCD (PQCD) methods [49–59]; the relevant formulas are summarized in Ref. [56]. Expressions for decay widths relevant to the $1D$ and $2P$ $c\bar{c}$ states are

$$\Gamma(3D_J \rightarrow ggg) = \frac{10\alpha_s^3}{9\pi} C_J \frac{|R_D''(0)|^2}{m_Q^6} \ln(4m_Q \langle r \rangle), \quad (7)$$

$$\Gamma(1D_2 \rightarrow gg) = \frac{2\alpha_s^2}{3} \frac{|R_D''(0)|^2}{m_Q^6}, \quad (8)$$

$$\Gamma(3P_2 \rightarrow gg) = \frac{8\alpha_s^2}{5} \frac{|R_P'(0)|^2}{m_Q^4}, \quad (9)$$

$$\Gamma(3P_1 \rightarrow q\bar{q}g) = \frac{8n_f\alpha_s^3}{9\pi} \frac{|R_P'(0)|^2}{m_Q^4} \ln(m_Q \langle r \rangle), \quad (10)$$

$$\Gamma(1P_1 \rightarrow ggg) = \frac{20\alpha_s^3}{9\pi} \frac{|R_P'(0)|^2}{m_Q^4} \ln(m_Q \langle r \rangle), \quad (11)$$

$$\Gamma(1P_1 \rightarrow gg\gamma) = \frac{36}{5} e_q^2 \frac{\alpha}{\alpha_s} \Gamma(1P_1 \rightarrow ggg), \quad (12)$$

$$\Gamma(3P_0 \rightarrow gg) = 6\alpha_s^2 \frac{|R_P'(0)|^2}{m_Q^4}, \quad (13)$$

where $C_J = \frac{26}{9}$, 1, 4 for $J=1, 2, 3$, and the number of light quarks is taken to be $n_f=3$. To obtain our numerical results for these partial widths we assumed $m_c=1.628$ GeV, $\alpha_s \approx 0.23$ (with some weak mass dependence), and used the wave functions of Ref. [23].

Considerable uncertainties arise in these expressions from the model dependence of the wave functions and possible relativistic and QCD radiative corrections (see for example the discussion in Ref. [23]). As one example of a likely in-

accuracy, the contact approximation for $c\bar{c}(1^1D_2) \rightarrow gg$ given above has been checked numerically, and overestimates the rate found with a full c -quark propagator by about two orders of magnitude [58]. Other problems are that the logarithm evident in some of these formulas is evaluated at a rather arbitrarily chosen scale, and that the PQCD radiative corrections to these processes are often found to be large, but are prescription dependent and so are numerically unreliable. Thus, we regard these formulas as rough estimates of the partial widths for these annihilation processes rather than accurate predictions, and they certainly merit more theoretical effort in the future. The numerical partial widths we find for these annihilation processes are given in Tables IV and V.

The final strong decays we consider are closed-flavor hadronic transitions of the type $(c\bar{c}) \rightarrow (c\bar{c}) + \pi\pi(\eta)$. There have been many theoretical estimates of these and related transitions [60–72]. Here we employ the multipole expansion of color gauge fields developed by Yan and collaborators [60–63] together with the Wigner-Eckart theorem to estimate the $E1$ - $E1$ transition rates [60]; the relevant expressions are summarized by Eichten and Quigg [71]. The recent BES Collaboration measurement of the $B(\psi(3770) \rightarrow J/\psi\pi^+\pi^-) = (0.59 \pm 0.26 \pm 0.16)\%$ [73] is used as input for the $c\bar{c}$ transitions of the type $(c\bar{c})_D \rightarrow (c\bar{c})_S\pi\pi$. One should be cautious about this result and the predictions we derive from it as CLEO- c has presented the smaller preliminary limit of $B(\psi(3770) \rightarrow J/\psi\pi^+\pi^-) < 0.26\%$ at 90% C.L. [45]. Furthermore, rescaling the $A_2(2,0) b\bar{b}$ amplitude needed for the $D \rightarrow S$ transitions gives $\Gamma(\psi(3770) \rightarrow J/\psi\pi^+\pi^-) \approx 58$ keV, which is consistent with the CLEO- c result but is about a factor of 2 smaller than the BES measurement. The hadronic transition rates, based on the BES measurement, are summarized with the other strong decays in Tables IV and V. We do not include decays of the type $2^3P_J \rightarrow 1^3P_{J'}$, as they are expected to be small compared to the decays considered here. Similarly, transitions with η and π^0 in the final state are also possible but are expected to have much smaller partial widths than the decays that we have included.

V. DISCUSSION OF $X(3872)$ $c\bar{c}$ ASSIGNMENTS

A summary of the strong and electromagnetic partial widths predicted for each $1D$ and $2P$ $c\bar{c}$ assignment for the $X(3872)$ is given in Table IV. The initial mass in all cases is taken to be 3872 MeV.

One may immediately eliminate the 2^3P_2 , 2^3P_0 and the “straw dog” assignment $3D_1$, due to the large theoretical total widths. The total width of a $3D_1(3872)$ state is predicted to be about two orders of magnitude larger than the experimental limit of 2.3 MeV (95% C.L.) for the $X(3872)$, and 2^3P_2 and 2^3P_0 states at 3872 MeV would have strong widths an order of magnitude larger than the experimental limit. (We note that the process $2^3P_0 \rightarrow D\bar{D}$ is accidentally near a node in the decay amplitude, which gives a suppressed rate for this S -wave decay. This may be an artifact of the decay model. In any case annihilation decays should ensure that the $2^3P_0 c\bar{c}$ is not a narrow state.)

A priori the most plausible $c\bar{c}$ assignments for the $X(3872)$ are 1^3D_2 and 1^1D_2 . Since the mode $D\bar{D}$ is for-

bidden, these states have no allowed open-charm decay mode, and must decay instead through the weaker short-distance $c\bar{c}$ annihilation processes, radiative decays, and closed-flavor hadronic transitions. We find that these decays lead to theoretical total widths of about 1 MeV for both these states.

These 2^- states should both have quite large $E1$ radiative branching fractions, in total $\approx 50\%$, and the final states are very characteristic. The spin-triplet 1^3D_2 will decay into $\chi_{c2}\gamma$ and $\chi_{c1}\gamma$ with a relative branching fraction of about 1:4, whereas the spin-singlet 1^1D_2 will decay into $h_c\gamma$, where h_c is the as yet unidentified spin-singlet P -wave state. Confirmation of a 1^1D_2 $c\bar{c}X(3872)$ assignment may therefore require the identification of the problematic 1^1P_1 $c\bar{c}$ state.

We find that the current Belle limit on the radiative decay of the $X(3872)$,

$$\frac{B(X(3872) \rightarrow \chi_{c1}\gamma)}{B(X(3872) \rightarrow J/\psi\pi^+\pi^-)} < 0.89, \quad 90\% \text{ C.L.}, \quad (14)$$

is only marginally a problem for the 1^3D_2 $c\bar{c}$ assignment, due to our larger scale (relative to Ref. [11]) and significant uncertainty in the $J/\psi\pi\pi$ branching fraction (see Table IV). However, the recent CLEO- c result would pose a problem for the prediction and we eagerly await more precise data from these experiments. With somewhat better experimental statistics we anticipate that the $\chi_{c1}\gamma$ and $\chi_{c2}\gamma$ modes will both be evident, if the $X(3872)$ is indeed a 1^3D_2 $c\bar{c}$ state.

Although the 1^3D_3 $c\bar{c}$ state does have an open-charm decay mode ($D\bar{D}$), we find that the centrifugal barrier actually implies a small total width of only a few MeV; given the uncertainties in the 3P_0 decay model, this state should also be considered a viable $X(3872)$ candidate. The 1^3D_3 assignment can also be tested by studying radiative decays; this state is predicted to have an 8% branching fraction to $\chi_{c2}\gamma$, but $\chi_{c1}\gamma$ in contrast is $M2$, and will have a much smaller partial width. Thus the $\chi_{c1}\gamma$ and $\chi_{c2}\gamma$ decay modes can be used to distinguish between 1^3D_2 and 1^3D_3 .

The 2^3P_1 and 2^1P_1 states if at 3872 MeV would have total widths of about 1–2 MeV, also consistent with the $X(3872)$ experimental limit. These states are notable in that they should *not* be clearly evident in radiative transitions; $E1$ branching fractions of only a few percent are expected, and unlike the $E1$ decays of D -wave charmonia, these $2P$ states do not populate the modes $\chi_c\gamma$ or $h_c\gamma$; instead an initial 2^3P_1 or 2^1P_1 leads to $(J/\psi, \psi')\gamma$ or $(\eta_c, \eta'_c)\gamma$ respectively. Problems with these $2P$ assignments are that we do not expect the $J/\psi\pi\pi$ final state to be prominent, and that the predicted masses are roughly 100 MeV higher than the $X(3872)$.

The search for a $J/\psi\pi^0\pi^0$ mode is a very important experimental task. If the $X(3872)$ is indeed a $c\bar{c}$ state, the presence or absence of this mode will select C parity ($-$) or ($+$) respectively [74]. Decays to $J/\psi\pi^0\pi^0$ imply that the initial state has $C=(-)$, whereas if the decay proceeds

through an isospin-violating transition to $J/\psi\rho^0$ followed by $\rho^0 \rightarrow \pi^+\pi^-$, the initial state has $C=(+)$. In the former case the $J/\psi\pi^0\pi^0$ mode should have approximately 1/2 the branching fraction of $J/\psi\pi^+\pi^-$ (expected for $I=0$). In contrast, the ρ^0 decays only to charged pions. The observation of this state in a $J/\psi\pi\pi$ mode and past experience with dipion decays suggest $C=(-)$, but this should be checked through a search for $J/\psi\pi^0\pi^0$. If a $J/\psi\pi^0\pi^0$ decay mode is confirmed with this strength, we are then left with the $C=(-)$ $c\bar{c}$ candidates 1^3D_3 , 1^3D_2 and 2^1P_1 . Conversely, if there is no significant $J/\psi\pi^0\pi^0$ mode relative to $J/\psi\pi^+\pi^-$, the $X(3872)$ would presumably be $C=(+)$, with $c\bar{c}$ candidates 1^1D_2 and 2^3P_1 . Studies of radiative decays can be used to test the remaining $c\bar{c}$ possibilities once the C parity is established. We note in passing that the pion invariant mass distribution has also been advocated as a discriminator between these assignments [15,60].

If we use the mass predictions of the Godfrey-Isgur model [instead of the $X(3872)$ mass] to calculate the properties of $1D$ and $2P$ $c\bar{c}$ states (Table V), we find that all of the $2P$ states are rather broad, making them more difficult to observe in B decay. In contrast all the $1D$ states remain relatively narrow, since the predicted Godfrey-Isgur masses are below the $X(3872)$ mass. We therefore expect that all members of the $1D$ multiplet will be observable in B meson decays, independent of the nature of the $X(3872)$.

VI. NON- $c\bar{c}$ ASSIGNMENTS: DD^* MOLECULE

The fact that the reported $X(3872)$ mass and the D^0D^{*0} threshold are equal to within the current errors of about 1 MeV has led to speculations that this state might actually be a weakly bound DD^* molecule, perhaps dominantly D^0D^{*0} [12–18]. The possibility of charm meson molecules has been discussed in several earlier references, especially regarding the $\psi(4040)$ as a D^*D^* candidate [54,75,76].

DD^* molecule assignments can be distinguished from $c\bar{c}$ through quantum numbers and decay modes. Since a molecular state would most likely be an S wave, $J^P=1^+$ is expected. Either C parity is possible *a priori*, and attractive forces do arise in each C -parity channel, due to strong virtual decay couplings to the (theoretically) higher-mass $C=(+)$ 2^3P_1 and $C=(-)$ 2^1P_1 $c\bar{c}$ states.

Assuming binding from one pion exchange forces, Törnqvist [12] argues that $C=(+)$, and in addition to the S -wave $J^{PC}=1^{++}$ state the P -wave combination with $J^{PC}=0^{-+}$ should also be bound. Since a $C=(+)$ state cannot decay to $J/\psi(\pi\pi)_S$, Törnqvist suggests that the observed $J/\psi\pi^+\pi^-$ final state may be due to a $J/\psi\rho^0$ decay, allowed by isospin mixing in the initial state [77]. (A dominantly D^0D^{*0} molecule for example has $I=0$ and $I=1$ components with comparable weights [13].) Swanson [18] finds that attraction from pion exchange alone is not sufficient to form a DD^* molecular bound state, but that a $J^{PC}=1^{++}$ bound state does form when short-ranged quark-gluon forces are included as well.

In a hypothetical very weakly bound, dominantly D^0D^{*0}

molecule, one would expect the decays and partial widths to be essentially those of the D^{*0} . This implies dominant decay modes $D^0 D^0 \pi^0$ and $D^0 D^0 \gamma$, in an approximately 1.5:1 ratio, and a total width equal to that of the D^{*0} , which is theoretically ≈ 50 keV. Swanson [18] in contrast finds that internal rescatter is important in the $D^0 D^{*0}$ bound system, which leads instead to dominant $J/\psi \rho^0$ and $J/\psi \omega$ decay modes, giving a total width of ≈ 2 MeV. This is essentially equal to the current experimental limit. A search for the $J/\psi \omega$ mode would be an important test of this molecule decay model.

There appears to be general agreement that the radiative transitions of a weakly bound molecule to any $(c\bar{c})\gamma$ channel should be highly suppressed, so establishing limits on these radiative partial widths would also provide useful tests of DD^* molecule models.

One should note that mixing between the DD^* and $c\bar{c}$ basis states will certainly be present at some level, so even in a dominantly molecular DD^* state, suppressed transitions from the $c\bar{c}$ component of the $X(3872)$ to $(c\bar{c})\gamma$ will occur. The observed radiative partial widths relative to predictions for pure $c\bar{c}$ states can be used to quantify the $(c\bar{c}) \leftrightarrow DD^*$ mixing.

VII. NON- $c\bar{c}$ ASSIGNMENTS: CHARMONIUM HYBRID

Charmonium hybrid states have been predicted to have masses in the range of 4.0 to 4.4 GeV, with the higher value preferred by recent LGT studies. The flux-tube decay model argues that these states will be narrow if they lie below the $S+P$ open-charm threshold DD_j^* , and hence will have a relatively large branching fraction to $J/\psi \pi \pi$. [Of course the large branching fraction reported for $\pi_1(1600) \rightarrow \rho \pi$ argues against dominance by high-mass $S+P$ decay modes.] Charmonium hybrids are also expected to have relatively small radiative widths. Although the reported properties of the $X(3872)$ are consistent with these expectations for 2^{+-} and 0^{+-} hybrids, the large discrepancy with the predicted LGT mass of 4.4 GeV makes this assignment appear unlikely. In addition, a recent lattice study finds that some hybrid closed-flavor decays have surprisingly large partial widths [78], which may also argue against a hybrid assignment for the $X(3872)$.

VIII. CONCLUSIONS

In this paper we have considered all possible $1D$ and $2P$ $c\bar{c}$ assignments for the recently discovered $X(3872)$, since these are the only $c\bar{c}$ states expected near the mass of the $X(3872)$. In particular we evaluated the strong and electromagnetic partial widths of all states in these multiplets, and compared the results to our current knowledge of the $X(3872)$.

Assuming a mass of 3872 MeV, the large predicted total widths eliminate the 1^3D_1 , 2^3P_2 and 2^3P_0 as candidates, leaving the 1^3D_3 , 1^3D_2 , 1^1D_2 , 2^3P_1 and 2^1P_1 . A search for the mode $J/\psi \pi^0 \pi^0$ will be important to discrimi-

nate between these remaining possibilities. The observation of a $J/\psi \pi^0 \pi^0$ mode with a relative $J/\psi \pi^+ \pi^-$ branching fraction of approximately 1:2 indicates a $C=(-)$ state, and would restrict the plausible $c\bar{c}X(3872)$ assignments to 1^3D_3 , 1^3D_2 , and 2^1P_1 . A limit on $J/\psi \pi^0 \pi^0$ well below this 1:2 ratio would imply $C=(+)$, leaving 1^1D_2 and 2^3P_1 as possible assignments. A unique assignment can then be established through studies of the final states populated in $X(3872)$ radiative transitions. The observation of a $J/\psi \pi^0 \pi^0$ signal with a strength comparable to $J/\psi \pi^+ \pi^-$ but significantly different from the 1:2 ratio would indicate that the initial state is not an I -spin eigenstate; depending on the value of this ratio, this might support a mixed-isospin DD^* molecule interpretation.

Radiative transitions have previously been advocated as important tests of the nature of the $X(3872)$ because the estimated rates vary widely for different types of initial states, and the radiative partial widths between pure $c\bar{c}$ basis states can be calculated with reasonable accuracy (of perhaps 30%). For pure $1D$ $c\bar{c}$ assignments for the $X(3872)$, we find that the relative branching fractions to the modes $\chi_{c1}\gamma$, $\chi_{c2}\gamma$ and $h_c\gamma$ depend strongly on the initial state, and can be used to distinguish between 1^3D_3 , 1^3D_2 and 1^1D_2 . We noted however that as the $X(3872)$ is essentially degenerate with the DD^* threshold, we expect a significant DD^* component in $X(3872)$, even if it is dominantly a $c\bar{c}$ state. Thus if $c\bar{c} \leftrightarrow DD^*$ mixing is significant, we would expect radiative transitions to $(c\bar{c})\gamma$ to be observed, but with partial widths that are suppressed relative to the predictions of the $c\bar{c}$ quark model. Similarly DD^* -molecule decay modes such as $D^0 D^0 \gamma$, $D^0 D^0 \pi^0$, $J/\psi \rho^0$ and $J/\psi \omega$ should also be present in a mixed $c\bar{c}$ - DD^* state, but at a suppressed rate relative to the partial width expected from a dominantly DD^* molecular bound state.

As an interesting final observation, we expect the 1^3D_3 ψ_3 to be rather narrow, and to have significant branching fractions to $J/\psi \pi \pi$ and $\chi_{c2}\gamma$. This suggests that the ψ_3 should be observable in B decay. The observation of all the members of the $1D$ $c\bar{c}$ multiplet would contribute very useful information to the study of spin-dependent forces in heavy quarkonia.

ACKNOWLEDGMENTS

We would like to acknowledge useful discussions with F. E. Close and E. S. Swanson, and communications from S. Conetti, G. Bali, T. Manke, C. Michael, A. Petrov, J. Rosner, and T. Skwarnicki. S.G. would also like to thank the organizers of the QWG2 workshop for providing a stimulating environment for discussions of the $X(3872)$. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada, the U.S. National Science Foundation through grant NSF-PHY-0244786 at the University of Tennessee, and the U.S. Department of Energy under contract DE-AC05-00OR22725 at Oak Ridge National Laboratory (ORNL).

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