High scale mixing unification and large neutrino mixing angles

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Starting with the hypothesis that quark and lepton mixings are identical at or near the grand unified theory scale, we show that the large solar and atmospheric neutrino mixing angles together with the small reactor angle U_{e3} can be understood purely as a result of renormalization group evolution provided the three neutrinos are quasidegenerate and have the same *CP* parity. The mechanism is found to work if the common Majorana mass for the neutrinos is larger than 0.1 eV, which falls right in the range reported recently and also in the range which will be probed in planned experiments.

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I. INTRODUCTION

The idea that disparate physical parameters describing forces and matter at low energies may unify at very short distances (or high mass scales) has been a very helpful tool in seeking a unified understanding of apparently unrelated phenomena [1]. In the context of supersymmetric grand unified theories, such an approach explains the weak mixing angle $\sin^2\theta_W$ and thereby the different strengths of the weak, electromagnetic, and strong forces. One of the key ingredients of the grand unified theories is the unification between quarks and leptons. One may therefore hope that, in a quark-lepton unified theory, the weak interaction properties of quarks and leptons parametrized by means of the flavor mixing matrices will become identical at high energies.

On the experimental side, recent measurements on atmospheric and solar neutrino fluxes and those at K2K and KamLAND, which are a manifestation of the phenomena of neutrino oscillations, suggest that two of the neutrino mixings, i.e., the mixings between ν_e and ν_μ and between ν_μ and ν_τ (to be denoted by θ_{12} and θ_{23}, respectively), are large [2–6] while the third mixing between the ν_e and ν_τ is bounded to be very small by the CHOOZ and Palo Verde reactor experiments, i.e., $\sin^2 2\theta_{13} < 0.15$ [7]. On the other hand, it is now quite well established that all observed quark mixing angles are very small. One may therefore ask whether there is any trace of quark-lepton unification in the mixing angles as we move to higher scales.

The first question in this connection is whether high scales have anything to do with neutrino masses or is it purely a weak scale phenomenon? One of the simplest ways to understand small neutrino masses is via the seesaw mechanism [8], according to which the neutrino mixing is indeed a high scale phenomenon, the new high scale being that of the right handed neutrino masses (M_R) in an appropriate extension of the standard model. Present data put the seesaw scale M_R very close to the conventional grand unified theory (GUT) scales. It is therefore tempting to speculate whether quark and lepton mixing angles are indeed unified at the GUT-seesaw scale. This would of course imply that all neutrino mixing angles at the high scale M_R are very small, whereas at the weak scale two of them are known to be large. In this paper we show that simple radiative correction effects embodied in the renormalization group evolution of parameters from the seesaw scale to the weak scale can indeed provide a complete understanding of all neutrino mixings at the weak scale, starting with very small mixings at the GUT-seesaw scale.

The fact that renormalization group evolution from the seesaw scale to the weak scale [9,10] can lead to drastic changes in the magnitudes of the mixing angles was pointed out in several papers [9,11–16], while enhancement of the two-neutrino mixing angle was also observed in [10]. In particular, it was shown in [11] that this dependence on renormalization group evolution can be exploited in simple seesaw extensions of the minimal supersymmetric standard model (MSSM) to explain the large value of the atmospheric mixing angle starting with a small mixing at the seesaw scale, provided two conditions are satisfied: (i) the two neutrino mass eigenstates have the same *CP* parity and (ii) they are very nearly degenerate in mass. In general, in gauge models that attempt to explain the large neutrino mixings [16], one needs to make many assumptions to constrain the parameters. In contrast, this class of "radiative magnification" models [11,12,14] provide an alternative approach which is relevant to understanding large neutrino mixings for the case of a quasidegenerate neutrino mass spectrum. In fact the main content of radiative magnification models is the quasidegeneracy assumption, and, since the value of the common Majorana mass m_0 for all neutrinos is required to be in the sub-eV range ($\geqslant 0.1$ eV), this assumption is experi-

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mentally testable in the ongoing and planned neutrinoless double beta decay searches [17,18].

It is well known that the radiative magnification technique requires fine-tuning of initial neutrino mass eigenvalues at the seesaw scale [9,11–14]. The degree of fine-tuning needed has been discussed at length by Casas, Espinosa, Ibarra, and Navarro (CEIN) [14], who also discussed the relevant magnification criteria in the two-flavor case. But in the three-flavor case it has been shown that the existence of infrared stable quasifixed points in the relevant renormalization group equations (RGEs) leads to vanishing mixing matrix elements at low energies [13,14]. Thus, magnification for mixing angles might be expected to occur in the three-flavor case [19] only if RG evolution is stopped before reaching the quasifixed point regime.

In this paper, we show that under the same conditions for radiative magnification as just outlined, if we start with the hypothesis that at the seesaw scale the quark and neutrino mixings are unified to a common set of values, i.e., the known extrapolated values of the well known Cabibbo-Kobayashi-Maskawa (CKM) angles, after renormalization group evolution to the weak scale, we can obtain solar and atmospheric mixing angles that are in agreement with observations without contradicting the CHOOZ–Palo Verde bound on θ_{13}.

This result has two important implications: (i) it would provide a very simple and testable way to understand the observed large neutrino mixings and (ii) if confirmed by the neutrinoless double beta decay experiments, it would provide a strong hint of quark-lepton unification at high scales.

One may wonder why we are addressing the question of unification of the mixing angles for neutrinos with those of quarks and not the unification of neutrino masses with quark masses. The answer is of course the well-known one, namely, neutrino masses have an origin (seesaw mechanism) that distinguishes them from the quark masses, which arise from electroweak symmetry breaking. Furthermore, within the seesaw mechanism neutrinos are Majorana fermions, whereas quarks are Dirac fermions. Thus, as far as the masses go, we have no reason to expect unification with quarks. In Sec. V, we take up the question of specific models where our scenario for neutrino masses and mixings is realized.

This paper is organized as follows. In Sec. II, we discuss the RGEs for the neutrinos in the mass basis. In Sec. III, we present the main result of our paper, i.e., the magnification of mixing angles at the weak scale. In Sec. IV, we discuss predictions of our approach for neutrinoless double beta decay and other processes. In Sec. V, we present a gauge model where approximate mixing unification hypothesis is realized and in Sec. VI we present our conclusions.

II. RENORMALIZATION GROUP EQUATIONS FOR MASSES AND MIXINGS

Our basic assumption is a seesaw type model which will lead to equal quark and lepton mixing angles at the seesaw scale as well as to a quasidegenerate neutrino spectrum. In Sec. V, we present a model where at the seesaw scale the neutrinos have this property. We then follow the "diagonalize and run" procedure for the neutrino parameters and use the RGEs directly for the physical observables, namely, the mass eigenvalues m_i and the mixing angles θ_{ij} $(i,j=1,2,3)$. We also assume the neutrino mass eigenstates to possess the same *CP* and ignore *CP* violating phases in the mixing matrix. Also, for simplicity, we adopt the mass ordering among the quasidegenerate eigenstates to be of type $m_3 \gtrsim m_2 \gtrsim m_1$. The real 3×3 mixing matrix is parametrized as

$$U=\begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}-c_{12}s_{13}s_{23} & c_{12}c_{23}-s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}s_{13}c_{23} & -c_{12}s_{23}-c_{23}s_{13}s_{12} & c_{13}c_{23} \end{bmatrix}, \tag{1}$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$ $(i,j=1,2,3)$. U diagonalizes the mass matrix M in the flavor basis with $U^TMU=\mathrm{diag}(m_1,m_2,m_3)$. The RGEs for the mass eigenvalues can be written as [13,14]

$$\frac{dm_i}{dt}=-2F_\tau m_i U_{\tau i}^2-m_iF_u \qquad (i=1,2,3). \tag{2}$$

For every $\sin\theta_{ij}=s_{ij}$, the corresponding RGEs are

$$\frac{ds_{23}}{dt}=-F_\tau c_{23}{}^2(-s_{12}U_{\tau1}D_{31}+c_{12}U_{\tau2}D_{32}), \tag{3}$$

$$\frac{ds_{13}}{dt}=-F_\tau c_{23}c_{13}{}^2(c_{12}U_{\tau1}D_{31}+s_{12}U_{\tau2}D_{32}), \tag{4}$$

$$\frac{ds_{12}}{dt}=-F_\tau c_{12}(c_{23}s_{13}s_{12}U_{\tau1}D_{31}-c_{23}s_{13}c_{12}U_{\tau2}D_{32} + U_{\tau1}U_{\tau2}D_{21}), \tag{5}$$

where $D_{ij}=(m_i+m_j)/(m_i-m_j)$ and, for the MSSM,

$$F_\tau=-h_\tau^2/(16\pi^2\cos^2\beta),$$

$$F_u=\left(\frac{1}{16\pi^2}\right)\left(\frac{6}{5}g_1^2+6g_2^2-6\frac{h_t^2}{\sin^2\beta}\right), \tag{6}$$

but, for the SM,

$$F_\tau=3h_\tau^2/(32\pi^2),$$

$$F_u=(3g_2^2-2\lambda-6h_t^2-6h_b^2-2h_\tau^2)/(16\pi^2). \tag{7}$$

RGEs of mixing angles in the three-flavor case have been shown to possess infrared stable quasifixed points leading to vanishing values of the mixing matrix elements [13,14]. Thus, as in the two-flavor case [11,12,14], the radiative magnification of two mixing angles, if at all feasible, could be realizable only if RG evolution is stopped before reaching the quasifixed point regime.

Since m_i and m_j are scale dependent, the initially chosen mass difference existing between them at $\mu=M_R$ is narrowed down during the course of RG evolution as we approach $\mu=M_{\rm SUSY}$. When the initially chosen fine-tuned mass difference between m_i and m_j tends to vanish, $D_{ij}\to\infty$, and the corresponding term in the right hand side (RHS) of Eqs. (3)–(5) predominantly drives the RG evolution for $\sin\theta_{ij}$, which might become large or even approach its maximal value anywhere between $\mu=M_{\rm SUSY}$ and M_R. This causes large magnification to the mixing angle due to radiative effects. Also F_τ is enhanced by a factor $\sim 10^3$ in the large $\tan\beta$ region in the case of the MSSM as compared to the SM, where such effects do not exist. Then the standard model evolution below $M_{\rm SUSY}$ causes negligible contribution to the magnified mixings for two reasons: (i) absence of $\tan^2\beta$ effects, and (ii) small range of RG evolution from $M_{\rm SUSY}$ to M_Z. We choose $M_{\rm SUSY}=1$ TeV and tune the input neutrino mass eigenvalues at M_R in such a way that bilarge neutrino mixings by radiative magnification are obtained at $M_{\rm SUSY}=1$ TeV. It is to be noted that for a given value of the seesaw scale, a different choice of $M_{\rm SUSY}$ would require slightly different values of initially chosen mass eigenvalues for the same set of mixing angle solutions. While mixing angles almost remain constant below $M_{\rm SUSY}$, the mass eigenvalues continue to decrease down to M_Z in this method.

The mixing unification hypothesis implies that we set all neutrino mixings at the seesaw scale equal to the corresponding quark mixings, which in the Wolfenstein parametrization are dictated by the parameter $\lambda_0=0.2$. We then have, at the seesaw scale, $s_{12}\simeq\lambda_0$, $s_{23}\simeq O(\lambda_0^2)$, and $s_{13}\simeq O(\lambda_0^3)$. These values get substantially magnified in the region around $M_{\rm SUSY}$. Using $|D_{31}|\simeq|D_{32}|\ll|D_{21}|$, we see from Eqs. (3)–(5) that the dominant contribution to RG evolution of $s_{23}(\mu)$ is due to the term $\sim\lambda_0^2F_\tau D_{32}$. Similarly, the terms contributing to the evolution of $s_{13}(\mu)$ are $\sim\lambda_0^3F_\tau D_{32}$ or $\sim\lambda_0^3F_\tau D_{31}$. On the other hand, the evolution of s_{12} is dominated by the term $\sim\lambda_0^5F_\tau D_{21}$ where the large enhancement likely to be caused by the largeness in $|D_{21}|$ is damped out due to the higher power of λ_0^5. Since the mixing angles change substantially only around $M_{\rm SUSY}$, such dominance in RG evolution holds approximately at all other lower scales below M_R.

If the neutrino mixing angles are to be compatible with experimental observations at low energies, we need at most the magnification factors $(\sin\theta_{23}/\sin\theta_{23}^0)\simeq 20$, $(\sin\theta_{13}/\sin\theta_{13}^0)\leqslant 60$, and $(\sin\theta_{12}/\sin\theta_{12}^0)\simeq 4$, where we have used the experimental neutrino mixings for θ_{ij} [2–7] and quark mixings for θ_{ij}^0 [20,21]. That the CHOOZ–Palo Verde bound can tolerate a magnification factor as large as 60 is crucial to achieve bilarge mixings by radiative magnification while keeping the magnified angle θ_{13} at low energies well below the upper bound. This is of course because of the smallness of λ_0^3, which is the starting value (order of magnitude) of the reactor angle. One can also observe that it is the smallness of the reactor angle that provides the "hidden" signal for the unification.

III. BILARGE NEUTRINO MIXINGS BY RG EVOLUTION

Starting from known values of gauge couplings, masses of quarks and charged leptons, and CKM mixings in the quark sector at low energies, first we use the bottom-up approach and all the relevant RGEs to obtain the corresponding quantities at higher scales with $M_R=10^{11}$ GeV–2×10^{18} GeV. Assuming the neutrino mixing at $\mu=M_R$ to be small and similar to quark mixing, we then expect the initial conditions at $\mu=M_R$ to be $\sin\theta_{23}^0\simeq 0.038$, $\sin\theta_{13}^0\simeq 0.0025$, and $\sin\theta_{12}^0\simeq 0.22$ [20,21]. Using these as input and the fine-tuned mass eigenvalues m_i^0 as unknown parameters at the high scale, we then follow the top-down approach though Eqs. (2)–(5) and other standard RGEs. The unknown parameters m_i^0 are tuned in such a way that the solutions obtained at low energies agree with mass squared differences and the mixing angles given by the experimental data within the 90% C.L. [2–7]:

$$\Delta m_{12}^2=(2\text{–}50)\times 10^{-5}\ \text{eV}^2,$$

$$\Delta m_{23}^2=(1.2\text{–}5)\times 10^{-3}\ \text{eV}^2,$$

$$\sin\theta_{23}=0.54\text{–}0.83,\quad \sin\theta_{12}=0.40\text{–}0.70,$$

$$\sin\theta_{13}\leqslant 0.16. \tag{8}$$

For numerical solutions we have chosen the same value of $M_{\rm SUSY}=1$ TeV in every case of M_R investigated here. Our model described in Sec. V is consistent with quasidegenerate mass eigenvalues over a wider range of the seesaw scale: $M_R=10^{11}$ GeV–10^{15} GeV. However, in view of the phenomenological importance of the results, we have explored the RG evolution to bilarge mixings including higher scales up to the reduced Planck scale (2×10^{18} GeV). In Table I we present fine-tuned input mass eigenvalues at the seesaw scale $M_R=10^{13}$ GeV and our solutions at M_Z in the large $\tan\beta$ ($=55$) region where we have chosen $M_{\rm SUSY}=1$ TeV.

The solutions clearly exhibit radiative magnification of both the mixing angles θ_{23} and θ_{12} for a wide range of input values of m_i^0. We find that, although enhancement due to RG evolution occurs in the ν_e-ν_τ sector also, $\sin\theta_{13}$ remains well within the CHOOZ–Palo Verde bound [7].

In Table II we present three sets of fine-tuned initial mass eigenvalues and our solutions for three different high scale values, $M_R=10^{11}$,10^{15}, and 2×10^{18} GeV. We find that, for the same value of $\tan\beta=55$ and $M_{\rm SUSY}=1$ TeV, the lowest possible mass eigenvalue at M_Z decreases slowly with increase of the seesaw scale. For example, the lowest mass eigenvalues at $\mu=M_Z$ are 0.27 eV, 0.22 eV, 0.209 eV, and 0.17 eV for $M_R=10^{11}$ GeV, 10^{13} GeV, 10^{15} GeV, and 2×10^{18} GeV, respectively.

A magnification formula has been derived by CEIN [14] for the product of the mixing matrix elements,

TABLE I. Radiative magnification to bilarge mixings at low energies for $\tan\beta=55$, $M_{\rm SUSY}=1$ TeV, and input values of $\sin\theta^0_{23}=0.038$, $\sin\theta^0_{13}=0.0025$, and $\sin\theta^0_{12}=0.22$ at the seesaw scale $M_R=10^{13}$ GeV.

m_1^0 (eV)	0.3682	0.5170	0.6168	0.7160	0.8160
m_2^0 (eV)	0.3700	0.5200	0.6200	0.7200	0.8200
m_3^0 (eV)	0.4210	0.5910	0.7050	0.8190	0.9330
m_1 (eV)	0.2201	0.3107	0.3719	0.4317	0.4920
m_2 (eV)	0.2223	0.3122	0.3723	0.4324	0.4926
m_3 (eV)	0.2244	0.3152	0.3759	0.4366	0.4973
Δm^2_{12} (eV^2)	1.2×10^{-4}	3.0×10^{-4}	3.5×10^{-4}	6.0×10^{-4}	5.9×10^{-4}
Δm^2_{23} (eV^2)	1.0×10^{-3}	1.8×10^{-3}	2.6×10^{-3}	3.6×10^{-3}	4.6×10^{-3}
$\sin\theta_{23}$	0.667	0.708	0.690	0.677	0.668
$\sin\theta_{13}$	0.090	0.104	0.097	0.096	0.090
$\sin\theta_{12}$	0.606	0.520	0.604	0.486	0.606

$$F_{\rm m}=\frac{U_{\tau i}U_{\tau j}(\mu)}{U_{\tau i}U_{\tau j}(M_R)} \simeq\left[1+\frac{h_\tau^2}{32\pi^2\cos^2\beta}D_{ij}(M_R)\ln\frac{M_R}{\mu}\right]^{-1}. \quad (9)$$

Using the values given in Tables I and II, we find that the magnification obtained from the formula matches reasonably well with our estimations for mixing between the second and third generations ($i,j=2,3$).

Our result on the approximate unification of quark and neutrino mixings at the high scale $M_R=10^{13}$ GeV is exhibited in Fig. 1, where we present the RG evolution of the sines of the three neutrino mixing angles starting from $M_R=10^{13}$ GeV down to M_Z for one set of input masses given in Table I: $m_1^0=0.2983$ eV, $m_2^0=0.2997$ eV, and $m_3^0=0.3383$ eV. The flatness of the curves below $M_{\rm SUSY}$ is due to the negligible renormalization effect from the SM, which evades the approach to the quasifixed points. The corresponding low energy solutions are $m_1=0.2201$ eV, $m_2=0.2223$ eV, $m_3=0.2244$ eV, $\Delta m^2_{12}=1.6\times10^{-4}$ eV^2, $\Delta m^2_{23}=1.0\times10^{-3}$ eV^2, $\sin\theta_{23}=0.667$, $\sin\theta_{13}=0.09$, and $\sin\theta_{12}=0.606$. The almost horizontal lines in the figure represent the sines of the CKM mixings, $\sin\theta^q_{ij}$ [20,21]. Unification of the neutrino mixings with the corresponding quark mixings is clearly demonstrated at the high scale.

TABLE II. Same as Table I but for higher and lower seesaw scales.

M_R(**GeV**)	10^{11}	10^{15}	2×10^{18}
m_1^0 (eV)	0.4083	0.3970	0.5150
m_2^0 (eV)	0.4100	0.400	0.5200
m_3^0 (eV)	0.4510	0.4730	0.668
m_1 (eV)	0.2723	0.2093	0.1714
m_2 (eV)	0.2726	0.2098	0.1718
m_3 (eV)	0.2745	0.2124	0.1750
Δm^2_{12} (eV^2)	1.6×10^{-4}	2.0×10^{-4}	1.36×10^{-4}
Δm^2_{23} (eV^2)	1.0×10^{-3}	1.1×10^{-3}	1.1×10^{-3}
$\sin\theta_{23}$	0.711	0.682	0.684
$\sin\theta_{13}$	0.103	0.098	0.094
$\sin\theta_{12}$	0.571	0.463	0.422

The evolution of mass eigenvalues corresponding to mixings given in Fig. 1 is shown in Fig. 2 for $M_R=10^{13}$ GeV. In contrast to sines of mixing angles that have negligible RG corrections below the SUSY scale, the mass eigenvalues are found to decrease until the lowest scale M_Z. The rate of decrease of the third eigenvalue is the highest, but the rates of decrease of the first and second eigenvalues are similar. The initially fine-tuned mass splittings chosen at the seesaw scale are narrowed down to match the experimental values at low energies due to cancellations caused by RG-generated splittings. At the same time, radiative magnification occurs to match the experimentally observed bilarge mixing and the CHOOZ–Palo Verde bound. When the seesaw scale is the reduced Planck scale, $M_R=2\times10^{18}$ GeV, with fine-tuned

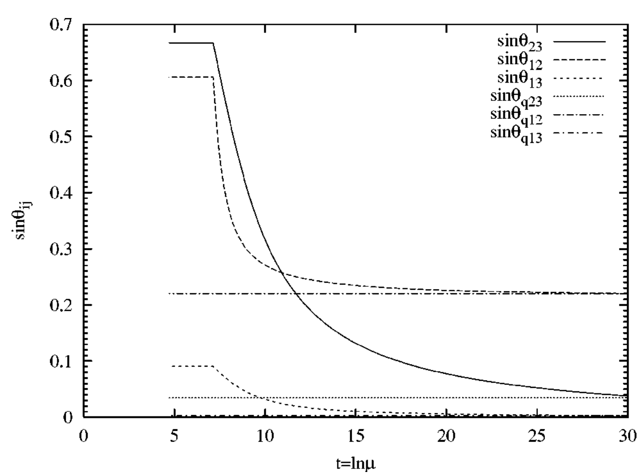

FIG. 1. Evolution of small quarklike mixings at the seesaw scale to bilarge neutrino mixings at low energies for the seesaw scale $M_R=10^{13}$ GeV with $\tan\beta=55$, $M_{\rm SUSY}=1$ TeV, and mass eigenvalues and mixing angles given in the first column of Table I. The solid, long-dashed, and short-dashed lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively, as defined in the text. The evolution of the sines of quark mixing angles, $\sin\theta_{qij}(i,j=1,2,3)$, is presented by the almost horizontal lines.

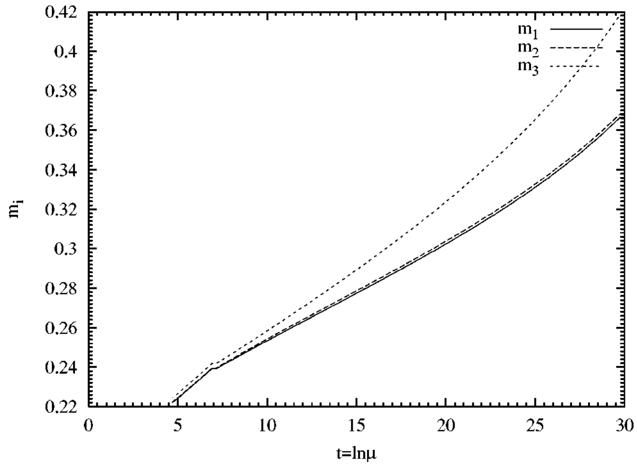

FIG. 2. Evolution of neutrino masses from the seesaw scale $M_R = 10^{13}$ GeV corresponding to $\tan\beta = 55$, $M_{\rm SUSY} = 1$ TeV, and initial values $m_1^0 = 0.3682$ eV, $m_2^0 = 0.37$ eV, and $m_3^0 = 0.4210$ eV, leading to the low scale eigenvalues $m_1 = 0.2201$ eV, $m_2^0 = 0.2223$ eV, and $m_3^0 = 0.2244$ eV at M_Z and bilarge mixings as shown in Fig. 1 and Table I.

input mass eigenvalues given in Table II, the lowest possible quasidegenerate mass eigenvalue at low energies is found to be 0.17 eV. For other values of M_R presented in Table II the evolutions are similar to Figs. 1 and 2. It is quite clear that radiative magnification to bilarge mixings is possible over a wide range of choices of M_R and correspondingly fine-tuned input mass eigenvalues at the seesaw scale.

We have noted that this radiative magnification mechanism of bilarge neutrino mixing leading to unification of quark mixings with the corresponding neutrino mixings also works when RG evolution of the Wolfenstein parameter A is taken into account [21]. As a result of such RG evolution, the extrapolated high scale values of the CKM matrix elements V_{ub}, V_{cb}, V_{td}, and V_{ts} [21] and the corresponding input values at the high scale are reduced by 6%–12% over the range of the seesaw scale investigated in this paper. Similarly, the mechanism also works with changes in the supersymmetry (SUSY) scale or few percent change in the input neutrino mass eigenvalues when more substantial threshold effects on the CKM elements observed in [22] are included. Such details, including threshold effects on CKM mixings and neutrino masses, will be reported elsewhere.

We have found that even if we choose $M_{\rm SUSY} = M_Z$ this mechanism of achieving bilarge neutrino mixing works for all values of M_R investigated here but with different sets of fine-tuned input mass eigenvalues. In this case the mixing angle solutions are energy scale dependent for all values of $\mu \geqslant M_Z$. However, the problem of fine-tuning in the input neutrino mass eigenvalues necessary at the seesaw scale to obtain the desirable RG solutions at low energies is not resolved by choosing different values of $M_{\rm SUSY}$. We have also noted that the radiative magnification mechanism leading to bilarge mixings works more easily if we start with $\sin\theta_{13}^0 = 0.0$, which could be relevant to certain neutrino mass textures. In this case the CHOOZ–Palo Verde bound is always protected.

It is worth reemphasizing that since we determine three input parameters (the three mass eigenvalues at high scale) to fit five experimentally known numbers as output parameters it is a overdetermined problem and there may be no solution. So there is a possibility of not being able to obtain correct mixing angles at the weak scale. But we have found that it is possible, thus showing that there is perhaps an element of truth in the unification hypothesis. It is also significant that the scale of 0.16–0.65 eV comes out as the range of allowed mass eigenvalues although such a scale was not put in at all *a priori*.

Finally, we change one of the input masses by 5% to see how much the final mixing angle predictions change. Out of three initial masses given in the second column in Table I when, for example, m_1^0 is decreased by 5% while keeping other input values unchanged, the predicted angle $\sin\theta_{23}$ remains unchanged, $\sin\theta_{13}$ increases to 0.14, but $\sin\theta_{12}$ decreases to 0.2 at the low scale M_Z. Similarly, although the prediction on Δm_{23}^2 does not change significantly, Δm_{12}^2 changes to 9×10^{-3} eV^2, upsetting the solar neutrino data at low energies. This gives an idea about the extent of fine-tuning needed to obtain the desired solutions.

IV. PREDICTIONS FOR BETA DECAY, DOUBLE BETA DECAY, U_{e3}, AND WMAP

Very recently the possibility of verifying our mixing unification hypothesis through lepton-flavor violating processes like $\mu\rightarrow e\gamma$ and $\tau\rightarrow\mu\gamma$ has been investigated [23]. We discuss here other possible experimental tests of the specific mechanism of radiative magnification.

Double beta and tritium beta decays. Our RG solutions are consistent with experimental data on Δm_{21}^2, Δm_{32}^2, and the mixing angles, if the input mass eigenvalues for $M_R = 10^{11}$–2×10^{18} GeV are in the range 0.35 eV–1.0 eV.

This corresponds to the low energy limits 0.16 eV $< m_i(M_Z) <$ 0.65 eV. Then, our choice of phases leads to the prediction

$$|\langle M_{ee}\rangle| = |\sigma_i m_i U_{ei}^2| = 0.16\ \text{eV–}0.65\ \text{eV}. \tag{10}$$

Recent searches for neutrinoless double beta decay have obtained the upper limit $|\langle M_{ee}\rangle| < (0.33$–$1.35)$ eV [17,18]. The range in Eq. (10) overlaps the one reported in [18] and the ones that will be covered in [24]. Thus a clear and testable prediction of the bilarge radiative magnification mechanism is that neutrinoless double beta decay should be observed in the next round of experiments.

Further, our low-energy limit on the quasidegenerate $m_i(M_Z)$ can be directly measured in tritium beta decay experiments. Although the present experimental bound on the mass is <2.2 eV, mass values as low as 0.35 eV can be reached by the KATRIN experiment [25].

Prediction for U_{e3}. Starting from the allowed range of high scale input values of the CKM mixing angle with $V_{ub} \simeq U_{e3}^0 \simeq 0.0025$–$0.004$, the RG evolutions predict enhancement of $\sin\theta_{13}$ at low energies,

$$U_{e3}=\sin\theta_{13}=0.08\text{–}0.10. \tag{11}$$

Although this prediction is well below the present experimental upper bound [7], it is accessible to several planned long-baseline neutrino experiments in the future, such as the NUMI-off-axis or JHF proposal.

Wilkinson Microwave Anisotropy Probe constraints on neutrino masses. Recently the Wilkinson Microwave Anisotropy Probe (WMAP) observations have provided very interesting constraints on the sum of neutrino masses [26,27]. The analysis depends on a number of cosmological parameters such as H_0, the bias parameter $b(k)$, and Ω_m from SN-Ia observations, etc. Depending on what values one chooses for the "priors," the constraint on the sum of all neutrino masses varies from 2.1 eV to 0.7 eV. Since we are proposing that the neutrino masses are degenerate, each individual mass will have an upper limit of 0.23 eV to 0.7 eV. Thus the radiative magnification hypothesis is consistent with WMAP observations [26] and also with the combined analysis of WMAP +2dFGRS data [27].

We have found that with $\tan\beta=55$ and due to RG effects alone the lowest allowed value of the neutrino mass eigenvalue at M_Z decreases slowly with increase in the seesaw scale. We obtain the lower bound to be 0.27 eV–0.16 eV for $M_R=10^{11}$–2×10^{18} GeV.

V. DEGENERATE NEUTRINOS FROM TYPE II SEESAW AND A MODEL FOR APPROXIMATE MIXING UNIFICATION

In this section, we address the question of how a quasidegenerate neutrino spectrum can arise within a gauge model that employs the seesaw mechanism for understanding neutrino masses [28].

To begin the discussion, let us present the different forms of the seesaw mechanism that provide a natural way to understand the small neutrino masses. Following the literature, we will call the two types of seesaw mechanism type I and type II. In the type I seesaw mechanism the neutrino mass matrix is given by the formula [8]

$$M_\nu=-M_D(fv_R)^{-1}M_D^T, \tag{12}$$

where f is the Majorana Yukawa coupling of the RH neutrinos, v_R is the $B-L$ symmetry breaking scale, and M_D is the Dirac neutrino mass matrix. In models where information about the $B-L$ symmetry is not given explicitly, fv_R is replaced by the mass matrix of the right handed neutrinos $M_R=fv_R$. Since one expects the pattern of M_D to be similar to the quark and lepton mass matrices, one expects the eigenvalues of M_R to be hierarchical and mixing angles to be small. Equation (12) then tells us that the neutrino masses are hierarchical. Clearly, in such models the radiative magnification of mixing angles does not occur via renormalization group evolution, as is clear from Eqs. (3)–(5) in the previous section.

The type I seesaw formula is generic to models which do not have any connection between the left and right handed fermions, such as in models where one extends the standard model by adding a right handed neutrino and mass terms for the RH neutrinos. Things undergo a drastic change, however, in models that have asymptotic parity invariance. In such models there are always Higgs fields that are parity partners of the RH Higgs fields which give mass to the RH neutrinos. Thus there are operators which give direct mass to the left handed neutrinos at the same time as the right handed neutrinos get mass. It turns out also that the direct neutrino mass term is seesaw suppressed, i.e., as the v_R scale goes to infinity, this contribution, like the right handed neutrino contribution, vanishes. This direct mass contribution leads to a modification of the seesaw formula to the following form (type II seesaw formula [29]):

$$M=fv_L-M_D(fv_R)^{-1}M_D^T. \tag{13}$$

Examples of models where the type II seesaw formula arises are left-right symmetric models or $SO(10)$ models with either $B-L=2$ triplet Higgs fields or $B-L=1$ doublet Higgs fields breaking the $B-L$ symmetry. Below we give an example of a model with triplet Higgs fields. It is important to note that the renormalization group equations hold for both the type I and type II seesaw formulas.

The Yukawa coupling matrix f in Eq. (13) that contributes to the first term in the seesaw formula, like the right handed neutrino mass matrix, depends on high scale physics and is therefore unconstrained by standard model results. We could therefore choose f to be close to the unit matrix. In this case, quark-lepton unification requires that the lepton mixing angles be very close to the quark mixing angles, but the neutrino mass spectrum, dominated by the first term in Eq. (13) in combination with the second term, can easily lead to a quasidegenerate spectrum of Majorana neutrinos as well as approximate mixing unification. In such schemes, radiative magnification works to provide an understanding of the large neutrino mixings. The question is whether there is some underlying symmetry of the theory for which one can write down a natural gauge model where $f=\mathbf{1}f_0$ as well as the near unification of quark and lepton mixings. Below we provide an example of this kind of model. An important point is that the renormalization group equations hold for this type II seesaw formula as long as we assume that the $SU(2)_L$ triplet Higgs field whose vacuum expectation value (VEV) is responsible for the first term in Eq. (13) is heavier than the seesaw scale. This is true in models realizing the type II seesaw.

We consider a nonsupersymmetric $SU(2)_L\times SU(2)_R\times SU(4)_{PS}$ gauge model with an S_4 global symmetry [30]. Before describing the model, a few words about S_4 symmetry may be helpful. This is a non-Abelian discrete symmetry group with 24 elements and has the irreducible representations $\mathbf{3},\mathbf{3}',\mathbf{2},\mathbf{1}',\mathbf{1}$. We will assign fundamental fermions to the $\mathbf{3}$ dimensional representation of S_4 and the Higgs fields ϕ_a and $B-L=2$ triplet fields to representations of S_4 as follows:

Fields	S_4 rep.
$\Psi_{L,R}(2,1,4)+(1,2,\overline{4})$	**3**
$\phi_0(2,2,1)$	**1**
$\phi_{1,2}(2,2,1)$	**2**
$\phi'_{1,2,3}(2,2,1)$	**3**
$\Delta_{L,R}(3,1,10)+(1,3,\overline{10})$	**1**

$$\Psi=\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}.$$

Let us now write down the S_4 invariant Yukawa couplings:

$$\begin{aligned}\mathcal{L}_Y=&f_0\left(\sum_a \psi^T_{L,a}\Psi_{L,a}\Delta_L+L\leftrightarrow R\right)+h_0\phi_0\left(\sum_a \overline{\psi}_{L,a}\Psi_{R,a}\right)\\&+h_2[(\overline{\psi}_{L,3}\Psi_{R,2}+\overline{\psi}_{L,2}\Psi_{R,3})\phi_1+(\overline{\psi}_{L,3}\Psi_{R,3}+\overline{\psi}_{L,2}\Psi_{R,2}\\&-2\overline{\psi}_{L,1}\Psi_{R,1})\phi_2]+h_3[(\overline{\psi}_{L,1}\Psi_{R,3}+\overline{\psi}_{L,3}\Psi_{R,1}\phi'_1\\&+(\overline{\psi}_{L,2}\Psi_{R,1}+\overline{\psi}_{L,1}\Psi_{R,2})\phi'_2+(\overline{\psi}_{L,3}\Psi_{R,3}-\overline{\psi}_{L,2}\Psi_{R,2})\phi'_3]\\&+\text{H.c.}\end{aligned}\tag{14}$$

To get the desired form of the seesaw formula, first note that $\langle\Delta^0_L\rangle=v_L\equiv v^2_{wk}/v_R$, $\Delta^0_R=v_R$, the bidoublet VEVs are of the form

$$\langle\phi_i\rangle=\begin{pmatrix}\kappa_i & 0\\ 0 & \kappa'_i\end{pmatrix},$$

and f_0 is the identity matrix.

One can break the S_4 symmetry softly so that all the ϕ's have different VEVs. Also note that the h_i's can be complex. Thus six ϕ's with independent VEVs give us 12 parameters, which are enough to fit the quark mixings and will predict all lepton mixings equal to quark mixings at the GUT scale. At the GUT scale, this would predict $m_b=m_\tau$ and $m_s=m_\mu$. For the b quark, this is the well-known b-τ unification. Using the Particle Data Group (PDG) values for $m_{b,s}$, we can run it up to the GUT scale to get $m_b(M_R)\simeq 0.98$–1.10 GeV, whereas the corresponding value of $m_\tau\sim 1.18$. However, we have for $m_s(M_R)\simeq 0.03$ GeV if we use the PDG values. This is about three times smaller than the muon mass at the seesaw scale [20]. So we have to add some terms that break quark-lepton symmetry.

To cure the m_s-m_μ problem, we invoke higher dimensional terms and add a new Higgs multiplet $\Sigma(1,1,15)$ that transforms as (1,1,15) under G_{224}. Also let us assume that $\Sigma(1,1,15)$ transforms like a **3** dimensional representation of S_4 with only $\langle\Sigma_3\rangle\neq 0$. The higher dimensional operators that involve Σ have the form $(\phi_0/M)[(\overline{\psi}_{L,1}\Psi_{R,3}+\overline{\psi}_{L,3}\Psi_{R,1}\Sigma_1+(\overline{\psi}_{L,2}\Psi_{R,1}+\overline{\psi}_{L,1}\Psi_{R,2})\Sigma_2+(\overline{\psi}_{L,3}\Psi_{R,3}-\overline{\psi}_{L,2}\Psi_{R,2})\Sigma_3]$. Since $\Sigma(1,1,15)$ has a VEV that breaks only $SU(4)_C$ symmetry, it gives different masses to quarks and leptons. For $\langle\Sigma_3\rangle/M\simeq 10^{-3}$, this has the right order of magnitude to lead to the difference between m_s and m_μ and not affect the off diagonal elements that are responsible for mixings. Since the mixing angles vary roughly as $M_{23}/(M_{33}-M_{22})$, they do not deviate too much from the symmetric values (since $M_{22}\ll M_{33}$).

As far as m_e and m_d go, we can again add nonrenormalizable Yukawa couplings such as $\overline{\Psi}_L\Psi_R\Delta^\dagger_R\Delta_R\phi$ type terms, which will modify only the first generation masses since their magnitude is of order $v^2_R/M^2_{P\ell}$ lower compared to the renormalizable terms. Again this contribution, being a purely diagonal contribution, will change the mixing angles only slightly. Therefore, we can get a model of the type we are considering with degenerate neutrinos and with quark and neutrino mixing angles approximately equal at the seesaw scale. This model can easily be supersymmetrized and all our conclusions go through.

Coming to the neutrino sector, we will first show how the type II seesaw emerges in this model. The complete Higgs field content of this model for the supersymmetric case is $\Psi(2,1,4)$; $\Psi^c(1,2,\overline{4})$, $\phi_0(2,2,1)$, $\phi_{1,2}(2,2,1)$, $\phi'_{1,2,3}(2,2,1)$, $\Delta(3,1,10)\oplus\overline{\Delta}(3,1,\overline{10})$, and $\Delta^c(1,3,\overline{10})\oplus\overline{\Delta}^c(1,3,10)$ as shown in the table in this section. In addition we add a Higgs field transforming as $\Omega(3,3,1)$.

The Higgs part of the superpotential can be written as

$$W'=\lambda\Omega(\Delta\Delta^c+\overline{\Delta\Delta^c}+\text{Tr}\,\phi_0^2+\cdots),\tag{15}$$

where the ellipsis denotes the S_4 singlet bilinears involving the other ϕ fields. Clearly, when we set $F_\Omega=0$ to maintain supersymmetry down to the weak scale, we find that $\langle\Delta^0\rangle\neq 0$. This leads to the type II seesaw, which is the cornerstone of our discussion.

The gauge group $SU(2)_L\times SU(2)_R\times SU(4)_{PS}$ $(=G_{224})$ is a subgroup of a number of GUTs like $SO(10)$, $SO(18)$, and E_6, etc. It also contains subgroups like $SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C(=G_{2213})$ and the standard model. Thus the model worked out in this section with $S_4\times G_{224}$ is equivalent to a number of underlying high scale models such as $S_4\times SO(10)$, $S_4\times SO(18)$, $S_4\times E_6$, etc. It also suggests the possibility of having $S_4\times G_{2213}$ as an approximate symmetry for quasidegeneracy.

In the absence of such symmetries as discussed in this section where a non-Abelian discrete symmetry S_4 occurs along with the gauge symmetry G_{224}, high scale unification of quark and neutrino mixings with quasidegenerate neutrinos but with hierachial quark masses would have been accidental. But the type II seesaw mechanism in the presence of $S_4\times G_{224}$ and its spontaneous breaking guarantees quasidegenerate neutrinos with almost equal mixings in the quark and lepton sectors at the high scale, while the model fits all the masses and mixings at low energies.

VI. CONCLUSION

In summary, we have shown that in the MSSM the hypothesis of quark-lepton mixing unification at the seesaw scale seems to generate the correct observed mixing pattern for neutrinos, i.e., the two large mixings needed for ν_e-ν_μ

and ν_μ-ν_τ and small mixing for U_{e3} at low energies. A quasidegenerate neutrino spectrum with a common mass for neutrinos $\geqslant 0.1$ eV is a testable prediction of the model. An important new result of our analysis is that, although magnification occurs for the U_{e3} parameter, it remains small due to the fact that V_{ub} is very small. The prediction for U_{e3} also provides another test of the model.

Throughout this paper we have treated all phases (Majorana and Dirac) as vanishingly small in the Maki-Nakagawa-Sakata (MNS) matrix. It would be interesting to investigate the effect of phases [31] and threshold effects on the implications of our mixing unification hypothesis.

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[1] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[2] M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).
[3] K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
[4] K. Scholderg, hep-ph/9905016; T. Kajita, invited talk at PASCOS 99; Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
[5] Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001); G.L. Fogli *et al.*, Phys. Rev. D **66**, 053010 (2002); J.N. Bahcall *et al.*, J. High Energy Phys. **07**, 054 (2002).
[6] M. Maltoni *et al.*, Phys. Rev. D **07**, 093003 (2003); A. Bandyopadhyay *et al.*, Phys. Lett. B **559**, 121 (2003); J.N. Bahcall *et al.*, J. High Energy Phys. **02**, 009 (2003).
[7] M. Appollonio *et al.*, Phys. Lett. B **466**, 415 (1999); F. Boehm *et al.*, Phys. Rev. D **64**, 112001 (2001).
[8] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by D. Freedman *et al.* (North-Holland, Amsterdam, 1980); T. Yanagida, in Proceedings of the KEK Workshop, 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
[9] K.S. Babu, C.N. Leung, and J. Pantaleone, Phys. Lett. B **319**, 191 (1993).
[10] P. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993).
[11] K.R.S. Balaji, A.S. Dighe, R.N. Mohapatra, and M.K. Parida, Phys. Rev. Lett. **84**, 5034 (2000); Phys. Lett. B **481**, 33 (2000).
[12] K.R.S. Balaji, R.N. Mohapatra, M.K. Parida, and E.A. Paschos, Phys. Rev. D **63**, 113002 (2001).
[13] P.H. Chankowski, W. Królikowski, and S. Pokorski, Phys. Lett. B **473**, 109 (2000); P.H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002).
[14] J.A. Casas, J.R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. **B569**, 82 (2000); **B573**, 652 (2000).
[15] J. Ellis and S. Lola, hep-ph/9904279; E. Ma, J. Phys. G **25**, L97 (1999); N. Haba, Y. Matsui, N. Okamura, and M. Sugiura, Prog. Theor. Phys. **103**, 145 (2000); P. Chankowski, A. Ioannisian, S. Pokorski, and J.W.F. Valle, Phys. Rev. Lett. **86**, 3488 (2001); M. Frigerio and A.Yu. Smirnov, J. High Energy Phys. **02**, 004 (2003); J.A. Casas, J.R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. **B556**, 3 (1999); S. Antusch, M. Drees, J. Kersten, M. Lindner, and M. Ratz, Phys. Lett. B **519**, 238 (2001).
[16] For recent reviews, see G. Altarelli and F. Feruglio, hep-ph/0206077; R.N. Mohapatra, hep-ph/0211252.
[17] H.V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. A **12**, 147 (2001); C. Aalseth *et al.*, Phys. Rev. D **65**, 092007 (2002).
[18] H.V. Klapdor-Kleingrothaus *et al.*, Mod. Phys. Lett. A **A 16**, 2409 (2001); Int. J. Mod. Phys. A **18**, 4113 (2003); H.V. Klapdor-Kleingrothaus, A. Dietz, and I.V. Krivosheina, Found. Phys. **32**, 1181 (2002).
[19] R.N. Mohapatra, G. Rajasekatran, and M.K. Parida, ''Bi-Large Neutrino Mixings by Radiative Magnification,'' in Proceedings of PASCOS'03 Conference, Bombay, 2003, edited by S. Banerjee *et al.*, Pramana (Proc. Suppl.) (to be published).
[20] C.R. Das and M.K. Parida, Eur. Phys. J. C **20**, 121 (2001); M.K. Parida and B. Purkayastha, *ibid.* **14**, 159 (2000); H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998); S.G. Neculich, *ibid.* **48**, 5293 (1993).
[21] S.R. Juraez, S.F. Herrera, P. Kielanowski, and G. Mora, Phys. Rev. D **66**, 116007 (2002); P. Kielanowski and S.R. Juraez, hep-ph/0301098.
[22] A.J. Buras, P.H. Chankowski, and J. Rosiek, Nucl. Phys. **B659**, 3 (2003); Phys. Lett. B **546**, 96 (2002).
[23] J.I. Illana and M. Masip, hep-ph/0307393.
[24] For a review, see O. Cremonesi, Nucl. Phys. B (Proc. Suppl.) **118**, 287 (2003).
[25] KATRIN Project, A. Osipowicz *et al.*, hep-ex/0109033.
[26] D. Spergel *et al.*, Astrophys. J., Suppl. Ser. **148**, 175 (2003).
[27] S. Hannestad, J. Cosmol. Astropart. Phys. **05**, 004 (2003); A. Pierce and H. Murayama, hep-ph/0302131; C. Giunti, Mod. Phys. Lett. A **A 18**, 1179 (2003); H.V. Klapdor-Kleingrothaus and U. Sarkar, *ibid.* **18**, 2243 (2003).
[28] For some earlier models with degenerate neutrinos, see D. Caldwell and R.N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); D.G. Lee and R.N. Mohapatra, Phys. Lett. B **329**, 463 (1994); P. Bamert and C. Burgess, *ibid.* **329**, 109 (1994); A. Ionissian and J.W.F. Valle, *ibid.* **332**, 93 (1994); A. Joshipura, Z. Phys. C **64**, 31 (1994); E. Ma and G. Rajasekaran, Phys. Rev. D **04**, 113012 (2001); K.S. Babu, E. Ma, and J.W.F. Valle, Phys. Lett. B **552**, 207 (2003).
[29] R.N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981); C. Wetterich, Nucl. Phys. **B187**, 343 (1981); G. Lazarides, Q. Shafi, and C. Wetterich, *ibid.* **B181**, 287 (1981).
[30] Lee and Mohapatra [28].
[31] For works subsequent to the PASCOS'03 Conference [19] and the first posting of this article in the arXiv, see J.A. Casas, J. Espinosa, and I. Navarro, J. High Energy Phys. **09**, 048 (2003); S. Antusch *et al.*, Nucl. Phys. **B674**, 401 (2003).