Precise prediction for the *W*-boson mass in the standard model

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The presently most accurate prediction for the *W*-boson mass in the standard model is obtained by combining the complete two-loop result with the known higher-order QCD and electroweak corrections. The numerical impact of the different contributions is analyzed in detail. A simple parametrization of the full result is presented, which approximates the full result for M_W to better than 0.5 MeV for 10 GeV$\leqslant M_H \leqslant$1 TeV if the other parameters are varied within their combined 2σ region around their experimental central values. The different sources of remaining theoretical uncertainties are investigated. Their effect on the prediction of M_W is estimated to be about 4 MeV for $M_H \lesssim 300$ GeV.

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The relation between the *W*-boson mass M_W, the *Z*-boson mass M_Z, the fine structure constant α, and the Fermi constant G_μ,

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\Delta r), \tag{1}$$

is of central importance for precision tests of the electroweak theory. Accordingly, a lot of effort has been devoted over more than two decades to accurately predict the quantity Δr, which summarizes the radiative corrections, within the standard model (SM) and extensions of it.

The one-loop result [1] can be written as

$$\Delta r^{(\alpha)}=\Delta\alpha-\frac{c_W^2}{s_W^2}\Delta\rho+\Delta r_{\mathrm{rem}}(M_H), \tag{2}$$

where $c_W^2=M_W^2/M_Z^2$, $s_W^2=1-c_W^2$. It involves large fermionic contributions from the shift in the fine structure constant due to light fermions, $\Delta\alpha\propto\log m_f$, and from the leading contribution to the ρ parameter $\Delta\rho$. The latter is quadratically dependent on the top-quark mass m_t as a consequence of the large mass splitting in the isospin doublet [2]. The remainder part, Δr_{rem}, contains in particular the dependence on the Higgs-boson mass, M_H. Higher-order QCD corrections to Δr are known at $O(\alpha\alpha_s)$ [3] and $O(\alpha\alpha_s^2)$ [4,5].

Recently the full electroweak two-loop result for Δr has been completed. It consists of the fermionic contribution [6–8], which involves diagrams with one or two closed fermion loops, and the purely bosonic two-loop contribution [9].

Beyond two-loop order the results for the pure fermion-loop corrections (i.e., contributions containing n fermion loops at n-loop order) are known up to four-loop order [10]. They contain in particular the leading contributions in $\Delta\alpha$ and $\Delta\rho$. Most recently, results for the leading three-loop contributions of $O(G_\mu^3 m_t^6)$ and $O(G_\mu^2\alpha_s m_t^4)$ have been obtained for arbitrary values of M_H (by means of expansions around $M_H=m_t$ and for $M_H\gg m_t$) [11], generalizing a previous result which was obtained in the limit $M_H=0$ [12].

Equation (1) is usually employed for predicting the *W*-boson mass,

$$M_W^2=M_Z^2\left\{\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha}{\sqrt{2}G_\mu M_Z^2}[1+\Delta r(M_W,M_Z,M_H,m_t,\ldots)]}\right\}, \tag{3}$$

which is done by an iterative procedure since Δr itself depends on M_W. Comparison of the prediction for M_W within the SM with the experimental value allows us to obtain indirect constraints on the Higgs-boson mass. These constraints are affected both by the experimental error of M_W and by the uncertainty of the theory prediction. The current experimental error of the *W*-boson mass is $\delta M_W^{\mathrm{exp}}=34$ MeV [13]. The accuracy in the measurement of the *W*-boson mass is expected to improve to about $\delta M_W^{\mathrm{expt,Tev/LHC}}=15$ MeV [14] from the measurements at run II of the Fermilab Tevatron and the CERN Large Hadron Collider (LHC), and to about $\delta M_W^{\mathrm{expt,LC}}=7$ MeV at a future linear collider (LC) running at the *WW* threshold [15]. The uncertainty of the theory prediction is caused by the experimental errors of the input parameters, e.g., m_t, and by the uncertainty from unknown higher-order corrections. In the global SM fit to all data [16] the

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TABLE I. The numerical values ($\times 10^4$) of the different contributions to Δr specified in Eq. (1) are given for different values of M_H and $M_W = 80.426$ GeV (the W and Z masses have been transformed so as to correspond to the real part of the complex pole). The other input parameters are listed in Eq. (5).

M_H/GeV	$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r^{(\alpha^2)}_{\rm ferm}$	$\Delta r^{(\alpha^2)}_{\rm bos}$	$\Delta r^{(G_\mu^2\alpha_s m_t^4)}$	$\Delta r^{(G_\mu^3 m_t^6)}$
100	283.41	35.89	7.23	28.56	0.64	−1.27	−0.16
200	307.35	35.89	7.23	30.02	0.35	−2.11	−0.09
300	323.27	35.89	7.23	31.10	0.23	−2.77	−0.03
600	353.01	35.89	7.23	32.68	0.05	−4.10	−0.09
1000	376.27	35.89	7.23	32.36	−0.41	−5.04	−1.04

highest sensitivity to M_H arises from the predictions for M_W and the effective weak mixing angle at the Z-boson resonance, $\sin^2\theta_{\rm eff}$.

In the present paper we combine the various pieces that are relevant for the prediction of M_W into a common result and analyze the numerical impact of the different contributions. Since in particular the electroweak two-loop result is very lengthy and involves numerical integrations of two-loop scalar integrals, it is not possible to present the full result in a compact analytic form. We therefore provide a simple parametrization of the full result which is easy to implement and should be accurate enough for practical applications. We discuss the sources of the remaining theoretical uncertainties and obtain an estimate for the uncertainty from unknown higher-order corrections.

We incorporate the following contributions into the result for Δr:

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha^2)}_{\rm ferm} + \Delta r^{(\alpha^2)}_{\rm bos} + \Delta r^{(G_\mu^2\alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}, \tag{4}$$

where $\Delta r^{(\alpha)}$ is the one-loop result, Eq. (2), $\Delta r^{(\alpha\alpha_s)}$ and $\Delta r^{(\alpha\alpha_s^2)}$ are the two-loop [3] and three-loop [4,5] QCD corrections, and $\Delta r^{(\alpha^2)}_{\rm ferm}$ [6–8] and $\Delta r^{(\alpha^2)}_{\rm bos}$ [9] are the fermionic and purely bosonic electroweak two-loop corrections, respectively. The contributions $\Delta r^{(G_\mu^2\alpha_s m_t^4)}$ and $\Delta r^{(G_\mu^3 m_t^6)}$ have been obtained from the leading three-loop contributions to $\Delta\rho$ given in Ref. [11].

We have not included the pure fermion-loop contributions at three-loop and four-loop order obtained in Ref. [10] because their contribution turned out to be small as a consequence of accidental numerical cancellations, with a net effect of only about 1 MeV in M_W (using the real-pole definition of the gauge-boson masses). Since the result given in Ref. [10] contains the leading contributions involving powers of $\Delta\alpha$ and $\Delta\rho$ beyond two-loop order, we do not make use of resummations of $\Delta\alpha$ and $\Delta\rho$ as it was often done in the literature in the past (see, e.g., Refs. [17]). Accordingly, the quantity Δr appears in Eq. (3) in fully expanded form. This means, for instance, that we do not include the $O(\alpha^3)$ term $3(\Delta\alpha)^2\Delta r^{(\alpha)}_{\rm bos}$, which can be inferred from the electric charge renormalization. It affects the prediction for M_W by about 1.5 MeV. This shift is, however, expected to partially cancel with the corresponding contributions proportional to $(\Delta\alpha)(\Delta\rho)\Delta r^{(\alpha)}_{\rm bos}$ and $(\Delta\rho)^2\Delta r^{(\alpha)}_{\rm bos}$ in an analogous way as for the pure fermion-loop contributions.

In Table I the numerical values of the different contributions to Δr are given for $M_W = 80.426$ GeV [13]. The other input parameters that we use in this paper are [13]

$$\begin{aligned} &m_t = 174.3\ \text{GeV}, \quad m_b = 4.7\ \text{GeV},\\ &M_Z = 91.1875\ \text{GeV}, \quad \Gamma_Z = 2.4952\ \text{GeV},\\ &\alpha^{-1} = 137.03599976, \quad \Delta\alpha = 0.05907,\\ &\alpha_s(M_Z) = 0.119,\\ &G_\mu = 1.16637\times 10^{-5}\ \text{GeV}^{-2}, \end{aligned} \tag{5}$$

where $\Delta\alpha \equiv \Delta\alpha_{\rm lept} + \Delta\alpha^{(5)}_{\rm had}$, and $\Delta\alpha_{\rm lept} = 0.0314977$ [18]. For $\Delta\alpha^{(5)}_{\rm had}$ we use the value given in Ref. [19], $\Delta\alpha^{(5)}_{\rm had} = 0.027572 \pm 0.000359$. The total width of the Z boson, Γ_Z, appears as an input parameter since the experimental value of M_Z in Eq. (5), corresponding to a Breit-Wigner parametrization with running width, needs to be transformed in our calculation into the mass parameter defined according to the real part of the complex pole, which corresponds to a Breit-Wigner parametrization with a constant decay width, see Ref. [7]. It is understood that M_W in this paper always refers to the conventional definition according to a Breit-Wigner parametrization with running width. The change of parametrization is achieved with the one loop QCD corrected value of the W-boson width as described in Ref. [7].

Table I shows that the two-loop QCD correction $\Delta r^{(\alpha\alpha_s)}$ and the fermionic electroweak two-loop correction $\Delta r^{(\alpha^2)}_{\rm ferm}$ are of similar size. They both amount to about 10% of the one-loop contribution $\Delta r^{(\alpha)}$, entering with the same sign. The most important correction beyond these contributions is the three-loop QCD correction $\Delta r^{(\alpha\alpha_s^2)}$, which leads to a shift in M_W of about -11 MeV. For large values of M_H also the contribution $\Delta r^{(G_\mu^2\alpha_s m_t^4)}$ becomes sizable. The purely bosonic two-loop contribution $\Delta r^{(\alpha^2)}_{\rm bos}$ and the leading electroweak three-loop correction $\Delta r^{(G_\mu^3 m_t^6)}$ give rise to shifts in M_W which are significantly smaller than the experimental error envisaged for a future linear collider, $\delta M_W^{\rm exp,LC} = 7$ MeV [15].

Since Δr is evaluated in Table I for a fixed value of M_W, the contributions $\Delta r^{(\alpha\alpha_s)}$ and $\Delta r^{(\alpha\alpha_s^2)}$ are M_H independent.

In the iterative procedure for evaluating M_W according to Eq. (3), on the other hand, these contributions also become M_H dependent through the M_H dependence of the inserted M_W value.

The result for M_W based on Eqs. (3), (4) can be approximated by the following simple parametrization (see Ref. [20] for an earlier parametrization of M_W),

$$M_W = M_W^0 - c_1 dH - c_2 dH^2 + c_3 dH^4 + c_4(dh-1) - c_5 d\alpha + c_6 dt - c_7 dt^2 - c_8 dH dt + c_9 dh dt - c_{10} d\alpha_s + c_{11} dZ, \tag{6}$$

where

$$dH = \ln\left(\frac{M_H}{100\ \text{GeV}}\right), \quad dh = \left(\frac{M_H}{100\ \text{GeV}}\right)^2,$$
$$dt = \left(\frac{m_t}{174.3\ \text{GeV}}\right)^2 - 1,$$
$$dZ = \frac{M_Z}{91.1875\ \text{GeV}} - 1, \quad d\alpha = \frac{\Delta\alpha}{0.05907} - 1,$$
$$d\alpha_s = \frac{\alpha_s(M_Z)}{0.119} - 1, \tag{7}$$

and the coefficients M_W^0, $c_1, \ldots, c_{11}$ take the following values:

$$M_W^0 = 80.3799\ \text{GeV}, \quad c_1 = 0.05429\ \text{GeV},$$
$$c_2 = 0.008939\ \text{GeV}, \quad c_3 = 0.0000890\ \text{GeV},$$
$$c_4 = 0.000161\ \text{GeV}, \quad c_5 = 1.070\ \text{GeV},$$
$$c_6 = 0.5256\ \text{GeV}, \quad c_7 = 0.0678\ \text{GeV},$$
$$c_8 = 0.00179\ \text{GeV}, \quad c_9 = 0.0000659\ \text{GeV},$$
$$c_{10} = 0.0737\ \text{GeV}, \quad c_{11} = 114.9\ \text{GeV}. \tag{8}$$

The parametrization given in Eqs. (6)–(8) approximates the full result for M_W to better than 0.5 MeV over the whole range of 10 GeV$\leqslant M_H \leqslant$1 TeV if all other experimental input values vary within their combined 2σ region around their central values given in Eq. (7).

In Table II the full result for M_W and the parametrization of Eqs. (6)–(8) are compared with each other. The table shows the shifts in M_W [relative to the value $M_W = 80.3799$ GeV, which is the result for $M_H = 100$ GeV and the central values of the other input parameters as specified in Eq. (5)] induced by varying M_H by 100 GeV and the other input parameters by 1σ around their experimental central values [13]. In the example of Table II, where only one parameter has been varied in each row and all others have been kept at their central values, the maximum deviation between the full result for M_W and the parametrization of Eqs. (6)–(8) is below 0.1 MeV.

TABLE II. Shifts in M_W caused by varying M_H by 100 GeV and the other input parameters by 1σ around their experimental central values [13]. The first column shows the full result for M_W, while the second column is based on the simple parametrization of Eqs. (6)–(8). The shifts δM_W are relative to the value $M_W = 80.3799$ GeV which is the result for $M_H = 100$ GeV and the central values of the other input parameters as specified in Eq. (5).

	δM_W (full result)/MeV	δM_W [Eqs. (6)–(8)]/MeV
$\delta M_H = 100$ GeV	−41.3	−41.4
$\delta m_t = 5.1$ GeV	31.0	31.0
$\delta M_Z = 2.1$ MeV	2.6	2.6
$\delta(\Delta\alpha_{\text{had}}^{(5)}) = 0.00036$	−6.5	−6.5
$\delta\alpha_s(M_Z) = 0.0027$	−1.7	−1.7

The parametrization of Eqs. (6)–(8) yields a good approximation of the full result for M_W even for values of M_H much smaller than the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_H = 114.4$ GeV [21]. If one restricts to the region $M_H > 100$ GeV, a slight readjustment of the coefficients in Eq. (8) yields an even more accurate parametrization of the full result. If Eqs. (6), (7) are used together with the following values of the coefficients:

$$M_W^0 = 80.3800\ \text{GeV}, \quad c_1 = 0.05253\ \text{GeV},$$
$$c_2 = 0.010345\ \text{GeV}, \quad c_3 = 0.001021\ \text{GeV},$$
$$c_4 = -0.000070\ \text{GeV}, \quad c_5 = 1.077\ \text{GeV},$$
$$c_6 = 0.5270\ \text{GeV}, \quad c_7 = 0.0698\ \text{GeV},$$
$$c_8 = 0.004055\ \text{GeV}, \quad c_9 = 0.000110\ \text{GeV},$$
$$c_{10} = 0.0716\ \text{GeV}, \quad c_{11} = 115.0\ \text{GeV}, \tag{9}$$

the full result for M_W is approximated to better than 0.2 MeV over the range of 100 GeV$\leqslant M_H \leqslant$1 TeV if all other experimental input values vary within their combined 2σ region around their central values given in Eq. (7).

From Table II one can read off the parametric theoretical uncertainties in the prediction for M_W being caused by the experimental errors of the input parameters. The dominant parametric uncertainty at present (besides the dependence on M_H) is induced by the experimental error of the top-quark mass. It is almost as large as the current experimental error of the W-boson mass, $\delta M_W^{\text{expt}} = 34$ MeV [13]. The uncertainty caused by the experimental error of m_t will remain the dominant source of theoretical uncertainty in the prediction for M_W even at the LHC, where the error on m_t will be reduced to $\delta m_t = 1-2$ GeV [22]. A further improvement of the parametric uncertainty of M_W will require the precise measurement of m_t at a future linear collider [23], where an accuracy of about $\delta m_t = 0.1$ GeV will be achievable [15].

We now turn to the second source of theoretical uncertainties in the prediction for M_W, namely the uncertainties from unknown higher-order corrections. Different approaches have been used in the literature for estimating the possible size of uncalculated higher-order corrections [7,24–26]. The "traditional Blue Band method" is based on the fact that the results of calculations employing different renormalization schemes or different prescriptions for including nonleading contributions in resummed or expanded form differ from each other by higher-order corrections. The deviations between the results of different codes in which the same corrections have been organized in a somewhat different way are used in this method as a measure for the size of unknown higher-order corrections [24]. In applying this method it is not easy to quantify how big the variety of different "options" and different codes should be in order to obtain a reasonable estimate of the higher-order uncertainties. As the method cannot account for genuine effects of irreducible higher-order corrections, it may lead to an underestimate of the theoretical uncertainties if at an uncalculated order a new source of potentially large corrections appears, e.g., a certain enhancement factor.

In Ref. [26] a different prescription has been proposed, in which for each type of unknown corrections the relevant enhancement factors are identified and the remaining coefficient arising from the actual loop integrals is set to unity. In Ref. [7] higher-order QCD corrections have been estimated in two different ways, from the renormalization scale dependence (in particular, taking into account the effect of switching from the on-shell to the $\overline{\text{MS}}$ definition of the top-quark mass) and from assuming that, for instance, the ratio of the $O(\alpha^2\alpha_s)$ and $O(\alpha^2)$ corrections is of the same size as the ratio of the $O(\alpha\alpha_s)$ and $O(\alpha)$ corrections.

Several of the corrections whose possible size had been estimated in Refs. [7,25,26] have meanwhile been calculated [9,11], and it turned out that the estimates agreed reasonably well with the actual size of the corrections. This adds confidence to applying the same kind of methods also for an estimate of the remaining higher-order uncertainties.

There are three sources of remaining uncertainties in the prediction for M_W from unknown higher-order corrections.

(i) *The corrections at* $O(\alpha^2\alpha_s)$ *beyond the known contribution of* $O(G_\mu^2\alpha_s m_t^4)$. The numerical effect of the $O(G_\mu^2\alpha_s m_t^4)$ correction was found to be up to 5 MeV in M_W for a light Higgs-boson mass, $M_H \lesssim 300$ GeV [11]. This contribution represents the leading term in an expansion for asymptotically large values of m_t. In the calculation of the electroweak two-loop corrections it was found that the formally next-to-leading order term of $O(G_\mu^2 m_t^2 M_Z^2)$ has approximately the same numerical effect as the formally leading term of $O(G_\mu^2 m_t^4)$ [27]. It can therefore be expected that the formally next-to-leading order term of $O(G_\mu^2\alpha_s m_t^2 M_Z^2)$ also may be of similar size as the leading $O(G_\mu^2\alpha_s m_t^4)$ term.

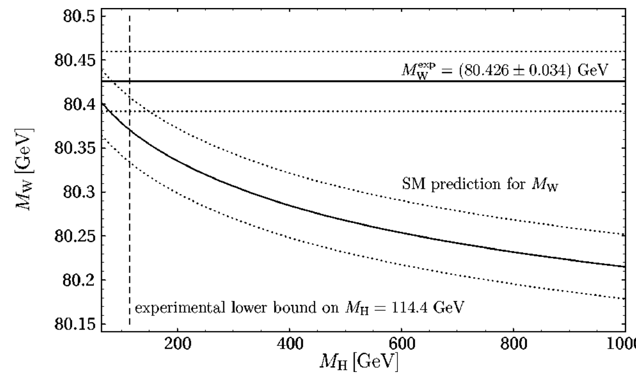

FIG. 1. Prediction for M_W in the SM as a function of M_H for $m_t = 174.3 \pm 5.1$ GeV. The current experimental value, $M_W^{\text{expt}} = 80.426 \pm 0.034$ GeV [13], and the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_H = 114.4$ GeV [21], are also indicated.

We therefore assign an uncertainty of about 3 MeV to the remaining theoretical uncertainties at $O(\alpha^2\alpha_s)$ (for $M_H \lesssim 300$ GeV).

(ii) *The unknown electroweak three-loop corrections*. The numerical effect of the $O(G_\mu^3 m_t^6)$ contribution was found to be small [11], shifting M_W by less than 0.3 MeV for $M_H \lesssim 300$ GeV. This shift is significantly smaller than the estimate in Ref. [26]. The pure fermion-loop corrections at three-loop order were found in Ref. [10] to shift M_W by about 1 MeV, which, however, involved an accidental numerical cancellation. It thus does not seem to be justified to assume that all other electroweak three-loop corrections are completely negligible. In Ref. [7] it was pointed out that reparametrizing the *W*-boson width, which enters the prediction for M_W at the two-loop level, by G_μ instead of α shifts the prediction for M_W by about 1 MeV, which is formally an effect of $O(\alpha^3)$. In order to take into account uncertainties of this kind [see also the discussion below Eq. (4)] we assign an uncertainty of 1–2 MeV to the unknown corrections at $O(\alpha^3)$.

(iii) *The four-loop QCD corrections of* $O(\alpha\alpha_s^3)$. The possible effect of the leading term of $O(G_\mu\alpha_s^3 m_t^2)$ was estimated in Ref. [26] to be about 1.3 MeV. Employing the known results at lower order of α_s and assuming a geometric progression yields a slightly larger result. We thus assign an uncertainty of 1–2 MeV for the $O(\alpha\alpha_s^3)$ corrections.

Adding the above estimates for the different kinds of unknown higher-order corrections in quadrature, we find as estimate of the remaining theoretical uncertainties from unknown higher-order corrections

$$\delta M_W^{\text{theo}} \approx 4 \ \text{MeV}. \tag{10}$$

This estimate holds for a relatively light Higgs boson, $M_H \lesssim 300$ GeV. For a heavy Higgs boson, i.e., M_H close to the TeV scale, the remaining theoretical uncertainty is significantly larger.

In Fig. 1 we have updated the comparison between the theory prediction for M_W within the SM and the experimental value, using the currently most accurate theory prediction based on Eqs. (3) and (4) and the most up-to-date experimental data [13]. For the theoretical uncertainty the estimate of Eq. (10) and the parametric uncertainties corresponding to 1σ variations of the input parameters (see Table II) have been used. As discussed above, at present the theoretical uncertainty is dominated by the effect of the experimental error of the top-quark mass.

Figure 1 confirms the well-known preference for a light Higgs-boson mass within the SM. If the 95% exclusion bound from the direct search for the SM Higgs is taken into account [21], the 1σ bands corresponding to the theory prediction and the experimental result for M_W show only a marginal overlap.

In summary, we have presented the currently most accurate prediction for M_W in the standard model. We have discussed the relative importance of the complete one-loop and two-loop contributions as well as the known corrections beyond two-loop order. We have summarized the present status of the theoretical uncertainties of M_W from the experimental errors of the input parameters, and we have obtained an estimate for the remaining theoretical uncertainties from unknown higher-order corrections. In the region of Higgs-boson mass values preferred by the electroweak precision data, $M_H \lesssim 300$ GeV, the uncertainty from unknown higher-order corrections amounts to about 4 MeV. This is much smaller than the present experimental error of M_W and even below the envisaged future experimental error at the next generation of colliders. Having reached this level of theoretical precision of M_W is important, however, for the precision test of the electroweak theory, in particular in view of the fact that M_W can be used as an input for calculating the effective weak mixing angle at the Z resonance, $\sin^2\theta_{\rm eff}$.

We have, furthermore, presented a simple parametrization of the full result containing all relevant corrections, which should be sufficiently accurate for practical applications. It approximates the full result for M_W to better than 0.5 MeV over the whole range of $10\ \text{GeV} \leqslant M_H \leqslant 1$ TeV if all other experimental input values vary within their combined 2σ region around their experimental central values. In view of the experimental exclusion bound on the Higgs-boson mass of $M_H > 114.4$ GeV it will normally be sufficient to restrict to the smaller range of $100\ \text{GeV} \leqslant M_H \leqslant 1$ TeV. For this case we provide a simple parametrization which approximates the full result for M_W even within 0.2 MeV.

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