Area spectrum of extremal Reissner-Nordström black holes from quasinormal modes

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Using the quasinormal mode frequency of extremal Reissner-Nordström black holes, we obtain the area spectrum for these types of black holes. We show that the area and entropy black hole horizon are equally spaced. Our results for the spacing of the area spectrum differ from that for Schwarzschild black holes.

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I. INTRODUCTION

The quantization of the black hole horizon area is one of the most interesting manifestations of quantum gravity. Since its first prediction by Bekenstein in 1974 [1], there has been much work on this topic [2–16]. Recently, the quantization of the black hole area has been considered [5,6] as a result of the absorption of a quasinormal-mode excitation. The quasinormal modes (QNMs) of black holes are the characteristic, ringing frequencies which result from their perturbations [17] and provide a unique signature of these objects [18], that might be observed in gravitational waves. In asymptotically flat spacetimes the idea of QNMs started with the work of Regge and Wheeler [19] where the stability of a black hole was tested, and they were first numerically computed by Chandrasekhar and Detweiler several years later [20]. The quasinormal modes now attract a lot of interest in different contexts: in AdS conformal field theory correspondence [21–31], when considering thermodynamic properties of black holes in loop quantum gravity [6–8], and in the context of the possible connection with critical collapse [21,32,33].

Bekenstein's idea for quantizing a black hole is based on the fact that its horizon area, in the nonextreme case, behaves as a classical adiabatic invariant [1,4]. In the spirit of the Ehrenfest principle, any classical adiabatic invariant corresponds to a quantum entity with a discrete spectrum; Bekenstein conjectured that the horizon area of a nonextremal quantum black hole should have a discrete eigenvalue spectrum. Moreover, the possibility of a connection between the quasinormal frequencies of black holes and the quantum properties of the entropy spectrum was first observed by Bekenstein [34], and further developed by Hod [5]. In particular, Hod proposed that the real part of the quasinormal frequencies, in the infinite damping limit, might be related via the correspondence principle to the fundamental quanta of mass and angular momentum. The proposed correspondence between quasinormal frequencies and the fundamental quantum of mass automatically leads to an equally spaced area spectrum. Remarkably, the spacing was such as to allow a statistical mechanical interpretation for the resulting eigenvalues for the Bekenstein-Hawking entropy. Dreyer [6] also used the large damping quasinormal mode frequency to fix the value of the Immirzi parameter γ in loop quantum gravity. He found that loop quantum gravity gives a correct prediction for the Bekenstein-Hawking entropy if the gauge group is SO(3) and not SU(2).

In this article our aim is to obtain the area and entropy spectrum of extremal Reissner-Nordström (RN) black holes in four-dimensional spacetime. Using the results of [35,36] for highly damped quasinormal modes, we show how the horizon area and entropy would be quantized. The authors of [36] noted that the variation of the mass of a RN black hole is not enough to determine the variation of its area, since the corresponding variation of the charge must be known. These authors, then, assumed the same area quantum as in the Schwarzschild case and deduced the corresponding quantum of charge emission. It would seem that in the case of an extremal RN black hole this issue does not arise, since M and Q are equal. We show that the results for the spacing of the area spectrum differ from the Schwarzschild black hole case. Conversely, if we assume that ΔA is indeed universal [36] and thus remains as in the Schwarzschild case, $\Delta A = 4\hbar \ln 3$, then the real part of the quasinormal frequency for an extremal RN black hole is different from the Schwarzschild black hole case.

II. EXTREMAL REISSNER-NORDSTRÖM BLACK HOLES

The RN black hole's (event and inner) horizons are given in terms of the black hole parameters by

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2}, \tag{1}$$

where M and Q are, respectively, the mass and charge of the black hole. In the extreme case these two horizons coincide:

$$r_{\pm} = M, \quad M = Q. \tag{2}$$

Accordingly, a very interesting conclusion follows [35] (see also the more recent paper [36]): the real part of the quasinormal frequency for extremal RN black holes coincides with the Schwarzschild value

$$\omega_R^{RN} = \frac{\ln 3}{4\pi R_H}, \tag{3}$$

where

$$R_H = 2M. \tag{4}$$

We assume that this classical frequency plays an important role in the dynamics of the black hole and is relevant to its

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quantum properties [5,6]. In particular, we consider ω_R^{RN} to be a fundamental vibrational frequency for a black hole of energy $E=M$. Given a system with energy E and vibrational frequency ω one can show that the quantity

$$I=\int \frac{dE}{\omega(E)} \tag{5}$$

is an adiabatic invariant [7], which via Bohr-Sommerfeld quantization has an equally spaced spectrum in the semiclassical (large n) limit:

$$I \approx n\hbar. \tag{6}$$

Now by taking ω_R^{RN} in this context, we have

$$I=\int \frac{dE}{\omega_R^{RN}}=\int \frac{4\pi R_H}{\ln 3}dM=\frac{4\pi}{\ln 3}\int 2M dM=\frac{4\pi}{\ln 3}M^2+c, \tag{7}$$

where c is a constant. On the other hand, the black hole horizon area is given by

$$A=4\pi r_+^2, \tag{8}$$

which, using Eq. (2), in the extremal case is as follows:

$$A=4\pi M^2. \tag{9}$$

The Boher-Sommerfeld quantization law and Eq. (7) then imply that the area spectrum is equally spaced,

$$A_n=n\hbar \ln 3. \tag{10}$$

We can obtain the above result by another method. From Eq. (9) we get

$$\Delta A=8\pi M \Delta M=8\pi M\hbar \omega_R^{RN}, \tag{11}$$

where we have associated the energy spacing with a frequency through $\Delta M=\Delta E=\hbar \omega_R^{RN}$. Now using Eqs. (3),(4) we have

$$\Delta A=\hbar \ln 3; \tag{12}$$

therefore the extremal RN black hole has the discrete spectrum

$$A_n=n\hbar \ln 3, \tag{13}$$

which is exactly the result of Eq. (10). Using the definition of the Bekenstein-Hawking entropy we have

$$S=\frac{A_n}{4\hbar}=\frac{n \ln 3}{4}. \tag{14}$$

The above results for the area spectrum and entropy are contradicted by results of Andersson and Howls [36] for extremal RN black holes. Andersson and Howls assumed that ΔA is universal and thus remains as in the Schwarzschild case, $\Delta A=4\hbar \ln 3$.

Now if we assume that ΔA is indeed universal [36] and thus remains as in the Schwarzschild case, $\Delta A=4\hbar \ln 3$, then the real part of the quasinormal frequency for an extremal RN black hole is

$$\omega_R^{RN}=\frac{\ln 3}{\pi R_H}. \tag{15}$$

In this case we have

$$I=\int \frac{dE}{\omega_R^{RN}}=\int \frac{\pi R_H}{\ln 3}dM=\frac{\pi}{\ln 3}\int 2M dM=\frac{\pi}{\ln 3}M^2+c; \tag{16}$$

Now Eqs. (5),(8),(9),(16) imply that the area spectrum is equally spaced as follows:

$$A_n=4n\hbar \ln 3. \tag{17}$$

III. CONCLUSION

Bekenestein's idea for quantizing a black hole is based on the fact that its horizon area, in the nonextremal case, behaves as a classical adiabatic invariant. It is interesting to investigate how extremal black holes would be quantized. Discrete spectra arise in quantum mechanics in the presence of a periodicity in the classical system, which in turn leads to the existence of an adiabatic invariant or action variable. Boher-Somerfeld quantization implies that this adiabatic invariant has an equally spaced spectrum in the semiclassical limit. In this article we have considered the extremal RN black hole in four-dimensional spacetime. Using the results for highly damped quasinormal modes, we obtained the area and entropy spectrum of the event horizon. Here we accept the proposed correspondence between the quasinormal mode frequencies and a transition energy ΔM and find that the quantum area should be $\Delta A=\hbar \ln 3$. Although the real parts of highly damped quasinormal modes for Schwarzschild and extremal RN black holes are equal [36], $\omega_R=(\ln 3)/8\pi M$, as one can see, for example, in [7,16,36], $\Delta A=4\hbar \ln 3$ for a Schwarzschild black hole. Therefore, in contrast with the claim of [36], ΔA is not universal for all black holes. Abdalla *et al.* [10] have also shown that the results for the spacing of the area spectrum for near extreme Kerr and near extreme Schwarzschild–de Sitter black holes differ from those for Schwarzschild as well as nonextreme Kerr black holes. Although this difference for the problem under consideration in [10], as the authors mentioned, may be justified due to the quite different nature of the asymptotic quasinormal mode spectrum of the near extreme black hole, in our problem the real parts of the highly damped quasinormal modes for Schwarzschild and extremal RN black holes are equal. According to Eq. (2) the location of the horizon for an extreme RN black hole is at $r=M$, but the Schwarzschild black hole horizon is located at $r=2M$; therefore the factor of 4 in the quantum area of a Schwarzschild black hole, $\Delta A=4\hbar \ln 3$, comes from the factor of 2 in $r=2M$.

On the other hand, if we assume that ΔA is indeed universal [36] and thus remains as in the Schwarzschild case, $\Delta A=4\hbar \ln 3$, then the real part of the quasinormal frequency for an extremal RN black hole is $\omega_R^{RN}=(\ln 3)/\pi R_H$, which is different from the Schwarzschild black hole case.

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