Fat gravitons, the cosmological constant and submillimeter tests

Raman Sundrum*
Department of Physics and Astronomy, Johns Hopkins University, 3400 North Charles Street, Baltimore, Maryland 21218-2686, USA
(Received 15 October 2003; published 20 February 2004)

We reexamine the proposal that the resolution of the cosmological constant problem involves a submillimeter breakdown of the point-particle approximation for gravitons. No fundamental description of such a breakdown, which simultaneously preserves the point-particle nature of matter particles, is yet known. However, basic aspects of the self-consistency of the idea, such as the preservation of the macroscopic equivalence principle while satisfying the quantum naturalness of the cosmological constant, are addressed in this paper within a soft graviton effective theory. It builds on Weinberg's analysis of soft graviton couplings and standard heavy particle effective theory, and minimally encompasses the experimental regime of soft gravity coupled to hard matter. A qualitatively distinct signature for short-distance tests of gravity is discussed, bounded by naturalness to appear above approximately 20 μm.

DOI: 10.1103/PhysRevD.69.044014 PACS number(s): 04.50.+h

I. INTRODUCTION

Imagine an alien species, sophisticated enough to know the overarching principles of quantum mechanics and relativity, but whose particle physics expertise (or funding) can engineer or observe momentum transfers only below 10^{-3} eV. While they have access to heavy, macroscopic sources, the only fundamental fields and particles they know are the metric of general relativity (GR), soft electromagnetic fields and perhaps some neutrinos. The alien theoreticians have nevertheless synthesized the various tools of quantum field theory from the big principles. Superstring theory is also flourishing. Phenomenologists have put in place a minimal effective field theory cut off at 10^{-3} eV, which accommodates the data below this scale while being agnostic about physics above.

The aliens have also run into the cosmological constant problem (CCP). (For a review see Ref. [1].) Actually, since the observed ''dark energy'' density of the cosmos is $\sim(10^{-3}\ \mathrm{eV})^4$ [2,3], their minimal effective theory is not presently fine-tuned. However, if new experiments above 10^{-3} eV continue to support the minimal effective theory, now with a larger UV cutoff, then the cosmological constant will be fine-tuned. Naturalness therefore predicts new physics just above 10^{-3} eV, acting to cut off the quartic divergences in the cosmological constant within the effective theory. The aliens are therefore quite excited about new short distance tests of gravity, $<0.1\mathrm{mm}\sim(10^{-3}\ \mathrm{eV})^{-1}$, along with other ''high-energy'' experiments. They imagine that they might discover submillimeter strings cutting off all of point-particle effective field theory, or supersymmetry enforcing cancellations in radiative corrections to the cosmological constant. Or perhaps something no one has thought of.

We, on the other hand, seem less excited that experiments have the answer. We already know too much. Our particle physicists have probed momentum transfers all the way up to a TeV without finding submillimeter supersymmetry or strings. Effective field theory of the standard model (SM) coupled to GR with a TeV cutoff beautifully accounts for all the data, but now the cosmological constant is unavoidably fine-tuned. There is a ''no-go'' theorem forbidding new light fields from relaxing the cosmological constant [1]. The door through which new submillimeter gravitational physics might enter into a solution of the CCP seems firmly shut.

The purpose of this paper is to pry open this door a little. An important first step is to notice that the TeV scale effective theory which leads to the CCP involves a tremendous extrapolation of standard GR to far shorter distances than gravity has been experimentally probed, in order to accommodate the wealth of SM data.[1] Naively, this observation has no bearing on the CCP, since SM corrections to the IR effective cosmological constant reside in the gravitational effective action, $\Gamma_{eff}[g_{\mu\nu}]$, evaluated for extremely soft gravitational fields. The SM fields are hard and off shell in general in such corrections, but then hard SM processes are well understood up to a TeV. Nevertheless, the central point of Ref. [5] was to argue that (virtual) high energy contributions to an effective action, $\Gamma_{eff}[A]$, of a sector, *A*, from integrating out a different sector, *B*, cannot be robustly determined (or even roughly estimated) without knowing the high energy dynamics and degrees of freedom of *both sectors, A and B*. This conclusion does not follow from standard Feynman diagram calculations, but rather by re-thinking whether certain diagrams are warranted at all. Reference [5] illustrated the general claim by studying an analogous system built out of QCD, where sector *A* undergoes a radical, but well hidden, change in its degrees of freedom, from light pions at low energies to quarks and gluons at high energies. The tentative conclusion drawn for the real CCP was that a drastic change in gravitational degrees of freedom above 10^{-3} eV, akin to

*Email address: sundrum@pha.jhu.edu

[1]Such an extrapolation should certainly not be taken for granted, as dramatically illustrated by noting that present data cannot distinguish a theory with a gravity-only large extra dimension with a size of order 0.1mm (similar to the well-known proposal of Ref. [4] with two extra dimensions) from the usual 4D theory.

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compositeness,[2] could suppress high energy SM contributions to $\Gamma_{eff}[g_{\mu\nu}]$. If this is correct, then even though we know much more than the aliens about the SM, we have every reason to be as excited about submillimeter tests of gravity [7,8] where the new degrees of freedom would be revealed.

The present paper will further discuss the case for such a resolution to the CCP by a ''fat graviton,'' and how it can be tested experimentally in the near future. We do not construct a fundamental theoretical model of fat gravitons and pointlike matter here. Instead we pursue a more modest goal. We argue that such a resolution to the CCP, and the relevance therefore of short-distance gravity experiments, is not ruled out by general considerations and principles, despite the fact that these considerations seem at first sight to strongly exclude any such scenario.

Many aspects of self-consistency are addressed by an effective field theory formalism we will call soft graviton effective theory (SGET). It blends together aspects of Weinberg's analysis of soft graviton couplings [9] with standard heavy particle effective theory [10]. It is hoped that the development of such an effective field theory description will make the ideas precise enough to pursue more fundamental model building, say within string theory, or to identify and pursue phenomenological implications. On the other hand, the more precise description of fat gravity may lead to falsification, either by experimental means or by proving ''no-go'' theorems. At least we will know for sure then that the door is shut.

The discussion of the CCP in this paper suffers from some significant limitations. There are issues related to the CCP which involve cosmological time evolution. The discussion here takes a rather static view of the problem, focusing on SM quantum corrections. A key diagnostic tool for any new mechanism for the CCP is to consider its behavior in cases where there are multiple (metastable) vacua. It is certainly very interesting to pursue these considerations in the case of the present proposal, but the result is not yet conclusive and a discussion is deferred for later presentation. There is undeniably a new scale in gravity provided by the observed dark energy density [2]. While in this paper it is related to the ''size'' of the fat graviton, this size is not predicted from other considerations but taken as input. Its constancy over cosmological times is also not determined.

The proposition that the small vacuum energy density might translate by naturalness into a submillimeter scale for new gravitational physics was made in Ref. [11] (although the primary subject of Ref. [11] is a quite different approach to the CCP). The idea that this new physics involves a submillimeter breakdown in pointlike gravity [5] has been further discussed in the extra-dimensional proposal of Ref. [12].

The layout of this paper is as follows. Section II reanalyzes the robustness of the CCP in standard effective field theory and how it rests on the presumption that the graviton is pointlike and able to mediate hard momentum transfers. Section III considers the consequences of rejecting this presumption, that is, entertaining the possibility of a ''fat graviton.'' We see that there is now a loophole in the CCP, but that the macroscopic consequences of GR and the equivalence principle are necessarily preserved. Section IV discusses experimental/observational contraints and predictions following from a fat graviton resolution of the CCP. In particular, naturalness predicts a non-zero cosmological constant, now however set only by the graviton ''size.'' There is rather a sharp prediction for where fat graviton modifications of Newton's law should appear, and the qualitative form they take. In Section V, we begin construction of a soft graviton effective theory which satisfies basic principles, captures the physics of hard SM processes as well as soft graviton exchanges between SM matter, but *does not extrapolate standard GR to short distances*. It is demonstrated that this effective theory, capable of minimally capturing our present experimental regimes, does not give robust contributions to the cosmological constant from heavy SM physics, thereby clarifying the fat-graviton loophole. Section VI provides conclusions.

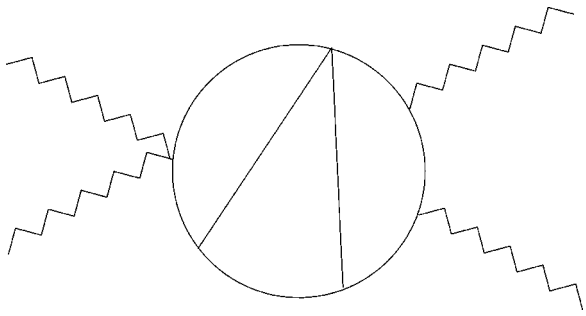

FIG. 1. Typical SM quantum contributions to $\Gamma_{eff}[g_{\mu\nu}]$. Jagged lines are gravitons and smooth lines are SM particles.

II. ROBUSTNESS OF THE CCP

The quantum contributions to the cosmological constant which dominate in standard effective field theory, and appear most robust, arise from Feynman diagrams such as Fig. 1, with SM matter loops and very soft graviton external lines. Diagrams with different numbers of external gravitons correspond to different terms in the expansion of the cosmological term about flat space,

$$\sqrt{-g}=1+\frac{h^{\mu}_{\mu}}{2M_{Pl}}+\ldots,\quad g_{\mu\nu}\equiv\eta_{\mu\nu}+\frac{h_{\mu\nu}}{M_{Pl}}. \qquad (2.1)$$

Of course the tadpole term implies that flat space is destabilized by a cosmological constant, but as long as it is small the flat space diagrams still provide a convenient way to expand the leading effects.

Diagrams such as Fig. 1 make $\mathcal{O}(\Lambda_{UV}^4/16\pi^2)$ contributions to the effective cosmological constant, when Λ_{UV} is taken to be a typical general coordinate invariant (GCI) cutoff. For $\Lambda_{UV}\geqslant$TeV, the contribution is many orders of mag-

[2]The graviton could not literally be a composite of a Poincaré invariant quantum field theory, by the theorem of Ref. [6]. However, the physical manifestations of compositeness are compatible with the graviton, as string theory perfectly illustrates.

nitude larger than the observed cosmological constant and the situation is highly unnatural. To search for the most robust contributions to the cosmological constant let us be more optimistic about the sensitivity to the true nature of the physics above a TeV which cuts off the diagrams (perhaps some type of stringiness). A simple way to do this is to calculate using dimensional regularization. However, this still results in finite contributions to the vacuum energy from known SM masses and interactions,

$$\sim \sum_{SM} \frac{(-1)^{F_{SM}}}{16\pi^2} m_{SM}^4 \ln m_{SM} + \mathcal{O}\left(\frac{\lambda_{SM}}{(16\pi^2)^2} m_{SM}^4\right), \tag{2.2}$$

which are still so large that the CCP is not much diminished. Here, the first term is due to just the free particle zero-point energies, while the second term is sensitive to SM couplings, λ_{SM}. These contributions of diagrams such as Fig. 1 seem theoretically very robust. After all, the couplings of the graviton lines are being evaluated for soft momenta, precisely where we are most confident about their couplings to SM matter given the experimental success of GR. We can therefore use Eq. (2.1) to evaluate the diagrams with any number of graviton legs, the one with no legs being the simplest way to compute the cosmological constant contribution of course. The remainder of the calculation involves the propagation and soft and hard quantum interactions of SM particles. Again, we have tested all this extensively in particle physics experiments up to a TeV. Yet it is the purpose of this paper to look for loopholes in the apparent robustness of the contributions to the cosmological constant from known SM physics.

Let us digress here from the main thrust of this paper to briefly discuss another well-known approach to the CCP which naively avoids the robustness of the contributions, Eq. (2.2). In this approach, GCI is replaced as the guardian symmetry of massless gravitons by special coordinate invariance (SCI) [13] (for a review see Ref. [1]), consisting of only coordinate transformations with unit Jacobian and metrics with $\sqrt{-g}=1$. In this approach, the cosmological constant appears as an extra integration constant of the (quantum) equations of motion, rather than being dynamically determined. In this way the CCP becomes an issue of *initial conditions* and does not relate to quantum corrections or, by naturalness, to any testable new gravitational physics (which would be a shame, but of course this is not an argument against the idea). Even with SCI, however there is a formal (and quite possibly physically relevant in a more fundamental description of gravity) objective meaning to Eq. (2.2). It follows by thinking of all m_{SM} as having their origins as the vacuum expectation values of some source external fields, which may even vary somewhat in different parts of the universe. We can formally write

$$m_{SM} = \langle \chi(x) \rangle. \tag{2.3}$$

Then Eq. (2.2) does contribute to dark energy, and if χ varies in different parts of the universe so do these contributions. In such a setting, the extra integration constant of SCI remains

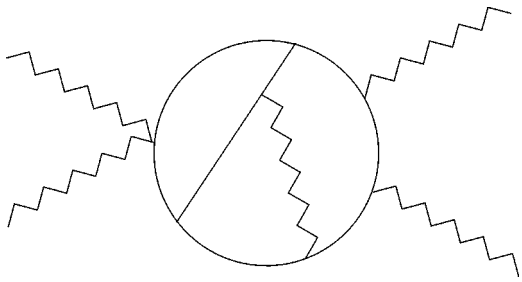

FIG. 2. Typical contribution to Γ_{eff} involving SM as well as quantum gravity corrections.

an exact spacetime constant which adds to the theoretically distinct and robust effects of Eq. (2.2). From this perspective, there is not much difference between the implications of GCI and SCI, except that the ability in effective field theory to simply add an arbitrary cosmological constant counterterm to any particular model is replaced by the ability to add the effect of an arbitrary integration constant. The need to fine-tune away quantum corrections from Eq. (2.2) remains intact, although with SCI this is done using the integration constant. From now on we return to taking GCI as the symmetry protecting the massless graviton.

There are also subleading contributions (for $\Lambda_{UV} < M_{Pl}$) from diagrams such as Fig. 2, with graviton lines in the quantum loops. These quantum gravity contributions to the cosmological constant of $\mathcal{O}(\Lambda_{UV}^6/(16\pi^2 M_{Pl}^2))$ are still significant from the point of view of naturalness (for $\Lambda_{UV} \geqslant$ TeV) but one might hope that our imperfect understanding of quantum gravity makes these contributions a less robust problem than diagrams such as Fig. 1. Further, they are certainly Planck suppressed, and it is consistent for us to first neglect these effects and tackle instead the leading quantum corrections. In this paper, for simplicity we will neglect quantum gravity corrections all together, deferring a treatment of this topic for later presentation.

Let us return to consider the dominant contributions coming from purely SM loops. When we allow general graviton momenta, Fig. 1 is a contribution to the gravitational effective action, $\Gamma_{eff}[g_{\mu\nu}]$, not just the cosmological constant. Let us ask, since the diagram is the cause of such concern, why we bother to include its contribution to $\Gamma_{eff}[g_{\mu\nu}]$ at all. A first response is that we are simply following the Feynman rules, but let us inquire more deeply what fundamental principles are at stake if we simply throw out these diagrams, but not the (well-tested) loop diagrams contributing to SM processes. Three principles stand out.

(I) Unitarity: Fig. 1 unitarizes lower-order tree and loop processes of the form gravitons $\rightarrow$ SM ($+$ gravitons). That is, Fig. 1 has imaginary parts for general momenta following from unitarity and these lower order processes. We have not yet seen such processes experimentally. Furthermore, when there are massive SM particles in the loops, the imaginary parts only exist once the external graviton momenta are above the SM thresholds. These are momenta for which quite generally we have not probed gravity (except that we know it is still so weak as to be invisible in experiments). If gravity is

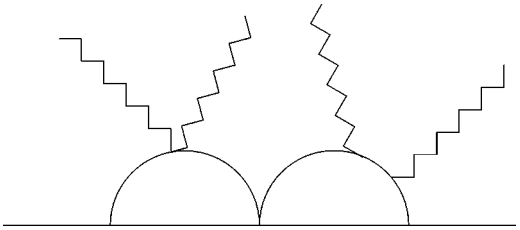

FIG. 3. Soft gravitons coupled to loop correction to SM self-energy.

radically modified below such SM thresholds then we would have to radically modify diagrams such as Fig. 1.

(II) GCI: There are diagrams without imaginary parts in any momentum regime, but which are required when we include diagrams with imaginary parts so as to maintain the GCI Ward identities, ultimately needed to protect theories of massless spin-2 particles. Note however that throwing out *all* SM contributions to $\Gamma_{eff}[g_{\mu\nu}]$ is a perfectly GCI thing to do.

(III) Locality: In standard effective field theory one also has non-vacuum SM diagrams with soft gravitons attached, such as Fig. 3, where soft gravitons couple to, and thereby measure, loop corrections to a SM self-energy. We certainly do not want to throw this away since these contributions are absolutely crucial in maintaining the precisely tested equivalence of gravitational and inertial masses of SM particles and their composites.

Now, when Figs. 3 and 1 are viewed as position space Feynman diagrams (or better yet as first quantized sums over particle histories) it is clear that they are *indistinguishable locally in spacetime*, only globally can we make out their topological difference. Locality of couplings of the point particles in the diagrams does not allow us to contemplate throwing away Fig. 1 which we do not want, while retaining Fig. 3 which we need. The gravitons cannot take a global view of which diagram they are entering into before ''deciding'' whether to couple or not. Thus our earlier argument for the robustness of the CCP hinges on locality. We can dissect diagrams contributing to $\Gamma_{eff}[g_{\mu\nu}]$ into small spacetime windows, and all the ingredient windows are well tested in other physical processes, albeit in globally different ways. We might contemplate dispensing with locality, but it seems to be the only way we explicitly know to have point-particle dynamics in a relativistic quantum setting. However, see Refs. [14] for an approach to the CCP with submillimeter non-locality, as well as Refs. [11] and [15] for extremely non-local approaches to the CCP.

III. ROOM FOR A FAT GRAVITON

A. Basic notions

If some particles appearing in the Feynman diagrams are secretly extended states then the constraints of locality, and the consequent robustness of the CCP, are weakened. Since we have tested the pointlike nature of SM particles to very short distances, the only candidate for an extended state is the graviton itself. Indeed, direct probes of the ''size'' of the graviton only bound it to be smaller than 0.2mm, following from short-distance tests of Newton's law [7]. Let us grant the graviton a size,

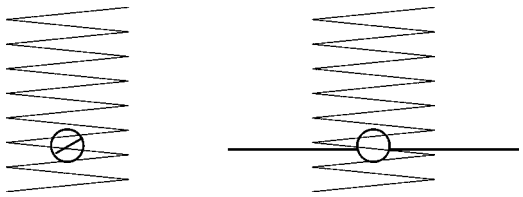

FIG. 4. A fat graviton can take a more global view of massive SM loop processes.

$$\ell_{grav} \equiv 1/\Lambda_{grav}\,. \tag{3.1}$$

Such a ''fat graviton'' does not have to couple with pointlike locality to SM loops, but rather with locality up to ℓ_{grav}. In particular for $m_{SM} \gg \Lambda_{grav}$ a fat graviton can couple to SM loops *globally*, thereby evading reason (III) for the robustness of the CCP. To see this, note that locality up to ℓ_{grav} corresponds to standard locality of SM loops when only graviton wavelengths $> \ell_{grav}$ are allowed. Of course, for $m_{SM} \gg \Lambda_{grav}$ and graviton momenta $< \Lambda_{grav}$, diagrams such as Fig. 1 can be expanded as a series in the external momenta, that is a set of local vertices in spacetime. A cartoon of all this is given in Fig. 4.

Thus a theory with a fat graviton could distinguish between Figs. 3 and 1, possibly suppressing Fig. 1 while retaining Fig. 3 needed for the equivalence principle. It is at least conceivable.

One naive objection is that among the diagrams contributing to the cosmological constant is the one with no graviton external legs, corresponding to the first term on the right-hand side of Eq. (2.1), that is, pure SM vacuum energy. Since the graviton does not appear in the diagram the size of the graviton appears irrelevant and incapable of suppressing the contribution. However, physically the cosmological term is a self-interaction of the graviton field (defined about flat space say). Once we trust pointlike diagrams we can use GCI to relate all of them for soft gravitons to the diagrams with no gravitons and then it becomes a mathematical convenience to compute this latter class of diagrams. They do not have any direct physical significance except as a short-hand for the diagrams with gravitons interacting. If the diagrams with graviton external lines are modified because gravitons are fat, there is no meaning to the diagram with no external lines. Indeed, notice that there is no physical consequence in an effective lagrangian if in addition to a cosmological constant multiplying $\sqrt{-g}$, we add a pure constant term with no gravitational field attached. When we expand about flat space the extra constant modifies the first term in Eq. (2.1). This shows that the cosmological term only has physical importance as a graviton coupling, and the size of the graviton is most certainly relevant to how SM loops can induce it.

Let us now return to the issue (I) of the need to unitarize processes, gravitons $\rightarrow$ SM states ($+$ gravitons). We can most easily deal with this by making a precise assumption for what is an intuitive property of fat objects. We assume

that hard momentum transfers $\gg\Lambda_{grav}$ via gravitational interactions are essentially forbidden, that is they are extremely highly suppressed even beyond the usual Planck suppressions of gravitational interactions. In particular for $m_{SM}\gg\Lambda_{grav}$, gravitons$\rightarrow$SM states (+ gravitons) is suppressed, and the related unitarizing SM loop contributions, as well as GCI-related diagrams, are not required.

Throwing out all massive SM loop contributions to $\Gamma_{eff}[g_{\mu\nu}]$ is entirely consistent with the GCI of the soft graviton effective field theory below Λ_{grav}.

B. Naturalness of the equivalence principle

While we have seen that the general considerations (I – III) for the robustness of massive SM loop contributions to the cosmological constant, and indeed the whole of $\Gamma_{eff}[g_{\mu\nu}]$, are evaded by a fat graviton in principle, one can ask whether it requires fine-tunings even more terrible than the original CCP in order to understand why the fat graviton manages to couple to self-energies of SM states in the precise way to maintain the equivalence between gravitational and inertial mass. After all, these self-energies, for example the mass of a proton or of hydrogen, are determined by short-distance physics $\ll\ell_{grav}$, which unlike a pointlike graviton, a fat graviton cannot probe. In fact we shall see that there is really no option but that soft fat gravitons couple according to the dictates of the equivalence principle. The only miracle is that the fat graviton has a mode which is massless with spin 2 and couples somehow to matter. The only way for soft massless spin-2 particles to consistently couple is under the protection of GCI [16], which in turn leads to the equivalence principle macroscopically. The energy and momentum of SM states are macroscopic features which the fat graviton could imaginably couple to. They are determined microscopically but are measurable macroscopically, just as one can measure the mass density of a chunk of lead macroscopically, even though this density has its origins in and is sensitive to microscopic physics. In fact in Section 5 we will show how the leading couplings of gravity to SM masses and interaction energies can be recovered as a consequence only of GCI below Λ_{grav}, forced on us once we accept that our fat object contains an interacting massless spin-2 mode.

C. Soft and hard SM effects

Until now, we have been careless of the complication that the SM contains particles which can be lighter than Λ_{grav}, such as the photon, as well as particles which are heavier. Even with the fat graviton we cannot throw away loops of these lighter SM fields contributing to $\Gamma_{eff}[g_{\mu\nu}]$ because the soft components of these fields are part of a standard effective field theory, including GR below Λ_{grav} (that is, the effective theory known to the aliens). Soft gravitons can therefore scatter into soft electromagnetic radiation, so there are unitarity-required loops in the gravitational effective action. Indeed, we know that soft massless SM loops are not local on the scale Λ_{grav}, that is such loops are not expandable as local vertices for soft graviton momenta. Therefore, even a fat graviton cannot couple globally to such loops and the arguments for their robustness from locality now apply. A further complication is that the soft SM particles can couple to hard or massive SM particles.

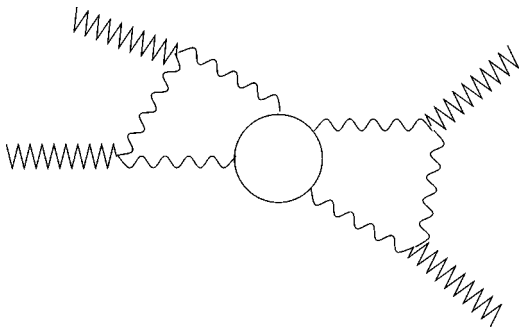

FIG. 5. A QED loop contribution to $\Gamma_{eff}[g_{\mu\nu}]$. Wavy lines are photons and solid lines are electrons.

Let us consider a concrete example from QED coupled to soft gravitons, Fig. 5. We cannot throw away this whole contribution to the gravity effective action because even for soft graviton external lines there are imaginary parts which unitarize soft gravitons $\rightarrow$ soft photons processes (and their reverse), as well as soft photon-photon scattering due to the electron loop. On the other hand if we keep this diagram, we get a contribution to the cosmological constant set by the electron mass, which is too big. The way too disentangle the hard and soft SM contributions is to simply do effective field theory below $\Lambda_{grav}\ll m_e$, where the soft photons ''see'' the electron loop as a local vertex, Fig. 6, the leading behavior being of the rough form

$$\mathcal{L}_{eff}\ni\alpha_{em}^2\frac{F^4}{m_e^4}, \tag{3.2}$$

$F_{\mu\nu}$ being the electromagnetic field strength. Thus we recover all the same soft graviton and photon imaginary parts of Fig. 5 with the diagram of Fig. 7, but now the contribution to the cosmological constant vanishes when we compute with dimensional regularization, since there is no mass scale in the propagators of the diagram itself, only in the overall coefficient.

Thus a more precise statement is that with fat gravitons, massive SM ($m_{SM}\gg\Lambda_{grav}$) and hard light SM pieces of loop contributions to $\Gamma_{eff}[g_{\mu\nu}]$ may consistently be suppressed while soft light SM contributions are not. ''Consistent'' re-

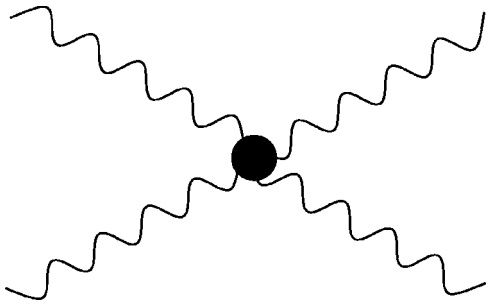

FIG. 6. Local effective light-by-light scattering vertex for soft photons.

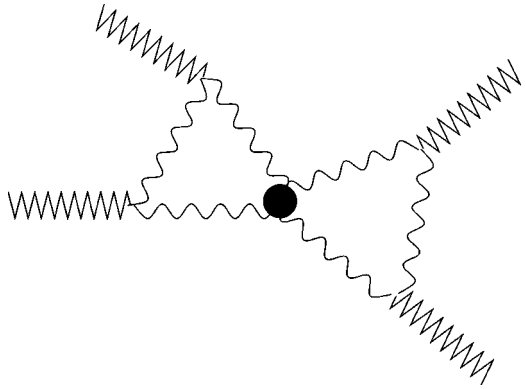

FIG. 7. An effective diagram with the same long distance physics as Fig. 5.

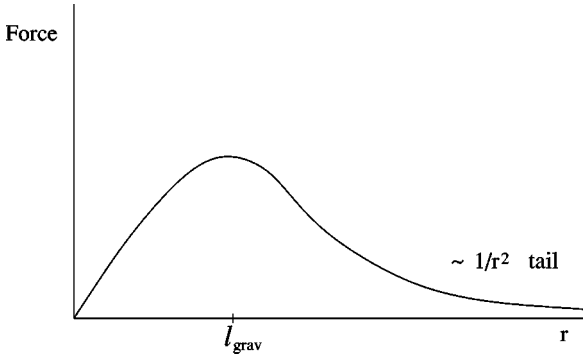

FIG. 8. The qualitative short-distance modification of Newton's law due to a fat graviton of size ℓ_{grav}.

fers to the principles we have discussed before, unitarity and GCI relations in the regime we trust GR as an effective field theory now, namely $<\Lambda_{grav}$, and locality down to ℓ_{grav}. There are no robust contributions to the cosmological constant from mass scales above Λ_{grav}.

D. The cosmological constant in fat gravity

Below Λ_{grav}, we have a standard effective field theory of GR coupled to SM light states. Here, *all* these states behave in a pointlike manner. At the edge of this effective theory there are Λ_{grav}-mass vibrational excitations of the fat graviton, since in relativistic theory an extended object cannot be rigid. At least in this standard effective field theory regime the quantum contributions to the cosmological constant should follow by the usual power counting, that is $\sim\mathcal{O}(\Lambda_{grav}^4/16\pi^2)$. The details however depend on the details of the fat graviton. Thus in a fat graviton theory naturalness implies that the lower bound on the full cosmological constant is $\sim\mathcal{O}(\Lambda_{grav}^4/16\pi^2)$.

IV. EXPERIMENTAL/OBSERVATIONAL CONSTRAINTS/ PREDICTIONS

When we apply the above naturalness bound for a fat graviton to the observed dark energy [2,3], we can derive a bound on the graviton size,

$$\ell_{grav}>20\ \mu\text{m}. \tag{4.1}$$

Of course since the naturalness bound involved an order of magnitude estimate for the fat graviton quantum corrections to the cosmological constant, the bound on ℓ_{grav} is not an exact prediction. However, it is a reasonably sharp prediction because much of the uncertainty is suppressed upon taking the requisite fourth root of the dark energy. If we can experimentally exclude ℓ_{grav} being in this regime we can falsify the idea that the fat graviton is (part of) the solution to the CCP.

How can we probe ℓ_{grav}? This is the finite range of the vibrational excitations of the fat graviton. They can therefore mediate deviations from Newton's law at or below ℓ_{grav}. The precise details of the deviations are sensitive to the unknown fat graviton theory, but the qualitative behavior follows from the essential feature of the fat graviton as unable to mediate momentum transfers harder than Λ_{grav}. In position space this implies Fig. 8, with the gravitational force being suppressed at distances below ℓ_{grav}. Of course this would be a striking signal to observe in submillimeter tests of gravity. It sharply contrasts with the rapid short distance enhancement expected in theories with (only) large extra dimensions for gravity [4]. Indeed there is no natural way (in the absence of supersymmetry [17]) to have such suppression within standard pointlike effective field theory. Present bounds from such tests [7] demonstrate that $\ell_{grav}<0.2$mm, so there is a fairly narrow regime to be explored in order to rule in or rule out a fat graviton approach to the CCP.

V. SOFT GRAVITON EFFECTIVE THEORY

A. Basic notions

Can the graviton really be fat when SM matter is pointlike? Of course the only way to confidently answer in the affirmative is to build a consistent fundamental theory of this type. It is presently not known how to do this, say within string theory, the only known fundamental theory of any type of quantum gravity, where $\ell_{grav}=\ell_{string}$. One might worry that there is some sort of theorem anwering in the negative, that in a regime of pointlike matter the graviton must be pointlike too. Such a theorem would have to show that our macroscopic tests of GR and microscopic tests of SM quantum field theory imply robust (pointlike) features of microscopic gravity. The best way to argue against such a theorem is to construct a consistent effective field theory description which encapsulates precisely the present asymmetric experimental regimes for gravity and matter, but which does not commit itself to the details of microscopic gravity such as whether the graviton is pointlike. In particular, it would not suffer the usual CCP. We now turn to such a construction, generalizing the methods of heavy particle effective theory [10]. Our discussion is closely related to the analysis of soft graviton couplings in Ref. [9]. Earlier discussions of heavy particle effective theory applied to GR appear in Refs. [18] and [5].

SGET is constructed from the fat graviton's ''point of view.'' While the fat graviton can itself only mediate soft

momentum transfers, it can ''witness'' and couple to hard momentum transfers taking place within the SM sector. The momentum of a freely propagating SM particle can be expressed in terms of its 4-velocity,

$$p_\mu = m v_\mu, \quad v^2 = 1, \; v_0 \geqslant 1. \tag{5.1}$$

Interactions with fat gravitons can change this momentum but by less than Λ_{grav},

$$p_\mu = m v_\mu + k_\mu, \quad |k_\mu| < \Lambda_{grav}. \tag{5.2}$$

The basic idea of the effective theory is to integrate out all components of the SM field which are not of this form, that is far off shell. The result is all that the fat graviton can ''see'' and couple to. In this regime, many SM loop effects which used to appear non-local in spacetime will appear local to the fat graviton. When coupling to gravity we will consider integrated out all massive vibrational excitations of the fat graviton, leaving only the soft massless graviton mode. We know that GCI is a necessary feature for basic consistency of the couplings of gravitons softer than Λ_{grav} [16]. The IR use of GCI will be enough to recapture the standard macroscopic tests of GR. Integrating out only SM fields which are far off shell will not eliminate hard but on-shell SM particles, and therefore when properly matched, the effective theory should reproduce the high energy *S* matrix of the SM as well. That is, the effective theory must reproduce what we have seen of nature, hard matter coupled to soft gravity.

A very important distinction should be made. One might consider for example a proton, propagating along, interacting only with gravity. In the effective field theory, we must treat the proton as an elementary particle, not a composite of far off-shell quarks and gluons. All these off-shell particles are integrated out but the elementary proton is integrated ''in'' to match the usual SM physics. Intuitively, the fat graviton cannot distinguish the substructure of the proton. In this manner, there will be many more ''elementary'' fields in the effective field theory than in the usual SM quantum field theory.

In this paper a simplified problem is considered, where the SM is replaced by a toy model consisting of a single massive real scalar field, ϕ, with $\lambda\phi^4$ coupling. The scalar has its own ''hierarchy'' fine-tuning problem, but we will ignore this irrelevant issue here. The central limitation is not the restriction to spin 0 (non-zero spin is tedious but straightforward), but rather the absence of light SM particles. We will rectify this omission elsewhere. We will refer to the renormalizable toy model as the ''SM'' or ''fundamental theory'' for the remainder of this section, hopefully without causing confusion. We will also defer here the consideration of soft graviton quantum corrections, where soft graviton lines carry loop momenta. Again this will be presented elsewhere.

B. Single heavy particle effective theory

To begin, let us consider a state consisting of a single massive ϕ particle. We describe it with an effective field $\phi_v(x)$ which respects the split of momentum in Eq. (5.2). The fact that the 4-velocity v is formally unaffected by gravity means that it is just an unchanging label for the field, while the Fourier components of $\phi_v(x)$ correspond to the ''residual'' momentum, $k_\mu < \Lambda_{grav}$, that alone can fluctuate with gravity interactions. However the split of Eq. (5.2) has an inherent redundancy formalized in terms of a symmetry known as reparametrization invariance (RPI) [19]:

$$\begin{aligned} v &\to v + \delta v, \\ k &\to k - m\,\delta v, \end{aligned} \tag{5.3}$$

where δv is an infinitesimal change in velocity, $\delta v \cdot v = 0$. Obviously this transformation results in the same physical momentum p and therefore the effective theory must identify the pairs (v,k) and $(v+\delta v, k - m\,\delta v)$.

Let us begin in flat space, without gravity. In terms of the effective field, RPI requires the identification

$$\phi_v(x) \leftrightarrow e^{im\,\delta v \cdot x}\phi_{v+\delta v}(x). \tag{5.4}$$

The simplest way to implement this is to treat this RPI as a ''gauge'' symmetry and ensure that the effective Lagrangian is invariant under it. One must then be careful to choose only one element of any ''gauge orbit'' when extracting physics. It is straightforward to see that a covariant derivative given by

$$D_\mu = \partial_\mu + i m v_\mu \tag{5.5}$$

is required in order to build RPI effective Lagrangians. For an isolated SM particle we need only consider quadratic Lagrangians. Using the fewest derivatives (corresponding to the fewest powers of $k_\mu/m < \Lambda_{grav}/m$) we have

$$\begin{aligned} \mathcal{L}_{eff} &= \frac{1}{2m}|D_\mu \phi_v|^2 - \frac{m}{2}\phi_v^\dagger \phi_v \\ &= \phi_v^\dagger \left[iv\cdot\partial + \frac{\partial^2}{2m} \right] \phi_v, \end{aligned} \tag{5.6}$$

where we integrated by parts to get the last line. In the first line we chose a particular linear combination of two RPI terms. The overall coefficient is just a conventional wave function renormalization, but the relative coefficient is chosen to satisfy the physical requirement that the propagator have a pole at $k = 0$, given the interpretation of Eq. (5.2).

Formally, in the derivative expansion (k/m expansion) the dominant term in $\mathcal{L}_{eff}$ is $\phi_v^\dagger iv\cdot\partial\phi_v$, while the subleading term, $\phi_v^\dagger(\partial^2/2m)\phi_v$ can be treated perturbatively, that is, as a higher derivative ''interaction'' vertex. The effective propagator is then given by

$$\frac{i}{k\cdot v + i\epsilon}. \tag{5.7}$$

This treatment will suffice for most of the examples given below. An important property of the effective propagator is that it contains a single pole, rather than the two poles at positive and negative energy of a standard field theory propagator. The reason is simple to understand. Given that (when

gravity is finally included) the maximum residual momentum k is $<\Lambda_{grav}$, even though the sign of k_0 is not fixed the sign of the total energy p_0 is clearly positive. Fat gravitons cannot impart the momentum transfers needed to go near the usual negative energy pole. Without negative total energies the effective field is necessarily complex, as indicated, and ϕ_v is purely a creation operator while $\phi_v^\dagger$ is purely a destruction operator. Such a split would look non-local in a fundamental quantum field theory, but not in the SGET ''seen'' by the fat graviton.

Sometimes one studies processes where components of k orthogonal to v are larger than $k.v$, such that one cannot treat $\phi_v^\dagger(\partial^2/2m)\phi_v$ perturbatively. It would seem then that if one uses all of Eq. (5.6) to determine the propagator one would find two poles again. However, in these circumstances one only needs to resum the $\partial_{\perp v}^2/2m$ part of the $\partial^2/2m$ term in the propagator. The other $(v\cdot\partial)^2/2m$ piece of $\partial^2/2m$ would then be of even higher order and could still be treated as a perturbtation. In this way the resulting propagator,

$$\frac{i}{k\cdot v + k_{\perp v}^2/2m + i\epsilon}, \tag{5.8}$$

again has a single pole. We will see such an example in what follows.

Let us now couple soft gravity to $\mathcal{L}_{eff}$. Interactions for soft massless spin-2 modes of the fat graviton only make sense if protected by GCI, so our effective theory must be exactly GCI. The only assumption is that there is a sensible theory of a fat object with a massless spin-2 mode coupling to matter. To determine the possible couplings we must first determine the spacetime transformation properties of $\phi_v(x)$. Naively, one would guess that since the particle has spin 0, in flat space $\phi_v(x)$ is a scalar field of Poincaré invariance, and becomes a GCI scalar field once we turn on gravity. However the first presumption is incorrect. To see this consider a flat space Poincaré transformation defined by

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu. \tag{5.9}$$

Restricting to an *infinitesimal* transformation of this type we have

$$\begin{aligned}\phi_v(x) &\to \phi_{\Lambda v}(\Lambda x + a)\\ &= \phi_{v+\delta v}(\Lambda x + a)\\ &\equiv e^{-im\delta v\cdot(\Lambda x+a)}\phi_v(\Lambda x + a),\end{aligned} \tag{5.10}$$

where in the second line we have used the fact that for an infinitesimal Lorentz transformation we can always write Λv in the form $v+\delta v$ where $v\cdot\delta v = 0$, and in the last line we have used the RPI equivalence relation, Eq. (5.4). This is not the transformation property of a Poincaré scalar field. But clearly $e^{imv\cdot x}\phi_v(x)$ *is* a Poincaré scalar. When we couple to gravity, it is this combination that remains a scalar field.

A generally covariant derivative of the scalar is easy to form,

$$\partial_\mu[e^{imv\cdot x}\phi_v(x)] = e^{imv\cdot x}(\partial_\mu + imv_\mu)\phi_v(x). \tag{5.11}$$

Thus the combination $D_\mu \equiv (\partial_\mu + imv_\mu)$ here is forced on us by both GCI and RPI. The leading GCI and RPI effective Lagrangian is then given by

$$\mathcal{L}_{eff} = \sqrt{-g}\left\{\frac{g^{\mu\nu}}{2m}D_\mu\phi_v^\dagger D_\nu\phi_v - \frac{m}{2}\phi_v^\dagger\phi_v\right\}. \tag{5.12}$$

To leading order in the expansions in k/m and m/M_{Pl}, this yields

$$\mathcal{L}_{eff} = \phi_v^\dagger\left\{iv\cdot\partial - \frac{m}{2M_{Pl}}v_\mu v_\nu h^{\mu\nu}\right\}\phi_v, \tag{5.13}$$

which reproduces the standard equivalence of gravitational and inertial mass.

Note that in our derivation of this equivalence we did not make use of the existence of standard GR at short distances, even though short distance physics may well contribute in complicated ways to the mass of the SM state.

Let us now ask what robust loop contribution the effective theory makes to the cosmological constant. Since the effective theory does have GCI we can just calculate the pure SM vacuum energy with no graviton external legs. Apparently this requires us to interpret the expression,

$$\int d^4k\ \ln(k\cdot v + i\epsilon). \tag{5.14}$$

Note that there is no physical SM mass scale in this expression so there is no robust contribution from known physics here at all even though the effective theory does reproduce the coupling of massive SM particles to gravity. We can simply set the above expression to zero. This can be thought of as normal ordering. The reason for having no robust contribution to the cosmological constant is because the effective theory of the soft gravitons does not know whether the graviton is fat or not, or indeed whether the heavy matter is highly composite, say the Earth (if the gravitons are very soft), or solitonic like a 0-brane in string theory. All these possibilites lead to and effective theory of the same form. The effective theory cannot commit to (even the rough size of) a cosmological constant contribution without knowing the differentiating physics which lies beyond itself.

C. Effective theory with SM interactions

Let us now consider how to generalize the effective theory for processes involving several interacting SM particles coupled to soft gravitons. The SGET general form can be compactly expressed,

$$\begin{aligned}\mathcal{L}_{eff} = \sqrt{-g}\sum_{v,v'}\frac{\kappa_{vv'}(D_\mu/m)}{m^{3/2(N+N')-4}}\phi_{v_1'}^\dagger\cdots\phi_{v_{N'}'}^\dagger\ \phi_{v_1}\cdots\\ \times\phi_{v_N}\exp^{[im(\Sigma v-\Sigma v')\cdot x]},\end{aligned} \tag{5.15}$$

where there are dimensionless coefficients $\kappa_{vv'}$ which in general can contain GCI and RPI derivatives acting on any

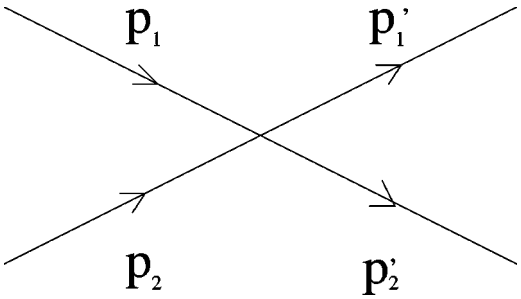

FIG. 9. Two-particle tree-level scattering. Arrows indicate incoming and outgoing nearly on-shell external states.

of the effective fields, contracted with the inverse metric, $g^{\mu\nu}$. Of course the non-trivial parts of such derivatives correspond to residual momenta, k, balanced by powers of $1/m$. Therefore they are relevant for subleading effects in Λ_{grav}/m.

The general procedure for specifying the (leading terms) of the SGET is to match the effective theory to the fundamental SM in the absence of gravity, and then to covariantize minimally with respect to (soft) gravity. The matched correlators are those with nearly on-shell external lines, Eq. (5.2). This will reproduce the soft graviton amplitudes of the fundamental SM directly coupled to gravity in the standard way, but now without any reference to pointlike graviton couplings. Therefore it is compatible with a having a fat graviton whose massless mode is protected only by an infrared GCI. Note that the hard SM momentum transfers are to be described in the effective theory by $v_i \to v_j'$, that is by a change of labels. Changes in effective field momenta, that is residual momenta, are necessarily soft. This is how a fat graviton sees SM processes, the hard momentum transfers are a given feature of such processes which the fat graviton cannot influence. See Ref. [20] for an effective field theory formalism exhibiting similar dynamical label changing in the context of non-relativistic QCD.

It is unusual to see explicit x dependence in (effective) Lagrangians such as appears in the phase factor. Here it is required by overall momentum conservation, rather than conservation of residual momenta. More formally, it is required by RPI, as well as GCI (recalling that it is $\phi_v e^{imv\cdot x}$ which is a scalar of GCI). It is possible, and perhaps prettier, to partially gauge fix the RPI, by reparametrizing (as can always be done) such that $\Sigma v = \Sigma v'$, so that the phase factor $\to 1$ without compromising GCI. We will call this the ''label-conserving gauge.'' However, when loops are considered we will find it convenient (but not necessary) to depart infinitesimally from this gauge fixing and consider an infinitesimal phase.

Below, we will work to leading order about the limits $M_{Pl}, m, \Lambda_{grav} \to \infty$, $\Lambda_{grav}/m \ll 1$, with m/M_{Pl} fixed. This formal limit simplifies the effective theory. It is similar to doing standard effective field theory calculations, including matching, without an explicit cutoff, even though physically one imagines new physics cutting off the effective theory at a finite scale.

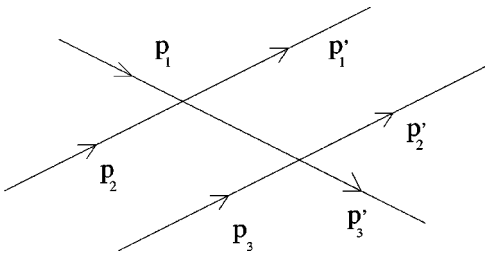

FIG. 10. A three-particle scattering diagram in the fundamental theory.

1. Tree-level matching

Let us follow the general procedure outlined above and first shut off gravity and work in flat space. Consider the tree level diagram for $2 \to 2$ scattering in the fundamental theory, Fig. 9, where the external momenta are nearly on shell. We can then express these external momenta in the form of Eq. (5.2), where we choose the label-conserving gauge,

$$v_1 + v_2 = v_1' + v_2'. \tag{5.16}$$

This amplitude is then straightforwardly matched by including a $2 \to 2$ effective vertex,

$$\mathcal{L}_{eff} \ni \frac{\lambda}{4m^2} \phi^\dagger_{v_1'} \phi^\dagger_{v_2'} \; \phi_{v_1} \phi_{v_2}. \tag{5.17}$$

The $1/4m^2$ factor arises only due to the different normalizations of the interpolating fields between the fundamental and effective theories.

Next, consider the $3 \to 3$ process in the fundamental theory, Fig. 10. Again, the external lines are nearly on shell, so we can express them as

$$p_i = m v_i + k_i, \quad p_i' = m v_i' + k_i', \quad i = 1,2,3, \tag{5.18}$$

with label conservation. The internal line has momentum,

$$p_{int} = m(v_1 + v_2 - v_1') + k_1 + k_2 - k_1'. \tag{5.19}$$

Generically in such hard SM collisions, in the limit $\Lambda_{grav}/m \ll 1$, the internal lines will be far off shell and to leading order we can drop the k's,

$$p_{int} \approx m(v_1 + v_2 - v_1'). \tag{5.20}$$

We can then match the fundamental diagram with an effective vertex, Fig. 11, given by

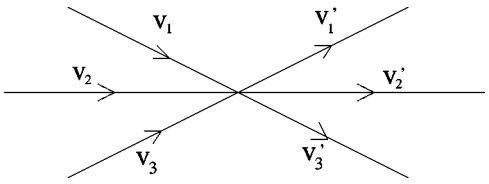

FIG. 11. Effective vertex obtained by integrating out the far off-shell internal line in Fig. 10.

$$\mathcal{L}_{eff} \ni \frac{\lambda^2}{8m^5[(v_1+v_2-v_1')^2-1]}\ \phi^\dagger_{v_1'}\phi^\dagger_{v_2'}\phi^\dagger_{v_3'}\ \phi_{v_1}\phi_{v_2}\phi_{v_3}. \tag{5.21}$$

Notice that what was a fundamentally non-local exchange requiring an off-shell internal line in the fundamental theory, that is with non-analytic dependence on the external total momenta, is replaced in the effective vertex by a local interaction with a non-analytic dependence only on the labels, v,v'. Physically, this is because the process is fundamentally non-local, but is local down to $\Lambda_{grav} \ll m$, that is local "enough" for a fat graviton.

If we had worked to higher order in Λ_{grav}/m, matching would have resulted in higher derivative effective vertices, corresponding to having retained higher powers of k/m in expanding $1/(p_{int}^2-m^2)$ for small $k<\Lambda_{grav}$.

For processes of the form of Fig. 10, there are also exceptional situations which result in p_{int} being nearly on shell. These arise when one considers experiments (in position space) where the interaction region for wave packets of particles 1 and 2 is greatly displaced from the trajectory of the wave packet for particle 3, compared with the size of the wave packets. Thus the three-particle scattering is dominated by a sequence of two-particle scatterings,

$$p_1+p_2\to p_1'+p_{int},$$
$$p_3+p_{int}\to p_2'+p_3', \tag{5.22}$$

where all momenta are nearly on shell. In these exceptional cases we can express $p_{int}=mv_{int}+k_{int}$ with label conservation:

$$v_1+v_2=v_1'+v_{int},$$
$$v_3+v_{int}=v_2'+v_3'. \tag{5.23}$$

In the limits we are considering, the fundamental internal propagator then approaches the effective propagator, Eq. (5.7), up to the convention-dependent field normalization,

$$\frac{1}{p_{int}^2-m^2+i\epsilon}=\frac{1}{2m}\ \frac{1}{k_{int}\cdot v_{int}+k_{int}^2/2m+i\epsilon}$$
$$\approx \frac{1}{2m}\ \frac{1}{k_{int}\cdot v_{int}+i\epsilon}. \tag{5.24}$$

Thus the exceptional fundamental diagram of the form of Fig. 10 is matched by an effective diagram of the same form, but where we use the effective 4-point vertices matched earlier,

$$\mathcal{L}_{eff} \ni \frac{\lambda}{4m^2}\phi^\dagger_{v_1'}\phi^\dagger_{v_{int}}\ \phi_{v_1}\phi_{v_2}+\frac{\lambda}{4m^2}\phi^\dagger_{v_2'}\phi^\dagger_{v_3'}\ \phi_{v_3}\phi_{v_{int}}, \tag{5.25}$$

and the effective propagator, Eq. (5.7), for the internal line.

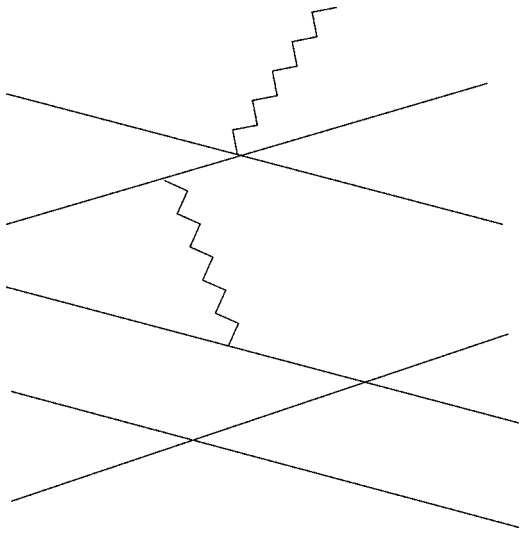

FIG. 12. A soft graviton + hard SM diagram in the effective theory.

More general tree level amplitudes are generally matched by a combination of the two procedures illustrated above, introducing new effective vertices and connecting effective vertices with effective propagators. Tree-level unitarity in the effective theory arises from the imaginary parts of amplitudes due to the $i\epsilon$ prescription when internal lines go on shell, precisely matching the fundamental theory.

There is a situation one can imagine for finite Λ_{grav} where we carefully tune the external momenta on Fig. 10 so that the internal line is intermediate between the two (more generic) situations we have considered of being far off shell or nearly on shell, that is, the internal line is of order Λ_{grav} off shell. In that case our simple procedure does not always allow the fundamental graph to be matched by a local effective vertex or an effective theory graph. It appears that there is a more complicated scheme for matching even these cases within SGET, and that they pose a technical rather than conceptual challenge. We will study these cases elsewhere.

2. Coupling the effective matter theory to soft gravity

Coupling the vertices of $\mathcal{L}_{eff}$ to gravity is very simple. For example, the minimal covariantization of Eq. (5.21) is given by multiplying by $\sqrt{-g}$. If we had worked to higher order in the derivative expansion we would have to also covariantize these derivatives with respect to GCI and contract them using $g^{\mu\nu}$.

We can now use the GCI effective theory to compute hard SM processes coupled to soft gravitons, such as say Fig. 12. The results automatically match with the leading behavior of the analogous diagrams if we coupled soft gravitons directly to the fundamental theory in the usual way. There is no extra tuning of couplings needed to recover standard gravitational results beyond imposing infrared GCI. The dominant couplings of soft gravitons therefore do not distinguish whether (a) the graviton is fat and GCI is only a guiding symmetry of the couplings in the far infrared, or (b) the gravitons are pointlike and GCI governs their couplings to the fundamental theory in the standard way usually assumed.

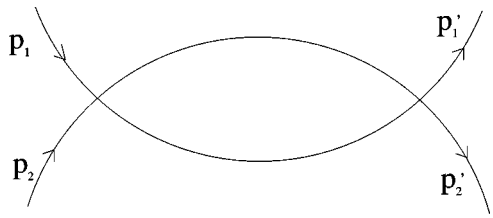

FIG. 13. A simple loop diagram of the fundamental SM theory with nearly on-shell external lines.

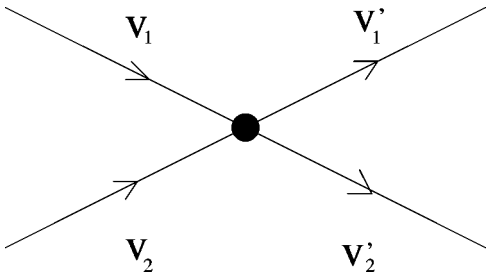

FIG. 14. The form of a one-loop correction to the four-point effective vertex.

3. Matching loops

Here, we will match the simplest fundamental non-trivial loop diagram, Fig. 13, in flat space. It illustrates the essential new complication that loops bring in the presence of a ''gauge'' symmetry like RPI, namely the need to gauge fix and determine the right integration measure. We will first consider the case where the incoming momenta are far away from the two-particle threshold. We can decompose the momenta as usual,

$$p_i = m v_i + k_i, \quad p_i' = m v_i' + k_i', \tag{5.26}$$

with label conservation. Denote the loop amplitude by $\Gamma_{fund}(p,p')$. There is a general result for Feynman diagrams [21], which is straightforward to explicitly check in this example, that Γ_{fund} is locally analytic in the external momenta except when near a threshold, which we assumed above is not the case. That is, $\Gamma_{fund}(mv+k, mv'+k')$ is analytic in k and k' and has a series expansion. Naively, we might try to match the whole diagram in the effective theory by Fourier transforming this series expansion into local operators in the effective theory. However, we cannot do this as it violates hermiticity of the effective lagrangian and ultimately unitarity. To see this focus on the leading term in the expansion and how it would appear as an effective vertex, depicted in Fig. 14, consider,

$$\mathcal{L}_{eff} \ni \sum_{v,v'} \frac{\Gamma_{fund}(v,v')}{4m^2} \phi^\dagger_{v_1'} \phi^\dagger_{v_2'} \phi_{v_1} \phi_{v_2}. \tag{5.27}$$

Hermiticity of the effective Lagrangian requires that the coupling $\Gamma_{fund}(v,v')$ be real, but by unitarity in the fundamental theory or direct calculation we know that $\Gamma_{fund}(v,v')$ has an imaginary part corresponding to the region of integration where the internal propagators are on-shell. Of course we are free to replace $\Gamma_{fund}(v,v') \to \mathrm{Re}\,\Gamma(v,v')$ in Eq. (5.27), but then the imaginary piece must be matched from another source.

Obviously, in the effective theory there is also a diagram of the form of Fig. 13, but where the vertices and propagators are replaced by effective vertices, Eq. (5.17), and propagators, Eq. (5.7). There are two internal momenta now,

$$p_{int} = m v_{int} + k_{int},$$
$$p_{int}' = m v_{int}' + k_{int}', \tag{5.28}$$

so naively the loop momentum integration measure has the form

$$\sum_{v_{int}} \sum_{v_{int}'} \int d^4 k_{int} \int d^4 k_{int}' \;\; \delta^4(m v_{int} + k_{int} + m v_{int}' + k_{int}' - p_1 - p_2). \tag{5.29}$$

This is ill defined, there are too many sums going on because we are multiple-counting combinations (v,k) that should be indentified by RPI. That is the correct measure has the form

$$\frac{\sum_{v_{int}} \sum_{v_{int}'} \int d^4 k_{int} \int d^4 k_{int}'}{\mathcal{R}} \delta^4(m v_{int} + k_{int} + m v_{int}' + k_{int}' - p_1 - p_2), \tag{5.30}$$

where the denominator means to identify RPI related combinations. Our job is to do this by gauge fixing RPI, so that we are summing just one representative of each RPI equivalence class. This is the central subtlety in computing with the effective theory at loop level.

We will fix the following gauge. The total incoming momentum p_1+p_2 is necessarily timelike. We will define our coordinates in its rest frame for convenience, not because the gauge fixing breaks manifest relativistic invariance (which in any case would not be a disaster if properly treated). In this frame we will gauge fix

$$\vec{k}_{int} = \vec{k}_{int}' = 0, \tag{5.31}$$

that is, only $k^0_{int}, k'^0_{int} \neq 0$. It is obvious that any nearly on-shell momenta like those in the internal lines of effective theory diagrams, p_{int}, p_{int}', can be decomposed in this gauge,

$$\vec{v} \equiv \frac{\vec{p}}{m},$$

$$v_0 \equiv \sqrt{1+\vec{v}^2},$$

$$k_0 \equiv p_0 - m v_0 . \tag{5.32}$$

Thus, an internal line is now specified by four real numbers $k_0, \vec{v}$ rather than seven, $k_\mu, v_\mu : v^2 = 1$. This is the right counting. We can get the correct RPI measure of integration for the internal momenta, by noting that obviously $\int d^4 p_{int}$ is a RPI measure, since the (v,k) split has not been made. Using Eq. (5.32) then leads to the RPI measure

$$m^3 \int d^3 \vec{v}_{int} \int dk^0_{int} . \tag{5.33}$$

Thus the effective theory version of Fig. 13 is given by

$$\begin{aligned}
\Gamma_{SGETloop} = & \frac{i\lambda^2}{(2\pi)^4} \frac{\sum_{v_{int}} \sum_{v'_{int}} \int d^4 k_{int} \int d^4 k'_{int}}{RPI} \\
& \times \delta^4 (m v_{int} + k_{int} + m v'_{int} + k'_{int} - p_1 - p_2) \\
& \times \frac{1}{k_{int} \cdot v_{int} + i\epsilon} \; \frac{1}{k'_{int} \cdot v'_{int} + i\epsilon} \\
= & \frac{i\lambda^2 m^6}{(2\pi)^4} \int d^3 \vec{v}_{int} \int dk^0_{int} \int d^3 \vec{v}'_{int} \\
& \times \int dk'^0_{int} \; \delta^3 (m\vec{v}_{int} + m\vec{v}'_{int} - \vec{p}_1 - \vec{p}_2) \\
& \times \delta (m v^0_{int} + m v'^0_{int} + k^0_{int} + k'^0_{int} - p^0_1 - p^0_2) \\
& \times \frac{1}{k^0_{int} v^0_{int} + i\epsilon} \; \frac{1}{k'^0_{int} \cdot v'^0_{int} + i\epsilon} .
\end{aligned} \tag{5.34}$$

Note that the two tree effective vertices here do not satisfy label conservation. This is because with our present gauge fixing it would be inconsistent to also insist on a label-conserving gauge. Therefore we relax the latter requirement, which in any case has only a cosmetic value.

We will work to leading (zeroth) order in the *external* residual momenta, so that we can simply take $p_i = m v_i, p'_i = m v'_i$. In our choice of frame we then have

$$\vec{v}_1 + \vec{v}_2 = 0,$$

$$E_{tot}/2 \equiv v^0_1 = v^0_2 . \tag{5.35}$$

Substituting this in and integrating the δ functions gives,

$$\begin{aligned}
\Gamma_{SGETloop} = & \frac{i\lambda^2 m^3}{(2\pi)^4} \int \frac{d^3 \vec{v}_{int}}{(v^0_{int})^2} \int dk^0_{int} \frac{1}{k^0_{int} + i\epsilon} \\
& \times \frac{1}{E_{tot} - 2m v^0_{int} - k^0_{int} + i\epsilon} .
\end{aligned} \tag{5.36}$$

The k^0_{int} integral is finite and done by contour integration,

$$\Gamma_{SGETloop} = \frac{\lambda^2 m^2}{2(2\pi)^3} \int \frac{d^3 \vec{v}_{int}}{(v^0_{int})^2} \frac{1}{v^0_{int} - E_{tot}/2m + i\epsilon} . \tag{5.37}$$

Now, the remaining $\vec{v}_{int}$ integral representation of $\Gamma_{SGETloop}$ is logarithmically divergent just like the familiar Γ_{fund}. However, it is straightforward to see that the imaginary parts, related by unitarity to tree-level two-particle scattering, are finite and agree (up to the usual difference in normalization of states),

$$\begin{aligned}
\mathrm{Im}\ \Gamma_{fund} &= \mathrm{Im}\ \Gamma_{SGETloop}/4m^2 \\
&= \frac{\lambda^2}{16\pi^2} \int \frac{d^3 \vec{v}_{int}}{(2 v^0_{int})^2} \delta (v^0_{int} - E_{tot}/2m).
\end{aligned} \tag{5.38}$$

This of course just corresponds to integrating over the phase space for on-shell 2-particle intermediate states. Thus the imaginary parts are matched between the effective and fundamental theories.

It is the real parts which diverge. In both cases the integrals converge in a $(4-\delta)$-dimensional spacetime, that is with dimensional regularization. The important point is that not just Re Γ_{fund}, discussed above, but also Re $\Gamma_{SGETloop}$ is a local analytic functions of the external momentum. The latter is easily seen by deformation of the integration contour for $|\vec{v}_{int}|$ to avoid the $E_{tot}/2m$ pole, as long as $E_{tot}/2m > 1$ as we assumed (that is we are above the two-particle threshold). Therefore $\Gamma_{SGETloop}$ is locally analytic in $E_{tot} = \sqrt{(m v_1 + k_1 + m v_2 + k_2)^2}$, that is, analytic in the k_i. Thus we can introduce a local and Hermitian effective vertex given by $4m^2 \mathrm{Re}\, \Gamma_{fund} - \mathrm{Re}\, \Gamma_{SGETloop}$, of the form of Fig. 14 to match the real parts, thereby completing the matching procedure.

Of course we can also reverse our starting assumption and reconsider matching if we chose $p_1 + p_2$ to be nearly at the two-particle threshold. In that case, we can decompose all the external particles of Fig. 13 to have a common 4-velocity label,

$$p_i = m v + k_i , \;\; p'_i = m v + k'_i . \tag{5.39}$$

The problem now is that even the real part of the diagram can be non-analytic in the k_i and therefore cannot be captured by a local effective vertex, but must emerge from an effective diagram of the form of Fig. 13 just as the imaginary part must. But in just this near-threshold case, this proves to be possible. This situation corresponds to a rather standard

case in heavy particle effective theory. For example, see Ref. [22]. Nevertheless, we will verify below that things work.

By a standard calculation we have

$$\Gamma_{fund} \equiv \frac{i\lambda^2}{(2\pi)^4}\int d^4p_{int}\ \frac{1}{p_{int}^2-m^2+i\epsilon} \times \frac{1}{(p_1+p_2-p_{int})^2-m^2+i\epsilon} = \frac{\lambda^2}{16\pi^2}\sqrt{1-4m^2/(p_1+p_2)^2} \times \ln\left\{\frac{\sqrt{1-4m^2/(p_1+p_2)^2}+1}{\sqrt{1-4m^2/(p_1+p_2)^2}-1}\right\} + \text{const.}, \tag{5.40}$$

where the constant term contains the usual divergence. Matching the constant term is trivial so we will focus on the term non-analytic in the external momenta. We will work to leading non-trivial order in k_i/m,

$$(p_1+p_2)^2 = 4m^2+2mv\cdot(k_1+k_2)+(k_1+k_2)^2 \approx 4m^2+2mv\cdot(k_1+k_2). \tag{5.41}$$

Substituting into Γ_{fund} and working to leading order yields

$$\Gamma_{fund} \ni \frac{i\lambda^2}{16\pi}\sqrt{\frac{(k_1+k_2)\cdot v}{m}}. \tag{5.42}$$

Because we are near threshold this is not analytic in even the residual momenta. This allows it to have the behavior required by unitarity, imaginary above threshold, $(k_1+k_2)\cdot v>0$, but real below, $(k_1+k_2)\cdot v<0$.

We now compute the analogous loop diagram in SGET of the form of Fig. 13. Again we must gauge fix. The simplest procedure is to write

$$p_{int} = mv+k_{int},\ p'_{int} = mv+k'_{int}, \tag{5.43}$$

with the *same* fixed velocity as the external lines. Thus the RPI integration measure is obvious, $\int d^4p_{int} = \int d^4k_{int}$. Let us first try to work strictly to leading order in the $1/m$ expansion. After integrating the momentum-conserving δ functions we have

$$\Gamma_{SGETloop} = \frac{i\lambda^2}{(2\pi)^4}\int d^4k_{int}\ \frac{1}{k_{int}\cdot v+i\epsilon} \times \frac{1}{(k_1+k_2-k_{int})\cdot v+i\epsilon} = \frac{\lambda^2}{(2\pi)^3}\ \frac{1}{(k_1+k_2)\cdot v}\ \int d^3k_{int\perp v}1 = 0, \tag{5.44}$$

where in the second line we have done the finite $\int d(k_{int}\cdot v)$ by contour integration. We see a cubic divergence. Formally, this corresponds to an $\mathcal{O}(m/(k_1+k_2)\cdot v)$ effect. However, in dimensional regularization $\int d^3k_{int\perp v}\,1 = 0$. Therefore we must work to higher order in $1/m$ by keeping the dominant subleading terms in the propagators, as in Eq. (5.8),

$$\Gamma_{SGETloop} = \frac{i\lambda^2}{(2\pi)^4}\int d^4k_{int}\ \frac{1}{k_{int}\cdot v+k_{int\perp v}^2/2m+i\epsilon} \times \frac{1}{(k_1+k_2-k_{int})\cdot v+(k_1+k_2-k_{int})_{\perp v}^2/2m+i\epsilon} \approx \frac{\lambda^2}{(2\pi)^3}\int d^3k_{int\perp v} \times \frac{1}{(k_1+k_2-k_{int})\cdot v+k_{int\perp v}^2/m+i\epsilon} = \frac{im^2\lambda^2}{4\pi}\sqrt{\frac{(k_1+k_2).v}{m}}, \tag{5.45}$$

where in the second line we have again done the finite $\int d(k_{int}\cdot v)$ by contour integration and kept only the leading terms in $(k_1+k_2)/m$, and in the last line we have evaluated the linearly divergent $\int d^3k_{int\perp v}$ using dimensional regularization. As can be seen, this precisely matches the non-analytic terms in Γ_{fund} (taking into account the different state normalization as usual).

D. The cosmological constant in SGET

Let us finally consider what robust contributions to the infrared cosmological constant emerge within SGET. Because of the procedure for obtaining the SGET by *first* matching to the fundamental flat space theory *and then* covariantizing with respect to gravity, there is no cosmological constant term in the effective Lagrangian, Eq. (5.15). Of course we could simply add one, but there is no robust reason to do so except quantum naturalness. Therefore let us consider loop contributions to the infrared cosmological constant within SGET. We can make use of the GCI enjoyed by SGET to simply compute the pure vacuum diagrams in the complete absence of graviton lines, that is, the coefficient of the "1" term in the expansion of $\sqrt{-g}$.

Let us consider some typical diagrams. We have already discussed the non-interacting diagram of Fig. 15 in the previous section and explained why it must be set to zero. Figure 16 is an example of a vacuum diagram involving propagators from a vertex to itself, which apparently requires us to make sense of expressions such as

$$\frac{\sum_v \int d^4k}{\mathcal{R}}\ \frac{1}{k\cdot v+i\epsilon}. \tag{5.46}$$

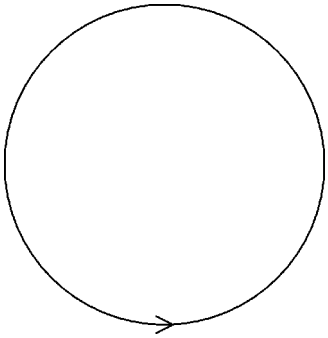

FIG. 15. Free particle vacuum energy diagram in SGET.

However, as was the case in Fig. 15, there is a physical reason why we must take the diagram to vanish. The reason is that Fig. 16 requires a vertex of the form $\phi_{v_1}\phi_{v_2}\,\phi^\dagger_{v_1}\phi^\dagger_{v_2}$, obtained by matching in situations where there is only a soft momentum transfer in two-particle scattering. As in the discussion of Fig. 15, under these circumstances the effective theory cannot distinguish whether the heavy particle is itself a composite of very light particles or solitonic, in which case it cannot make robust large loop contributions. We can summarize the vanishing of diagrams like Figs. 15 and 16 by saying that in the SGET we must normal order.

Figure 17 is a vacuum diagram one can draw despite normal ordering. We will compute it by first gauge fixing in a similar manner to the non-vacuum loop diagram we examined above. However, unlike that case where there was a natural frame selected by the incoming (or outgoing) momenta, we must simply choose a frame arbitrarily, and fix the gauge $\vec{k}_{int}=0$ in that frame. One might worry that this will lead to a Lorentz non-invariant answer, but it will not as we will see below. There are originally four k^0_{int} residual energies for the four internal lines, but after integrating the energy-conserving δ function we are left with three residual energy integrals to be done. Let us focus on any single one of these. It clearly has the form

$$\int dk^0_{int}\ \frac{1}{k^0_{int}v^0_{int}+i\epsilon}\ \frac{1}{k^0_{int}v'^0_{int}+\cdots+i\epsilon}=0, \quad (5.47)$$

where the ellipsis refers to a (real) combination of other energies external to this integral. This integral is finite and evaluates to zero by contour integration. To see this note that

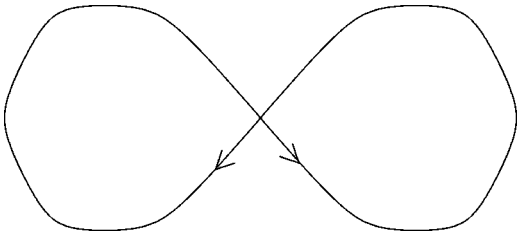

FIG. 16. A vacuum diagram with propagators from a vertex to itself.

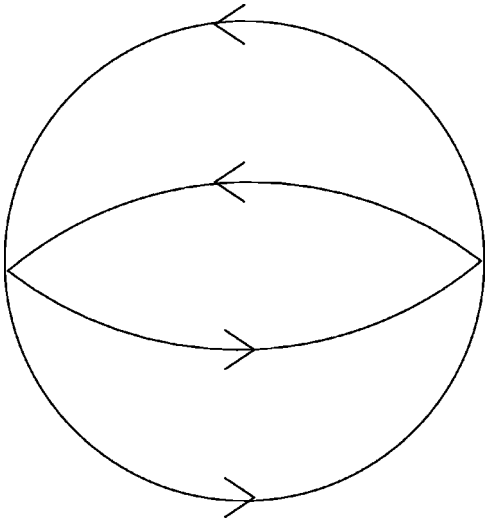

FIG. 17. A vacuum diagram in SGET which survives normal ordering, but not evaluation.

$v^0_{int}, v'^0_{int}>0$ and therefore both poles lie on the same side of the real k^0_{int} axis, so the contour can be closed at infinity without enclosing any poles.

If we repeated this same type of analysis for Fig. 18, we would get residual energy integrals of the form

$$\int dk^0_{int}\ \frac{1}{k^0_{int}v^0_{int}+i\epsilon}\ \frac{1}{-k^0_{int}v'^0_{int}+\ \ldots\ +i\epsilon}, \quad (5.48)$$

which does not vanish. However, the diagram does not exist in the SGET because it requires vertices of the form $\phi\phi\phi\phi$ rather than the $\phi^\dagger\phi^\dagger\phi\phi$ vertices appearing in Fig. 17. Recalling that SGET vertices arise from matching to fundamental correlators with nearly on-shell external lines, we see that no effective vertex such as $\phi\phi\phi\phi$ could have arisen upon matching. Therefore Fig. 18 simply does not exist. Recalling that $\phi^\dagger_v$ and ϕ_v are creation and destruction operators, one might wonder whether the presence of $\phi^\dagger\phi^\dagger\phi\phi$ is correlated

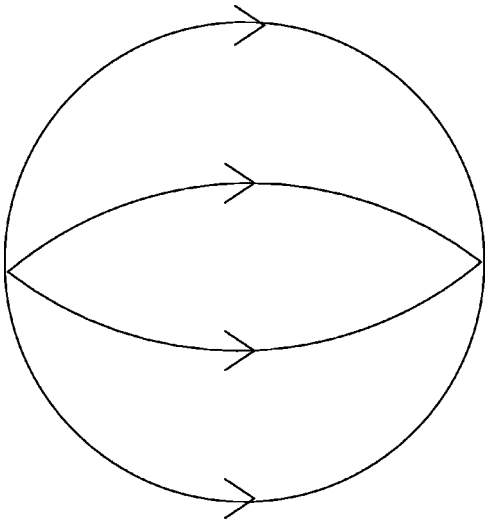

FIG. 18. A non-existent vacuum diagram in SGET because of the absence of the requisite vertices.

with that of $\phi\phi\phi\phi$ as it is in fundamental field theories. In fundamental theories this is a consequence of locality, there is no local way of separating positive and negative energy operators. However, as we have seen above, locality only down to $\Lambda_{grav} \ll m$ does not imply such a correlation.

It is straightforward to check that the examples of Figs. 15 to 18 exhaust all the possible cases arising in general vacuum diagrams. In SGET there are simply no robust contributions from heavy matter to the cosmological constant.

For a fat graviton there is new gravitational physics at Λ_{grav}, its vibrational excitations. We do not explicitly know this physics but we can estimate its loop contributions to the cosmological constant by standard power counting, $\mathcal{O}(\Lambda_{grav}^4/16\pi^2)$. This sets the minimal natural size of the cosmological constant.

VI. CONCLUSIONS

The soft graviton effective theory demonstrates a clear qualitative distinction between (a) loop effects of heavy SM physics on SM processes, (b) soft graviton exchanges between such ongoing SM processes, and (c) loop effects of heavy SM physics on the low-energy gravitational effective action, and in particular the infrared cosmological constant. The effective theory can match (a) to the fundamental SM theory (which is of course very well tested), and describe (b) constrained only by general coordinate invariance in the infrared, such as even a fat graviton must have. In this manner the effective theory captures the two pillars of our experimental knowledge, soft gravitation and the SM of high-energy physics. However, none of this implies any robust contributions to (c).

There is therefore a loophole in the cosmological constant problem for a fat graviton which is absent for a pointlike graviton, and it makes sense to vigorously hunt for its realization experimentally and within more fundamental theories such as string theory. Quantum naturalness related to the contributions to the cosmological constant of the vibrational excitations of the fat graviton, as well as from soft gravitons, photons and neutrinos, implies that Newton's Law should yield to a suppression of the gravitational force below distances of *roughly* 20 μm, as illustrated in Fig. 8. Of course the onset of such modifications of Newton's law may be seen at somewhat larger distances.

Future work will focus on generalizing the effective theory to include massless or light SM particles as well as quantum gravity corrections, and to looking for potential signals for experiment and observation due to higher order (in the derivative expansion) effects, that are allowed by the effective theory but forbidden by the standard theory where point-like gravity is extrapolated to at least a TeV. The compatibility of fat graviton ideas with multiple matter vacua will also be investigated.

ACKNOWLEDGMENTS

In thinking about fat gravitons, I greatly benefited from comments, criticisms, leads, and help from Nima Arkani-Hamed, Tom Banks, Andy Cohen, Savas Dimopoulos, Michael Douglas, Gia Dvali, Adam Falk, Shamit Kachru, Mike Luke, Markus Luty, Ann Nelson, Dan Pirjol, Joe Polchinski, Martin Schmaltz, Matt Strassler, and Andy Strominger. This research was supported in part by NSF Grant P420D3620434350 and in part by the DOE Outstanding Junior Investigator Program through Grant P442D3620444350.

[1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[2] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); P. M. Garnavich *et al.*, Astrophys. J. **509**, 74 (1998), S. Perlmutter *et al.*, *ibid.* **517**, 565 (1999).
[3] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[4] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998).
[5] R. Sundrum, J. High Energy Phys. **9907**, 001 (1999).
[6] S. Weinberg and E. Witten, Phys. Lett. **96B**, 59 (1980).
[7] C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner, and H. E. Swanson, Phys. Rev. Lett. **86**, 1418 (2001); EOT-WASH Group Collaboration, E. G. Adelberger hep-ex/0202008.
[8] J. C. Long, H. W. Chan, and J. C. Price, Nucl. Phys. **B539**, 23 (1999); for a review see J. C. Long, and J. C. Price, hep-ph/0303057.
[9] S. Weinberg, Phys. Rev. **140**, B515 (1965).
[10] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); H. Georgi, *ibid.* **240**, 447 (1990).
[11] T. Banks, Nucl. Phys. **B309**, 493 (1988).
[12] G. Dvali, G. Gabadadze, and M. Shifman, Phys. Rev. D **67**, 044020 (2003).
[13] J. J. van der Bij, H. van Dam, and Y. J. Ng, Physica A **116**, 307 (1982); S. Weinberg, remarks at the ''Workshop on Problems in Unification and Supergravity,'' La Jolla Institute, 1983; F. Wilczek and A. Zee, unpublished work quoted by Zee, in *High Energy Physics: Proceedings of the Annual Orbis Scientiae*, edited by S. L. Mintz and A. Perlmutter (Plenum, New York, 1985); W. Buchmller and N. Dragon, University of Hannover Report No. ITP-UH 1/88, 1988; Phys. Lett. B **207**, 292 (1988).
[14] J. W. Moffat, hep-ph/0102088.
[15] A. Adams, J. McGreevy, and E. Silverstein, hep-th/0209226; N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, hep-th/0209227.
[16] S. Weinberg, Phys. Rev. **135**, B1049 (1964); **138**, B988 (1965).
[17] J. Scherk, Phys. Lett. **88B**, 265 (1979).
[18] J. F. Donoghue and T. Torma, Phys. Rev. D **54**, 4963 (1996).
[19] M. E. Luke and A. V. Manohar, Phys. Lett. B **286**, 348 (1992).
[20] M. E. Luke, A. V. Manohar, and I. Z. Rothstein, Phys. Rev. D **61**, 074025 (2000).
[21] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, (McGraw-Hill, New York, 1964).
[22] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl. Phys. **B534**, 329 (1998).