Multidimensional cosmological models: Cosmological and astrophysical implications and constraints

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We investigate four-dimensional effective theories which are obtained by dimensional reduction of multidimensional cosmological models with factorizable geometry and we consider the interaction between conformal excitations of the internal space (geometrical moduli excitations) and Abelian gauge fields. It is assumed that the internal space background can be stabilized by minima of an effective potential. The conformal excitations over such a background have the form of massive scalar fields (gravitational excitons) propagating in the external spacetime. We discuss cosmological and astrophysical implications of the interaction between gravexcitons and four-dimensional photons as well as constraints arising on multidimensional models of the type considered in our paper. In particular, we show that due to the experimental bounds on the variation of the fine-structure constant, gravexcitons should decay before nucleosynthesis starts. For a successful nucleosynthesis, the masses of the decaying gravexcitons should be $m \gtrsim 10^4$ GeV. Furthermore, we discuss the possible contribution of gravexcitons to ultrahigh-energy cosmic rays. It is shown that, at energies $E \sim 10^{20}$ eV, the decay length of gravexcitons with masses $m \gtrsim 10^4$ GeV is very small, but that for $m \lesssim 10^2$ GeV it becomes much larger than the Greisen-Zatsepin-Kuzmin cutoff distance. Finally, we investigate the possibility for gravexciton-photon oscillations in strong magnetic fields of astrophysical objects. The corresponding estimates indicate that even the high-magnetic-field strengths B of magnetars (special types of pulsars with $B > B_{\text{critical}} \sim 4.4 \times 10^{13}$ G) are not sufficient for an efficient and copious production of gravexcitons.

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I. INTRODUCTION

The multidimensionality of our Universe is one of the most intriguing assumptions in modern physics. It is a natural ingredient of theories which unify different fundamental interactions with gravity, such as string and M theory [1]. The idea has received a great deal of renewed attention over the past few years within the "brane-world" description of the universe. In this approach, the $SU(3)\times SU(2)\times U(1)$ standard model (SM) fields, related to the usual four-dimensional physics, are localized on a three-dimensional spacelike hypersurface (brane), whereas the gravitational field propagates in the whole (bulk) spacetime. The geometry can be factorizable, as in the standard Kaluza-Klein (KK) approach, or nonfactorizable, as in M-theory-inspired Randall-Sundrum (RS) scenarios [2]. For factorizable KK geometries, the topology is the product

$$M = M_0 \times M_1 \times \cdots \times M_n \tag{1}$$

of a nonwarped manifold M_0, which constitutes the external spacetime, and compact manifolds M_i, $i=1,\ldots,n$, as internal spaces which are warped with functions depending on the external coordinates. Factorizable models of this type will be the subject of the present consideration.[1] For these models, the four-dimensional Planck scale $M_{\text{Pl}(4)}$ and the fundamental mass scale $M_{*(4+D')}$ are connected by the relation

$$M^2_{\text{Pl}(4)} \sim V_{D'} M^{2+D'}_{*(4+D')}, \tag{2}$$

where $V_{D'} = \Pi^n_{i=1} \text{vol}(M_i)$ denotes the volume of the compactified $D' = \Sigma^n_{i=1} d_i$ extra dimensions $[d_i = \dim(M_i)]$. Relation (2) is valid for models with factorizable geometry regardless of SM matter localized on branes; i.e., it is valid in KK scenarios as well as in brane-world scenarios. It reflects the fact that gravitons propagate in the whole multidimensional bulk M. The gravity force law in the product manifold M depends strongly on the considered length scale. Introducing the characteristic sizes b_i of the internal factor spaces as $b_i \sim \text{vol}(M_i)^{1/d_i}$ and denoting the largest size among the b_i as $b_l = \max b_i$ and the smallest one by $b_s = \min b_i$, it changes from r^{-2} at scales $r > b_l$ in an effectively $(d_0=4)$-dimensional spacetime M_0 to $r^{-(2+D')}$ at scales $r < b_s$, where all $D = d_0 + D'$ dimensions of the complete manifold M contribute to the dilution of the effective gravity

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[1]In contrast, the nonfactorizable geometries of RS-type models consist of two or more patches of 5D anti–de Sitter space glued together along branes (with one of them identified as our "world-brane"). The four-dimensional spacetimes are then warped with factors which depend on the extra $D'=1$ dimension.

force (see, e.g., Refs. [3,4]). Physically acceptable values for the compactification scales b_i range from the four-dimensional quantum gravity scale $M_{\mathrm{Pl}(4)}^{-1} < b_s$ up to the recently established experimental bound $b_l < 10^{-1}$ cm (see Refs. [5,6]).

For models with internal spaces all of the same characteristic size b, and the fundamental energy scale set at $M_{*(4+D')} \sim 1$ TeV, this leads to ADD brane-world scenarios [3] with compactification scales defined from Eq. (2) as

$$b \sim V_{D'}^{1/D'} \sim 10^{(32/D')-17} \text{ cm.} \tag{3}$$

In these scenarios, physically acceptable values are constrained by experiments [5,6] to $D' \geq 3$ so that, e.g., for $D' = 3$ one arrives at a submicrometer compactification scale $b \sim 10^{-5}$ mm of the internal space. The fixing of $M_{*(4+D')}$ at the scale 1–30 TeV near the electroweak (EW) scale $M_{\mathrm{EW}} \sim 246$ GeV [7] provides an elegant resolution of the hierarchy problem. At the same time, a scale hierarchy between b and 1 $\mathrm{TeV}^{-1} \sim 10^{-17}$ cm remains which only disappears in the limit $D' \to \infty$ when $b \to 1$ $\mathrm{TeV}^{-1} \sim 10^{-17}$ cm.

In contrast to the 4D gravity force law, which is experimentally tested at length scales above 0.1 mm, the four-dimensionality of the SM gauge interactions is tested down to scales of about $M_{\mathrm{EW}}^{-1} \sim l_{\mathrm{EW}} \sim 10^{-16}$ cm. For models with large extra dimensions, this requires a localization of the SM fields on a world-brane. In general, this can be a higher-dimensional p-brane with $p \geq 3$. The $d_{\parallel} = p-3$ longitudinal dimensions of this p-brane should then be compactified at sufficiently small scales $b < l_{\mathrm{EW}}$ [8].

According to observational data, the extra-dimensional space components should be static or nearly static at least from the time of primordial nucleosynthesis. Otherwise, the fundamental physical constants would vary (see, e.g., [9,10]). Equation (2) shows, for example, that if $V_{D'}$ is a dynamical function[2] which varies with time, then the effective four-dimensional gravitational constant will vary as well. This means that, at the present evolutionary stage of the Universe, the compactification scale of the internal space should either be stabilized and trapped at the minimum of some effective potential, or it should vary sufficiently slowly [similar to the slowly varying cosmological constant in the quintessence scenario [12] (see also the reviews [13,14])] so that the variations of derived parameters, like the variation of the fine-structure constant α, would meet their observational bounds. In both cases, small fluctuations over stabilized or slowly varying compactification scales (conformal scales/geometrical moduli) are still possible.

The stabilization of extra dimensions (moduli stabilization) in models with large extra dimensions (ADD models) was the subject of numerous investigations (see, e.g., Refs. [11,15–18]).[3] In the corresponding considerations, the product topology of a $(4+D')$-dimensional bulk spacetime was constructed from Einstein spaces with scale (warp) factors depending only on the coordinates of the external four-dimensional factor space. As a consequence, the conformal excitations of the extra-dimensional space components have the form of massive scalar fields living in the external (our) spacetime. Within the framework of multidimensional cosmological models (MCM), such excitations were investigated in [19–24].[4] In Ref. [19], they were called gravitational excitons. Later, since the ADD compactification approach, these geometrical moduli excitations have been known as radions [11,15].

In the present paper, we study the interaction of gravitational excitons with Abelian gauge fields, and in particular with electromagnetic (EM) fields. A possible observation of reactions in this interaction channel would be of great interest because it could provide strong evidence for the existence of extra dimensions. The corresponding interaction term of the four-dimensional effective theory has the form

$$\Delta S_{\mathrm{EM}} \sim \kappa_0 \int_{M_0} d^4x \sqrt{|\tilde{g}^{(0)}|}\, \psi F_{\mu\nu} F^{\mu\nu}, \tag{4}$$

where F denotes the EM field strength tensor, and the massive scalar field $\kappa_0 \psi < 1$ describes small scale factor (warp factor) excitations (gravexcitons) of the extra-dimensional space components. The interaction term (4) follows from a KK-like dimensional reduction scheme, where the scale factors, their excitations ψ, and the EM field strength F are considered in a zero-mode approximation. As a result, they will only depend on the coordinates of the external spacetime M_0. In this scheme, the extra-dimensional space components should be compactified at scales $b_i < l_{\mathrm{EW}}$. (In a brane-world context, one could interpret such a setup as a rough approximation of a p-brane endowed with an internal warped product structure where three dimensions and the time are represented by M_0, the remaining p-3 longitudinal dimensions are mimicked by extra-dimensional warped factor spaces $M_1 \times \cdots \times M_n$, and where any gravity contributions of the large transverse extra dimensions are neglected.) Furthermore, it is assumed that the scale factors are stabilized and frozen in one of the minima of an effective potential with ψ as fluctuations over this minimum. The main goal of our paper consists in the investigation of interaction (4) and its cosmological and astrophysical implications.

[2] It is clear that a dynamical behavior of the volume of the extra dimensions allows for several different implications of Eq. (2): (i) one can assume $M_{*(4+D')} \sim M_{\mathrm{EW}}$ as the fundamental relation and keep it fixed for varying $M_{\mathrm{Pl}(4)}$ [11], (ii) the four-dimensional Planck scale $M_{\mathrm{Pl}(4)}$ is the fundamental scale and the multidimensional/electroweak scale $M_{*(4+D')} \sim M_{\mathrm{EW}}$ is varying when $V_{D'}$ varies, (iii) all three scales $M_{\mathrm{Pl}(4)}$, $M_{*(4+D')}$, and M_{EW} are varying before nucleosynthesis and their present values are defined by string dynamical processes, and (iv) for fixed $M_{\mathrm{Pl}(4)}$ and $M_{*(4+D')}$, the relation $M_{*(4+D')} \sim M_{\mathrm{EW}}$ is the result of an earlier dynamics and not a fundamental one.

[3] In most of these papers, the moduli stabilization was considered without regard to the energy-momentum localized on the brane. A brane matter contribution was taken into account, e.g., in [17].

[4] See also Ref. [25], where a decoupling of scale factor excitations and inflaton was observed for a special solution subset of the Einstein equations.

The paper is structured as follows. In Sec. II, we explain the general setup of our model and give a basic description of gravitational excitons from extra dimensions. A consideration of the interaction between gravitational excitons and four-dimensional photons is presented in Sec. III. It is followed by a discussion of cosmological and astrophysical implications of this interaction (Sec. IV). Due to the Planck-scale suppression $\kappa_0 \sim 1/M_{\mathrm{Pl}(4)}$ and a decay rate $\Gamma \sim m^3/M^2_{\mathrm{Pl}(4)}$, gravitational excitons with mass m are WIMPs (weakly interacting massive particles [26]) similar to other moduli fields, Polonyi fields [27–31], and scalarons [32]. We investigate gravexcitons from a cosmological perspective by taking into account experimental bounds on the variation of the fine-structure constant α. Additionally, we discuss in this section a possible gravexciton contribution to ultrahigh-energy cosmic rays (UHECR) as well as possible gravexciton-photon oscillations in strong magnetic fields of magnetars. The main results are summarized in the Conclusion section.

II. GRAVITATIONAL EXCITONS

In this section, we present a sketchy outline of the basics of gravitational excitons from extra dimensions. A more detailed description can be found, e.g., in our paper [19].

Let us consider a multidimensional spacetime manifold M with warped product topology (1) and metric

$$g = g_{MN}(X) dX^M \otimes dX^N = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}, \qquad (5)$$

where x are some coordinates of the $(D_0 = 4)$-dimensional manifold M_0 and

$$g^{(0)} = g^{(0)}_{\mu\nu}(x) dx^\mu \otimes dx^\nu. \qquad (6)$$

Let further the internal factor manifolds M_i be d_i-dimensional warped Einstein spaces with warp factors $e^{\beta^i(x)}$ and metrics $g^{(i)} = g^{(i)}_{m_i n_i}(y_i)\, dy_i^{m_i} \otimes dy_i^{n_i}$, i.e.,

$$R_{m_i n_i}[g^{(i)}] = \lambda^i g^{(i)}_{m_i n_i}, \quad m_i, n_i = 1,\ldots,d_i, \qquad (7)$$

and

$$R[g^{(i)}] = \lambda^i d_i \equiv R_i \sim k b_i^{-2} \qquad (8)$$

with $k = 0, \pm 1$. b_i is the characteristic size of the Einstein space M_i [modulo the dimensionless warp factor $a_i \equiv \exp(\beta^i)$], i.e., b_i is the effective scale factor of the space M_i with metric $g^{(i)}$ and corresponding volume[5]

$$V_{d_i} \equiv \int_{M_i} d^{d_i} y \sqrt{|g^{(i)}|} \sim b_i^{d_i}, \quad i = 1,\ldots,n, \qquad (9)$$

where V_{d_i} has dimensions cm^{d_i}. Without loss of generality, we set the compactification scales of the internal spaces at the present time at $\beta^i = 0 \to a_i = 1$ $(i = 1,\ldots,n)$. This means that at the present time the total volume of the internal spaces is completely defined by the characteristic scale factors b_i,

$$V_{D'} \equiv \prod_{i=1}^{n} \int_{M_i} d^{d_i} y \sqrt{|g^{(i)}|} \sim \prod_{i=1}^{n} b_i^{d_i}. \qquad (10)$$

With total dimension $D = D_0 + D' = D_0 + \Sigma_{i=1}^{n} d_i$, $\kappa_D^2 = 8\pi / M_{*(4+D')}^{2+D'}$ a D-dimensional gravitational constant, Λ a D-dimensional bare cosmological constant, and S_{YGH} the standard York-Gibbons-Hawking boundary term, we consider an action of the form

$$S = \frac{1}{2\kappa_D^2} \int_M d^D X \sqrt{|g|} \{R[g] - 2\Lambda\} + S_m + S_{\mathrm{YGH}}. \qquad (11)$$

Here the action S_m corresponds to possible matter fields, e.g., gauge fields and scalar fields.[6] In some models, these matter fields can be considered phenomenologically as a perfect fluid with energy density ρ and corresponding equation of state [20,21,34,35]. This provides, for example, an efficient way to take into account the Casimir effect [36], Freund-Rubin monopoles [24,37,38], or other hypothetical potentials [20,39]. All these cases can be described by an additional potential term

$$S_m = -\int_M d^D X \sqrt{|g|} \rho(x). \qquad (12)$$

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[5]The volume is well defined for positive curvature spaces $(k = +1)$. For compact negative and zero curvature spaces $(k = -1, 0)$, i.e., compact hyperbolic spaces (CHSs) $M_i = H^{d_i}/\Gamma_i$ and tori $T_j = R^{d_j}/\Gamma_j$, we interpret this volume as the scaled volume of the corresponding fundamental domain (“elementary cell”) $V_{d_i} \sim b_i^{d_i} \times V_{\mathrm{FD}(i)}$ (see, e.g., [33] and references therein). Here H^{d_i}, R^{d_j} are hyperbolic and flat universal covering spaces, and Γ_i, Γ_j are appropriate discrete groups of isometries. Furthermore, we assume for the scale factors of the metrics $\gamma^i \sim b_i \hat{\gamma}^i$, with $\hat{\gamma}^i$ scaled in such a way that $V_{\mathrm{FD}(i)} \sim O(1)$. Thus, the volume V_{d_i} is mainly defined by b_i.

[6]Here the action S_m is treated as an action component of matter fields in a standard KK scenario. In an ADD-like large-scale compactification scenario, such an ansatz can be used in the case when the energy momentum localized on the brane can be ignored in comparison with the energy momentum of the matter in the bulk (see, e.g., Refs. [22–24]). Then S_m can provide an approximate description of the bulk matter components.

For some specific classes of models, e.g., for those describing the Casimir effect or Freund-Rubin monopoles, the energy density ρ depends on the external coordinates only through the scale factors $a_i(x)=e^{\beta^i(x)}$ $(i=1,...,n)$ of the internal factor spaces M_i. Obviously, the functional dependence on these scale factors is highly model-dependent. Throughout the present section, we will not specify this dependence, using instead the general expression (12). An explicit consideration of matter fields is left for Secs. III and IV.

After dimensional reduction and conformal transformation

$$g^{(0)}_{\mu\nu}=\Omega^2\tilde{g}^{(0)}_{\mu\nu}, \tag{13}$$

$$\Omega=\exp\left(-\frac{1}{D_0-2}\sum_{i=1}^{n} d_i\beta^i\right) \tag{14}$$

from the intermediate Brans-Dicke frame to the final Einstein frame, action (11) reads [19,40]

$$S[\tilde{g}^{(0)},\beta]=\frac{1}{2\kappa_0^2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{\tilde{R}[\tilde{g}^{(0)}]-\bar{G}_{ij}\tilde{g}^{(0)\mu\nu}\partial_\mu\beta^i\partial_\nu\beta^j-2U_{\text{eff}}\}, \tag{15}$$

where

$$\kappa_0^2=\kappa_D^2/V_{D'}=8\pi/M_{\text{Pl}}^2 \Rightarrow M_{\text{Pl}}^2=V_{D'}M_{*(4+D')}^{(2+D')} \tag{16}$$

is the D_0-dimensional gravitational constant (hereafter, $M_{\text{Pl}}\equiv M_{\text{Pl}(4)}$), and formula (16) reproduces Eq. (2). The tensor components of the midisuperspace metric (target space metric on $\mathbb{R}_T^n$) $\bar{G}_{ij}$ $(i,j=1,...,n)$, its inverse metric $\bar{G}^{ij}$, and the effective potential are given as

$$\bar{G}_{ij}=d_i\delta_{ij}+\frac{1}{D_0-2}d_id_j, \quad \bar{G}^{ij}=\frac{\delta^{ij}}{d_i}+\frac{1}{2-D}, \tag{17}$$

and

$$U_{\text{eff}}(\beta)=\left(\prod_{i=1}^{n}e^{d_i\beta^i}\right)^{-2/(D_0-2)}\times\left[-\frac{1}{2}\sum_{i=1}^{n}R_ie^{-2\beta^i}+\Lambda+\kappa_D^2\rho\right]. \tag{18}$$

We recall that ρ depends on the scale factors of the internal spaces: $\rho=\rho(\beta^1,\dots,\beta^n)$. Thus, action (15) describes a self-gravitating σ model with flat target space $(\mathbb{R}_T^n,\bar{G})$ (17) and self-interaction potential (18). Accordingly, the internal spaces can stabilize if the effective potential (18) has at least one minimum with respect to the scale factors β^i. Because the conformal transformation (13) was performed only with respect to the external metric $g^{(0)}$, the stability of the internal space configurations does not depend on the concrete choice of the frame (Einstein or Brans-Dicke).

In the following, we consider models with a constant scale factor background which is localized in a minimum of the effective potential U_{eff}. The values of the scale factors in this minimum are rescaled in such a way that at the present time it holds that $\vec{\beta}=0$, $\partial U_{\text{eff}}/\partial\beta^i|_{\vec{\beta}=0}=0$. In general, the effective potential U_{eff} can have more than one minimum so that transitions between these minima should be possible. For simplicity, we leave effects related to such transitions out of the scope of our present investigation. Instead, we will concentrate on small scale factor fluctuations $\beta^i<1$ in the vicinity of one minimum only. The action functional (15) can then be rewritten in terms of decoupled normal modes $\kappa_0\psi^i<1$ (for details, we refer to [19–21]),

$$S[\tilde{g}^{(0)},\psi]=\frac{1}{2\kappa_0^2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{\tilde{R}[\tilde{g}^{(0)}]-2\Lambda_{\text{eff}}\}+\sum_{i=1}^{n}\frac{1}{2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{-\tilde{g}^{(0)\mu\nu}\psi^i_{,\mu}\psi^i_{,\nu}-m_i^2\psi^i\psi^i\}, \tag{19}$$

where $\Lambda_{\text{eff}}\equiv U_{\text{eff}}(\vec{\beta}=0)$ plays the role of a D_0-dimensional effective cosmological constant. The normal modes and their masses squared m_i^2 are obtained by a simultaneous diagonalization of the σ-model metric (17) and the Hessian

$$\left.\frac{\partial^2U_{\text{eff}}}{\partial\beta^i\partial\beta^\kappa}\right|_{\vec{\beta}=0}. \tag{20}$$

In the special case of only one internal space $(n=1)$, this procedure reduces to a simple rescaling

$$\beta^1=-\kappa_0\sqrt{\frac{D_0-2}{d_1(D-2)}}\,\psi^1 \tag{21}$$

and

$$m_1^2=\frac{D_0-2}{d_1(D-2)}\left.\frac{\partial^2U_{\text{eff}}}{\partial(\beta^1)^2}\right|_{\beta^1=0}. \tag{22}$$

Summarizing this section, we conclude that conformal zero-mode excitations of the internal factor spaces M_i have the form of massive scalar fields developing on the background of the external spacetime M_0. In analogy with excitons in solid-state physics (excitations of the electronic subsystem of a crystal), we called these conformal excitations of the internal spaces gravitational excitons [19]. Later, since Refs. [11,15], these particles have been known as radions.

III. ABELIAN GAUGE FIELDS

In this section, we study the interaction of gravitational excitons with Abelian gauge fields, and in particular with the standard electromagnetic field of $U(1)_{\text{EM}}$ symmetry. Strictly speaking, the photon will not exist as a separate gauge boson at temperatures higher than the electroweak scale $M_{\text{EW}}\sim 246$ GeV, where the full electroweak $SU(2)\times U(1)$ model should be considered. Nevertheless, our results should reproduce the correct coupling term between the gravexciton sector and the EM gauge field sector of the theory. In the next section, we will use this coupling term for estimating the strength of cosmological and astrophysical effects related to the corresponding interaction channel.

In order to derive the concrete form of the coupling term in the dimensionally reduced, four-dimensional effective theory, we start from the simplified toy model ansatz

$$S_{\rm EM}=-\frac{1}{2}\int_M d^D X\sqrt{|g|}F_{MN}F^{MN}, \tag{23}$$

where the gauge field is assumed Abelian also in the higher-dimensional setup. Additionally, we work in the zero-mode approximation for these fields, i.e., we keep only the zero modes of the harmonic expansion in mass eigenstates of the higher-dimensional fields[7] [42,43]. In this case, the Abelian vector potential depends only on the external coordinates, $A_M=A_M(x)$ $(M=1,...,D)$, and the corresponding nonzero components of the field strength tensor are $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ $(\mu,\nu=1,...,D_0)$ and $F_{\mu m_i}=\partial_\mu A_{m_i}-\partial_{m_i}A_\mu=\partial_\mu A_{m_i}$ $(m_i=1,...,d_i;i=1,...,n)$.

Dimensional reduction of the gauge field action (23) yields

$$S_{\rm EM}=-\frac{1}{2}\int_{M_0} d^{D_0}x\sqrt{|g^{(0)}|}\prod_{i=1}^n e^{d_i\beta^i}\Bigg\{F_{\mu\nu}F^{\mu\nu}+2g^{(0)\mu\nu}\times\sum_{i=1}^n e^{-2\beta^i(x)}\overline{g}^{(i)m_in_i}\partial_\mu A_{m_i}\partial_\nu A_{n_i}\Bigg\}, \tag{24}$$

where we introduced the metric integral

$$\overline{g}^{(i)m_in_i}:=\frac{1}{V_{d_i}}\int_{M_i} d^{d_i}y\sqrt{|g^{(i)}|}g^{(i)m_in_i}(y^i) \tag{25}$$

and included the factor $\sqrt{V_{D'}}$ into A_M for convenience: $\sqrt{V_{D'}}A_M\rightarrow A_M$. Due to this redefinition, the field strength tensor $F_{\mu\nu}$ acquires the usual dimensionality $\mathrm{cm}^{-D_0/2}$ (in geometrical units $\hbar=c=1$). In Eq. (24), we assumed $F^{\mu\nu}=g^{(0)\mu\kappa}g^{(0)\nu\delta}F_{\kappa\delta}$.

It is easily seen that the A_{m_i} components play the role of scalar fields in the D_0-dimensional spacetime. In what follows, we will not investigate the dynamics of these fields. Instead, we will concentrate on the interaction between gravexcitons and the 2-form field strength $F=dA$, $A=A_\mu dx^\mu$ which is described by the first term of the action functional (24). The corresponding truncated action (without A_{m_i} terms) will be denoted by $\overline{S}_{\rm EM}$.

The exact field strength 2-form $F=dA$ with components $F_{\mu\nu}$ is invariant under gauge transformations $A\mapsto A^f=A+df$, $F^f=dA+d^2f=dA=F$, with $f(x)$ any smooth and single-valued function. Accordingly, $\overline{S}_{\rm EM}$ is gauge-invariant too [see Eq. (24)].

The action functional (24) is written in a Brans-Dicke frame. For passing by the conformal transformation (13), (14) to the Einstein frame, we choose an ansatz

$$A=\Omega^k\widetilde{A} \tag{26}$$

for the vector potential and introduce the auxiliary field strength $\overline{F}$ by the relation

$$F=dA=d(\Omega^k\widetilde{A})=\Omega^k\overline{F},$$
$$\overline{F}=d(\ln\Omega^k)\wedge\widetilde{A}+d\widetilde{A}. \tag{27}$$

The conformally transformed effective action then reads

$$\overline{S}_{\rm EM}=-\frac{1}{2}\int_{M_0} d^{D_0}x\sqrt{|\widetilde{g}^{(0)}|}\{\Omega^{2(k-1)}\overline{F}_{\mu\nu}\overline{F}^{\mu\nu}\}, \tag{28}$$

where the external space indices are raised and lowered by the metric $\widetilde{g}^{(0)}$. With $\widetilde{F}=d\widetilde{A}$, we have in Eq. (28) explicitly

$$\overline{F}_{\mu\nu}\overline{F}^{\mu\nu}=\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}-2\widetilde{F}^{\mu\nu}[\widetilde{A}_\mu\partial_\nu(\ln\Omega^k)-\widetilde{A}_\nu\partial_\mu(\ln\Omega^k)]+2\{\widetilde{g}^{(0)\mu\kappa}\partial_\mu(\ln\Omega^k)\partial_\kappa(\ln\Omega^k)\widetilde{A}^\nu\widetilde{A}_\nu-[\widetilde{A}^\mu\partial_\mu(\ln\Omega^k)]^2\}. \tag{29}$$

In order to fix the conformal weight k of the vector potential in Eq. (26), we require the effective external field strength tensor $\overline{F}_{\mu\nu}$ in Eq. (28) to be gauge-invariant, i.e., to be invariant under $\widetilde{A}\mapsto\widetilde{A}^f=\widetilde{A}+df$. From Eq. (27), we have for this transformation

$$\overline{F}\mapsto\overline{F}^f=d\widetilde{A}+d^2f+d(\ln\Omega^k)\wedge(\widetilde{A}+df)$$
$$=\overline{F}+d(\ln\Omega^k)\wedge df \tag{30}$$

so that for nontrivial $\Omega\neq 1$ the gauge invariance $\overline{F}=\overline{F}^f$ is only achieved for zero conformal weight $k=0$. The same result follows also directly from the gauge invariance of the field strength tensor F in Eq. (24) and the ansatz (26): One checks immediately that $\overline{F}$ is invariant under a transformation $\widetilde{A}\mapsto\check{A}=\widetilde{A}+\Omega^{-k}df$, which only for $k=0$ is a gauge transformation.

This means that in order to preserve the gauge invariance of the action functional, when passing from the Brans-Dicke frame to the Einstein frame, we have to keep the vector potential unchanged, i.e., we have to fix the conformal weight at $k=0$. As a result, we arrive at the action functional

$$\overline{S}_{\rm EM}=-\frac{1}{2}\int_{M_0} d^{D_0}x\sqrt{|\widetilde{g}^{(0)}|}\{e^{[2/(D_0-2)]\Sigma_{n=1}^n d_i\beta^i(x)}F_{\mu\nu}F^{\mu\nu}\} \tag{31}$$

with dilatonic coupling of the Abelian gauge fields to the gravitational excitons.

[7]The excitation of Kaluza-Klein modes of Abelian gauge fields was considered, e.g., in Ref. [41].

For completeness, we note that for $k=1$, according to Eqs. (28) and (29), we obtain a theory with a pure free action term $\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$ without any prefactor ($\Omega^{2(k-1)}=1$) but with explicitly destroyed gauge invariance. The corresponding effective action reads

$$\bar{S}_{\rm EM}=-\frac{1}{2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}-4\tilde{F}^{\mu\nu}\tilde{A}_{[\mu}\partial_{\nu]}(\ln\Omega)+2[\tilde{g}^{(0)\mu\kappa}\partial_\mu(\ln\Omega)\partial_\kappa(\ln\Omega)\tilde{A}^\nu\tilde{A}_\nu-(\tilde{A}^\mu\partial_\mu\ln\Omega)^2]\}. \quad (32)$$

Obviously, the localization of the scale factors β^i at their present values $\beta^i=0$ results in $\Omega\equiv1$. Then, both approaches (31) and (32) coincide with each other. However, the presence of small scale factor fluctuations above this background will restore the dilatonic coupling of Eq. (31) (see also the next section).

IV. GRAVITATIONAL EXCITONS AND THEIR COSMOLOGICAL AND ASTROPHYSICAL IMPLICATIONS

In this section, we discuss some cosmological and astrophysical implications related to the possible existence of gravitational excitons. We suppose that the scale factor background of the internal spaces is localized in one of the minima of the effective potential (see Sec. II) and that gravexcitons are present as small fluctuations above this static background. Our analysis is based on the dilatonic coupling (31) which describes the interaction between gravexcitons and zero-mode photons.[8] Hereafter, we treat these KK zero-mode photons as our usual SM matter photons. In particular, the vector potential $A_\mu(x)$ of the previous section corresponds to our 4D photons. In the following, we consider the simplest example—the interaction between gravitational excitons and photons in a system with only one internal space ($n=1$) with its scale factor β^1 localized in one of the minima of the effective potential (18). We rescale the size of the corresponding factor space in such a way that the background component takes the value $\beta^1=0$ at the present time.

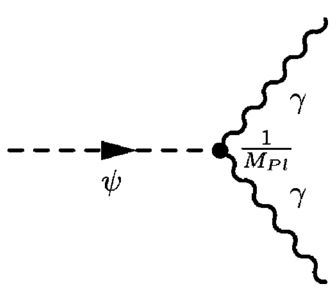

FIG. 1. Planck-scale suppressed gravexciton decay: $\psi\to2\gamma$.

[8]Brane-world models with on-brane dilatonic coupling terms have been considered, e.g., in Refs. [44,45]. In a rough approximation, the results of the present section will also hold for these models.

Then, for small scale factor fluctuations $\beta^1<1$, action (19) together with Eq. (31) reads

$$S=\frac{1}{2\kappa_0^2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{\tilde{R}[\tilde{g}^{(0)}]-2\Lambda_{\rm eff}\}+\frac{1}{2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\{-\tilde{g}^{(0)\mu\nu}\psi_{,\mu}\psi_{,\nu}-m_\psi^2\psi\psi\}-\frac{1}{2}\int_{M_0}d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{F_{\mu\nu}F^{\mu\nu}-2\sqrt{\frac{d_1}{(D_0-2)(D-2)}}\kappa_0\psi F_{\mu\nu}F^{\mu\nu}\right\}+\cdots . \quad (33)$$

We used the notations of Eq. (19), $m_\psi:=m_1$, and relation (21) between β^1 and the rescaled fluctuational component $\psi\equiv\psi^1$. As mentioned above, $\kappa_0^2=8\pi/M_{\rm Pl}^2$ is the D_0-dimensional (usually $D_0=4$) gravitational constant. The last term describes the interaction between gravitational excitons and photons. In the lowest-order tree-level approximation, this term corresponds to the vertex of Fig. 1 and describes the decay of a gravitational exciton into two photons. The probability of this decay is easily estimated as[9]

$$\Gamma\sim\left(\frac{1}{M_{\rm Pl}}\right)^2m_\psi^3=\left(\frac{m_\psi}{M_{\rm Pl}}\right)^3\frac{1}{t_{\rm Pl}}\ll m_\psi, \quad (34)$$

which results in a lifetime τ of the gravitational excitons with respect to this decay channel of

$$\tau=\frac{1}{\Gamma}\sim\left(\frac{M_{\rm Pl}}{m_\psi}\right)^3t_{\rm Pl}. \quad (35)$$

Similar to Polonyi particles in spontaneously broken supergravity [27,28], scalarons in the $(R+R^2)$ fourth-order theory of gravity [32], or moduli fields in the hidden sector of supersymmetry (SUSY) [29–31], gravitational excitons are WIMPs (weakly interacting massive particles [26]) because their coupling to the observable matter is suppressed by powers of the Planck scale.

A. Light gravexcitons

We first consider light excitons with masses $m_\psi\lesssim10^{-21}M_{\rm Pl}\sim10^{-2}$ GeV$\sim20m_e$ (where m_e is the electron mass). Their lifetime τ is greater than the age of the universe $\tau\gtrsim10^{19}$ sec$>t_{\rm univ}\sim10^{18}$ sec. From this estimate it follows that they can be considered as dark matter (DM). The type of the DM depends on the masses of the particles which constitute it. It is hot for $m_{\rm DM}\lesssim50$–100 eV, warm for 100 eV $\lesssim m_{\rm DM}\lesssim10$ keV, and cold for $m_{\rm DM}\gtrsim10$–50 keV.

Concerning the gravexciton decay processes, we note that there exists a characteristic time t_D within the evolution of the Universe when these processes are most efficient [see, e.g., Eq. (43) below]. The time can be estimated as $H(t_D)\sim\Gamma\Rightarrow t_D\sim\tau$, where $H=\dot{a}/a$ is the Hubble constant, a is the scale factor of the external space in the Einstein frame, and dots denote derivatives with respect to the synchronous

[9]Exact calculations give $\Gamma=[2d_1/(d_1+2)]\,(m_\psi^3/M_{\rm Pl}^2)$.

time t in the Einstein frame. It is clear that for gravexcitons with masses $m_\psi \ll 10^{-2}$ GeV, we can neglect their decay during the evolution of the Universe because for these masses the characteristic time t_D is much greater than the age of our Universe. Such gravexcitons up to the present time undergo coherent oscillations according to the approximate equation

$$\ddot{\psi}+3H\dot{\psi}+m_\psi^2\psi=0, \tag{36}$$

and did not convert into radiation with subsequent reheating of the Universe.[10] Assuming that at the present time the gravexciton energy density ρ_ψ is less than or similar to the critical density $\rho_\psi \lesssim \rho_c$, we obtain an upper bound for the gravexciton masses

$$m_\psi \lesssim \sqrt{\frac{3}{8\pi}} H_{\rm eq}\left(\frac{M_{\rm Pl}}{\psi_{\rm in}}\right)^4 \sim 10^{-56} M_{\rm Pl}\left(\frac{M_{\rm Pl}}{\psi_{\rm in}}\right)^4 \tag{37}$$

if $m_\psi > 10^{-56} M_{\rm Pl}$, so that the oscillations began at the radiation-dominated (RD) stage[11] (the consistency condition is $\psi_{\rm in} \lesssim M_{\rm Pl}$). Similarly, we obtain for all masses $m_\psi < 10^{-56} M_{\rm Pl}$ (then the oscillations started at the beginning of the matter-dominated (MD) stage) the bound

$$\psi_{\rm in} \lesssim M_{\rm Pl} \tag{38}$$

Here $\psi_{\rm in}$ denotes the amplitude of the initial field oscillations of ψ above the minimum position of the effective potential. Usually, it is assumed that $\psi_{\rm in} \sim O(M_{\rm Pl})$, although it depends on the form of $U_{\rm eff}$ and can be considerably less than $M_{\rm Pl}$. Particles with masses $m_\psi \sim 10^{-33}$ eV $\sim 10^{-61} M_{\rm Pl}$ are of special interest because via $\Lambda_{\rm eff} \sim m_\psi^2$ [19–22] they are related to the recently observed value of the effective cosmological constant (dark energy) $\Lambda_{\rm eff} \sim 10^{-123} \Lambda_{\rm Pl(4)} \sim 10^{-57}$ cm^{-2}. These ultralight particles have a period of oscillations $t \sim 1/m_\psi \sim 10^{18}$ sec which is of order of the Universe age. Thus, for these particles a splitting of the scale factor of the internal space into a background component and gravexcitons makes no sense. A more adequate interpretation of the scale factor dynamics would be in terms of a slowly varying background in the sense of a quintessence scenario [12,13].

Another very strong restriction on light gravexcitons follows from experiments on the time variation of the fine-structure constant α. It is well known (see, e.g., [49]) that the interaction between a scalar field $\varphi \equiv \kappa_0 \psi$ and an electromagnetic field F of the form $f(\varphi)F^2$ results in a variation of the fine-structure constant α,

$$\left|\frac{\dot{\alpha}}{\alpha}\right| = \left|\frac{\dot{f}}{f}\right|. \tag{39}$$

(The dot denotes differentiation with respect to time.) This relation has its origin in the observation that a theory with $L=f(\varphi)F^2$ can be interpreted as a theory of an electromagnetic field in a dielectric medium with permittivity $\epsilon_d = f(\varphi)$ and permeability $\mu = f^{-1}(\varphi)$. On the other hand, it is equivalent to a field theory in vacuum with $L=\tilde{F}^2$ [an analog of $\tilde{F}$ in our Eq. (32)] and a variable electric charge $e = f^{-1/2}(\varphi)e_0$. Thus, the fine-structure constant is also a dynamical function, $\alpha = e^2/(\hbar c) = e_0^2/(f\hbar c)$.

Let φ_0 denote the value of φ for a stable configuration at the present time, and $\varphi - \varphi_0 = \kappa_0 \eta = \eta/\bar{M}_{\rm Pl} < 1$ (here, $\bar{M}_{\rm Pl} := M_{\rm Pl}/\sqrt{8\pi}$) be small fluctuations in its vicinity. Then the

[10]Strictly speaking, the decay Γ of gravexcitons as well as their production due to interaction with other matter (with high-temperature radiation at the radiation-dominated (RD) stage, in particular) have to be taken into account here, too. The latter process can be described by a source term on the rhs of the effective equation of motion

$$\ddot{\psi}+(3H+\Gamma)\dot{\psi}+m_\psi^2\psi=I_m,$$

where I_m is proportional either to the trace of the EMT of matter on the world-brane (in brane-world scenarios [17]), or to the energy density of matter in our 4D Universe (in the Kaluza-Klein approach [9,21,34,46]). The behavior of the source term I_m is defined not only by the redshifting EMT of matter but also by gravexcitons' interactions with it, e.g., by gravexciton decay into radiation as well as by reactions of the type gravexciton$+\gamma \to$everything. The thermal production and balance of gravexcitons is described by the Boltzmann equation for their number density n_ψ,

$$\frac{dn_\psi}{dt}+3Hn_\psi=-\langle\sigma v\rangle n_\psi(n_\psi-n_{\psi({\rm eq})})-\Gamma(n_\psi-n_{\psi({\rm eq})}),$$

where $\langle\sigma v\rangle \sim M_{\rm Pl}^2$ is the typical cross section of these reactions and Γ is given by Eq. (34). $n_{\psi({\rm eq})}$ denotes the gravexciton number density in thermal equilibrium (the subscript is not to be confused with that of the characteristic time of matter/radiation equality). However, due to extremely weak interaction of gravexcitons with matter, it is very difficult for them to reach the thermal equilibrium. For particles which never dominate the Universe, ψ usually starts to oscillate when $n_\psi \ll n_{\psi({\rm eq})}$ and, as was shown in [47], the contribution of thermally produced gravexcitons to the total energy density is negligible. Thus, in this case the source term (that effectively reflects the efficiency of interaction between gravexcitons and ordinary matter) does not play any important role in Eq. (36). On the other hand, in scenarios where gravexcitons dominate the early dynamics of the Universe, their number density can initially be $n_\psi \gg n_{\psi({\rm eq})}$ and it will decrease until it reaches its thermal equilibrium at $n_{\psi({\rm eq})}$. In such a scenario, the initial amplitude can be assumed as $\psi_{\rm in} \sim M_{\rm Pl}$. Then the number density of γ quanta will be $n_\gamma \sim T^3 \ll n_\psi$, so that $n_\psi \sim e^{-\Gamma t} a^{-3}$. Therefore, thermal production of gravexcitons may be neglected. The scenario when gravexcitons have enough time to decay into radiation with subsequent reheating of the Universe is considered below, in Sec. IV B.

[11]In Eq. (37), $H_{\rm eq}$ denotes the Hubble parameter at that evolutionary stage when the energy densities of matter and radiation are of the same order (matter/radiation equality). Using the WMAP data for the ΛCDM model [48], we obtain $H_{\rm eq} \sim 10^{-56} M_{\rm Pl}$. Because the field oscillations start when the Hubble parameter becomes less than the mass of the particles ($H \lesssim m$), particles with masses greater or less than $H_{\rm eq}$ start to oscillate at the RD stage or at the MD stage, respectively.

interaction term of the Lagrangian reads $\gamma(\eta/\bar{M}_{\rm Pl})F^2$, where $\gamma := df/d\varphi|_{\varphi_0}$. In our case, we have $\varphi_0=0$, and $f(\varphi)$ and γ are defined by Eqs. (31) and (33) as

$$f(\varphi) = -\tfrac{1}{2}e^{-2\sqrt{[d_1/(D_0-2)(D-2)]}\varphi}$$

$$\Rightarrow \gamma = \sqrt{\frac{d_1}{(D_0-2)(D-2)}}. \tag{40}$$

The experimental bounds on the time variation of α have been considerably refined during recent years (see, e.g., [50–54] and references therein). Different experiments give different bounds on $|\dot{\alpha}/\alpha|$ (see Table II in [54]), from $\lesssim 10^{-12}$ yr^{-1} (following from the data analysis of the observed cosmic microwave background [50]) to $\lesssim 10^{-17}$ yr^{-1} (following from the Oklo experiment [55]). Estimates on primordial nucleosynthesis require $|\Delta\alpha/\alpha| \lesssim 10^{-4}$ at a redshift of order $z=10^9-10^{10}$ [56], i.e., $|\dot{\alpha}/\alpha| \lesssim 10^{-14}$ yr^{-1}. The WMAP data analysis [57] gives upper bounds on the variation of α during the time from reionization/recombination ($z\sim 1100$) until today: $|\Delta\alpha/\alpha| \lesssim (2\times 10^{-2})-(6\times 10^{-2})$, i.e., $|\dot{\alpha}/\alpha| \lesssim (2\times 10^{-12})-(6\times 10^{-12}$ yr$^{-1})$. In all these estimates, $\dot{\alpha}=\Delta\alpha/\Delta t$ is the average rate of change of α during the time interval Δt (corresponding to a redshift z). For our calculations, we use the estimate $|\dot{\alpha}/\alpha| \lesssim 10^{-15}$ yr^{-1} [52], which follows from observations of the spectra of quasars at a Hubble time scale $\Delta t \sim H^{-1} \sim 10^{10}$ yr. For this bound, we obtain from Eq. (39)

$$\left|\frac{\dot{f}}{f}\right| = \left|\frac{1}{f}\frac{df}{d\varphi}\frac{\dot{\psi}}{\bar{M}_{\rm Pl}}\right| = \left|\frac{\gamma}{f}\frac{\Delta\psi}{\Delta t}\frac{1}{\bar{M}_{\rm Pl}}\right| = \left|\frac{\dot{\alpha}}{\alpha}\right| \lesssim 10^{-15}\ \text{yr}^{-1}, \tag{41}$$

which leads to the following restriction on the parameter γ:

$$|\gamma| \approx \Delta t\left|\frac{\dot{\alpha}}{\alpha}\right|\frac{\bar{M}_{\rm Pl}}{\Delta\psi} \Rightarrow |\gamma| \lesssim 10^{-5}. \tag{42}$$

Here we took into account the present value of $f(0)=1$, supposed for the time interval a value of $\Delta t \sim H^{-1} \sim 10^{10}$ yr, and assumed $\Delta\psi \sim \bar{M}_{\rm Pl}$. However, Eq. (40) shows that in our model we have $\gamma|_{D_0=4} = \sqrt{d_1/[2(2+d_1)]} \sim O(1)$. This obvious contradiction means that light gravexcitons with masses $m_\psi \lesssim 10^{-2}$ GeV should have decayed at sufficiently early times of the evolution of the Universe in order not to contradict the experimental bounds on the variation of the fine-structure constant.[12] From this point of view, the presence of such light gravexcitons is unacceptable for the time after the end of primordial nucleosynthesis. Additionally, ultralight gravexcitons can lead to the appearance of a fifth force with characteristic length scale $\lambda \sim 1/m_\psi$. Recent gravitational (Cavendish-type) experiments (see, e.g., [6,59]) exclude fifth force particles with masses $m_\psi \lesssim 1/(10^{-2}$ cm$) \sim 10^{-3}$ eV. This sets an additional restriction on the allowed mass region of gravexcitons.

B. Heavy gravexcitons

Let us now consider gravexcitons with masses $m_\psi \gtrsim 10^{-2}$ GeV. For such gravexcitons, the decay (34) plays an important role. If there is no broad parametric resonance ("preheating") (see [60–62] for the details of the derivation), then for $H \ll m_\psi$ as a result of this decay the factor $e^{-\Gamma t}$ becomes dominant in the energy density and the number density

$$\rho_\psi \sim e^{-\Gamma t}a^{-3}, \quad n_\psi \sim e^{-\Gamma t}a^{-3}, \tag{43}$$

and due to the gravexciton decay the Universe undergoes a reheating (RH) up to the temperature [22,28,29]

$$T_{\rm RH} \sim \sqrt{\frac{m_\psi^3}{M_{\rm Pl}}}. \tag{44}$$

For a successful nucleosynthesis, a temperature $T_{\rm RH} \gtrsim 1$ MeV is needed so that the gravexciton masses should satisfy $m_\psi \gtrsim 10^4$ GeV. This result is obtained under the only assumption that gravexcitons dominate at the time of their decay. Moreover, for a successful hot baryogenesis,[13] it should hold that $m_\psi \gtrsim 10^{14}$ GeV [28]. Thus, either the decaying particles should have masses m_ψ which satisfy these lower bounds (and will decay before nucleosynthesis due to the interaction channel shown in Fig. 1) or, for lighter gravexcitons, we should find other scenarios which would allow us to get rid of such particles before nucleosynthesis starts. The latter can be achieved if the decay rate becomes larger. However, a fast decay in the regime of a broad parametric resonance [62] may not be realized for gravexcitons since it requires the condition $\Gamma \gg H$ at the beginning of the decay ($H \sim m_\psi$), which is not satisfied [see estimate (34)].

Thus, in order to avoid the aforementioned problems, gravexcitons should be sufficiently heavy and decay before nucleosynthesis starts. However, as was pointed out in Sec. II, in models with stabilized internal spaces the minimum of the effective potential plays the role of the effective cosmological constant, which usually (for simplified cosmological models) is connected with the gravexciton masses by a relation $\Lambda_{\rm eff} \sim m_\psi^2$. Hence, heavy gravexcitons result in $\Lambda_{\rm eff} \gg 10^{-57}$ cm^{-2}, which is in obvious contradiction to recent observations [14]. For such particles, a mechanism should be found which could provide a reduction of $\Lambda_{\rm eff}$ to its observable value. A similar problem with a large cosmological constant exists also in superstring modular cosmology [31]. [For a discussion of the cosmological constant problem (CCP) within the framework of string theory, we refer to [65].] A possible resolution of this problem could consist in a consideration of more or less realistic models which contain differ-

[12]In general, the performed estimates can be refined by accounting for viralization and accretion processes during the formation of overdense spacetime regions, such as galaxies [58]. In this case, if the gravexcitons are also sufficiently clustered, the study of time variations of α should be supplemented by an analysis of spatial variations of α.

[13]A low-temperature mechanisms for the baryogenesis can considerably lower this mass value [28,63] (see also [64]).

ent types of matter fields. Some of these fields should violate the null energy condition (NEC) and the weak energy condition (WEC). As shown in our recent paper [24], in this case one can obtain large gravexciton masses $m_\psi \sim M_{*(4+D')}$ and a small positive $\Lambda_{\rm eff}$ which is in agreement with the observed value.[14]

Most probably, primordial cosmological gravexcitons decayed already at the early stages of the evolution of the Universe. Nevertheless, it is of great interest to consider different mechanisms which could lead to a gravexciton production at the present time. Obviously, due to the Planck-scale suppression, it is hardly possible to observe the interaction between gravexcitons and photons in laboratory-scale experiments. Thus, we should look for regions in our Universe where conditions could exist which are suitable enough for such reactions to occur. For example, a thermal gravexciton production would become possible at temperatures $T \gtrsim m_\psi$ when the exponential suppression in the creation probability is switched off: $P \sim \exp(-m_\psi/T) \sim O(1)$. Then, the creation rate (per unit time per unit volume) can be estimated as $\nu \sim T^6/M_{\rm Pl}^2$ (see, e.g., Ref. [67], where a similar estimate was given for the production of KK gravitons). In our case, a thermal production of heavy gravexcitons with masses $m_\psi \gtrsim 10^4$ GeV would take place if $T > 10^{17}$ K. The maximal temperatures which can be reached in any known astrophysical objects[15] (supernovas, neutron stars, pulsars) are defined by nuclear reactions and do not exceed $10^{10}-10^{12}$ K $\sim 1-10^2$ MeV. Hence, a thermal gravexciton production in the cores of these objects is absent. Such a production could be possible at early evolution stages of the hot Universe, but the corresponding cosmological gravexcitons would have decayed before nucleosynthesis started.

C. Gravexcitons and UHECR

The observation of cosmic rays with ultrahigh energies (UHECR) $E \gtrsim 10^{20}$ eV (see, e.g., review [69]) shows that our Universe contains astrophysical objects where particles with energies $E \gg m_\psi \gtrsim 10^4$ GeV can be produced. Obviously, these energies should have a nonthermal origin and should be large enough for the creation of heavy gravexciton, e.g., in reactions $2\gamma \rightarrow \psi$ (γ quanta conformally excite internal spaces). Then, if a gravexciton acquires the energy $E > m_\psi$, it propagates through the Universe without decay a distance [70],

$$l_D \sim \frac{E}{\Gamma m_\psi} \sim \frac{E}{10^{20}\ \mathrm{eV}} \left(\frac{10^{17}\ \mathrm{GeV}}{m_\psi} \right)^4 l_{\rm Pl}, \tag{45}$$

where $\Gamma \sim m_\psi^3/M_{\rm Pl}^2$ is defined by Eq. (34). Substituting $E \sim 10^{20}$ eV and $m_\psi \sim 10^4$ GeV, we obtain $l_D \sim 10$ pc, which is considerably less than the Greisen-Zatsepin-Kuzmin (GZK) cutoff distance $r_{\rm GZK} \sim 50$ Mpc. Thus, such heavy gravexcitons cannot be used for the explanation of the UHECR problem. However, it can be easily seen that for $m_\psi \lesssim 10^2$ GeV the decay length exceeds the cutoff distance, $l_D \gg r_{\rm GZK}$. On the other hand, the very weak coupling constant $\kappa_0 \sim M_{\rm Pl}^{-1}$ leads to a negligible scattering of gravexcitons at the CMBR [70]. Thus, if a mechanism could be found to reconcile the presence of gravexcitons of mass $m_\psi \sim 10^2$ GeV with a successful nucleosynthesis, then such gravexcitons could be helpful in solving the UHECR problem.

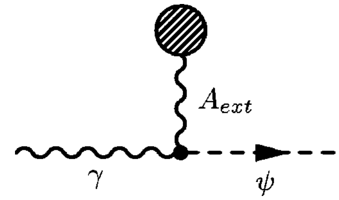

FIG. 2. Photon-induced gravexciton creation in a strong magnetic background field.

D. Gravexcitons and magnetars

Another possible source for the production of gravexcitons could be the strong magnetic fields of some astrophysical objects (e.g., neutron stars, pulsars or magnetars). In analogy with effects described for axions [71,72], a strong external electric/magnetic field can lead to oscillations between gravitational excitons and photons in accordance with the diagram of Fig. 2. It corresponds to an interaction term $L_{\rm eff} \sim \kappa_0 \psi F_{\rm ext}^{\mu\nu} f_{\mu\nu}$, where $F_{\rm ext}^{\mu\nu}$ is the field strength of the external electric/magnetic field and $f_{\mu\nu}$ denotes the photon wave function. In the presence of a strong external field, the probability for an electric/magnetic conversion of gravitational excitons into photons (and vice versa) could be much greater than Eq. (34) and would result in observable lines in the spectra of astrophysical objects. Usually, the electric fields of astrophysical objects are very small. However, the magnetic fields can be very strong. For example, magnetars as special types of pulsars can possess magnetic fields of strengths $B > B_{\rm critical} \sim 4.4 \times 10^{13}\ \mathrm{G} \sim 3 \times 10^{-6}\ \mathrm{GeV}^2$. So, it is of interest to estimate the magnetic-field strengths which would provide an efficient and copious gravexciton production. For this purpose, we note that for the creation of a gravexciton the energy conservation law in a stationary external field requires an energy of the incident photon of $E \gtrsim m_\psi$. Furthermore, an efficient gravexciton production takes place for magnetic-field strengths $B^2 \sim m_\psi^2 \kappa_0^{-2} \sim m_\psi^2 M_{\rm Pl}^2$. Thus, for a gravexciton of mass $m_\psi \sim 10^4$ GeV we obtain $B \sim 10^{23}\ \mathrm{GeV}^2$, which is much larger than $B_{\rm critical}$. Hence, we have to conclude that the magnetic-field strengths of magnetars are not sufficient for an efficient production of gravexcitons. The reason for this is clear. First, gravexcitons are heavy particles and, second, the gravexciton-photon interaction itself is Planck-scale-suppressed.

To conclude this section, we note that, similar to the results on dilatonic couplings $\sim e^{-\phi}F^2$ of electromagnetic

[14]For a possible resolution of the CCP in codimension-2 brane-world scenarios, we refer the reader to the recent work [66].

[15]Concerning the maximal possible temperature in the Universe, see the paper [68] by Sakharov in which this temperature was estimated as $T_{\max} \sim T_{\rm Pl} \sim 10^{32}$ K.

fields in inflation models [73] and string cosmology [74], we can expect in theory (31) an amplification of electromagnetic vacuum fluctuations due to the presence of a dynamical gravexciton background. Such amplification processes can generate sufficiently strong magnetic seed fields to start galactic dynamo effects which maintain the intergalactic magnetic field at the present time. Furthermore, they can result in contributions to the observable anisotropy of the cosmic microwave background.

V. CONCLUSION

In the present paper, we investigated interactions of small conformal scale factor fluctuations (gravitational excitons) of extra-dimensional space components with 4D Abelian gauge fields. The considered model was based on a factorizable background geometry.

With the help of a toy model ansatz, we demonstrated that the 4D gauge-invariant electromagnetic sector of the theory is dilatonically coupled to the scale factor fluctuations (gravexcitons). For a static background with background scale factors frozen in the minimum of an effective potential, the interaction term of gravexcitons and 4D photons has the form $(\psi/M_{\mathrm{Pl}})F^{\mu\nu}F_{\mu\nu}$. This Planck scale suppressed coupling leads for gravexcitons with mass m_ψ to a decay rate $\Gamma\sim m_\psi^3/M_{\mathrm{Pl}}^2$. Accordingly, gravexcitons are WIMPs with respect to this interaction channel. Depending on the concrete gravexciton mass, different physical effects will become relevant.

(i) Light gravexcitons with masses $m_\psi<10^{-2}$ GeV have a lifetime which exceeds the age of the Universe so that they can be considered as dark matter (DM). In this case, the inequality (37) is saturated, and in order that gravexcitons constitute the present DM, each mass value m_ψ should be connected with a corresponding special value of the initial fluctuation amplitude ψ_{in}. Additionally, laboratory tests of the gravitational inverse-square law limit the gravexciton mass from below: $m>10^{-3}$ eV. Furthermore, such gravexcitons lead to a temporal variability of the fine-structure constant above the experimentally established value (even with the Webb *et al.* data [52] taken as upper limits on the variability of α). Thus, this case seems to be excluded.[16]

(ii) Ultralight gravexcitons with masses $m_\psi\sim 10^{-33}$ eV are closely related to the observable cosmological constant/dark energy: $m_\psi^2\sim\Lambda_{\mathrm{obs}}\sim 10^{-57}\ \mathrm{cm}^{-2}$. The oscillation period of these particles exceeds the age of the Universe, and a splitting of the scale factor of the internal space into a background component and gravexcitons makes no sense. A more adequate interpretation of the scale factor dynamics would be in terms of a slowly varying background similar to the scalar field dynamics in a quintessence scenario. However, any dark energy model based on ultralight gravexcitons should first address the problem of the absence of a ''fifth force.''

(iii) For gravexcitons with masses $m_\psi>10^{-2}$ GeV, decay processes play an important role during the evolution of the Universe. These processes can contribute considerably to reheating. We demonstrated that heavy gravexcitons with masses $m_\psi\gtrsim 10^4$ GeV can best meet the existing cosmological restrictions. On the one hand, the reheating temperature of these particles is sufficiently high for a successful nucleosynthesis. On the other hand, they intensively decay already before nucleosynthesis starts, which prevents a too large variation of the fine-structure constant afterwards.

The Planck-scale suppression of the interaction and the high energy $E\gtrsim m_\psi$ needed for the creation of heavy gravexcitons with masses $m_\psi\gtrsim 10^4$ GeV will make it very difficult, or even impossible, to observe such reactions in future laboratory-scale experiments. So, it is of interest to clarify whether reactions with intensive gravexciton production may take place in some astrophysical objects.

As a first example, we discussed a possible contribution of gravexcitons to UHECR with energies $E\sim 10^{20}$ eV. It is clear that, due to their weak interaction with ordinary matter, gravexcitons will propagate in the Universe without significant scattering on CMBR. This makes them attractive as a possible candidate for UHECR. Our estimates show that the decay length of gravexcitons with masses $m_\psi\gtrsim 10^4$ GeV is very short, but for masses $m_\psi\lesssim 10^2$ GeV it becomes much longer than the Greisen-Zatsepin-Kuzmin cutoff distance.

As a second example, we considered the interaction between gravexcitons and photons in the strong magnetic background fields of magnetars. The presence of the background fields will strongly enhance the reaction and, in analogy to axion-photon oscillations, one might expect the occurrence of gravexciton-photon oscillations. However, our estimates show that even the strong magnetic fields of magnetars with field strengths $B>B_{\mathrm{critical}}\sim 4.4\times 10^{13}$ G are not strong enough for an efficient gravexciton production.

It remains to clarify whether other astrophysical objects or other interaction channels could lead to an efficient gravexciton production with directly observable phenomena.

Note added. After finishing this paper, we noticed that the paper [47] appeared in which cosmological implications of extra dimensions are also discussed.

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[16]Apart from the recent radical hypothesis that α is significantly spatially dependent so that its measured value is characteristic for our spatial vicinity only [58].

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