Inflation from supersymmetric quantum cosmology

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We propose a realization of inverted hybrid inflation scenario in the context of $n=2$ supersymmetric quantum cosmology. The spectrum of density fluctuations is calculated in the de Sitter regimen as a function of the gravitino and the Planck mass, and explicit forms for the wave function of the Universe are found in the WKB regimen for FRW closed and flat universes.

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I. INTRODUCTION

Inflation is a theoretically attractive idea for solving many classical problems of standard big bang cosmology. Recently, observations have confirmed its predictions of a flat universe with a nearly scale invariant perturbation spectrum [1,2].

On the other hand, despite its many successes, there are still not completely natural inflation models known in particle physics. Inflation generally requires small parameters in particle theory to provide a flat potential, needed for sufficient inflation and for the correct density fluctuations [1,3]. Probably the most attractive models of inflation are the hybrid inflation [4–7], extranatural inflation [8,9] and inverted hybrid inflation [10,11] models. In the hybrid inflation scenario proposed by Linde [4,5], the slowly rolling inflaton field ϕ is not responsible for most of the energy density. That role is played by another field ψ, which is held in place by its interaction with the inflaton field until the latter falls below a critical value; thus, ψ is destabilized, rolling to its true vacuum, and inflation ends. The hybrid inflation models are based on particle physics motivations such as supersymmetry, supergravity, and superstrings. In the context of the last two of these, we do not expect inflation to be possible for field values exceeding a Planck mass, regardless of whether the potential energy there is larger than the Planck energy, because supergravity corrections tend to generate a steep potential that is unable to sustain inflation. Given the success of hybrid inflation, it was subsequently suggested that the hybrid mechanism could be adapted to create an inverted model in which the inflaton field ϕ has a negative mass squared and rolls away from the origin, predicting a spectral index which can be significantly below 1 in contrast to virtually all other hybrid inflation models.

Supersymmetry may play an important role for inflation. Many models have been proposed describing the inflationary phase transition in globally supersymmetric theories [3,6,12–15] and locally supersymmetric theories [6,16–19]. These models have been analyzed without and with supergravity corrections. Supergravity corrections spoil the flatness of the inflaton potential because supersymmetry is broken during inflation.

The study of supersymmetric minisuperspace models has lead to important and interesting results [20]. In works [21–23] a new approach has been proposed to the study of supersymmetric quantum cosmology. The main idea is to extend the group of local time reparametrization of the cosmological models to the local conformal time supersymmetry, which is a subgroup of the four dimensional spacetime supersymmetry. This scheme allowed us to formulate, in the superfield representation, the supersymmetric action to study homogeneous models. The Grassmann superpartners of the scale factor and the homogeneous scalar fields at the quantum level are elements of the Clifford algebra. In this level, these models are specific supersymmetric quantum mechanics models with spontaneous breaking of supersymmetry when the vacuum energy is zero. In this paper we consider the inverted hybrid inflation models in the context of local conformal $n=2$ supersymmetry and we find some simple solutions (WKB solutions) to the Wheeler–De Witt equation.

The plan of this paper is as follows. In Sec. II we introduce the $n=2$ local conformal supersymmetry formulation of the FRW model interacting with a set of spatially homogeneous real scalar matter superfields. In Sec. III we introduce the WKB procedure to obtain classical and quantum solutions, and also we shall analyze the potential proposed in our model, in the case of the inflationary potential where this is modified by the presence of the local supersymmetry breaking sector (via the scalar φ). Also, if we consider the semiclassical solutions to the WDW equation for the inflationary phase (not zero potential and zero superpotential), and the potential is zero but the superpotential is not zero, both cases are related with the two minimums that have the potential, local and global minimums, respectively. Besides, the density fluctuation produced by the inflaton field ϕ is shown. Section IV is devoted to conclusions.

II. SUPERSYMMETRIC LAGRANGIAN AND SUSY BREAKING

The most general superfield action for a homogeneous scalar supermultiplet interacting with the scale factor in the supersymmetric $n=2$ FRW model has the form

$$S=\int\left[6\left(-\frac{1}{2\kappa^2}\frac{\mathcal{R}}{\mathcal{N}}D_{\bar\eta}\mathcal{R}D_{\eta}\mathcal{R}+\frac{\sqrt{k}}{2\kappa^2}\mathcal{R}^2\right)+\frac{1}{2}\frac{\mathcal{R}^3}{\mathcal{N}}D_{\bar\eta}\Phi^i D_{\eta}\Phi^i-2\mathcal{R}^3 g(\Phi^i)\right]d\eta d\bar\eta dt, \quad (1)$$

where $k=0,1$ denotes flat and closed space and $\kappa^2=8\pi G_N=1/M_p^2$; G_N is the Newton's gravitational constant ($\hbar=c$

$=1$). The units for the constants and fields are the following: $[\kappa^2]=l^2, [\mathcal{N}]=l^0, [\mathcal{R}]=l^1, [\Phi^i]=l^{-1}, [g(\Phi^i)]=l^{-3}$; here l correspond to units of length. $D_\eta=\partial/\partial\eta+i\bar{\eta}(\partial/\partial t)$ and $D_{\bar{\eta}}=-\partial/\partial\bar{\eta}-i\eta(\partial/\partial t)$, $i=1,2,3$, are the supercovariant derivatives of the superconformal supersymmetry $n=2$, which have dimensions of $[D_\eta]=[D_{\bar{\eta}}]=l^{-1/2}$.

Its series expansion for the one-dimensional gravity superfield $\mathcal{N}(t,\eta,\bar{\eta})$ is

$$\mathcal{N}=N(t)+i\eta\bar{\psi}'(t)+i\bar{\eta}\psi'(t)+\eta\bar{\eta}V'(t), \tag{2}$$

in which $N(t)$ is the lapse function, and we have also introduced the reparametrization $\psi'(t)=N^{1/2}(t)\psi(t)$ and $V'(t)=N(t)V(t)+\bar{\psi}(t)\psi(t)$.

The Taylor series expansion for the superfield $\mathcal{R}(t,\eta,\bar{\eta})$ has a similar form

$$\mathcal{R}=R(t)+i\eta\bar{\lambda}'(t)+i\bar{\eta}\lambda'(t)+\eta\bar{\eta}B'(t), \tag{3}$$

where $\lambda'(t)=\kappa N^{1/2}(t)\lambda(t)$ and $B'(t)=\kappa N(t)B(t)+\frac{1}{2}\kappa(\bar{\psi}\lambda-\psi\bar{\lambda})$.

The real scalar matter superfields Φ^i may be written as

$$\Phi^i=\phi^i(t)+i\eta\bar{\chi}'^i(t)+i\bar{\eta}\chi'^i(t)+\eta\bar{\eta}F'^i(t), \tag{4}$$

and $\chi'^i(t)=N^{1/2}(t)\chi^i(t)$, $F'^i(t)=NF^i+\frac{1}{2}(\bar{\psi}\chi^i-\psi\bar{\chi}^i)$.

Integrating over the Grassmann variables and making the following redefinition of the odd fields $\lambda\to\frac{1}{3}R^{-1/2}\lambda$ and $\chi^i\to R^{-3/2}\chi^i$, we find the Lagrangian

$$\begin{aligned}L=&-\frac{3R(DR)^2}{\kappa^2N}+\frac{2}{3}i\bar{\lambda}D\lambda+\frac{\sqrt{k}}{\kappa}R^{1/2}(\bar{\psi}\lambda-\psi\bar{\lambda})+\frac{1}{3}NR^{-1}\sqrt{k}\bar{\lambda}\lambda+\frac{3k}{\kappa^2}NR+\frac{R^3(D\phi)^2}{2N}-i\bar{\chi}^iD\chi^i\\&-\frac{3}{2}\sqrt{k}NR^{-1}\bar{\chi}^i\chi^i-\kappa^2Ng(\phi^i)\bar{\lambda}\lambda-6\sqrt{k}Ng(\phi^i)R^2-NR^3V(\phi^i)+\frac{3}{2}\kappa^2Ng(\phi^i)\bar{\chi}^i\chi^i+\frac{i\kappa}{2}D\phi^i(\bar{\lambda}\chi^i+\lambda\bar{\chi}^i)\\&-2N\frac{\partial^2g(\phi^i)}{\partial\phi^i\phi^j}\bar{\chi}^i\chi^j-\kappa N\frac{\partial g(\phi^i)}{\partial\phi^i}(\bar{\lambda}\chi^i-\lambda\bar{\chi}^i)+\frac{\kappa}{4}R^{-3/2}(\psi\bar{\lambda}-\bar{\psi}\lambda)\bar{\chi}^i\chi^i-\kappa R^{3/2}(\bar{\psi}\lambda-\psi\bar{\lambda})g(\phi^i)\\&+R^{3/2}\frac{\partial g(\phi^i)}{\phi^i}(\bar{\psi}\chi^i-\psi\bar{\chi}^i)+\frac{3\kappa^2N}{8R^3}\chi^i\bar{\chi}^i\chi^j\bar{\chi}^j,\end{aligned} \tag{5}$$

(eliminating the auxiliary fields), where $DR=\dot{R}-(i\kappa/6)R^{-1/2}(\psi\bar{\lambda}+\bar{\psi}\lambda)$ and $D\phi^i=\dot{\phi}^i-(i/2)R^{-3/2}(\bar{\psi}\chi^i+\psi\bar{\chi}^i)$ are the supercovariant derivatives, and $D\lambda=\dot{\lambda}+(i/2)V\lambda$, $D\chi^i=\dot{\chi}^i+(i/2)V\chi^i$ are the $U(1)$ covariant derivatives.

In the usual models of hybrid inflation ϕ is rolling towards zero. The potential is typically of the form

$$V(\phi)=V_0+\frac{1}{2}m^2\phi^2+\cdots,$$

and is dominated by the term V_0. When ϕ fall below some critical value ϕ_c, the other field rolls to its vacuum value so that V_0 disappears and inflation ends, but in our scenario we have the opposite case of inverted hybrid inflation, where ϕ roll away from the origin and the scalar potential for the homogeneous scalar fields in our scenario become

$$\begin{aligned}V(\phi^i)&=2\left(\frac{\partial g(\phi^i)}{\partial\phi^i}\right)^2-3\kappa^2g^2(\phi^i)\\&=\frac{1}{2}F_i^2-\frac{3}{\kappa^2R^2}B^2.\end{aligned} \tag{6}$$

The first of them is the potential for the scalar fields in the case of global supersymmetry; the second term is the contribution of the local character of supersymmetry, where the bosonic auxiliary fields F^i and B are

$$F^i=2\frac{\partial g(\phi^i)}{\partial\phi^i},\quad B=\kappa^2Rg(\varphi^i). \tag{7}$$

The potential is not positive semidefinite in contrast with standard supersymmetric quantum mechanics. Unlike the standard supersymmetric quantum mechanics, this model allows supersymmetry breaking when the vacuum energy is equal to zero.

The selection rules for the ocurrence of spontaneous supersymmetry breaking are

$$\begin{aligned}\frac{\partial V(\phi^i)}{\partial\phi^i}&=4\left[\frac{\partial g(\phi^i)}{\partial\phi^j}\left(\frac{1}{2}\frac{\partial^2g(\phi^i)}{\partial\phi^i\partial\phi^j}\right)-\left(\frac{\partial g(\phi^i)}{\partial\phi^i}\right)\left(\frac{3\kappa^2}{2}g(\phi^i)\right)\right]\\&=0,\quad\text{at}\quad\phi^i=\phi_0^i,\end{aligned}$$

$$V(\phi_0^i)=0\Rightarrow\left[\left(\frac{\partial g(\phi^i)}{\partial\phi^i}\right)^2-\frac{3\kappa^2}{2}g^2(\phi^i)\right]=0,$$

$$F^i = 2\frac{\partial g(\phi^i)}{\partial \phi^i} \neq 0, \quad \text{at} \quad \phi^i = \phi_0^i. \tag{8}$$

The first condition implies the existence of a minimum in the scalar potential; the second condition is the absence of the cosmological constant, and the third condition is for the breaking of supersymmetry. From the Lagrangian (5) we can identify $m_{3/2} = \kappa^2 g(\phi_0^i)$ as the gravitino mass in the effective supergravity theory. The factor R in the kinetic term of the scalar factor $-(3/\kappa^2)R\dot{R}^2$ plays the role of a metric tensor in the Lagrangian (it is the metric tensor in the minisuperspace generated by this model). Now, we need to contruct the corresponding supersymmetric quantum mechanics, from which the quantum Hamiltonian operator emerges, and becomes the central piece in our study. The quantization procedure must take into account the nature of the Grassmann variables, antisymmetrize them and write the bilinear combinations in the form of the commutators, this leads to the following quantum Hamiltonian:

$$\begin{aligned}\mathcal{H} = &-\frac{\kappa^2}{12R}\pi_R^2 - \frac{3k}{\kappa^2}R - \frac{1}{6}\sqrt{k}[\overline{\lambda},\lambda] + \frac{\pi_{\phi^i}^2}{2R^3} - \frac{i\kappa\pi_{\phi^i}}{4R^3}([\overline{\lambda},\chi^i] + [\lambda,\overline{\chi^i}]) - \frac{\kappa^2}{16R^3}[\overline{\lambda},\lambda][\overline{\chi^i},\chi^i] \\ &+ \frac{3\sqrt{k}}{4R}[\overline{\chi^i},\chi^i] + \frac{\kappa^2}{2}g(\phi^i)[\overline{\lambda},\lambda] + 6\sqrt{k}g(\phi^i)R^2 + R^3V(\phi^i) - \frac{3\kappa^2}{4}g(\phi^i)[\overline{\chi^i},\chi^i] \\ &+ \frac{1}{2}\frac{\partial^2 g(\phi^i)}{\partial\phi^i\partial\phi^j}[\overline{\chi^i},\chi^j] + \frac{\kappa}{2}\frac{\partial g(\phi^i)}{\partial\phi^i}([\overline{\lambda},\chi^i] - [\lambda,\overline{\chi^i}]) - \frac{3\kappa^2}{32R^3}[\overline{\chi^i},\chi^i][\overline{\chi^j},\chi^j].\end{aligned} \tag{9}$$

We are going to use the following matrix representation for the operators $\lambda, \overline{\lambda}, \chi^i, \overline{\chi^i}$:

$$\lambda = \sqrt{\frac{3}{2}}\sigma_- \otimes 1 \otimes 1 \otimes 1, \quad \chi^1 = \sigma_3 \otimes \sigma_- \otimes 1 \otimes 1,$$

$$\chi^2 = \sigma_3 \otimes \sigma_3 \otimes \sigma_- \otimes 1, \quad \chi^3 = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_-,$$

$$\overline{\lambda} = -\sqrt{\frac{3}{2}}\sigma_+ \otimes 1 \otimes 1 \otimes 1, \quad \overline{\chi^1} = \sigma_3 \otimes \sigma_+ \otimes 1 \otimes 1,$$

$$\overline{\chi^2} = \sigma_3 \otimes \sigma_3 \otimes \sigma_+ \otimes 1, \quad \overline{\chi^3} = \sigma_3 \otimes \sigma_3 \otimes \sigma_3 \otimes \sigma_+, \tag{10}$$

where $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$ and the commutators involved in the quantum Hamiltonian can be written as

$$[\overline{\lambda},\lambda] = -\frac{3}{2}\sigma_3 \otimes 1 \otimes 1 \otimes 1, \quad [\overline{\chi^1},\chi^1] = 1 \otimes \sigma_3 \otimes 1 \otimes 1,$$

$$[\overline{\chi^2},\chi^2] = 1 \otimes 1 \otimes \sigma_3 \otimes 1, \quad [\overline{\chi^3},\chi^3] = 1 \otimes 1 \otimes 1 \otimes \sigma_3.$$

III. WKB TYPE SOLUTIONS

The usual hybrid inflation models are based on a superpotential of the type

$$g_{inf} = \lambda(M^2 - \psi^2)\phi. \tag{11}$$

To this superpotential we add a supersymmetry breaking part [22]

$$g_{SB} = m_{3/2}M_p^2\left(1 + \sqrt{\frac{3}{2}}\frac{\varphi}{M_p} + \frac{3}{4}\frac{\varphi^2}{M_p^2}\right). \tag{12}$$

The "total superpotential" g is the sum of these two contributions

$$g(\phi^i) = \lambda(M^2 - \psi^2)\phi + m_{3/2}M_p^2\left(1 + \sqrt{\frac{3}{2}}\frac{\varphi}{M_p} + \frac{3}{4}\frac{\varphi^2}{M_p^2}\right) \tag{13}$$

and then the scalar potential $V(\phi_i)$ $(\phi_1 = \phi, \phi_2 = \psi, \phi_3 = \varphi)$ can be calculated with the help of the relation (6)

$$\begin{aligned}V(\phi,\psi,\varphi) = &2\lambda^2(M^2 - \psi^2)^2 + 8\lambda^2\psi^2\phi^2 \\ &+ 2m_{3/2}^2M_p^4\left(\sqrt{\frac{3}{2}}\frac{1}{M_p} + \frac{3}{2}\frac{\varphi}{M_p^2}\right)^2 \\ &- \frac{3}{M_p^2}\lambda^2(M^2 - \psi^2)^2\phi^2 \\ &- 3m_{3/2}^2M_p^2\left(1 + \sqrt{\frac{3}{2}}\frac{\varphi}{M_p} + \frac{3}{4}\frac{\varphi^2}{M_p^2}\right)^2 \\ &- 6m_{3/2}\lambda\phi(M^2 - \psi^2)\left(1 + \sqrt{\frac{3}{2}}\frac{\varphi}{M_p} + \frac{3}{4}\frac{\varphi^2}{M_p^2}\right).\end{aligned} \tag{14}$$

This scalar potential possesses two minimums: a global minimum and a local one. The global minimum is localized in $\phi_0=0,\psi_0^2=M^2,\varphi_0=0$, implying that $V(\phi_0,\psi_0,\varphi_0)=0$ and $g(\phi_0,\psi_0,\varphi_0)=m_{3/2}M_p^2$. The local minimum is found in $\phi^*=-m_{3/2}M_p^2/2\lambda M^2, \psi^*=0, \varphi^*=-\sqrt{\frac{2}{3}}M_p$; with these values we have $V(\phi^*,\psi^*,\varphi^*)=2\lambda^2M^4$ and the superpotential in the local minimum $g(\phi^*,\psi^*,\varphi^*)=0$.

Using the usual representation for the momentum operators

$$\Pi_R=-\frac{6}{\kappa^2}R\dot{R}, \quad \hat{\Pi}_R=-i\frac{\partial}{\partial R}, \tag{15}$$

$$\Pi_{\phi_i}=R^3\dot{\phi}_i, \quad \hat{\Pi}_{\phi_i}=-i\frac{\partial}{\partial \phi_i}. \tag{16}$$

The corresponding Wheeler–De Witt equation has the form

$$\begin{aligned}\tilde{\mathcal{H}}\Psi=R\mathcal{H}\Psi=\bigg[&-\frac{\kappa^2}{12}\pi_R^2-\frac{3k}{\kappa^2}R^2-\frac{1}{6}\sqrt{k}[\bar{\lambda},\lambda]R\\&+\frac{\pi_{\phi^i}^2}{2R^2}-\frac{i\kappa\pi_{\phi^i}}{4R^2}([\bar{\lambda},\chi^i]+[\lambda,\bar{\chi}^i])\\&-\frac{\kappa^2}{16R^2}[\bar{\lambda},\lambda][\bar{\chi}^i,\chi^i]+\frac{3\sqrt{k}}{4}[\bar{\chi}^i,\chi^i]\\&+\frac{\kappa^2R}{2}g(\phi^i)[\bar{\lambda},\lambda]+6\sqrt{k}g(\phi^i)R^3+R^4V(\phi^i)\\&-\frac{3\kappa^2}{4}Rg(\phi^i)[\bar{\chi}^i,\chi^i]+\frac{1}{2}\frac{\partial^2g(\phi^i)}{\partial\phi^i\partial\phi^j}[\bar{\chi}^i,\chi^j]R\\&+\frac{\kappa}{2}R\frac{\partial g(\phi^i)}{\partial\phi^i}([\bar{\lambda},\chi^i]-[\lambda,\bar{\chi}^i])\bigg]\Psi=0. \end{aligned}\tag{17}$$

In the matrix realization to the operators λ,χ^i on the wave function $\Psi(R,\phi^i)$ (that have 16 components), the particular Wheeler–De Witt equation that we are going to consider now is for the case $R\gg l_{pl}$, and $k=0,1$. Under this situation the term of the scalar potential in the Wheeler–De Witt equation is dominant, and we take the scalar potential to be evaluated in the local minimum. For this case the superpotential is zero and the Wheeler–De Witt equation that governs the quantum behavior is[1]

$$\begin{aligned}&\left[-\frac{\kappa^2}{12}\hat{\Pi}_R^2+R^4V(\phi^*,\psi^*,\varphi^*)\right]\Psi_{16}\\&\quad=\left[-\frac{1}{12M_p^2}\frac{\partial^2}{\partial R^2}+\frac{1}{2}R^4F_\phi^2(\phi^*,\psi^*,\varphi^*)\right]\Psi_{16}=0.\end{aligned}\tag{18}$$

[1]Due to the matrix realization for the operators, there are two not null components for the wave function, i.e., Ψ_1 and Ψ_{16}, but both components have the same WDW equation; we only write one of them.

Using the ansatz for the wave function $\Psi=e^{-if}$ and taking the WKB approximation $\partial^2f/\partial R^2\ll(\partial f/\partial R)^2$, Eq. (18) leads to the following form for the wave function:

$$\Psi=e^{-i(\sqrt{6}/3)M_pF_\phi R^3}. \tag{19}$$

In the semiclassical regimen it is well known that

$$\Pi_R=\Psi^*\hat{\Pi}_R\Psi\Rightarrow-6M_p^2R\dot{R}=-\sqrt{6}M_pF_\phi R^2. \tag{20}$$

From this, we can obtain the functional form of the scale factor

$$R=C_0e^{(\sqrt{6}M^2/3M_p)\lambda t}, \tag{21}$$

and the Hubble parameter then takes the form

$$H=\frac{\sqrt{6}M^2\lambda}{3M_p}. \tag{22}$$

Now we are in a position to calculate the density fluctuations. The density fluctuations are produced by the inflaton field, the other fields not have a role here because they are fixed to the origin. Then we have a polynomial scalar potential in the inflaton field, assuming that V_0 dominates,

$$V=2\lambda^2M^4-6m_{3/2}\lambda M^2\phi-\frac{3\lambda^2M^4}{M_p^2}\phi^2. \tag{23}$$

The wave function in the semiclassical region is given by Eq. (19), with $V=\frac{1}{2}F_\phi^2$. With the help of the relation

$$\Pi_\phi=R^3\dot{\phi}=\Psi^*\hat{\Pi}_\phi\Psi, \tag{24}$$

it is possible to obtain $\dot{\phi}$

$$\dot{\phi}=-\frac{\sqrt{6}}{3}M_p\frac{\partial F_\phi}{\partial\phi}=-\frac{1}{\sqrt{3}}M_pV^{-1/2}\frac{\partial V}{\partial\phi}, \tag{25}$$

explicitly

$$\dot{\phi}=\sqrt{\frac{3}{2}}m_{3/2}M_p. \tag{26}$$

In this way, we can obtain the following expression for the density fluctuations:

$$\frac{\delta\rho}{\rho}\simeq\frac{H^2}{\dot{\phi}}=\frac{2\sqrt{6}\lambda^2M^4}{9m_{3/2}M_p^3}, \tag{27}$$

which depends on the gravitino and Planck masses values.

On the other hand, using the global minimum in the potential $V(\phi_0,\varphi_0,\psi_0)=0$, the eigenstates of the Hamiltonian (17) have sixteen components in the matrix representation that we have chosen at the end of the previous section. Using the matrix representation for $\lambda,\bar{\lambda},\chi$ and $\bar{\chi}$, one finds that Ψ_{16} can have the right behavior when $R\to\infty$, being the following partial differential equation in both cases to $k=0,1$:

$$\left(-\frac{1}{12M_p^2}\hat{\Pi}_R^2+\frac{9}{4}m_{3/2}R\right)\Psi_{16}=0,\quad \text{for } k=0 \tag{28}$$

and

$$\left(-\frac{1}{12M_p^2}\hat{\Pi}_R^2+6m_{3/2}M_p^2R^3\right)\Psi_{16}=0,\quad \text{for } k=1. \tag{29}$$

The semiclassical solutions of these equations become

$$\Psi_{16}=e^{-i2\sqrt{2}\sqrt{m_{3/2}}M_pR^{3/2}},\quad k=0, \tag{30}$$

$$\Psi_{16}=e^{(-12i/5)\sqrt{2m_{3/2}}M_p^2R^{5/2}},\quad k=1. \tag{31}$$

The behavior of the scale factor corresponding to the situations of a flat and a closed universe are, respectively,

$$R\sim t^{2/3},\quad k=0, \tag{32}$$

$$R\sim t^2,\quad k=1. \tag{33}$$

Then we obtain, for a flat universe, a scale factor as a dust dominated universe. The last case corresponds to a scenario like power law inflation [24–26], where the scale factor evolves as a power of time. This type of solution is perhaps the most prominent example of an exact solution to the full equations of motion (not slow roll approximation); it has the extra advantage that the equations for the generation of density perturbations also can be solved exactly. The inflaton field acts as a perfect fluid with $\omega=\frac{1}{3}$, considering the state equation $p=(\omega-1)\rho$.

IV. CONCLUSIONS

The inverted hybrid inflation process appears in a natural way in the supersymmetric theory given in [21–23]. Under this scheme, it was possible to find the behavior for the scale factor R. When the scalar potential is evaluated in the local minimum, the behavior was inflationary and the density fluctuations depend on the inverse of the gravitino mass; see Eq. (27). Also, we find that for the global minimum in the scalar potential, we obtain, for a flat universe, a scale factor as a dust dominated universe; and for a closed universe, the behavior corresponds to a scenario like power law inflation, where the scale factor evolves as a power of time and the inflaton field acts as a perfect fluid.

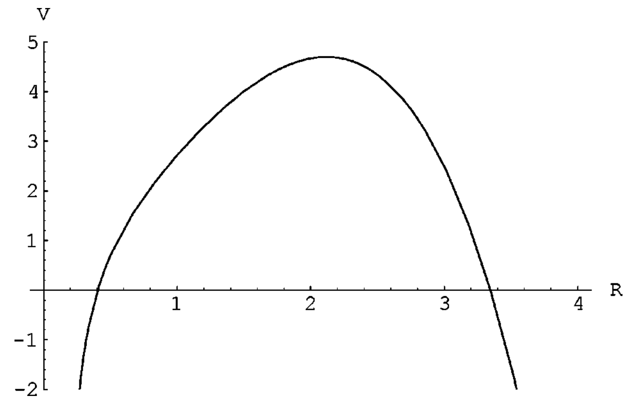

FIG. 1. The structure of the potential $V(R)=-0.08R^4-0.2R^{-2}+3R$, with the coefficient of R^4 smaller than the other terms.

Our solutions for the scale factor are very robust, in the sense that, even keeping the next contribution to the WDW equation (quadratic terms in R, for instance in $k=1$ case), the functional form of the scale factor is retained.

On the other hand, an interesting situation is obtained as a consequence of the contribution of the fermionic sector. A tunneling wave function for a flat universe is possible. The wave equation describing the process is

$$\left[\hat{\Pi}_R^2-\frac{27}{32}\frac{1}{R^2}+\frac{9m_{3/2}}{4\kappa^2}R-\frac{12V_0}{\kappa^2}R^4\right]\Psi_{16}=0,$$

for which we can identify the potential

$$V(R)=-\frac{27}{32}\frac{1}{R^2}+\frac{9m_{3/2}}{4\kappa^2}R-\frac{12V_0}{\kappa^2}R^4.$$

A schematic form of this potential is shown in Fig. 1.

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