Non-Gaussianity in the curvaton scenario

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Since a positive future detection of nonlinearity in the cosmic microwave background anisotropy pattern might allow us to descriminate among different mechanisms giving rise to cosmological adiabatic perturbations, we study the evolution of the second-order cosmological curvature perturbation on superhorizon scales in the curvaton scenario. We provide the exact expression for the non-Gaussianity in the primordial perturbations including gravitational second-order corrections which are particularly relevant in the case in which the curvaton dominates the energy density before it decays. As a by-product, we show that in the standard scenario where cosmological curvature perturbations are induced by the inflaton field, the second-order curvature perturbation is conserved even during the reheating stage after inflation.

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I. INTRODUCTION

One of the basic ideas of modern cosmology is that there was an epoch early in the history of the Universe when potential, or vacuum, energy associated with a scalar field, the inflaton, dominated other forms of energy density such as matter or radiation. During such a vacuum-dominated era the scale factor grew exponentially (or nearly exponentially) in time. During this phase, dubbed inflation [1,2], a small, smooth spatial region of size less than the Hubble radius could grow so large as to easily encompass the comoving volume of the entire presently observable Universe. If the Universe underwent such a period of rapid expansion, one can understand why the observed Universe is so homogeneous and isotropic to a high accuracy.

Inflation has also become the dominant paradigm for understanding the initial conditions for structure formation and for cosmic microwave background (CMB) anisotropy. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations ''redshifted'' out of the horizon during an early period of superluminal expansion of the Universe, where they are ''frozen'' [3–7]. Perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB, which was first detected by the cosmic background explorer (COBE) satellite [8–10]. The last and most impressive confirmation of the inflationary paradigm has been recently provided by the data of the Wilkinson Microwave Anistropy Probe (WMAP) mission which has marked the beginning of the precision era of the CMB measurements in space [11]. The WMAP Collaboration has produced a full-sky map of the angular variations of the CMB, with unprecedented accuracy. WMAP data confirm the inflationary mechanism as responsible for the generation of curvature (adiabatic) superhorizon fluctuations.

Despite the simplicity of the inflationary paradigm, the mechanism by which cosmological adiabatic perturbations are generated is not yet fully established. In the standard picture, the observed density perturbations are due to fluctuations of the inflaton field itself. When inflation ends, the inflaton oscillates about the minimum of its potential and decays, thereby reheating the Universe. As a result of the fluctuations each region of the Universe goes through the same history but at slightly different times. The final temperature anisotropies are caused by the fact that inflation lasts different amounts of time in different regions of the Universe leading to adiabatic perturbations. Under this hypothesis, the WMAP dataset already allows one to extract the parameters relevant for distinguishing among single-field inflation models [12].

An alternative to the standard scenario is represented by the curvaton mechanism [13–16] where the final curvature perturbations are produced from an initial isocurvature perturbation associated to the quantum fluctuations of a light scalar field (other than the inflaton), the curvaton, whose energy density is negligible during inflation. The curvaton isocurvature perturbations are transformed into adiabatic ones when the curvaton decays into radiation much after the end of inflation.[1] Contrary to the standard picture, the curvaton mechanism exploits the fact that the total curvature perturbation (on uniform density hypersurfaces) ζ can change on arbitrarily large scales due to a nonadiabatic pressure perturbation which may be present in a multifluid system [18–22]. While the entropy perturbations evolve independently of the curvature perturbation on large scales, the evolution of the large-scale curvature is sourced by entropy perturbations.

Fortunately, the standard and the curvaton scenarios have different observational signatures. The curvaton scenario allows one to generate the observed level of density perturbations with a much lower scale of inflation and thus generi-

[1] Recently, another mechanism for the generation of cosmological perturbations has been proposed [17]. It acts during the reheating stage after inflation if superhorizon spatial fluctuations in the decay rate of the inflaton field are induced during inflation, causing adiabatic perturbations in the final reheating temperature in different regions of the Universe.

cally predicts a smaller level of gravitational waves. More interestingly, density perturbations generated through the curvaton scenario could be highly non-Gaussian and the level of non-Gaussianity in the primordial perturbations, which is usually parametrized by a dimensionless nonlinear parameter $f_{\rm NL}$, depends upon an unknown parameter r indicating the fraction of energy density contributed by the curvaton field at the epoch of its decay. For tiny values of r, it has been estimated that the non-Gaussianity can be large enough to be detectable by present CMB experiments [14,16]; the current WMAP [23] bound on non-Gaussianity, $|f_{\rm NL}|\lesssim 10^2$, already requires r to be larger than about 10^{-2}.

This curvaton prediction has to be contrasted to that predicted within the traditional one-single field model of inflation where the initially tiny nonlinearity in the cosmological perturbations generated during the inflationary epoch [24,25] gets enhanced in the postinflationary stages giving rise to a well-defined prediction for the nonlinearity in the gravitational potentials [26].

Since a positive future detection of nonlinearity in the CMB anisotropy pattern might allow one to descriminate among the mechanisms by which cosmological adiabatic perturbations are generated, it is clear that the precise determination of the non-Gaussianity predicted by the curvaton mechanism is of primary interest.

The goal of this paper is to provide an exact expression for the nonlinear parameter $f_{\rm NL}$ within the curvaton scenario including second-order corrections from gravity which are particularly relevant in the case in which the curvaton dominates the energy density before it decays. We perform a fully relativistic analysis of the dynamics of second-order perturbations taking advantage of the second-order gauge-invariant curvature perturbation introduced in Refs. [27,28] (see also [24,29,30]) and showing how it evolves on arbitrarily large scales in the presence of two fluids, matter (the curvaton) and radiation. Our results generalize the estimates given in Refs. [14,16] and confirm their findings in the limit $r\ll 1$.

The paper is organized as follows. In Sec. II we briefly summarize the properties of the curvaton scenario and how the primordial curvature perturbations are created at first-order. In Sec. III we compute the second-order curvature perturbation from the curvaton fluctuations and determine the exact expression for the nonlinear parameter $f_{\rm NL}$ as a function of the unknown parameter r. In Sec. IV we show that our findings can be easily generalized to the standard scenario where adiabatic perturbations are provided by the same field driving inflation and prove that the second-order curvature perturbation is conserved even during the reheating stage after inflation when the inflaton field decays to give birth to the standard radiation phase. Finally, Sec. V contains our conclusions.

II. GENERATING THE CURVATURE PERTURBATION AT LINEAR ORDER

During inflation the curvaton field σ is supposed to give a negligible contribution to the energy density and to be an almost free scalar field, with a small effective mass $m_\sigma^2 = |\partial^2 V/\partial\sigma^2| \ll H_I^2$ [14,16], where $H_I=\dot{a}/a$ is the Hubble rate during inflation, a is the scale factor, and a dot denotes derivative with respect to cosmic time.

The unperturbed curvaton field satisfies the equation of motion

$$\sigma'' + 2\mathcal{H}\sigma' + a^2\frac{\partial V}{\partial\sigma} = 0, \tag{1}$$

where a prime denotes differentation with respect to the conformal time $d\tau = dt/a$ and $\mathcal{H}=a'/a$ is the Hubble parameter in conformal time. It is also usually assumed that the curvaton field is very weakly coupled to the scalar fields driving inflation and that the curvature perturbation from the inflaton fluctuations is negligible [14,16]. Thus if we expand the curvaton field up to first-order in the perturbations around the homogeneous background as $\sigma(\tau,\mathbf{x}) = \sigma(\tau) + \delta^{(1)}\sigma$, the linear perturbations satisfy on large scales

$$\delta^{(1)}\sigma'' + 2\mathcal{H}\delta^{(1)}\sigma' + a^2\frac{\partial^2 V}{\partial\sigma^2}\,\delta^{(1)}\sigma = 0. \tag{2}$$

As a result on superhorizon scales its fluctuations $\delta\sigma$ will be Gaussian distributed and with a nearly scale-invariant spectrum given by

$$P^{1/2}_{\delta\sigma}(k) \approx \frac{H_*}{2\pi}, \tag{3}$$

where the subscript asterisk denotes the epoch of horizon exit $k=aH$. Once inflation is over the inflaton energy density will be converted to radiation (γ) and the curvaton field will remain approximately constant until $H^2 \sim m_\sigma^2$. At this epoch the curvaton field begins to oscillate around the minimum of its potential which can be safely approximated to be quadratic $V \approx \frac{1}{2}m_\sigma^2\sigma^2$. During this stage the energy density of the curvaton field just scales as nonrelativistic matter $\rho_\sigma \propto a^{-3}$. The energy density in the oscillating field is

$$\rho_\sigma(\tau,\mathbf{x}) \approx m_\sigma^2\sigma^2(\tau,\mathbf{x}), \tag{4}$$

and it can be expanded into a homogeneous background $\rho_\sigma(\tau)$ and a first-order perturbation $\delta^{(1)}\rho_\sigma$ as

$$\rho_\sigma(\tau,\mathbf{x}) = \rho_\sigma(\tau) + \delta^{(1)}\rho_\sigma(\tau,\mathbf{x}) = m_\sigma^2\sigma + 2m_\sigma^2\sigma\delta^{(1)}\sigma. \tag{5}$$

As it follows from Eqs. (1) and (2) for a quadratic potential the ratio $\delta^{(1)}\sigma/\sigma$ remains constant and the resulting relative energy density perturbation is

$$\frac{\delta^{(1)}\rho_\sigma}{\rho_\sigma} = 2\left(\frac{\delta^{(1)}\sigma}{\sigma}\right)_*, \tag{6}$$

where the asterisk stands for the value at horizon crossing.

Such perturbations in the energy density of the curvaton field produce in fact a primordial density perturbation well after the end of inflation. The primordial adiabatic density perturbation is associated with a perturbation in the spatial curvature ψ and it is usually characterized in a gauge-invariant manner by the curvature perturbation ζ on hypersurfaces of uniform total density ρ. At linear order the quantity ζ is given by the gauge-invariant formula [31]

$$\zeta^{(1)} = -\psi^{(1)} - \mathcal{H}\frac{\delta^{(1)}\rho}{\rho'}, \tag{7}$$

and on large scales it obeys the equation of motion [31,21]

$$\zeta^{(1)\prime} = -\frac{\mathcal{H}}{\rho+P}\delta^{(1)}P_{\rm nad}, \tag{8}$$

where $\delta^{(1)}P_{\rm nad} = \delta^{(1)}P - c_s^2\delta^{(1)}\rho$ is the nonadiabatic pressure perturbation, $\delta^{(1)}P$ being the pressure perturbation and $c_s^2 = P'/\rho'$ the adiabatic sound speed. In the curvaton scenario the curvature perturbation is generated well after the end of inflation during the oscillations of the curvaton field because the pressure of the mixture of matter (curvaton) and radiation produced by the inflaton decay is not adiabatic. A convenient way to study this mechanism is to consider the curvature perturbations ζ_i associated with each individual energy density components, which to linear order are defined as [21]

$$\zeta_i^{(1)} \equiv -\psi^{(1)} - \mathcal{H}\left(\frac{\delta^{(1)}\rho_i}{\rho_i'}\right). \tag{9}$$

Therefore during the oscillations of the curvaton field, the total curvature perturbation in Eq. (7) can be written as a weighted sum of the single curvature perturbations [21,16]

$$\zeta^{(1)} = (1-f)\zeta_\gamma^{(1)} + f\zeta_\sigma^{(1)}, \tag{10}$$

where the quantity

$$f = \frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma} \tag{11}$$

defines the relative contribution of the curvaton field to the total curvature perturbation. From now on we shall work under the approximation of sudden decay of the curvaton field. Under this approximation the curvaton and the radiation components ρ_σ and ρ_γ satisfy separately the energy conservation equations

$$\begin{aligned}\rho_\gamma' &= -4\mathcal{H}\rho_\gamma,\\ \rho_\sigma' &= -3\mathcal{H}\rho_\sigma,\end{aligned} \tag{12}$$

and the curvature perturbation ζ_i remains constant on superhorizon scales until the decay of the curvaton. Therefore from Eq. (10) it follows that the first-order curvature pertubation evolves on large scales as

$$\zeta^{(1)\prime} = f'(\zeta_\sigma^{(1)} - \zeta_\gamma^{(1)}) = \mathcal{H}f(1-f)(\zeta_\sigma^{(1)} - \zeta_\gamma^{(1)}), \tag{13}$$

and by comparison with Eq. (8) one obtains the expression for the nonadiabatic pressure perturbation at first order [14,16]

$$\delta^{(1)}P_{\rm nad} = \rho_\sigma(1-f)(\zeta_\gamma^{(1)} - \zeta_\sigma^{(1)}). \tag{14}$$

Since in the curvaton scenario it is supposed that the curvature perturbation in the radiation produced at the end of inflation is negligible,

$$\zeta_\gamma^{(1)} = -\psi^{(1)} + \frac{1}{4}\frac{\delta^{(1)}\rho_\gamma}{\rho_\gamma} = 0. \tag{15}$$

Similarly the value of $\zeta_\sigma^{(1)}$ is fixed by the fluctuations of the curvaton during inflation,

$$\zeta_\sigma^{(1)} = -\psi^{(1)} + \frac{1}{3}\frac{\delta^{(1)}\rho_\sigma}{\rho_\sigma} = \zeta_{\sigma I}^{(1)}, \tag{16}$$

where I stands for the value of the fluctuations during inflation. From Eq. (10) the total curvature perturbation during the curvaton oscillations is given by

$$\zeta^{(1)} = f\zeta_\sigma^{(1)}. \tag{17}$$

As it is clear from Eq. (17) initially, when the curvaton energy density is subdominant, the density perturbation in the curvaton field $\zeta_\sigma^{(1)}$ gives a negligible contribution to the total curvature perturbation, thus corresponding to an isocurvature (or entropy) perturbation. On the other hand during the oscillations $\rho_\sigma \propto a^{-3}$ increases with respect to the energy density of radiation $\rho_\gamma \propto a^{-4}$, and the perturbations in the curvaton field are then converted into the curvature perturbation. Well after the decay of the curvaton, during the conventional radiation and matter dominated eras, the total curvature perturbation will remain constant on superhorizon scales at a value which, in the sudden decay approximation, is fixed by Eq. (17) at the epoch of curvaton decay

$$\zeta^{(1)} = f_D\zeta_\sigma^{(1)}, \tag{18}$$

where D stands for the epoch of the curvaton decay.

Going beyond the sudden decay approximation it is possible to introduce a transfer parameter r defined as [16,22]

$$\zeta^{(1)} = r\zeta_\sigma^{(1)}, \tag{19}$$

where $\zeta^{(1)}$ is evaluated well after the epoch of the curvaton decay and $\zeta_\sigma^{(1)}$ is evaluated well before this epoch. The numerical study of the coupled perturbation equations has been performed in Ref. [22] showing that the sudden decay approximation is exact when the curvaton dominates the energy density before it decays $(r=1)$, while in the opposite case

$$r \approx \left(\frac{\rho_\sigma}{\rho}\right)_D. \tag{20}$$

III. SECOND-ORDER CURVATURE PERTURBATION FROM THE CURVATON FLUCTUATIONS

Here we generalize to second-order in the density perturbations the results of the previous section.

As it has been shown in Ref. [28] it is possible to define the second-order curvature perturbation on uniform total density hypersurfaces by the quantity (up to a gradient term)

$$\zeta^{(2)}=-\psi^{(2)}-\mathcal{H}\frac{\delta^{(2)}\rho}{\rho'}+2\mathcal{H}\frac{\delta^{(1)}\rho'}{\rho'}\frac{\delta^{(1)}\rho}{\rho'}+2\frac{\delta^{(1)}\rho}{\rho'}\times(\psi^{(1)'}+2\mathcal{H}\psi^{(1)})-\left(\frac{\delta^{(1)}\rho}{\rho'}\right)^2\left(\mathcal{H}\frac{\rho''}{\rho'}-\mathcal{H}'-2\mathcal{H}^2\right), \tag{21}$$

where the curvature perturbation ψ has been expanded up to second order as $\psi=\psi^{(1)}+\frac{1}{2}\psi^{(2)}$ and $\delta^{(2)}\rho$ corresponds to the second-order perturbation in the total energy density around the homogeneous background $\rho(\tau)$,

$$\rho(\tau,\mathbf{x})=\rho(\tau)+\delta\rho(\tau,\mathbf{x})=\rho(\tau)+\delta^{(1)}\rho(\tau,\mathbf{x})+\frac{1}{2}\delta^{(2)}\rho(\tau,x). \tag{22}$$

The quantity $\zeta^{(2)}$ is gauge-invariant and, as its first-order counterpart defined in Eq. (7), it is sourced on superhorizon scales by a second-order nonadiabatic pressure perturbation [28].

In Ref. [26] the conserved quantity $\zeta^{(2)}$ has been used in the standard scenario where the generation of cosmological perturbations is induced by fluctuations of the inflaton field (and there is no curvaton) in order to follow the evolution on large scales of the primordial nonlinearity in the cosmological perturbations from a period inflation to the matter dominate era. In the present scenario the conversion of the curvaton isocurvature perturbations into a final curvature perturbation at the epoch of the curvaton decay can be followed through the sum (10) of the individual curvature perturbations weighted by the ratio f of Eq. (11).

Let us now extend such a result at second order in the perturbations. As we shall see in Sec. III A this result will enable us to compute in an exact way the level of non-Gaussianity produced by the nonlinearity of the perturbations in the curvaton energy density.

Since the quantities $\zeta_i^{(1)}$ and $\zeta_i^{(2)}$ are gauge-invariant, we choose to work in the spatially flat gauge $\psi=0$ if not otherwise specified. Note that from Eqs. (6) and (16) the value of $\zeta_\sigma^{(1)}$ is thus given by

$$\zeta_\sigma^{(1)}=\frac{1}{3}\frac{\delta^{(1)}\rho_\sigma}{\rho_\sigma}=\frac{2}{3}\frac{\delta^{(1)}\sigma}{\sigma}=\frac{2}{3}\left(\frac{\delta^{(1)}\sigma}{\sigma}\right)_*, \tag{23}$$

where we have used the fact that $\zeta_\sigma^{(1)}$ (or equivalently $\delta^{(1)}\sigma/\sigma$) remains constant, while from Eq. (16) in the spatially flat gauge

$$\zeta_\gamma^{(1)}=\frac{1}{4}\frac{\delta^{(1)}\rho_\gamma}{\rho_\gamma}. \tag{24}$$

During the oscillations of the curvaton field the first-order energy conservation equations in the spatially flat gauge $\psi=0$ yield on large scales[2]

$$\delta^{(1)}\rho'=\delta^{(1)}\rho'_\sigma+\delta^{(1)}\rho'_\gamma=-3\mathcal{H}\delta^{(1)}\rho_\sigma-4\mathcal{H}\delta^{(1)}\rho_\gamma, \tag{25}$$

and hence using Eqs. (12), (23), (24), and (25)

$$\frac{\delta^{(1)}\rho'}{\rho'}=3f\zeta_\sigma^{(1)}+4(1-f)\zeta_\gamma^{(1)},$$
$$\mathcal{H}\frac{\delta^{(1)}\rho}{\rho'}=-f\zeta_\sigma^{(1)}-(1-f)\zeta_\gamma^{(1)}. \tag{26}$$

We can thus rewrite the total second-order curvature perturbation $\zeta^{(2)}$ as

$$\zeta^{(2)}=-\mathcal{H}\frac{\delta^{(2)}\rho}{\rho'}-[f\zeta_\sigma^{(1)}+(1-f)\zeta_\gamma^{(1)}]\times[f^2\zeta_\sigma^{(1)}+(1-f)(2+f)\zeta_\gamma^{(1)}]. \tag{27}$$

In a similar manner to the linear order, let us introduce now the curvature perturbations $\zeta_i^{(2)}$ at second order for each individual component. Such quantities will be given by the same formula as Eq. (21) relatively to each energy density ρ_i,

$$\zeta_i^{(2)}=-\psi^{(2)}-\mathcal{H}\frac{\delta^{(2)}\rho_i}{\rho_i'}+2\mathcal{H}\frac{\delta^{(1)}\rho_i'}{\rho_i'}\frac{\delta^{(1)}\rho_i}{\rho_i'}+2\frac{\delta^{(1)}\rho_i}{\rho_i'}\times(\psi^{(1)'}+2\mathcal{H}\psi^{(1)})-\left(\frac{\delta^{(1)}\rho_i}{\rho_i'}\right)^2\left(\mathcal{H}\frac{\rho_i''}{\rho_i'}-\mathcal{H}'-2\mathcal{H}^2\right). \tag{28}$$

Using the same procedure described above it follows that in the spatially flat gauge

$$\zeta_\sigma^{(2)}=\frac{1}{3}\frac{\delta^{(2)}\rho_\sigma}{\rho_\sigma}-(\zeta_\sigma^{(1)})^2, \tag{29}$$

$$\zeta_\gamma^{(2)}=\frac{1}{4}\frac{\delta^{(2)}\rho_\gamma}{\rho_\gamma}-2(\zeta_\gamma^{(1)})^2. \tag{30}$$

Such quantities are gauge-invariant and, in the sudden decay approximation they are separately conserved until the curvaton decay. Using Eqs. (29) and (30) to express the second-order perturbation in the total energy density $\delta^{(2)}\rho=\delta^{(2)}\rho_\sigma+\delta^{(2)}\rho_\gamma$, and after some algebra, one finally obtains the following expression for the total curvature perturbation $\zeta^{(2)}$:

$$\zeta^{(2)}=f\zeta_\sigma^{(2)}+(1-f)\zeta_\gamma^{(2)}+f(1-f)(1+f)(\zeta_\sigma^{(1)}-\zeta_\gamma^{(1)})^2. \tag{31}$$

[2]Here and in the following we neglect gradient terms which, upon integration over time, may give rise to nonlocal operators which are not necessarily suppressed on large scales being of the form $\nabla^{-2}[\nabla(\cdot)\nabla(\cdot)]$ or $\nabla^{-2}[(\cdot)\nabla^2(\cdot)]$. However, note that these gradient terms will not affect the gravitational potential bispectrum on large scales.

Equation (31) is one of our main results. It generalizes to second-order in the perturbations the weighted sum of Eq. (10). In particular notice that in the limit where one of the two fluids is completely subdominant ($f\to 0$ or $f\to 1$) the corresponding curvature perturbation $\zeta_i^{(2)}$ turns out to coincide with the total one $\zeta^{(2)}$.

Under the sudden decay approximation of the curvaton field the individual curvature perturbations are separately conserved on large scales, and thus from Eq. (31) it follows that $\zeta^{(2)}$ evolves according to the equation

$$\zeta^{(2)\prime}=f'(\zeta_\sigma^{(2)}-\zeta_\gamma^{(2)})+f'(1-3f^2)(\zeta_\sigma^{(1)}-\zeta_\gamma^{(1)})^2. \quad (32)$$

Note that Eq. (32) can be rewritten as [28]

$$\zeta^{(2)\prime}=-\frac{\mathcal{H}}{\rho+P}\widehat{\delta^{(2)}}P-\frac{2}{\rho+P}[\delta^{(1)}P_{\rm nad}-2(\rho+P)\zeta^{(1)}]\zeta^{(1)\prime}, \quad (33)$$

with $\delta^{(1)}P_{\rm nad}$ given by Eq. (14) and

$$\widehat{\delta^{(2)}}P=\rho_\sigma(1-f)[(\zeta_\gamma^{(2)}-\zeta_\sigma^{(2)})+(f^2+6f-1)\times(\zeta_\sigma^{(1)}-\zeta_\gamma^{(1)})^2+4\zeta_\gamma^{(1)}(\zeta_\sigma^{(1)}-\zeta_\gamma^{(1)})] \quad (34)$$

is the gauge-invariant nonadiabatic pressure perturbation on uniform density hypersurfaces on large scales which can be checked to coincide with the generic expression provided in Ref. [28]:

$$\widehat{\delta^{(2)}}P=\delta^{(2)}P-\frac{P'}{\rho'}\delta^{(2)}\rho+P'\left[2\left(\frac{\delta^{(1)\prime}\rho}{\rho'}-\frac{\delta^{(1)\prime}P}{P'}\right)\frac{\delta^{(1)}\rho}{\rho'}+\left(\frac{P''}{P}-\frac{\rho''}{\rho}\right)\left(\frac{\delta^{(1)}\rho}{\rho'}\right)^2\right]. \quad (35)$$

The second-order curvature perturbation in the standard radiation or matter eras will remain constant on superhorizon scales and, in the sudden decay approximation, it is thus given by the quantity in Eq. (31) evaluated at the epoch of the curvaton decay,

$$\zeta^{(2)}=f_D\zeta_\sigma^{(2)}+f_D(1-f_D^2)(\zeta_\sigma^{(1)})^2, \quad (36)$$

where we have used the curvaton hypothesis that the curvature perturbation in the radiation produced at the end of inflation is negligible so that $\zeta_\gamma^{(1)}\approx 0$ and $\zeta_\gamma^{(2)}\approx 0$. The curvature perturbation $\zeta_\sigma^{(1)}$ is given by Eq. (23), while $\zeta_\sigma^{(2)}$ in Eq. (29) is obtained by expanding the energy density of the curvaton field, Eq. (4), up to second order in the curvaton fluctuations

$$\rho_\sigma(\mathbf{x},t)=\rho_\sigma(\tau)+\delta^{(1)}\rho_\sigma(\tau,x^i)+\frac{1}{2}\delta^{(2)}\rho_\sigma(\tau,x^i)$$
$$=m_\sigma^2\sigma+2m_\sigma^2\ \sigma\ \ \delta^{(1)}\sigma+m_\sigma^2(\delta^{(1)}\sigma)^2. \quad (37)$$

It follows that

$$\frac{\delta^{(2)}\rho_\sigma}{\rho_\sigma}=\frac{1}{2}\left(\frac{\delta^{(1)}\rho_\sigma}{\rho_\sigma}\right)^2=\frac{9}{2}(\zeta_\sigma^{(1)})^2, \quad (38)$$

where we have used Eq. (23), and hence from Eq. (29) we obtain

$$\zeta_\sigma^{(2)}=\frac{1}{2}(\zeta_\sigma^{(1)})^2=\frac{1}{2}(\zeta_\sigma^{(1)})_I^2, \quad (39)$$

where we have emphasized that also $\zeta_\sigma^{(2)}$ is a conserved quantity whose value is determined by the curvaton fluctuations during inflation. Plugging Eq. (39) into Eq. (36) the curvature perturbation during the standard radiation or matter dominated eras turns out to be

$$\zeta^{(2)}=f_D\left(\frac{3}{2}-f_D^2\right)(\zeta_\sigma^{(1)})^2. \quad (40)$$

Non-Gaussianity of the curvaton perturbations

Let us now focus on the calculation of the nonlinear parameter $f_{\rm NL}$ which is usually adopted to characterize the level of non-Gaussianity of the Bardeen potential [32]. In order to compute $f_{\rm NL}$ in the curvaton scenario, we switch from the spatially flat gauge $\psi=0$ to the longitudinal or Poisson gauge [33]. Such a procedure is possible since the curvature perturbations $\zeta_i^{(2)}$ are gauge-invariant quantities. In particular this is evident from the expression found in Eq. (40). During the matter dominated era from Eq. (21) it turns out that [26]

$$\zeta^{(2)}=-\psi^{(2)}+\frac{1}{3}\frac{\delta^{(2)}\rho}{\rho}+\frac{5}{9}\left(\frac{\delta^{(1)}\rho}{\rho}\right)^2$$
$$=-\psi^{(2)}+\frac{1}{3}\frac{\delta^{(2)}\rho}{\rho}+\frac{20}{9}(\psi^{(1)})^2, \quad (41)$$

where in the last step we have used that on large scales $\delta^{(1)}\rho/\rho=-2\psi^{(1)}$ in the Poisson gauge [26]. Equation (41) combined with Eq. (40), which gives the constant value on large scales of the curvature perturbation $\zeta^{(2)}$ during the matter dominated era, yields

$$\psi^{(2)}-\frac{1}{3}\frac{\delta^{(2)}\rho}{\rho}=\frac{1}{9}\left[20-\frac{75}{2f_D}+25f_D\right](\psi^{(1)})^2, \quad (42)$$

where we have used $f_D\zeta_\sigma^{(1)}=-\frac{5}{3}\psi^{(1)}$ from Eq. (18) and the usual linear relation between the curvature perturbation and the Bardeen potential $\zeta^{(1)}=-\frac{5}{3}\psi^{(1)}$ during the matter dominated era. Since on large scales [from the second-order (0 -0] and $(i-j)$ components of Einstein equations, see Eqs.

(A.39) and (A.42-43) in [24]] the following relations hold during the matter-dominated phase:

$$\phi^{(2)}=-\frac{1}{2}\frac{\delta\rho^{(2)}}{\rho}+4(\psi^{(1)})^2,$$
$$\psi^{(2)}-\phi^{(2)}=-\frac{2}{3}(\psi^{(1)})^2+\frac{10}{3}\nabla^{-2}(\psi^{(1)}\nabla^2\psi^{(1)})-10\nabla^{-2}[\partial^i\partial_j(\psi^{(1)}\partial_i\partial^j\psi^{(1)})], \tag{43}$$

we conclude that

$$\phi^{(2)}=\left[\frac{10}{3}+\frac{5}{3}f_D-\frac{5}{2f_D}\right](\psi^{(1)})^2-2\nabla^{-2}(\psi^{(1)}\nabla^2\psi^{(1)})+6\nabla^{-2}[\partial^i\partial_j(\psi^{(1)}\partial_i\partial^j\psi^{(1)})]. \tag{44}$$

The total curvature perturbation will then have a non-Gaussian (χ^2) component. The lapse function $\phi=\phi^{(1)}+\frac{1}{2}\phi^{(2)}$ can be expressed in momentum space as

$$\phi(\mathbf{k})=\phi^{(1)}(\mathbf{k})+\frac{1}{(2\pi)^3}\int d^3k_1 d^3k_2\,\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\times f_{\rm NL}^{\phi}(\mathbf{k}_1,\mathbf{k}_2)\,\phi^{(1)}(\mathbf{k}_1)\,\phi^{(1)}(\mathbf{k}_2), \tag{45}$$

where we have defined an effective "momentum-dependent" nonlinearity parameter $f_{\rm NL}^{\phi}$. Here the linear lapse function $\phi^{(1)}=\psi^{(1)}$ is a Gaussian random field. The gravitational potential bispectrum reads

$$\langle\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\rangle=(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\times[2f_{\rm NL}^{\phi}(\mathbf{k}_1,\mathbf{k}_2)\mathcal{P}_\phi(k_1)\mathcal{P}_\phi(k_2)+\text{cyclic}], \tag{46}$$

where $\mathcal{P}_\phi(k)$ is the power spectrum of the gravitational potential. From Eq. (44) we read the nonlinearity parameter

$$f_{\rm NL}^{\phi}=\left[\frac{7}{6}+\frac{5}{6}r-\frac{5}{4r}\right]+g(\mathbf{k}_1,\mathbf{k}_2), \tag{47}$$

where

$$g(\mathbf{k}_1,\mathbf{k}_2)=\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k^2}\left(1+3\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k^2}\right), \tag{48}$$

with $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$ and we have replaced f_D with r to go beyond the sudden approximation. Notice that in the final bispectrum expression, the diverging terms arising from the infrared behavior of $f_{\rm NL}^{\phi}(\mathbf{k}_1,\mathbf{k}_2)$ are automatically regularized once the monopole term is subtracted from the definition of ϕ (by requiring that $\langle\phi\rangle=0$).

As far as the momentum-independent part is concerned, we note that in the limit $r\ll1$ we obtain $f_{\rm NL}^{\phi}=-(5/4r)$ which reproduces the estimate provided in [14,16], while, in the limit $r\simeq1$, we obtain $f_{\rm NL}^{\phi}=\frac{3}{4}$ for $r\simeq1$.[3] These values have to be compared to the value $f_{\rm NL}^{\phi}=-\frac{1}{2}$ [26] obtained for perturbations whose wavelengths reenter the horizon during the matter-dominated phase for the standard scenario in which curvature perturbations are induced by fluctuations of the inflaton field. We conclude that if $r\ll1$ the non-Gaussianity in the curvaton scenario is larger than the one predicted in the standard scenario. Finally, we point out that additional non-Gaussianity will be generated after horizon-crossing, due to known Newtonian and relativistic second-order contributions which are relevant on subhorizon scales, such as the Rees-Sciama effect [34], whose detailed analysis has been given in Ref. [35]. It is important to consider also these effects when making a comparison with observations.

IV. A COMMENT ON THE EVOLUTION OF THE CURVATURE PERTURBATION DURING THE REHEATING PHASE IN THE STANDARD SCENARIO

Note that Eq. (31) is indeed valid in the general framework of an oscillating scalar field and a radiation fluid, the curvaton scenario being only a particular case. Thus in this section we shall indicate the generic scalar field by φ instead of σ. From Eq. (31) it is possible to derive the following equation of motion on large scales for the second-order curvature perturbation $\zeta^{(2)}$:

$$\zeta^{(2)\prime}=f'(\zeta_\varphi^{(2)}-\zeta_\gamma^{(2)})+f'(1-3f^2)(\zeta_\varphi^{(1)}-\zeta_\gamma^{(1)})^2, \tag{49}$$

where we have used the fact that, in the approximation of sudden decay of the scalar field φ, the individual curvature perturbations at first and second-order are separately conserved. Using Eqs. (10) and (31) it is possible to rewrite $\zeta^{(2)\prime}$ in terms only of $\zeta^{(2)},\zeta_\varphi^{(2)}$ and $\zeta^{(1)},\zeta_\varphi^{(1)}$ as

$$\zeta^{(2)\prime}=-\mathcal{H}f(\zeta^{(2)}-\zeta_\varphi^{(2)})+\mathcal{H}f(1+2f)(\zeta^{(1)}-\zeta_\varphi^{(1)})^2. \tag{50}$$

Here we want to make a simple but important observation. Besides the curvaton scenario, the most interesting case where there is an oscillating scalar field and a radiation fluid is just the phase of reheating following a period of inflation in the standard scenario for the generation of cosmological perturbations on large scales. In such a situation the oscillating scalar field is just the inflaton field φ whose fluctuations induce curvature perturbations. Therefore it is possible to see in a straightforward way that during the reheating phase, when the inflaton field finally decays into radiation, a solution of Eq. (50) is the one corresponding to a total curvature perturbation which is indeed fixed by the inflaton curvature perturbation during inflation $\zeta^{(1)}=\zeta_\varphi^{(1)}=\zeta_{\varphi I}^{(1)}$, $\zeta^{(2)}=\zeta_\varphi^{(2)}=\zeta_{\varphi I}^{(2)}$.

[3] Notice that the formula (36) in [16] for the estimate of the nonlinear parameter contains a sign misprint and should read $f_{\rm NL}^{\phi}\simeq-(5/4r)$, giving $f_{\rm NL}^{\phi}\simeq-\frac{5}{4}$ for $r\simeq1$.

V. CONCLUSIONS

In this paper we have determined the evolution on large scales of the second-order curvature perturbation within the curvaton scenario where two fluids are present, the curvaton and radiation. We have computed the nonlinear parameter $f_{\rm NL}$ measuring the level of non-Gaussianity in the primordial cosmological perturbations and provide its exact expression as a function of the parameter r. Our findings are particularly interesting if one wishes to extract from a positive future detection of nonlinearity in the CMB anisotropy pattern a way to descriminate among the mechanisms by which cosmological adiabatic perturbations are generated. It would be interesting to extend our results to those models which can accommodate for a primordial value of $f_{\rm NL}$ larger than unity. This is the case, for instance, of a large class of multifield inflation models which leads to either non-Gaussian isocurvature perturbations [36] or cross-correlated non-Gaussian adiabatic and isocurvature modes [37] and the so-called ‘‘inhomogeneous reheating'' mechanism where the curvature perturbations are generated by spatial variations of the inflaton decay rate [17].

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