Higher-order QED corrections to *W*-boson mass determination at hadron colliders

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The impact of higher-order final-state photonic corrections on the precise determination of the W-boson mass at the Fermilab Tevatron and CERN LHC colliders is evaluated. In the presence of realistic selection criteria, the shift in the W mass from a fit to the transverse mass distribution is found to be about 10 MeV in the $W \to \mu\nu$ channel and a few MeV in the $W \to e\nu$ channel. The calculation, which is implemented in a Monte Carlo event generator for data analysis, can contribute to reduce the uncertainty associated with the W mass measurement at future hadron collider experiments.

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Precision tests of the standard model require more and more accurate knowledge of the basic parameters of the theory. In particular, future measurements of the W-boson and top quark masses at the Fermilab Tevatron and CERN Large Hadron Collider (LHC) are expected to considerably improve the present indirect bound on the Higgs-boson mass from electroweak precision data. As recently discussed [1], a precision of 27 MeV (16 MeV) for the W-boson mass M_W is the target value for run IIa (run IIb) of the Tevatron. An accuracy of 15 MeV is the goal of the LHC [2].

In order to measure M_W with such a high precision in a hadron collider environment, it is mandatory to keep under control higher-order QCD and electroweak radiative corrections to the W and Z production processes. The status of QCD corrections to weak boson production in hadronic collisions is reviewed in Ref. [3], while recent progress in the calculation of electroweak corrections, as achieved by means of independent calculations [4–7], is summarized in Ref. [8]. As shown in Refs. [2,4–8], the relevant distributions to extract M_W (e.g., the transverse mass spectrum) are mainly modified by photon radiation and, in particular, by final-state photon radiation, which gives rise to collinear logarithms of the form $(\alpha/\pi)\log(\hat{s}/m_l^2)$, where $\hat{s}$ is the effective center of mass (c.m.) energy and m_l is the mass of the final-state lepton. This poses the question of the impact of higher-order (i.e., beyond order α) leading logarithmic corrections due to multiphoton radiation. A first attempt toward the inclusion of $O(\alpha^2)$ QED corrections was the calculation of the double-bremsstrahlung matrix elements $q\overline{q'} \to W \to l\nu\gamma\gamma$ and $q\overline{q} \to \gamma, Z \to l^+l^-\gamma\gamma$ $(l=e,\mu)$ performed in Ref. [9]. The aim of the present work is to evaluate the impact of higher-order final-state QED corrections on the W mass determination at hadron colliders, by including both real bremsstrahlung and virtual corrections. To this end, a parton shower (PS) approach in QED [10] is employed to simulate multiphoton radiation effects. An independent calculation of multiphoton radiative corrections in leptonic W decays has appeared very recently [11], but without quantifying their impact on the W mass measurement. The uncertainty in the W mass due to higher-order QED effects is presently estimated by the Collider Detector at Fermilab (CDF) Collaboration at the Tevatron to be 20 MeV in the $W \to e\nu$ channel, and 10 MeV in the $W \to \mu\nu$ channel [12]. An uncertainty of 12 MeV is assigned by the DØ Collaboration to the $W \to e\nu$ channel [13].

An appropriate theoretical tool to compute photonic radiative corrections in the leading log approximation is the QED PS approach [10]. It consists of a numerical solution of the QED Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation for the charged lepton structure function $D(x,Q^2)$ in the non-singlet channel. The solution can be cast in the form [10]

$$
\begin{aligned}
D(x,Q^2) = {} & \Pi(Q^2,m^2)\,\delta(1-x) \\
& + \left(\frac{\alpha}{2\pi}\right)\int_{m^2}^{Q^2}\Pi(s,s')\frac{ds'}{s'}\Pi(s',m^2) \\
& \times \int_0^{x_+} dy\, P(y)\,\delta(x-y) \\
& + \left(\frac{\alpha}{2\pi}\right)^2\int_{m^2}^{Q^2}\Pi(s,s')\frac{ds'}{s'}\int_{m^2}^{s'}\Pi(s',s'')\frac{ds''}{s''} \\
& \times \Pi(s'',m^2)\int_0^{x_+}dx_1\int_0^{x_+}dx_2 P(x_1)P(x_2) \\
& \times \delta(x-x_1x_2) + \cdots ,
\end{aligned}
\tag{1}
$$

where

$$
\Pi(s_1,s_2) = \exp\left[-\frac{\alpha}{2\pi}\int_{s_2}^{s_1}\frac{ds'}{s'}\int_0^{x_+}dz P(z)\right]
$$

is the Sudakov form factor, $P(z)$ is the $e \to e+\gamma$ splitting function and x_+ is an infrared regulator. Equation (1) allows us to compute $D(x,Q^2)$ by means of a Monte Carlo (MC) algorithm, which, as shown in Ref. [10], simulates the emission of a shower of (real and virtual) photons by a charged fermion and accounts for exponentiation of soft photons and resummation of collinear logarithms due to multiple hard bremsstrahlung. A remarkable advantage of the PS algorithm is the possibility of generating transverse momentum p_T of fermions and photons at each branching. The generation of transverse degrees of freedom can be performed according to different recipes, as described in detail in Ref. [10]. Here, we generate photon angular variables according to the leading

TABLE I. Comparison between the present calculation (HORACE) and WGRAD [5,17] for the $p\overset{(-)}{p}\to W\to l\nu$, $l=e,\mu$ cross sections (in pb), at the Tevatron run II and the LHC.

	$\sqrt{s}=2$ TeV		$\sqrt{s}=14$ TeV	
	e	μ	e	μ
Born WGRAD	441.7(1)		1906(1)	
WGRAD	418.3(4)	429.4(3)	1800(2)	1845(2)
final-state WGRAD	419.7(1)	430.0(1)	1808(1)	1854(1)
Born HORACE	441.6(1)		1905(1)	
$O(\alpha)$ HORACE	419.4(1)	429.9(1)	1806(1)	1853(1)
exponentiated HORACE	419.5(1)	430.0(1)	1808(1)	1853(1)

pole behavior $1/(1-\beta_l\cos\vartheta_{l\gamma})$, where β_l is the lepton velocity and $\vartheta_{l\gamma}$ is the relative lepton-photon angle.

A simple recipe to evaluate final-state corrections to $p\overset{(-)}{p}\to W\to\nu l$ consists of attaching a single structure function $D(x,Q^2)$ to the lepton coming from the W decay. Needless to say, this amounts to the neglect of photonic corrections due to initial-state radiation, initial-state–final-state interference, and W-boson emission. However, it is known that radiation from an internal off-shell particle cannot contribute to leading logarithmic corrections, which are the main concern of the present study. On the other hand, initial-state QED corrections need an appropriate treatment, as carefully discussed in Refs. [4–8,14] and as very recently reanalyzed in Ref. [15]. Actually, radiation off quarks gives rise to quark mass singularities that must be reabsorbed in parton distribution functions (PDF), in analogy to gluon emission in QCD. This requires the inclusion of QED corrections into the DGLAP evolution of PDFs and into their fit to experimental data. Currently, available PDF sets do not incorporate such effects. However, it is known [2,14,15] that the inclusion of QED contributions in PDF evolution results in a modified scale dependence of PDFs, which was found to be negligible in the x range relevant for W-boson production at hadron colliders [6]. Further, photonic initial-state and initial-state–final-state interference corrections have only a small and uniform effect on the M_T distribution, while QED final-state radiation significantly distorts the M_T shape, thus considerably affecting the extracted value of M_W [5].

Even if the treatment of final-state photon radiation alone is not gauge invariant, it can be easily checked by comparing the PS spectrum with the gauge-invariant factor for collinear photon emission by a fermion [16]. It is found that gauge violations are confined to the nonlogarithmic corrections, which are beyond the approximation of the present analysis.

In order to quantify the effect of higher-order final-state QED corrections on the W mass determination, we developed the MC event generator HORACE (higher-order radiative corrections) following the approach described above and performed a number of MC experiments. Before the phenomenological analysis, we performed a tuned comparison between the predictions of HORACE and those of WGRAD [5,17], to verify the accuracy of our calculation. The results of such a comparison are shown in Table I, using default PDFs, input parameters, and cuts as in Ref. [17]. WGRAD includes the $O(\alpha)$ electroweak radiative corrections to W production in the pole approximation (second line in Table I) and it also gives the possibility for selecting the effect of a gauge-invariant subset due to final-state corrections (final-state WGRAD in Table I). Therefore, the difference, at a few 0.1% level, between WGRAD and final-state WGRAD points out, when comparing with the Born predictions, the dominance of final-state radiation within the full set of $O(\alpha)$ electroweak corrections.

On the other hand, it can be seen that the predictions by the final-state WGRAD are in very good agreement with our results by $O(\alpha)$ HORACE, which is an order-α expansion of the complete PS algorithm. Since the differences are well below the 0.1% level, this comparison demonstrates that the gauge-invariance violations present in our approach are numerically negligible. The contribution of higher-order effects can be seen by comparing $O(\alpha)$ HORACE with our complete predictions given by exponentiated HORACE. Their effect on the integrated cross section is tiny, within 0.1%.

Having established the physical and technical accuracy of our calculation, we move to the analysis of the W mass shift due to higher-order corrections. The input parameters used in the simulations are

$$\begin{aligned}
&m_{\nu_l}=0,\quad m_e=0.511\times10^{-3}\ \text{GeV},\\
&m_\mu=0.10565836\ \text{GeV},\quad \alpha^{-1}=137.03599976,\\
&G_\mu=1.16639\times10^{-5}\ \text{GeV}^{-2},\quad \alpha_s=0.1185,\\
&M_W=80.423\ \text{GeV},\quad M_Z=91.1882\ \text{GeV},\\
&\sin^2\theta_W=1-\frac{M_W^2}{M_Z^2},\quad \Gamma_W=\frac{3G_\mu M_W^3}{2\sqrt{2}\pi}\left(1+\frac{2\alpha_s}{3\pi}\right).
\end{aligned}\tag{2}$$

We adopt the G_μ scheme and fixed-width scheme in our calculation. At the parton level, we consider the processes

$$\begin{aligned}
&u+\bar{d}\to W^+\to l^++\nu_l,\quad u+\bar{s}\to W^+\to l^++\nu_l,\\
&c+\bar{d}\to W^+\to l^++\nu_l,\quad c+\bar{s}\to W^+\to l^++\nu_l,
\end{aligned}\tag{3}$$

and their charge conjugate, with $l=e,\mu$ and CKM matrix elements according to Ref. [18]. The results for the processes $p\bar{p}\to W\to l+\nu$ (Tevatron) and $pp\to W\to l+\nu$ (LHC) are obtained by convoluting the parton-level matrix element with

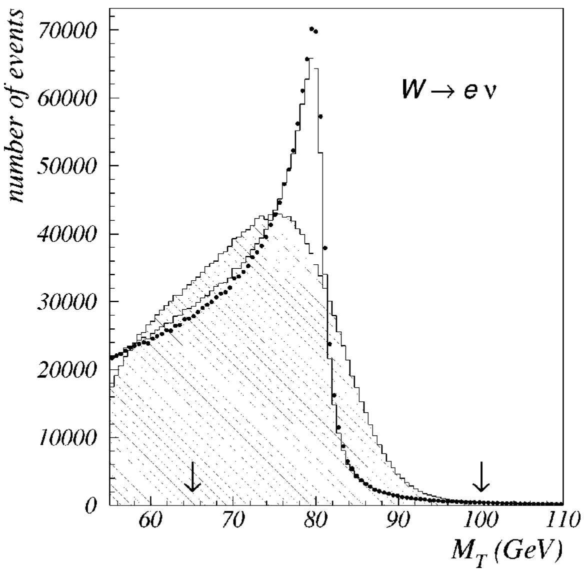

FIG. 1. The transverse mass distribution without lepton identification criteria and detector resolutions (solid histogram), with lepton identification criteria (markers) and with detector resolutions (shaded histogram), in the $W \to e\nu$ channel at $\sqrt{s}=2$ TeV. Arrows indicate the fit region.

CTEQ6 PDFs [19]. The virtuality scale Q^2 is set to be $Q^2 = \hat{s}$, $\hat{s}$ being the effective c.m. energy after gluon radiation, in both PDFs and lepton structure function. The c.m. energies considered are $\sqrt{s}=2$ TeV for the Tevatron and $\sqrt{s}=14$ TeV for the LHC.

To model the acceptance cuts used by the CDF and DØ Collaborations in their W mass analyses, we impose the following transverse momentum (p_T) and pseudorapidity (η) cuts: $p_T(l)>25$ GeV, $|\eta(l)|<1.2$, $\not{p}_T>25$ GeV. However, in order to perform a realistic phenomenological analysis and study the dependence of the W mass shift from detector effects, we implement, in addition to the above cuts, the lepton identification requirements quoted in Table I of Ref. [5]. Furthermore, we simulate uncertainties in the energy and momentum measurements of the charged leptons in the detector by means of a Gaussian smearing of the particle four-momenta, using as standard deviation values the specifications relative to electrons and muons for the run II DØ detector [20].

The strategy followed by the CDF and DØ Collaborations to extract M_W from the data is to perform a maximum likelihood fit to the transverse mass distribution of the final-state lepton pair or to the transverse momentum of the charged lepton. Here we consider the transverse mass, which is the preferred quantity to determine the W mass and is defined as

$$M_T = \sqrt{2 p_T(l) p_T(\nu)(1-\cos\phi^{l\nu})}, \tag{4}$$

where $p_T(l)$ and $p_T(\nu)$ are the transverse momentum of the lepton and neutrino, and $\phi^{l\nu}$ is the angle between the lepton and the neutrino in the transverse plane. The transverse mass distribution, as obtained by our simulation, is shown in Fig. 1 at $\sqrt{s}=2$ TeV. The distribution without lepton identification requirements and smearing effects (solid histogram) is compared to the distribution including lepton identification criteria (markers) and detector resolutions (shaded histogram). The shape of the M_T spectrum is considerably modified by detector resolution effects, in agreement with the results shown in Refs. [12,13]. The arrows in Fig. 1 select the range 65 GeV$<M_T<$100 GeV, which is used by the CDF Collaboration in its W mass analysis and one that we also adopt in the fitting procedure described below.

To evaluate the shift induced by higher-order corrections on the W mass, we perform binned χ^2 fits and binned maximum likelihood fits to the M_T distribution, in complete analogy with the experimental fitting procedure. Here we show only the results of the χ^2 fits, because the results of the maximum likelihood fits are in perfect agreement with the former. Using HORACE, we generate a sample of pseudodata and calculate with high numerical precision the m_T spectrum (binned into 100 bins) at the Born level and for a fixed, "physical" value of the W mass, i.e., $M_W^{\rm ref}=80.423$ GeV. Next, we compute the m_T spectrum including $O(\alpha)$ leading-log corrections for 20 hypothesized W mass values, with a spacing of 5 MeV for the $W\to e\nu$ channel and 10 MeV for the $W\to\mu\nu$ channel. We then normalize the spectra within the fit interval and we calculate, for each M_W value, the χ^2 as $\chi^2=\sum_i (\sigma_{i,\alpha}-\sigma_{i,\rm Born})^2/(\Delta\sigma^2_{i,\alpha}+\Delta\sigma^2_{i,\rm Born})$, where $\sigma_{i,\rm Born}$ and $\sigma_{i,\alpha}$ are the MC predictions for the ith bin at the Born and $O(\alpha)$ level, respectively, and $\Delta\sigma_{i,\rm Born}, \Delta\sigma_{i,\alpha}$ the corresponding statistical errors due to numerical integration. This allows to quantify the mass shift due to $O(\alpha)$ corrections. The shift due to higher-order corrections is derived according to the same procedure, by generating a sample of pseudodata for the M_T distribution at $O(\alpha)$ and fitting them in terms of the M_T spectrum obtained including higher-order corrections for 10 hypothesized W mass values. In this case, we use 1 MeV spacing between masses. Figure 2 shows the $\Delta\chi^2=\chi^2-\chi^2_{\rm min}$ distributions as a function of $\Delta M_W \equiv M_W - M_W^{\rm ref}$, for the fit with $O(\alpha)$ corrections (left) and the fit with higher-order corrections (right). The mass shift observed for $O(\alpha)$ corrections amounts to about 20 MeV for the $W\to e\nu$ decay (dashed line) and to 110 MeV for the $W\to\mu\nu$ decay (solid line), as a consequence of the different identification requirements. It is worth noticing that, due to $O(\alpha)$ corrections, events are shifted to lower values of M_T and then we expect that the best fit value of M_W is greater than $M_W^{\rm ref}$, as observed in Fig. 2. These shifts are in reasonable agreement with the results of the CDF and DØ Collaborations, even in the absence of a complete detector simulation. The mass shift due to higher-order effects is about 10 MeV for the $W\to\mu\nu$ channel (solid line) and a few MeV (dashed line) for the $W\to e\nu$ channel. As expected, higher-order contributions slightly reduce the effect due to $O(\alpha)$ corrections. We performed the same analysis for the LHC collider (using the cuts and pseudodetector simulation of the Tevatron collider) and found that the same conclusions do apply to the LHC.

In conclusion, we have evaluated the impact of higher-order final-state QED corrections on the determination of the

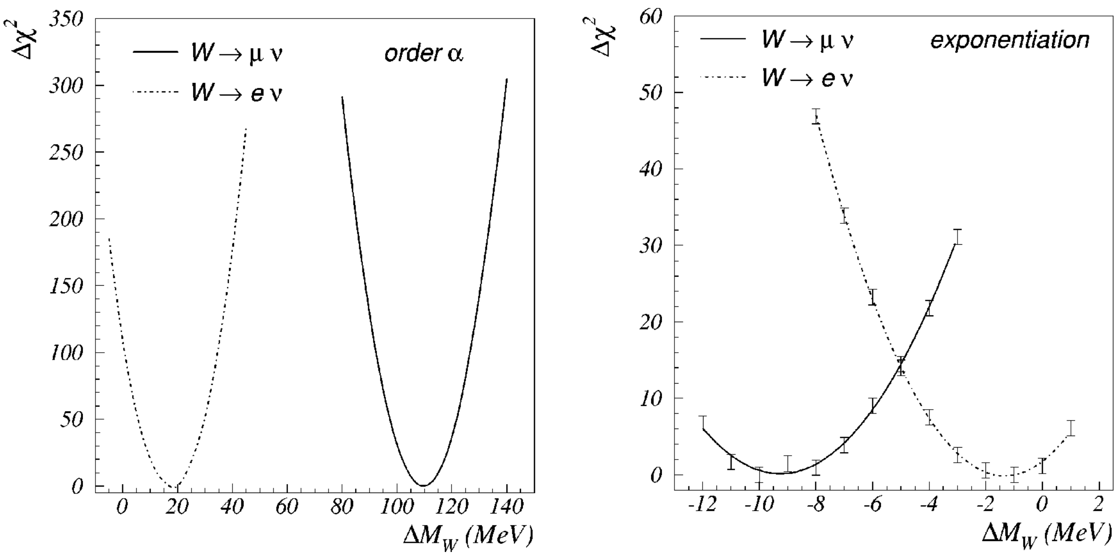

FIG. 2. The $\Delta\chi^2=\chi^2-\chi^2_{\rm min}$ distributions from a fit to the M_T distribution, including $O(\alpha)$ QED corrections (left) and higher-order QED corrections (right), as a function of the W mass shift, at $\sqrt{s}=2$ TeV. The results for the $W\to e\nu$ and $W\to\mu\nu$ channels are shown.

W mass at hadron colliders, in view of future improved measurements with an accuracy of 15–30 MeV. In the presence of realistic selection criteria, we have found that the shift due to these corrections is about 10 MeV in the $W\to\mu\nu$ channel and a few MeV in the $W\to e\nu$ channel. The calculation, if included in future experimental analyses, would reduce the uncertainty in the precision measurement of the *W* mass at hadron colliders. To this end, the MC program HORACE is available for data analysis. A more realistic analysis would require a full detector simulation, which is beyond the scope of the present paper.

A merging of fixed-order electroweak and QCD calculations with higher-order QED effects would be highly desirable for extracting the *W* mass from data at the aimed level of accuracy. The study of the neutral-current process $p\overset{(-)}{p}\to\gamma,Z\to l^+l^-$ is left to a future work.

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