Little Higgs models and custodial $SU(2)$

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In this paper we present a little Higgs model that has custodial $SU(2)$ as an approximate symmetry. This theory is a simple modification of the ''minimal moose'' model with $SO(5)$ global symmetries protecting the Higgs boson mass. This allows for a simple limit where TeV physics makes small contributions to precision electroweak observables. The spectrum of particles and their couplings to standard model fields are studied in detail. At low energies this model has two Higgs doublets and it favors a light Higgs boson from precision electroweak bounds, though for different reasons than in the standard model. The limit on the breaking scale, f, is roughly 700 GeV, with a top partner of 2 TeV, W' and B' of 2.5 TeV, and heavy Higgs partners of 2 TeV. These particles are easily accessible at hadron colliders.

I. INTRODUCTION

Recently the little Higgs mechanism has been proposed as a way to stabilize the weak scale from the radiative corrections of the standard model. In little Higgs models the standard model Higgs boson is a pseudo Goldstone boson and is kept light by approximate nonlinear symmetries [1–7], see [8,9] for summaries of the physics and [13–16] for more detailed phenomenology. The little Higgs mechanism requires that two separate couplings communicate to the Higgs sufficient breaking of the nonlinear symmetry to generate a Higgs boson mass. The weak scale is radiatively generated two loop factors beneath the cutoff $\Lambda \sim 10$–30 TeV. Little Higgs models predict a host of new particles at the TeV scale that cancel the low energy quadratic divergences to the Higgs boson mass from standard model fields. The little Higgs mechanism has particles of the *same* spin cancel the quadratic divergences to the Higgs boson mass; i.e., a fermion cancels a quadratic divergence from a fermion. In models described by ''theory space,'' such as the minimal moose model, particles of the same spin and quantum numbers cancel quadratic divergences, for example, a TeV scale vector that transforms as a $SU(2)_L$ triplet cancels the W quadratic divergence. To avoid fine-tuning the Higgs potential by more than $O(20\%)$ the top quark one loop quadratic divergence should be cutoff by roughly 2 TeV, the quadratic divergence from $SU(2)_L$ should be cutoff by 5 TeV, while the quadratic divergence from the Higgs quartic coupling should be cutoff by 8 TeV.

These TeV scale particles are heavier than the current experimental limits on direct searches; however, these particles may have effects at low energy by contributing to higher dimension operators in the standard model after integrating them out. The effects of integrating out the TeV scale partners have been considered in [10–12] and have provided constraints on some little Higgs models from precision electroweak observables. Understanding what constraints are placed on each little Higgs model is a detailed question but their themes are the same throughout. The arguments for the most severe constraints on the ''littlest Higgs'' model discussed in [11,12] arise from the massive vector bosons interactions because they can contribute to low energy four Fermi

operators and violate custodial $SU(2)$. Consider the B' which cancels the quadratic divergence of the B, the gauge eigenstates are related to the physical eigenstates by

$$B = \cos\theta' B_1 + \sin\theta' B_2, \quad B' = \cos\theta' B_2 - \sin\theta' B_1,$$
$$(1.1)$$

where the mixing angles are related to the high energy gauge couplings through

$$g_1' = \frac{g'}{\cos\theta'}, \quad g_2' = \frac{g'}{\sin\theta'},$$
$$(1.2)$$

where g' is the low energy $U(1)_Y$ gauge coupling. With the standard model fermions charged only under $U(1)_1$, the coupling to the B' is

$$\mathcal{L}_{B'\text{F Int}} = g' \tan\theta' B'_\mu j^\mu_{U(1)_Y},$$
$$(1.3)$$

where $j^\mu_{U(1)_Y}$ is the $U(1)_Y$ current. The mass of the B' goes as

$$m_{B'}^2 \sim \frac{g'^2 f^2}{\sin^2 2\theta'},$$
$$(1.4)$$

where f is the breaking scale. After integrating out the B' there is a four Fermi coupling of the form

$$\mathcal{L}_{4\text{Fermi}} \sim \frac{\sin^4\theta'}{f^2} (j^\mu_{U(1)_Y})^2.$$
$$(1.5)$$

The coefficient of this operator needs to be roughly less than $(6 \text{ TeV})^{-2}$ and can be achieved keeping f fixed as $\theta' \to 0$.

The little Higgs boson also couples to the B' through the current

$$\mathcal{L}_{B'\text{H Int}} \sim g' \cot 2\theta' B'_\mu (ih^\dagger \overleftrightarrow{D}^\mu h).$$
$$(1.6)$$

Integrating out the B' induces several dimension 6 operators including

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$$\mathcal{L}_{(h^\dagger Dh)^2} \sim \frac{\cos^2 2\theta'}{f^2}[(h^\dagger Dh)^2 + \text{H.c.}]. \qquad (1.7)$$

This operator violates custodial $SU(2)$ and after electroweak symmetry breaking it lowers the mass of the Z^0 and gives a positive contribution to the T parameter. This operator needs to be suppressed by $(5 \text{ TeV})^{-2}$. Thus the Higgs coupling prefers the limit $\theta' \to \pi/4$. There are additional contributions to the T parameter that can negate this effect, this argument shows the potential tension in little Higgs models that could push the limits on f to 3–5 TeV.

The reason why the B' contributes to an $SU(2)_C$ violating operator is because it, like the B, couples as the T^3 generator of $SU(2)_r$,[1] and its interactions explicitly break $SU(2)_C$. The most straightforward way of softening this effect is to complete the B' into a full triplet of $SU(2)_C$.[2] This modification adds an additional charged vector boson $W^{r\pm}$. By integrating out these charged gauge bosons there is another dimension 6 operator that gives a mass to the $W^\pm$ compensating for the effect from the B'. This can be implemented by gauging $SU(2)_r$ instead of $U(1)_2$. At the TeV scale $SU(2)_r \times U(1)_1 \to U(1)_Y$. With these additional vector bosons, it is possible to take the $\theta' \to 0$ limit without introducing large $SU(2)_C$ violating effects while simultaneously decoupling the standard model fermions from the B' and keeping the breaking scale f fixed. Thus the limits on the model will roughly reduce to limits on the $SU(2)_r$ coupling and the breaking scale.

It is not necessary to have a gauged $SU(2)_r$ for the little Higgs mechanism to be viable because the constraining physics is not crucial for stabilizing the weak scale. The B' is canceling the $U(1)_Y$ quadratic divergence that is only borderline relevant for a cutoff $\Lambda \lesssim 10$–15 TeV but is providing some of the main limits through its interactions with the Higgs and the light fermions. The light fermions play no role in the stability of the weak scale, therefore the limits from their interactions can be changed without altering the little Higgs mechanism. It is straightforward to avoid the strongest constraints [17]. The easiest possibility is to only gauge $U(1)_Y$ and accept its quadratic divergence with a cutoff at 10–15 TeV. Another way of dealing with this issue is to have the fermions charged equally under both $U(1)$ gauge groups. With this charge assignment the fermions decouple from the B' when $\theta' \to \pi/4$ which also decouples the little Higgs boson from the B'. There are other ways of decoupling the B' by mixing the standard model fermions with multi-TeV Dirac fermions in a similar fashion as [7]. However having a

gauged $SU(2)_r$ allows for a particularly transparent limit where TeV scale physics is parametrically safe and does not add significant complexity.

In this paper a new little Higgs model is presented that has the property that it has custodial $SU(2)$ as an approximate symmetry of the Higgs sector by gauging $SU(2)_r$ at the TeV scale. To construct a little Higgs theory with an $SU(2)_C$ symmetry we can phrase the model building issue as: "Find a little Higgs theory that has the Higgs boson transforming as a **4** of $SO(4)$." This is precisely the same challenge as finding a little Higgs theory that has a Higgs transforming as a $\mathbf{2}_{1/2}$ of $SU(2)_L \times U(1)_Y$. In the latter case it was necessary to find a group that contained $SU(2) \times U(1)$ and where the adjoint of the group had a field transforming as a $\mathbf{2}_{1/2}$ and the simplest scenario is $SU(3)$ where $\mathbf{8} \to \mathbf{3}_0 + \mathbf{2}_{1/2} + \mathbf{1}_0$. For a **4** of $SO(4)$ the simplest possibility is $SO(5)$ where an adjoint of $SO(5)$ decomposes into $\mathbf{10} \to \mathbf{6} + \mathbf{4}$. The generators of $SO(5)$ are labeled as T^l, T^r, and T^v for the $SU(2)_l$, $SU(2)_r$, and $SO(5)/SO(4)$ generators, respectively.

The model presented in this paper is a slight variation of the "minimal moose" [3] that has four nonlinear sigma model fields, X_i

$$X_i = \exp(ix_i/f), \qquad (1.8)$$

where x_i is the linearized field and f is the breaking scale associated with the nonlinear sigma model. The minimal moose has an $[SU(3)]^8$ global symmetry associated with transformations on the fields

$$X_i \to L_i X_i R_i^\dagger \qquad (1.9)$$

with $L_i, R_i \in SU(3)$. To use the $SO(5)$ group theory replace the $SU(3) \to SO(5)$ keeping the "minimal moose module" of four links with an $[SO(5)]^8$. The minimal moose had an $SU(3) \times [SU(2) \times U(1)]$ gauged where the $[SU(2) \times U(1)]$ was embedded inside $SU(3)$ while this model has an $SO(5) \times [SU(2) \times U(1)]$ gauge symmetry, using the T^{la} generators for $SU(2)$ and T^{r3} generator for $U(1)$.

The primary precision electroweak constraints arise from integrating out the TeV scale vector bosons. In this model there is a full adjoint of $SO(5)$ vector bosons. Under $SU(2)_l \times SU(2)_r$ they transform as

$$W^l \sim (\mathbf{3}_l, \mathbf{1}_r) \quad W^r \sim (\mathbf{1}_l, \mathbf{3}_r) \quad V \sim (\mathbf{2}_l, \mathbf{2}_r). \quad (1.10)$$

Because only $U(1)_Y$ is gauged inside $SU(2)_r$, the W^{ra} split into $W^{r\pm}$ and W^{r3}. The W^{r3} is the mode that is responsible for canceling the one loop quadratic divergence of the $U(1)_Y$ gauge boson and is denoted as the B'. Finally the V has the same quantum numbers as the Higgs boson but has no relevant interactions to standard model fields.

In the limit where the $SO(5)$ gauge coupling becomes large the standard model W and B gauge bosons become large admixtures of the $SU(2) \times U(1)$ vector bosons. This means that the orthogonal combinations, the W' and B', are dominantly admixtures of the $SO(5)$ vector bosons. The standard model fermions are charged only under $SU(2) \times U(1)$ which means that the TeV scale vector bosons decouple from the standard model fermions in this limit.

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[1]Recall that in the limit that $g' \to 0$ there is an $SU(2)_l \times SU(2)_r$ symmetry of the Higgs and gauge sector. Only the T^3 generator is gauged inside $SU(2)_r$ and g' can be viewed as a spurion parameterizing the breaking. After electroweak symmetry breaking $SU(2)_l \times SU(2)_r \to SU(2)_C$.

[2]The W' transforms as a triplet of $SU(2)_C$ so no $SU(2)_C$ violating operators are generated by its interactions.

In the remaining portion of the paper the explicit model is presented and the spectrum is calculated along with the relevant couplings for precision electroweak observables in Sec. II. This model has two light Higgs doublets with the charged Higgs boson being the heaviest of the physical Higgs states because of the form of the quartic potential. This potential is different than the quartic potential of the MSSM and has the property that it forces the Higgs vacuum expectation values to be complex, breaking $SU(2)_C$ in the process. This will result in the largest constraint on the model. In Sec. III, the TeV scale particles are integrated out and their effects discussed in terms of the dimension 6 operators that are the primary precision electroweak observables. For an $SO(5)$ coupling of $g_5 \sim 3$ and $f \sim 700$ GeV and for $\tan \beta \lesssim 0.3$ the model has no constraints placed on it. The limit on $\tan \beta$ ensures a light Higgs boson with mass in the 100–200 GeV range. With the rough limits on the parameters, the masses for the relevant TeV scale fields are roughly 2.5 TeV for the gauge bosons, 2 TeV for the top partner, and 2 TeV for the Higgs partners. Finally in Sec. IV the outlook for this model and the state of little Higgs models in general is discussed.

II. $SO(5)$ MINIMAL MOOSE

Little Higgs models are theories of electroweak symmetry breaking where the Higgs boson is a pseudo-Goldstone boson and can be described as gauged nonlinear sigma models. In this model there is an $SO(5) \times [SU(2) \times U(1)]$ gauge symmetry with standard gauge kinetic terms with couplings g_5 and g_2, g_1, respectively. There are four nonlinear sigma model fields, X_i, that transform under the global $[SO(5)]^8 = [SO(5)_L]^4 \times [SO(5)_R]^4$ as

$$X_i \rightarrow L_i X_i R_i^\dagger. \tag{2.1}$$

Under a gauge transformation the nonlinear sigma model fields transform as

$$X_i \rightarrow G_{2,1} X_i G_5^\dagger, \tag{2.2}$$

where G_5 is an $SO(5)$ gauge transformation and $G_{2,1}$ is an $SU(2) \times U(1)$ gauge transformation with $SU(2) \times U(1)$ embedded inside $SO(4) \simeq SU(2)_l \times SU(2)_r$, see the Appendix for a summary of the conventions. The gauge symmetries explicitly break the global $[SO(5)]^8$ symmetry and the gauge couplings g_5 and $g_{2,1}$ can be viewed as spurions. Notice that g_5 only breaks the $[SO(5)_R]^4$ symmetry, while $g_{2,1}$ only breaks the $[SO(5)_L]^4$ symmetry.

The nonlinear sigma model fields, X_i, can be written in terms of linearized fluctuations around a vacuum $\langle X_i \rangle = \mathbb{1}$,

$$X_i = \exp(ix_i/f), \tag{2.3}$$

where f is the breaking scale of the nonlinear sigma model and x_i are adjoints under the diagonal global $SO(5)$. The interactions of the nonlinear sigma model become strongly coupled at roughly $\Lambda \simeq 4\pi f$ where new physics must arise. The kinetic term for the nonlinear sigma model fields is

$$\mathcal{L}_{\text{nl}\sigma\text{m Kin}} = \frac{1}{2} \sum_i f^2 \text{Tr} \, D_\mu X_i D^\mu X_i^\dagger, \tag{2.4}$$

where the covariant derivative is

$$\begin{aligned} D_\mu X_i = \partial_\mu X_i - i g_5 X_i T^{[mn]} W_{SO(5)\,\mu}^{[mn]} \\ + i (g_2 T^{la} W_\mu^{la} + g_1 T^{r3} W_\mu^{r3}) X_i, \end{aligned} \tag{2.5}$$

where $W_{SO(5)}^{[mn]}$ are the $SO(5)$ gauge bosons, W^{la} are the $SU(2)$ gauge bosons, and W^{r3} is the $U(1)$ gauge boson. One linear combination of linearized fluctuations is eaten

$$\rho \propto x_1 + x_2 + x_3 + x_4 \tag{2.6}$$

leaving three physical pseudo-Goldstone bosons in adjoints of the global $SO(5)$ that decompose under $SU(2)_l \times SU(2)_r$ as

$$\phi^l \sim (\mathbf{3}_l, \mathbf{1}_r), \quad \phi^r \sim (\mathbf{1}_l, \mathbf{3}_r), \quad h \sim (\mathbf{2}_l, \mathbf{2}_r). \tag{2.7}$$

Under $U(1)_Y$, ϕ^r splits into $\phi^{r\,0}$ and $\phi^{r\,\pm}$.

A. Radiative corrections

There are no one loop quadratic divergences to the masses of the pseudo-Goldstone bosons from the gauge sector because all the nonlinear sigma model fields are bifundamentals of the gauge groups. This occurs because the g_5 gauge couplings break only the $SO(5)_{R_i}$ global symmetries, while the $g_{2,1}$ couplings only break the $SO(5)_{L_i}$ symmetries. To generate a mass term it must arise from an operator $|\text{Tr} \, X_i X_j^\dagger|^2$ and needs to simultaneously break both the left and right global symmetries. This requires both the g_5 and $g_{2,1}$ gauge couplings which cannot appear as a quadratic divergence until two loops. This can be verified with the Coleman-Weinberg potential [18]. In this case the mass squared matrix is

$$\begin{aligned} (W_5^A \quad & W_{2,1}^{A'}) \\ & \times \begin{pmatrix} g_5^2 f^2 \text{Tr} \, T^A X_i X_i^\dagger T^B & g_5 g_{2,1} f^2 \text{Tr} \, T^A X_i T^{B'} X_i^\dagger \\ g_5 g_{2,1} f^2 \text{Tr} \, T^{A'} X_i^\dagger T^B X_i & g_{2,1}^2 f^2 \text{Tr} \, T^{A'} X_i^\dagger X_i T^{B'} \end{pmatrix} \\ & \times \begin{pmatrix} W_5^B \\ W_{2,1}^{B'} \end{pmatrix}. \end{aligned} \tag{2.8}$$

Because the fields are unitary matrices, the entries along the diagonal are independent of the background field, x_i, and so is the trace of the mass squared. Therefore

$$V_{1\,\text{loop CW}\,\Lambda^2} = \frac{3}{32\pi^2} \Lambda^2 \text{Tr} \, M^2[x_i] = \text{const.} \tag{2.9}$$

There are one loop logarithmically divergent, one loop finite, and two loop quadratic divergences from the gauge sector. All these contributions result in masses for the pseudo-Goldstone bosons that are parametrically two loop factors down from the cutoff and are $\mathcal{O}(g^2 f/4\pi)$ in size.

B. Vector bosons: Masses and couplings

The masses for the vector bosons arise as the lowest order expansion of the kinetic terms for the nonlinear sigma model fields. The $SO(5)$ and $SU(2)$ W^l vector bosons mix as do the $SO(5)$ and $U(1)$ $W^{r\,3}$ vector bosons. They can be diagonalized with the following transformations:

$$B = \cos\theta' W^{r3} - \sin\theta' W^{r3}_{SO(5)},$$

$$B' = W'^{r3} = \sin\theta' W^{r3} + \cos\theta' W^{r3}_{SO(5)},$$

$$W^a = \cos\theta W^{la} - \sin\theta W^{la}_{SO(5)},$$

$$W'^a = W'^{la} = \sin\theta W^{la} + \cos\theta W^{la}_{SO(5)},$$

where the mixing angles are related to the couplings by

$$\cos\theta' = g'/g_1, \quad \sin\theta' = g'/g_5,$$

$$\cos\theta = g/g_2, \quad \sin\theta = g/g_5. \tag{2.10}$$

The angles θ and θ' are not independent and are related through the weak mixing angle by

$$\tan\theta_w = \frac{\sin\theta'}{\sin\theta} \tag{2.11}$$

and since $\theta_w \simeq 30°$, $\sin\theta \approx \sqrt{3}\sin\theta'$.

The masses for the vectors can be written in terms of the electroweak gauge couplings and mixing angles

$$m^2_{W'} = \frac{16g^2 f^2}{\sin^2 2\theta}, \quad m^2_{B'} = \frac{16g'^2 f^2}{\sin^2 2\theta'},$$

$$m^2_{W^r \pm} = \frac{16g'^2 f^2}{\sin^2 2\theta'}\cos^2\theta'. \tag{2.12}$$

These can be approximated in the $\theta' \to 0$ limit as

$$m^2_{B'} \simeq m^2_{W'}\left(1 - \frac{2}{3}\sin^2\theta\right) \quad m^2_{W^r \pm} \simeq m^2_{W'}(1-\sin^2\theta). \tag{2.13}$$

Note that the B', the mode that is canceling the quadratic divergence of the B, is not anomalously light.[3] The $U(1)_Y$ quadratic divergence is borderline relevant for naturalness and could be neglected if the cutoff $\Lambda \lesssim 10$–15 TeV. The corresponding mode is contributing to electroweak constraints but doing little to stabilize the weak scale quantitatively.

The Higgs boson couples to these vector bosons through the currents

$$j^{\mu\,a}_{W'} = g\cot 2\theta\, j^{\mu\,a}_H = \frac{g\cos 2\theta}{2\sin 2\theta}ih^\dagger \sigma^a \overset{\leftrightarrow}{D}{}^\mu h,$$

[3]The B' in the "littlest Higgs" boson is a factor of $\sqrt{5}$ lighter and in the $SU(3)$ minimal moose it is a factor of $\sqrt{3}$ lighter.

$$j^\mu_{B'} = g'\cot 2\theta'\, j^\mu_H = -\frac{g'\cos 2\theta'}{2\sin 2\theta'}ih^\dagger \overset{\leftrightarrow}{D}{}^\mu h, \tag{2.14}$$

where D_μ is the standard model covariant derivative and $j^{\mu\,a}_H$ is the $SU(2)_L$ current that the Higgs couples to and j^μ_H for $U(1)_Y$.

The Higgs boson also couples to the charged $SU(2)_r$ vector bosons through

$$j^\mu_{W^r\,+} = -\frac{g'\cos\theta'}{\sqrt{2}\sin 2\theta'}ihD^\mu h,$$

$$j^\mu_{W^r\,-} = j^{\mu\,\dagger}_{W^r\,+}, \tag{2.15}$$

where the $SU(2)_L$ indices are contracted with the alternating tensor. Notice that this interaction is not invariant under rephasing of the Higgs: $h \to e^{i\phi}h$ sends $j_{W^r\,+} \to e^{2i\phi}j_{W^r\,+}$.

C. Scalar masses and interactions

In order to have viable electroweak symmetry breaking there must be a significant quartic potential among the light fields. It is useful to define the operators

$$\mathcal{W}_i = X_i X^\dagger_{i+1} X_{i+2} X^\dagger_{i+3}, \tag{2.16}$$

where addition in i is modulo 4. There is a potential for the nonlinear sigma model fields

$$\mathcal{L}_{\text{Pot.}} = \lambda_1 f^4 \text{Tr}\,\mathcal{W}_1 + \lambda_2 f^4 \text{Tr}\,\mathcal{W}_2 + \text{H.c.} \tag{2.17}$$

There is a $\mathbb{Z}_4$ symmetry where the link fields cycle as $X_i \to X_{i+j}$ that forces $\lambda_1 = \lambda_2$. This is an approximate symmetry that is kept to $O(10\%)$. This potential gives a mass to one linear combination of linearized fields

$$u_H = \frac{1}{2}(x_1 - x_2 + x_3 - x_4). \tag{2.18}$$

The other two physical modes are the little Higgs boson and are classically massless

$$u_1 = \frac{1}{\sqrt{2}}(x_1 - x_3) \quad u_2 = \frac{1}{\sqrt{2}}(x_2 - x_4). \tag{2.19}$$

The potential in Eq. (2.17) can be expanded out in terms of these physical eigenmodes using the Baker-Campbell-Hausdorff formula

$$\mathcal{L}_{\text{Pot.}} = \lambda_1 f^4 \text{Tr}\exp\left(2i\frac{u_H}{f} + \frac{1}{2}\frac{[u_1,u_2]}{f^2} + \cdots\right)$$

$$+ \lambda_2 f^4 \text{Tr}\exp\left(-2i\frac{u_H}{f} + \frac{1}{2}\frac{[u_1,u_2]}{f^2} + \cdots\right) + \text{H.c.} \tag{2.20}$$

The low energy quartic coupling is related to the previous couplings through

$$\lambda^{-1}=\lambda_1^{-1}+\lambda_2^{-1}, \quad \lambda_1=\lambda/\cos^2\vartheta_\lambda, \quad \lambda_2=\lambda/\sin^2\vartheta_\lambda.$$

The approximate $\mathbb{Z}_4$ symmetry sets $\theta_\lambda\approx\pi/4$ and the symmetry breaking parameter is $\cos 2\vartheta_\lambda\sim O(10^{-1})$. The mass of the heavy scalar is

$$m_{u_H}^2=\frac{16\lambda f^2}{\sin^2 2\vartheta_\lambda}. \tag{2.21}$$

After integrating out the massive mode the resulting potential for the little Higgs model is the typical commutator potential

$$V(u_1,u_2)=-\lambda\,\mathrm{Tr}[u_1,u_2]^2+\cdots. \tag{2.22}$$

In order to have stable electroweak symmetry breaking it is necessary to have a mass term $ih_1^\dagger h_2+\mathrm{H.c.}$ This can arise from a potential of the form

$$\mathcal{L}_{T^r\,3\mathrm{Pot.}}=i\epsilon f^4\mathrm{Tr}\,T^{r\,3}(\mathcal{W}_1+\mathcal{W}_2+\mathcal{W}_3+\mathcal{W}_4)+\mathrm{H.c.}, \tag{2.23}$$

where $T^{r\,3}$ is the $U(1)$ generator. The size of the effects are radiatively stable and they are set to be a loop factor less than λ, $\epsilon\sim 10^{-2}\lambda$. The coefficients are taken to be pure imaginary because the imaginary coefficient will be necessary to ensure stable electroweak symmetry breaking while the real parts are small $SO(5)$ splittings among the various modes. Expanding this out to quadratic order

$$V_{T^{r3}\;\mathrm{Pot.}}=4\epsilon f^2\mathrm{Tr}\,T^{r3}i[u_1,u_2]+\cdots. \tag{2.24}$$

In terms of the Higgs doublets, $h_{1,2}\in u_{1,2}$, the potentials are

$$V(h_1,h_2)\simeq\frac{\lambda}{2}(|h_1^\dagger h_2-h_1^\dagger h_2|^2+4|h_1 h_2|^2)$$
$$+(4i\epsilon f^2 h_1^\dagger h_2+\mathrm{H.c.}), \tag{2.25}$$

where the $h_1 h_2$ term is contracted with the $SU(2)$ alternating tensor. This potential is not the same as the MSSM potential and will lead to a different Higgs sector.[4] There are radiative corrections to this potential whose largest effect gives soft masses of $O(100\text{ GeV})$ to the doublets

$$V_{\mathrm{eff}}\simeq\frac{\lambda}{2}(|h_1^\dagger h_2-h_2^\dagger h_1|^2+4|h_1 h_2|^2)$$
$$+[(ib+m_{12}^2)h_1^\dagger h_2+\mathrm{H.c.}]+m_1^2|h_1|^2+m_2^2|h_2|^2, \tag{2.26}$$

where $b\approx 4\epsilon f^2$. Typically m_{12}^2 is taken to be small to simplify the phenomenology so that the Higgs states fall into CP eigenstates.

Radiative corrections

There are no one loop quadratic divergences to the Higgs mass from the scalar potential.[5] The symmetry breaking pattern in the potential is more difficult to see, but notice that if either λ_1 or λ_2 vanished then there is a nonlinear symmetry acting on the fields

$$\delta_{\epsilon_1}u_1=\epsilon_1+\cdots, \quad \delta_{\epsilon_1}u_2=\epsilon_1+\cdots,$$

$$\delta_{\epsilon_1}u_H=-\frac{i}{4f}[\epsilon_1,u_1-u_2]+\cdots,$$

$$\delta_{\epsilon_2}u_1=\epsilon_2+\cdots, \quad \delta_{\epsilon_2}u_2=\epsilon_2+\cdots,$$

$$\delta_{\epsilon_2}u_H=+\frac{i}{4f}[\epsilon_2,u_1-u_2]+\cdots. \tag{2.27}$$

$\mathrm{Tr}\,\mathcal{W}_1$ preserves the first nonlinear symmetry but breaks the second, while $\mathrm{Tr}\,\mathcal{W}_2$ preserves the second but breaks the first. Either symmetry is sufficient to keep u_1 and u_2 as exact Goldstones bosons, this is why $\lambda\to 0$ as λ_1 or $\lambda_2\to 0$.

There are one loop logarithmically divergent contributions to the masses of the little Higgs boson as well as one loop finite and two loop quadratic divergences. These are all positive and parametrically give masses of the order of $\lambda^2 f/4\pi$.

D. Electroweak symmetry breaking

At this point electroweak symmetry can be broken. The little Higgs boson are classically massless but pick up $O(100\text{ GeV})$ masses from radiative corrections to the tree-level Lagrangian. The gauge and scalar corrections to the little Higgs masses give positive contributions to the mass squared of the little Higgs boson while fermions give negative contributions. The mass matrix for the Higgs sector is of the following form:

$$\mathcal{L}_{\mathrm{soft\ mass}}=(h_1^\dagger\quad h_2^\dagger)\begin{pmatrix} m_1^2 & \mu^2 \\ \mu^{*\,2} & m_2^2 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{2.28}$$

where $\mu^2=m_{12}^2+ib$. To have viable electroweak symmetry breaking requires

$$m_1^2>0,\;\; m_2^2>0,$$

$$m_1^2 m_2^2-m_{12}^4>0,$$

$$m_1^2 m_2^2-m_{12}^4-b^2<0. \tag{2.29}$$

The vacuum expectation values are

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[4]In the $SU(3)$ minimal moose the Higgs potential was identical to the MSSM because of the close relation between little Higgs theories and orbifolded extra dimensions, see [4] for the precise relation.

[5]More generally potentials that only contain any nonlinear sigma model field at most once can only give a quadratically divergent contribution to themselves.

$$\langle h_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \cos\beta \end{pmatrix}, \quad \langle h_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \sin\beta e^{i\phi} \end{pmatrix}.$$
(2.30)

The potential has a flat direction when $\beta = 0, \pi/2$ and when $\phi = 0$. Unfortunately when $\phi \neq 0$ custodial $SU(2)$ is broken.[6] The phase can be solved for in terms of the soft masses as

$$\cos\phi = \frac{m_{12}^2}{m_1 m_2}.$$
(2.31)

The breaking of $SU(2)_C$ by the Higgs sector provides one of the strongest limits on the model. For simplicity $\mu^2 = ib$ is taken to be pure imaginary forcing $\phi = \pi/2$. Taking $\phi = \pi/2$ is clearly the worst-case scenario for $SU(2)_C$ and not generic because there is no reason for m_{12} to be significantly smaller than any of the other masses.

The parameters of electroweak symmetry breaking can be solved for readily in the limit $\phi = \pi/2$ in terms of the masses

$$2\lambda v^2 = (m_1^2 + m_2^2)\left(\frac{|b|}{m_1 m_2} - 1\right),$$

$$\tan\beta = \frac{m_1}{m_2},$$

$$\tan 2\alpha = \left(1 - \frac{2m_1 m_2}{|b|}\right)\tan 2\beta,$$
(2.32)

where α is the mixing angle for the $h^0 - H^0$ sector. The soft masses should not be much larger than v otherwise it either requires some tuning of the parameters so that $b \simeq m_1 m_2$ or λ becoming large. These arguments will change when $m_{12}^2 \neq 0$. The masses for the five physical Higgs boson are

$$m_{A^0}^2 = m_1^2 + m_2^2,$$

$$m_{H^\pm}^2 = m_1^2 + m_2^2 + 2\lambda v^2 = x m_{A^0}^2,$$

$$m_{h^0}^2 = m_{H^\pm}^2 \frac{(1 - \sqrt{1 - m_0^2/m_{H^\pm}^2})}{2},$$

$$m_{H^0}^2 = m_{H^\pm}^2 \frac{(1 + \sqrt{1 - m_0^2/m_{H^\pm}^2})}{2}$$

$$= m_{H^\pm}^2 - m_{h^0}^2,$$
(2.33)

where

$$x = |b|/m_1 m_2, \quad m_0^2 = \frac{8\lambda v^2 \sin^2 2\beta}{x}.$$
(2.34)

---

[6]This can be seen by going back to the $SO(4)$ description. By having a phase it is the same as having two $SO(4)$ vectors acquire vacuum expectation values in different directions leaving only $SO(2) \simeq U(1)_Y$ unbroken.

The heaviest Higgs boson is the charged $H^\pm$ and this has consequences for precision electroweak observables. The mass of the lightest Higgs is bounded by

$$\frac{1}{4}m_0^2 \leq m_{h^0}^2 \leq \frac{1}{2}m_0^2,$$
(2.35)

where the lower bound is saturated as $m_{H^\pm}^2 \to \infty$ and the upper bound is saturated as $m_{H^\pm}^2 \to m_0^2$.

E. Fermions

The standard model fermions are charged only under the $SU(2) \times U(1)$ gauge group. Since all the fermions except the top quark couple extremely weakly to the Higgs sector, the standard Yukawa coupling to the linearized Higgs doublets can be used without destabilizing the weak scale. These small Yukawa couplings are spurions that simultaneously break flavor symmetries as well as the chiral symmetries of the nonlinear sigma model. There are many ways to covariantize these couplings but they only differ by irrelevant operators.

$$\mathcal{L}_{\text{Yuk}} = y_u q h u^c + y_d q h^\dagger d^c + y_e l h^\dagger e^c.$$
(2.36)

There is no symmetry principle that prefers type I or type II models. This can have significant implications for Higgs searches.

The couplings of the standard model fermions to the heavy gauge bosons is

$$\mathcal{L}_{\text{Int}} = g \tan\theta W'^{a}_{\mu} j^{\mu}_{F\,a} + g' \tan\theta' B'_{\mu} j^{\mu}_{F},$$
(2.37)

where $j^{\mu\,a}_{F}$ is the $SU(2)_L$ electroweak current involving the standard model fermions and j^{μ}_{F} is the $U(1)_Y$ electroweak current involving the standard model fermions. In the limit $g_5 \to \infty$ both $\theta, \theta' \to 0$ and the TeV scale gauge bosons decouple from the standard model fermions.

1. Top Yukawa

The top quark couples strongly to the Higgs and how the top Yukawa is generated is crucial for stabilizing the weak scale. The top sector must preserve some of the $[SO(5)]^8$ global symmetry that protects the Higgs mass. There are many ways of doing this but generically the mechanisms involve adding additional Dirac fermions. To couple the nonlinear sigma model fields to the quark doublets it is necessary to transform the bivector representation to the bi-spinor representation, see the Appendix. The linearized fields are re-expressed as

$$\tilde{x}_{i\,\alpha}{}^{\beta} = x_{i\,[mn]} \sigma^{[mn]}{}_{\alpha}{}^{\beta},$$
(2.38)

where m, n are $SO(5)$ vector indices running from 1 to 5, α, β are $SO(5)$ spinor indices running from 1 to 4 and $\sigma^{[mn]}{}_{\alpha}{}^{\beta}$ are generators of $SO(5)$ in the spinor representation. The exponentiated field, $\tilde{X}_i = \exp(i\tilde{x}_i/f)$, has well-defined transformation properties under the global $SO(5)$'s

and the operator, $\mathcal{X}=(\tilde{X}_1 \tilde{X}_3^\dagger)$, transforms only under the $SU(2) \times U(1)$ gauge symmetry

$$\mathcal{X} \rightarrow \tilde{G}_{2,1} \mathcal{X} \tilde{G}_{2,1}^\dagger, \qquad (2.39)$$

where $\tilde{G}_{2,1}$ is an $[SU(2) \times U(1)] \subset SO(5)$ gauge transformation in the spinor representation of $SO(5)$.

It is necessary to preserve some of the global $SO(5)$ symmetry in order to remove the one loop quadratic divergence to the Higgs mass from the top. As in the minimal moose, it is necessary to add additional fermions to fill out a full representation, in this case a **4** of $SO(5)$ for either the q_3 or the u_3^c. The large top coupling is a result of mixing with this TeV scale fermion. The most minimal approach is to complete the q_3 into

$$\mathcal{Q}=(q_3, \tilde{u}, \tilde{d}), \quad \mathcal{U}^c=(0_2, u_3^c, 0), \qquad (2.40)$$

where $\tilde{u} \sim (\mathbf{3}_c, \mathbf{1}_{+2/3})$ and $\tilde{d} \sim (\mathbf{3}_c, \mathbf{1}_{-1/3})$ with charge conjugate fields $\tilde{u}^c$ and $\tilde{d}^c$ canceling the anomalies. The top Yukawa coupling is generated by

$$\mathcal{L}_{\text{top}}=y_1 f \mathcal{U}^c \mathcal{X} \mathcal{Q}+y_2 f \tilde{u} \tilde{u}^c+\tilde{y}_2 f \tilde{d} \tilde{d}^c+\text{H.c.} \qquad (2.41)$$

The $\tilde{u}$ and u_3^c mix with an angle ϑ_y and after integrating out the massive combination the low energy top Yukawa is given by

$$y_{\text{top}}^{-2}=2(|y_1|^{-2}+|y_2|^{-2}) \quad \tan \vartheta_y=\frac{|y_1|}{|y_2|}. \qquad (2.42)$$

After electroweak symmetry breaking the top quark and the top partner pick up a mass

$$m_t=\frac{y_{\text{top}} v \cos \beta}{\sqrt{2}} \quad m_{t'}=\frac{2\sqrt{2} y_{\text{top}} f}{\sin 2\vartheta_y}\left(1-\frac{v^2 \cos^2\beta \sin^2 2\vartheta_y}{32f^2}\right). \qquad (2.43)$$

The decoupling limit is the $y_2 \rightarrow \infty$ limit where $\vartheta_y \rightarrow 0$.

2. Radiative corrections

The top coupling respects a global $SO(5)$ symmetry. This ensures that there are no one loop quadratically divergent contributions to the Higgs mass and can be seen through the Coleman-Weinberg potential. The one loop quadratic divergence is proportional to $\text{Tr}\, MM^\dagger$, where $M \sim P_{\mathcal{U}^c} \mathcal{X}$ is the mass matrix for the top sector in the background of the little Higgs boson and $P_{\mathcal{U}^c}=\text{diag}(0,0,1,0)$ is a projection matrix from the $\mathcal{U}^c$. Expanding this out

$$V_{1 \text{ loop CW}\Lambda^2}=-\frac{12\Lambda^2}{32\pi^2}\text{Tr}\, P_{\mathcal{U}^c} \mathcal{X} \mathcal{X}^\dagger P_{\mathcal{U}^c}$$

$$\sim \text{Tr}\, P_{\mathcal{U}^c}=\text{const}, \qquad (2.44)$$

which gives no one loop quadratic divergences to any of the x_i fields. One loop logarithmically divergent, one loop finite, and two loop quadratically divergent masses are generated at

the order $O(y_{\text{top}}^2 f/4\pi)$. Since the top only couples to h_1 among the light fields, it only generates a negative contribution to m_1^2. This drives $\tan \beta$ to be small since this is the only interaction that breaks the $h_1 \leftrightarrow h_2$ symmetry explicitly.

Note that the $\tilde{d}$ can be decoupled without affecting naturalness. This is because there is an accidental $SU(3)$ symmetry that is identical to the $SU(3)$ symmetry of the minimal moose.

$$\mathcal{L}_{\text{top}}=y_1 f u^c \tilde{u}+\frac{i}{\sqrt{2}} y_1 u^c h_1 q-\frac{1}{4}\frac{y_1}{f} u^c h_1^\dagger h_1 \tilde{u}+\cdots \qquad (2.45)$$

is invariant under

$$\delta h_1=\epsilon, \quad \delta q=\frac{i\sqrt{2}}{f}\epsilon^* \tilde{u}, \quad \delta \tilde{u}=\frac{i\sqrt{2}}{f}\epsilon q. \qquad (2.46)$$

This can be seen by imagining an $SU(4)$ symmetry acting on $\mathcal{X}$. With only the $\tilde{u}$ there is an $SU(3)$ acting in the upper components. The $SU(4)$ symmetry is just the $SO(6) \supset SO(5)$. The $SU(3)$ is not exact but to quadratic order in h it is an accidental symmetry. This means that in principle it is possible to send $\tilde{y}_2 \rightarrow 4\pi$ without affecting naturalness and therefore it is safe to ignore this field. Performing the same calculation as above, the charged singlet, $\phi_1^{r\,\pm}$, gets a quadratically divergent mass and is lifted to the TeV scale.

III. PRECISION ELECTROWEAK OBSERVABLES

Throughout this note the scalings of the contributions of TeV scale physics to precision electroweak observables have been discussed. The contributions to the higher dimension operators of the standard model are calculated in this section. The most physically transparent way of doing this is to integrate out the heavy fields and then run the operators down to the weak scale. The most difficult contribution to calculate is the custodial $SU(2)$ violating operator because there are several sources. Beyond that there are four Fermi operators and corrections to the Z^0 and $W^\pm$ interactions. There are no important contributions to the S parameter besides the contributions from the Higgs that turn out to be small. In Sec. III D, we summarize the constraints on the model from precision electroweak observables and state the limits on the masses.

A. Custodial $SU(2)$

Custodial $SU(2)$ provides limits on beyond the standard model physics. When written in terms of the electroweak chiral Lagrangian, violations of $SU(2)_C$ are related to the operator [20]

$$O_4=c_4 v^2 (\text{Tr}\, T_3 \omega^\dagger D_\mu \omega)^2, \qquad (3.1)$$

where ω are the Goldstone bosons associated with electroweak symmetry breaking. The coefficient of this operator is calculated in this section. This is directly related to $\delta\rho$.

However, typically limits are stated in terms of the T parameter which is related to $\delta\rho_*$ which differs from $\delta\rho$ when there are modifications to the $W^\pm$ and Z^0 interactions with standard model fermions. In Sec. III D, this difference is accounted for.

There are typically five new sources of custodial $SU(2)$ violation in little Higgs models. The first is from the nonlinear sigma model structure itself. By expanding the kinetic terms to quartic order there are operators that give the $W^\pm$ and Z^0 masses. If $SU(2)_C$ had not been broken by the vacuum expectation values of the Higgs, then there could not be any operators that violate $SU(2)_C$. Custodial $SU(2)$ is only broken with the combination of the two vacuum expectation values which means that the only possible operator that could violate $SU(2)_C$ must be of the form $(h_2^\dagger D h_1)^2$. However, the kinetic terms for the nonlinear sigma model fields never contain h_1 and h_2 simultaneously meaning that any operator of this form is not present.

1. Vector bosons

The second source of custodial $SU(2)$ violation is from the TeV scale gauge bosons. The massive W' never gives any $SU(2)_C$ violating contributions to the $W^\pm$ and Z^0 mass. The B' typically gives an $SU(2)_C$ violating contribution to the electroweak gauge boson masses but the additional contributions from the $W^{r\,\pm}$ vector bosons largely cancel this. Summing the various contributions

$$\delta\rho = -\frac{v^2}{64f^2}\sin^2 2\theta' + \frac{v^2}{64f^2}\sin^2 2\beta \sin^2\phi. \quad (3.2)$$

The second term is a result of the phase in the Higgs vacuum expectation value that breaks the $SU(2)_C$ and arises because the $W^{r\,\pm}$ interactions are not invariant under rephasing of the Higgs. The phase is generally taken to be $\pi/2$ to have the Higgs states fall into CP eigenstates. This is not generic and requires tuning m_{12}^2 to be small. Numerically this contribution is

$$\alpha^{-1}\delta\rho \simeq \frac{1}{8}\sin^2 2\beta \frac{(1 \text{ TeV})^2}{f^2}, \quad (3.3)$$

where the $\sin^2 2\theta'$ term has been dropped because it cancels in the conversion to ρ_* as will be shown in Sec. III D. This prefers β to be small which is the direction that is radiatively driven by the top sector. For instance at $\sin 2\beta \sim \frac{1}{3}$, this contribution to $\delta\rho$ is negligibly small for $f \sim 700$ GeV. By going to small $\tan\beta$ the mass of the lightest Higgs boson becomes rather light, for instance, for $\sin 2\beta \simeq \frac{1}{3}$ the mass of the lightest Higgs boson is bounded by $m_{h^0} \lesssim v$ with most of the parameter space dominated by $m_{h^0} \lesssim 150$ GeV.

2. Triplet VEV

Another possible source of $SU(2)_C$ violation is from a triplet vacuum expectation value (VEV). The form of the plaquette potential in Eq. (2.20) ensures that the trilinear couplings are of the following form:

$$h_1^\dagger \phi_H^l h_2 - h_2^\dagger \phi_H^l h_1. \quad (3.4)$$

There are two equivalent ways of calculating the effect, either integrating out ϕ_H^l to produce higher dimension operators or by calculating its vacuum expectation value. The operator appears as

$$\mathcal{L}_{u_H u_1 u_2} = \lambda \cot 2\vartheta_\lambda f i \operatorname{Tr} u_H [u_1, u_2]. \quad (3.5)$$

After integrating out u_H the leading derivative interaction is

$$\mathcal{L}_{\text{eff}} = -\frac{\cos^2 2\vartheta_\lambda}{16f^2} \operatorname{Tr} D_\mu[u_1, u_2] D^\mu[u_1, u_2], \quad (3.6)$$

where D_μ are the standard model covariant derivatives. Expanding this out there is a term that gives a contribution to ρ

$$\delta\rho = \frac{v^2}{4f^2}\cos^2 2\vartheta_\lambda \sin^2 2\beta \sin^2\phi. \quad (3.7)$$

The approximate $\mathbb{Z}_4$ symmetry of the scalar and gauge sectors that sets $\vartheta_\lambda \simeq \pi/4$ with $\cos 2\vartheta_\lambda \sim 10^{-1}$ meaning that this contribution is adequately small.

One might also worry that the light triplets in $u_{1,2}$ get tadpoles after electroweak symmetry breaking (through radiatively generated $h^\dagger \phi h$ terms), which due to their relatively light masses could lead to phenomenologically dangerous triplet VEVs.[7] However, these light scalars are not involved in canceling off the quadratic divergences to the higgs masses. Thus these triplets can be safely raised to the TeV scale by introducing "Ω plaquettes" as described in [4], where $\Omega = \exp(2\pi i T^{r3}) = \operatorname{diag}(-1, -1, -1, -1, 1)$. These operators suitably suppress the magnitudes of the light triplet VEVs and do not affect naturalness.

3. Two Higgs doublets

The ρ parameter also receives contributions from integrating out the Higgs bosons. It is known that this contribution can be either positive or negative. It is positive generically if the $H^\pm$ states are either lighter or heavier than all the neutral states, while it is negative if there are neutral Higgs states lighter and heavier than it. The Higgs potential of this theory generically predicts that the charged Higgs boson is the heaviest Higgs boson. There are four parameters of the Higgs potential: m_1^2, m_2^2, b, and λ where one combination determines $v = 247$ GeV. If $\phi \neq \pi/2$ then this analysis becomes much more complicated. The contribution to ρ_* from vacuum polarization diagrams is

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[7]We thank C. Csaki for pointing out that integrating out heavy quarks might generate these terms.

$$\delta\rho_* = \frac{\alpha}{16\pi \sin^2\theta_w m_{W^\pm}^2}\{F(m_{A^0}^2, m_{H^\pm}^2)$$

$$+ \sin^2(\alpha - \beta)[F(m_{H^\pm}^2, m_{h^0}^2) - F(m_{A^0}^2, m_{h^0}^2)$$

$$+ \delta\hat{\rho}_{SM}(m_{H^0}^2)] + \cos^2(\alpha - \beta)[F(m_{H^\pm}^2, m_{H^0}^2)$$

$$- F(m_{A^0}^2, m_{H^0}^2) + \delta\hat{\rho}_{SM}(m_{h^0}^2)]\}, \tag{3.8}$$

where

$$F(x,y) = \frac{1}{2}(x+y) - \frac{xy}{x-y}\log\frac{x}{y}, \tag{3.9}$$

$$\delta\hat{\rho}_{SM}(m^2) = F(m^2, m_{W^\pm}^2) - F(m^2, m_{Z^0}^2)$$

$$+ \frac{4m^2 m_{W^\pm}^2}{m^2 - m_{W^\pm}^2}\log\frac{m^2}{m_{W^\pm}^2}$$

$$- \frac{4m^2 m_{Z^0}^2}{m^2 - m_{Z^0}^2}\log\frac{m^2}{m_{Z^0}^2}. \tag{3.10}$$

In two Higgs doublet models setting an upper limit on the lightest Higgs mass from precision electroweak measurements is less precise. There can be cancelations but it appears as though the T parameter is quadratically sensitive to the mass of the heaviest Higgs boson. The spectrum of Higgs generated by the Higgs potential keeps the splittings between the masses of the Higgs bosons constant

$$m_{H^\pm}^2 - m_{A^0}^2 = 2\lambda v^2 \quad m_{H^\pm}^2 - m_{H^0}^2 = m_{h^0}^2$$

with $m_{h^0}^2 \leq 4\lambda v^2 \sin^2 2\beta$. This means that if λ is kept small then the T parameter is insensitive to the overall mass scale of the Higgs. With $\alpha - \beta = \pi/4$ the contribution to ρ_* goes as

$$\alpha^{-1}\delta\rho_* \simeq \frac{1}{10} - \frac{m_{h^0}^2}{(500 \text{ GeV})^2} - \frac{1}{4}\frac{m_{h^0}^2}{m_{H^\pm}^2}$$

$$- \frac{1}{30}\log\frac{m_{H^\pm}^2}{(500 \text{ GeV})^2}, \quad \lambda = \frac{1}{2},$$

$$\simeq \frac{1}{3} - \frac{m_{h^0}^2}{(500 \text{ GeV})^2} - \frac{1}{2}\frac{m_{h^0}^2}{m_{H^\pm}^2}$$

$$- \frac{1}{30}\log\frac{m_{H^\pm}^2}{(500 \text{ GeV})^2}, \quad \lambda = 1. \tag{3.11}$$

As λ becomes larger the contributions to the T parameter typically become larger, positive, and favoring heavier Higgs with smaller mass splittings to satisfy precision electroweak fits. Notice that even for $\lambda = \frac{1}{2}$ where the contributions to $\delta\rho_*$ are quite small the mass of the lightest Higgs boson is only bounded by $m_{h^0} \leq 350$ GeV. However the contributions

to ρ from the gauge boson sector prefer a small β to keep the contributions small, thus favoring a light Higgs boson.

4. Top partners

The top partners provide another source of $SU(2)_C$ violating operators arising from integrating out the partners to the top quark: $\tilde{u}$ and $\tilde{u}^c$. Since this is a Dirac fermion it decouples in a standard fashion as y_2 becomes large [19]. The contribution after subtracting off the standard model top quark contribution is

$$\delta\rho_{t'*} = \frac{N_c \sin^2\theta_L}{8\pi^2 v^2}[\sin^2\theta_L F(m_{t'}^2, m_t^2) + F(m_{t'}^2, m_b^2)$$

$$- F(m_t^2, m_b^2) - F(m_t^2, m_t^2)]$$

$$\simeq \frac{N_c \sin^2\theta_L}{16\pi^2 v^2}\left[\sin^2\theta_L m_{t'}^2\right.$$

$$\left. + 2\cos^2\theta_L \frac{m_{t'}^2 m_t^2}{m_{t'}^2 - m_t^2}\log\frac{m_{t'}^2}{m_t^2} - (2 - \sin^2\theta_L)m_t^2\right], \tag{3.12}$$

where θ_L is the t' and t mixing angle after electroweak symmetry breaking and can be expressed in terms of the original Yukawa and the mixing angle ϑ_y

$$\sin\theta_L \simeq \frac{v \sin^2\vartheta_y \cos\beta}{2f}. \tag{3.13}$$

Using this and the expressions for the mass of the t and t' in Eq. (2.43) the expression for the $\delta\rho_{t'*}$ parameter reduces to

$$\delta\rho_{t'*} \simeq \frac{3y_{top}^2 v^2 \sin^4\vartheta_y \cos^4\beta}{128\pi^2 f^2}$$

$$\times\left[\tan^2\vartheta_y - 2\left(\log\frac{v^2 \sin^2\vartheta_y \cos^2\vartheta_y \cos^2\beta}{4f^2} + 1\right)\right]. \tag{3.14}$$

This contribution vanishes as $\vartheta_y \to 0$ which is the limit $y_1 \to 0$ while keeping y_{top} fixed. In the limit of $\vartheta_y = \pi/4 - \delta\vartheta_y$ near where $m_{t'}$ is minimized, the contribution for small β goes as

$$\alpha^{-1}\delta\rho_{t'*} \simeq \frac{(1 - 4.4\delta\vartheta_y + 7.5\delta\vartheta_y^2)}{25}(1 - 1.8\sin^2\beta$$

$$+ 0.7\sin^4\beta)\frac{(1 \text{ TeV})^2}{f^2}. \tag{3.15}$$

This is adequately small for any β and the contribution quickly drops with $\delta\vartheta_y$. For instance, with $\delta\vartheta_y \simeq 0.1$, $\delta\rho_{t'*}$ drops by 40% while $m_{t'}$ only rises by 2%. This means that this contribution can be taken to be a subdominant effect.

B. S parameter

The main source for contributions to the S parameter is from integrating out the physical Higgs bosons. As for the case with the ρ parameter, a two Higgs doublet spectrum leaves a great deal of room for even a heavy spectrum where all the states are above 200 GeV. Generically the S parameter does not lead to any constraints in the Higgs spectrum because of cancelations

$$
S = \frac{1}{12\pi}\left[\sin^2(\beta-\alpha)\log\frac{m_{H^0}^2}{m_{h^0}^2} - \frac{11}{6}\right.
$$
$$
+ \cos^2(\beta-\alpha)G(m_{H^0}^2, m_{A^0}^2, m_{H^\pm}^2)
$$
$$
\left. + \sin^2(\beta-\alpha)G(m_{h^0}^2, m_{A^0}^2, m_{H^\pm}^2)\right], \qquad (3.16)
$$

where

$$
G(x,y,z) = \frac{x^2+y^2}{(x-y)^2} + \frac{(x-3y)x^2\log\frac{x}{z}-(y-3x)y^2\log\frac{y}{z}}{(x-y)^3}.
$$
$$
(3.17)
$$

This can be approximated by expanding around large $m_{H^\pm}^2$ masses and taking $\alpha-\beta=\pi/4$

$$
S = S_{\mathrm{SM}} - \frac{5}{144\pi} - \frac{1}{16\pi}\frac{2\lambda v^2}{m_{H^\pm}^2}
$$
$$
+ \frac{1}{48\pi}\frac{m_{h^0}^2}{m_{H^\pm}^2} + \frac{1}{24\pi}\log\frac{m_{H^\pm}^2}{m_{h^0}^2}. \qquad (3.18)
$$

These are adequately small in general for all reasonable values of λ and $m_{h^0}^2$.

C. Electroweak currents

The last source of electroweak constraints comes from the modifications to electroweak currents and four Fermi operators at low energies. These come from two primary sources, the Higgs-Fermion interactions from the current interactions in Eqs. (2.14) and (2.37)

$$
\mathcal{L}_{\mathrm{HF}} = -\frac{j_{\mu\,W'\mathrm{H}}^a j^{\mu\,a}{}_{W'\mathrm{F}}}{M_{W'}^2} - \frac{j_{\mu\,B'\mathrm{H}} j^{\mu}{}_{B'\mathrm{F}}}{M_{B'}^2}
$$
$$
= -\frac{\sin^2\theta\cos 2\theta}{8f^2}j_{\mathrm{H}}^a{}^\mu j_{\mathrm{F}a\ \mu} - \frac{\sin^2\theta'\cos 2\theta'}{8f^2}j_{\mathrm{H}}{}^\mu j_{\mathrm{F}\mu}
$$
$$
(3.19)
$$

and the direct four Fermi interactions

$$
\mathcal{L}_{\mathrm{FF}} = -\frac{(j_{\mu\,W'\mathrm{F}}^a)^2}{2M_{W'}^2} - \frac{(j_{\mu\,B'\mathrm{F}})^2}{2M_{B'}^2}
$$
$$
= -\frac{\sin^4\theta}{8f^2}j_{\mathrm{F}}^a{}^\mu j_{\mathrm{F}a\ \mu} - \frac{\sin^4\theta'}{8f^2}j_{\mathrm{F}}{}^\mu j_{\mathrm{F}\mu}. \qquad (3.20)
$$

It requires a full fit to know what the limits on these interactions are, but to first approximation these interactions are fine if they are suppressed by roughly $\Lambda_{\mathrm{lim}} \sim 6$ TeV [22]. Since $\sin\theta \approx \sqrt{3}\sin\theta'$, the biggest constraints come from the effects of the W'. The constraints reduce to a limit on the g_5-f plane of

$$
\frac{2\sqrt{2}f}{\sin\theta} \gtrsim \Lambda_{\mathrm{lim}}. \qquad (3.21)
$$

Clearly for $f \sim 2.5$ TeV there are no limits on g_5, for $f \sim 1.5$ TeV, $g_5 \sim 1.5$ and for $f \sim 0.7$ TeV, $g_5 \sim 3$.[8] These are clearly all in the natural regime for the little Higgs mechanism to be stabilizing the weak scale. This limit is very closely related to the mass of the W'

$$
M_{W'} \gtrsim \frac{g}{\sqrt{2}\cos\theta}\Lambda_{\mathrm{lim}}. \qquad (3.22)
$$

Thus, the mass of the $W' \gtrsim \frac{2}{5}\Lambda_{\mathrm{lim}}$. This sets a lower limit on the mass of the W' of 2.5 TeV.

D. Summary of limits

To state the limits it is necessary to convert ρ to ρ_* which is related to the T parameter. While ρ is related to custodial $SU(2)$, ρ_* is related to physical results and differs from ρ when there are modifications to electroweak current interactions. The difference is due to the discrepancy between the pole mass of the $W^\pm$ and the way that the mass of the $W^\pm$ is extracted through muon decay.

In this model the standard model fermions couple to the W' and B' and integrating out the heavy gauge bosons generates both four Fermi interactions and corrections to the J_Y, J_W fermionic currents after electroweak symmetry breaking. Following the analysis in [12,21], the Fermi constant is corrected by

$$
\frac{1}{G_F} = \sqrt{2}v^2\left(1 + \frac{\delta M_W^2}{M_{W_0}^2} - \frac{v^2}{64f^2}\sin^2 2\theta\right). \qquad (3.23)
$$

To determine ρ_*, it is necessary to integrate out the Z^0 and express the four Fermi operators as

$$
-\frac{4G_F}{\sqrt{2}}\rho_*(J_3 - s_*^2 J_Q)^2 + \alpha J_Q^2, \qquad (3.24)
$$

---

[8]It is not possible to push g_5 much larger than 3 because perturbativity is lost when the loop factor suppression $T_2(A)g_5^2/8\pi^2$ becomes roughly 1. This requires $g_5 \lesssim 5$.

which gives us to order (v^2/f^2)

$$\delta\rho_* = \alpha T = \frac{\delta M_W^2}{M_{W_0}^2} - \frac{\delta M_Z^2}{M_{Z_0}^2} + \frac{v^2}{64f^2}\sin^2 2\theta'$$

$$= \delta\rho + \frac{v^2}{64f^2}\sin^2 2\theta'. \tag{3.25}$$

Because all the other contributions to ρ are small, the primary limit on the theory comes from the $SU(2)_C$ violation in the gauge sector.

At this point the limits can be summarized for the masses of the particles. The limit on the breaking scale, f, is roughly 700 GeV from the contributions to T from the gauge bosons. The Higgs contributions to ρ_* could have been large, but because $\tan\beta$ is small it turns out to be subdominant. The mass of the lightest Higgs is bounded to be less than 250 GeV with most of the parameter space dominated by masses less than 150 GeV. The TeV scale vector bosons are all roughly degenerate with masses greater than 2.5 TeV. The mass of the top partner is roughly 2 TeV. While the mass of the heavy Higgs boson are roughly 2 TeV from the limits on f.

If we chose to exclude the A_b^{FB} measurement as an outlier, the implications for this model are significant. Discarding this measurement might be reasonable since it deviates from other standard model measurements by roughly 3σ. This model does not significantly alter the physics of A_b^{FB} from the standard model. This measurement is not generally excluded because doing so pulls the fit for the T parameter positive which favors a very light Higgs boson in the standard model and is excluded by direct searches. However there are additional positive contributions that mimic a light Higgs boson in this model. On a general principle, the connection between a light Higgs boson and a positive contribution to the T parameter does not hold in two Higgs doublet models and it is quite easy to have the Higgs sector produce $\delta T \sim 0.2$. By ignoring A_b^{FB} the best fit for the $S-T$ plane moves to $T \sim 0.15 \pm 0.1$. See [23,24] for more details. This significantly reduces the constraints on this model because all TeV scale physics pulls towards positive T. The contribution from the gauge bosons becomes roughly about the best fit for T even with $\tan\beta \sim 1$ and $f \sim 700$ GeV. This in turn can lower the limit on $m_{t'}$ and also remove the preference for lighter Higgs.

IV. CONCLUSIONS AND OUTLOOK

In this paper we have found a little Higgs model with custodial $SU(2)$ symmetry that is easily seen to be consistent with precision electroweak constraints. This demonstrated that little Higgs models are viable models of TeV scale physics that stabilize the weak scale and that the breaking scale, f, can be as low as 700 GeV without being in contradiction to precision electroweak observables. This theory is a small modification to the minimal moose having global $SO(5)$ symmetries in comparison to $SU(3)$. Most of the qualitative features of the minimal moose carried over

into this model including that it is a two Higgs doublet model with a colored Dirac fermion at the TeV scale that cancels the one loop quadratic divergence of the top and several TeV scale vector bosons. By having custodial $SU(2)$ it is possible to take the simple limit where the g_5 coupling is large where the contributions from TeV scale physics to precision electroweak observables become small. In the model presented, a breaking scale as low as $f = 700$ GeV was allowed by precision electroweak observables. The limits on the W' and B' are around 2.5 TeV and the mass of the top partner is roughly 2 TeV. These are the states that cancel the one loop quadratic divergences from the standard model's gauge and top sectors and their masses are where naturalness dictates. The charged Higgs boson was typically the heaviest among the light Higgs scalars, this resulted in a positive contribution to T. The limits from custodial $SU(2)$ violating operators favored a light Higgs boson coming not from the standard oblique corrections from the Higgs boson, but indirectly from integrating out the TeV scale gauge bosons. These already mild limits might be reduced by going away from a maximal phase. Changing this phase would also require recalculating the contributions to $\delta\rho$ from the Higgs sector when the states do not fall into CP eigenstates. There are additional scalars that could be as light as 100 GeV that came as the $SO(5)$ partners to the Higgs. As mentioned earlier in the section on triplet VEVs, these states can be lifted by "Ω plaquettes" to the multi-TeV scale and therefore their relevance for phenomenology is model dependent.

This model predicts generically a positive contribution to T mimicking the effect of a light Higgs boson in the standard model. This is interesting because if one excludes the A_b^{FB} measurement as an outlier then the fit to precision electroweak observables favors a positive $T \sim 0.15 \pm 0.1$. This is generally stated as the standard model has a best fit for a Higgs mass of 40 GeV if the A_b^{FB} measurement is excluded.

There has been recent interest in the phenomenology of the Higgs bosons inside little Higgs models. Most of the recent work we believe carries over qualitatively including the suppression of $h \to gg, \gamma\gamma$ [15,16]. The LHC should be able to produce copious numbers of the TeV scale partners in the top and vector sectors [13].

Another possible way of removing limits arising from the phase in the Higgs vacuum expectation value is to construct a model that has only one Higgs doublet. All "theory space" models automatically have two Higgs doublets so one possibility would be to follow the example of the "littlest Higgs" model and construct a coset model such as $SO(9)/[SO(5) \times SO(4)]$ [25]. There may be other two Higgs doublet models that have a gauged $SU(2)_r$ that do not force the Higgs vacuum expectation value to break $SU(2)_C$.

To summarize the larger context of this model, it provides a simple realistic little Higgs theory that is parametrically safe from precision electroweak measurements. While it is not necessary to have a gauged $SU(2)_r$, it allows for transparent limits to be taken where the TeV scale physics decouples from the physics causing constraints while still cutting off the low energy quadratic divergences. There are other ways of avoiding large contributions to electroweak

precision observables without a gauged $SU(2)_r$. The important issue is that the physics that is stabilizing the weak scale from the most important interactions is not providing significant constraints on little Higgs models. This is the deeper reason why the model presented worked in such a simple fashion. Precision electroweak constraints are coming from the interactions of either the B' or the interactions of the light fermions. The quadratic divergence from $U(1)_Y$ only becomes relevant at a scale of 10–15 TeV and is oftentimes above the scale of strong coupling for little Higgs models. The interactions of the light fermions with the TeV scale vector bosons is not determined by electroweak gauge symmetry and can be altered by either changing the charge assignments or by mixing the fermions with multi-TeV scale Dirac fermions.

In a broader view little Higgs models offer a rich set of models for TeV scale physics that stabilize the weak scale. Each little Higgs model has slightly different contributions to precision electroweak observables, but they do not have parametric problems fitting current experimental measurements. In the next five years the LHC will provide direct probes of TeV scale physics and determine whether little Higgs models play a role in stabilizing the weak scale.

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APPENDIX: GENERATORS

The $SO(5)$ commutation relations are

$$[T^{mn}, T^{op}] = \frac{i}{\sqrt{2}} (\delta^{mo} T^{np} - \delta^{mp} T^{no} - \delta^{no} T^{mp} + \delta^{np} T^{mo})$$
(A1)

where m,n,o,p run from $1, \ldots, 5$. These generators can be broken up into

$$T^{la} = \frac{1}{2\sqrt{2}} \epsilon^{abc} T^{bc} + \frac{1}{\sqrt{2}} T^{a4},$$

$$T^{ra} = \frac{1}{2\sqrt{2}} \epsilon^{abc} T^{bc} - \frac{1}{\sqrt{2}} T^{a4},$$

$$T^{v0} = T^{45}, \quad T^{va} = T^{a5}.$$
(A2)

The commutation relations in this basis are of $SO(5)$ are

$$[T^{la}, T^{lb}] = i\epsilon^{abc} T^{lc}, \quad [T^{ra}, T^{rb}] = i\epsilon^{abc} T^{rc},$$

$$[T^{la}, T^{rb}] = 0, \quad [T^{v0}, T^{la}] = -[T^{v0}, T^{ra}] = \frac{i}{2} T^{va},$$

$$[T^{v0}, T^{va}] = \frac{i}{2} (T^{ra} - T^{la}),$$

$$[T^{va}, T^{lb}] = -\frac{i}{2} T^{v0} \delta^{ab} + \frac{i}{2} \epsilon^{abc} T^{vc},$$

$$[T^{va}, T^{rb}] = \frac{i}{2} T^{v0} \delta^{ab} + \frac{i}{2} \epsilon^{abc} T^{vc},$$

$$[T^{va}, T^{vb}] = \frac{i}{2} \epsilon^{abc} (T^{lc} + T^{rc}).$$
(A3)

1. Vector representation

The vector representation of $SO(5)$ can be realized as

$$T^{mn \ op} = \frac{-i}{\sqrt{2}} (\delta^{mo} \delta^{np} - \delta^{no} \delta^{mp}),$$
(A4)

where m,n,o,p again run over $1, \ldots, 5$ and m,n label the $SO(5)$ generator while o,p are the indices of the vector representation. In this representation

$$\text{Tr}\, T^A T^B = \delta^{AB}.$$
(A5)

2. Spinor representation

The spinor representation is given by the form

$$\sigma^{la} = \begin{pmatrix} \sigma^a/2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma^{ra} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^a/2 \end{pmatrix},$$

$$\sigma^{v0} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^{va} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & i\sigma^a \\ -i\sigma^a & 0 \end{pmatrix}.$$
(A6)

In this representation

$$\text{Tr}\, T^A T^B = \frac{1}{2} \delta^{AB}.$$
(A7)

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