Study of $q\bar{q}$ states in transverse lattice QCD using alternative fermion formulations

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(Received 30 September 2003; published 23 February 2004)

In this work we investigate $q\bar{q}$ spectra and wave functions of light front transverse lattice Hamiltonians that result from different methods of formulating fermions on the transverse lattice. We adopt the one link approximation for the transverse lattice and discrete light cone quantization (DLCQ) to handle longitudinal dynamics. We perform a detailed study of the continuum limit of DLCQ and associated techniques to manage severe light front infrared divergences. We explore the effects of various parameters of the theory, especially the strength of the helicity-flip interaction and the link mass on spectra and wave functions.

DOI: 10.1103/PhysRevD.69.034502 PACS number(s): 11.15.Ha, 11.10.Ef, 11.15.Tk, 12.38.Gc

I. INTRODUCTION

A promising method to calculate observables in QCD is the transverse lattice formulation [1–3]. In this method, one keeps $x^{\pm}=x^0\pm x^3$ continuous and discretizes the transverse space spanned by coordinates $x^{\perp}=(x^1,x^2)$. With the gauge choice $A^+=0$, A^- becomes a constrained variable which can be eliminated in favor of dynamical gauge variables. So far very encouraging results have been obtained in the pure gauge and meson sectors [4–6].

Because of the doubling phenomena, fermions on the lattice pose challenging problems. To date, calculations of meson properties using transverse lattice have employed Wilson fermions [7]. It is well known that the Wilson term explicitly breaks chiral symmetry and makes it difficult to explore the consequences of spontaneous chiral symmetry breaking in the chiral limit. In this limit, in the one link approximation on the transverse lattice, the Wilson term can be adjusted to produce the desirable level splitting between π and ρ. This, however, results in the undesirable consequence that the splitting of the ρ multiplet is almost as large as the π-ρ splitting. Because of the doubling problem, one cannot keep the Wilson term very small. Thus it is desirable to explore other formulations of fermions on the transverse lattice that may have different chiral properties.

In a recent work [8] we have addressed the problems of fermions on the light front transverse lattice. We proposed and numerically investigated different approaches of formulating fermions on the transverse lattice. In one approach, which uses forward and backward derivatives, fermion doubling is absent and the helicity flip term which is proportional to the fermion mass in light front QCD becomes an irrelevant term in the free field limit. In the literature, symmetric derivatives have been used which lead to fermion doubling due to the decoupling of even and odd lattices. Using the light front staggered fermion formulation and the Wilson fermion formulation, we studied the removal of doublers from the spectrum. Our investigations lead to the identification of an even-odd helicity flip symmetry of the light front transverse Hamiltonian, the absence of which means the removal of doublers in all the cases that we studied.

In this work we make a detailed comparison of various light front QCD Hamiltonians that result from different ways of formulating fermions on the transverse lattice. As the first step in our calculations, we adopt the one link approximation in the meson sector which has been widely used in the literature. (Only very recently, the effect of additional links in the meson sector has been investigated [6].) Since the one link approximation is very crude, our aim is not to reproduce physical observables. Rather, we explore the effects of various coupling strengths on the low-lying spectra and wave functions and compare two different formulations.

We use discretized light cone quantization (DLCQ) [9] to address longitudinal dynamics. Because of the presence of severe light front infrared divergences, a major concern here is the reliability of DLCQ results when calculations are done at finite resolution K and results are extrapolated to the continuum $(K\rightarrow\infty)$. In meson calculations so far, $K\leqslant 20$ have been chosen. In this work we perform a detailed study of the continuum limit of DLCQ by performing calculations at larger values of K.

In the meson sector, in the zero link approximation, at each transverse location we have a two-dimensional field theory which in the large N_c limit (where N_c is the number of colors) is nothing but the 't Hooft model. In this well-studied model, excited states are simply excitations of the $q\bar{q}$ pair, which contain nodes in the wave functions. The picture changes when one link is included, thereby allowing fermions to hop. The admixture of $q\bar{q}$ link states with $q\bar{q}$ states is controlled by the strengths of the particle number changing interactions and the mass of the link field. One link approximation is *a priori* justified for very massive links and/or weak particle changing interaction since in this case low-lying excited states are also $q\bar{q}$ excitations. Likewise, for large particle changing interaction strength and/or light link

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mass, low-lying excited states are $q\bar{q}$ link states. We explore the spectra and wave functions resulting from the choice of various regions of parameter space.

The plan of this paper is as follows. In Sec. II we present the details of the light front transverse lattice Hamiltonian resulting from the use of forward and backward derivatives and the resulting effective Hamiltonian when the unitary link variables are replaced by general complex matrices. In this section we also present the canonical transverse lattice QCD Hamiltonian resulting from the addition of the Wilson term. Section III contains comparisons of numerical results for the two Hamiltonians. Finally Sec. IV contains our summary and conclusions. Typical terms in the Hamiltonian with forward and backward derivatives in the Fock representation in DLCQ is presented in Appendix A. Explicit expressions for the states are given in Appendix B. For completeness, explicit expressions for the matrix elements in the forward-backward case and the Wilson case are presented in Appendixes C and D.

II. HAMILTONIANS

Due to the constraint equation in the light front theory, different methods are possible to put fermions on the transverse lattice. In this section we present the detailed structure of the resulting QCD Hamiltonians for two methods studied in Ref. [8], namely, forward and backward derivatives and symmetric derivatives together with the Wilson term.

A. Hamiltonian with forward and backward derivatives

Details of the derivation of the fermionic part of the Hamiltonian are already given in Ref. [8]. Here we give the details of the gauge field part of the Hamiltonian. Nonlinear constraints on the unitary link variables make it difficult to perform canonical quantization. We also present the effective Hamiltonian when nonlinear unitary variables are replaced by linear variables.

1. Gauge field part of the Lagrangian density

The gauge field part of the Lagrangian density in the continuum is

$$\mathcal{L}_G=\frac{1}{2g^2}\,\mathrm{Tr}\,F_{\rho\sigma}F^{\rho\sigma}, \tag{2.1}$$

where $F^{\rho\sigma}=\partial^\rho A^\sigma-\partial^\sigma A^\rho+[A^\rho,A^\sigma]$ with $A^\rho=igA^{\rho\alpha}T^\alpha$. Here $\rho,\sigma=0,1,2,3$ and $\alpha=1,2,\ldots,8$. For ease of notation we suppress the dependence of field variables on the longitudinal coordinate in this section. With the gauge choice $A^+=0$, the Lagrangian density can be separated into three parts,

$$\mathcal{L}_G=\mathcal{L}_T+\mathcal{L}_L+\mathcal{L}_{LT}. \tag{2.2}$$

Here $\mathcal{L}_T$ depends entirely on the lattice gauge field $U_r(\mathbf{x})$,

$$\mathcal{L}_T=\frac{1}{g^2a^4}\sum_{r\neq s}\{\mathrm{Tr}[U_r(\mathbf{x})U_s(\mathbf{x}+a\hat{\mathbf{r}})\times U_{-r}(\mathbf{x}+a\hat{\mathbf{r}}+a\hat{\mathbf{s}})U_{-s}(\mathbf{x}+a\hat{\mathbf{s}})-1]\}. \tag{2.3}$$

The purely longitudinal part $\mathcal{L}_L$ depends on the constrained gauge field A^-,

$$\mathcal{L}_L=\frac{1}{8}(\partial^+A^{-\alpha})^2 \tag{2.4}$$

and the mixed part $\mathcal{L}_{LT}$ depends both on lattice gauge field and the constrained gauge field,

$$\mathcal{L}_{LT}=\frac{1}{g^2a^2}\mathrm{Tr}[\partial_\mu U_r(\mathbf{x})\partial^\mu U_r^\dagger(\mathbf{x})]+\frac{1}{2a^2}gA^{-\alpha}J^{+\alpha}_{LINK}. \tag{2.5}$$

Here the link current

$$J^{+\alpha}_{LINK}(\mathbf{x})=\sum_r\frac{1}{g^2}\mathrm{Tr}\{T^\alpha[U_r(\mathbf{x})i\overleftrightarrow{\partial^+}U_r^\dagger(\mathbf{x})+U_r^\dagger(\mathbf{x}-a\hat{\mathbf{r}})i\overleftrightarrow{\partial^+}U_r(\mathbf{x}-a\hat{\mathbf{r}})]\}. \tag{2.6}$$

Substituting back the expression for $A^{-\alpha}$ from the constraint equation

$$(\partial^+)^2A^{-\alpha}=\frac{2g}{a^2}[J^{+\alpha}_{LINK}-J^{+\alpha}_q] \tag{2.7}$$

with

$$J^{+\alpha}_q(\mathbf{x})=2\,\eta^\dagger(\mathbf{x})T^\alpha\eta(\mathbf{x}), \tag{2.8}$$

where η is the dimensionless two-component lattice fermion field, in the $A^{-\alpha}$ dependent terms in the Lagrangian density, namely,

$$-\frac{1}{2}\frac{g}{a^2}A^{-\alpha}J^{+\alpha}_q+\frac{1}{8}(\partial^+A^{-\alpha})^2+\frac{1}{2}\frac{g}{a^2}A^{-\alpha}J^{+\alpha}_{LINK} \tag{2.9}$$

we generate the terms

$$\frac{g^2}{2a^4}J^{+\alpha}_{LINK}\left(\frac{1}{\partial^+}\right)^2J^{+\alpha}_{LINK}+\frac{g^2}{2a^4}\eta^\dagger T^\alpha\eta\left(\frac{1}{\partial^+}\right)^2\eta^\dagger T^\alpha\eta-\frac{g^2}{a^4}J^{+\alpha}_{LINK}\left(\frac{1}{\partial^+}\right)^2\eta^\dagger T^\alpha\eta. \tag{2.10}$$

Collecting all the terms, the canonical Lagrangian density for transverse lattice QCD is

$$\begin{aligned}\mathcal{L}=&\frac{1}{a^2}\eta^\dagger(\mathbf{x})i\partial^-\eta(\mathbf{x})+\frac{1}{a^4g^2}\mathrm{Tr}[\partial_\mu U_r(\mathbf{x})\partial^\mu U_r^\dagger(\mathbf{x})]\\&-\frac{m^2}{a^2}\eta^\dagger(\mathbf{x})\frac{1}{i\partial^+}\eta(\mathbf{x})+im\frac{1}{a^2}\eta^\dagger(\mathbf{x})\hat{\sigma}_s\frac{1}{a}\frac{1}{\partial^+}[U_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})-\eta(\mathbf{x})]\\&+im\frac{1}{a^2}[\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})U_r^\dagger(\mathbf{x})-\eta^\dagger(\mathbf{x})]\hat{\sigma}_r\frac{1}{a}\frac{1}{\partial^+}\eta(\mathbf{x})\\&-\frac{1}{a^4}[\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})U_r^\dagger(\mathbf{x})-\eta^\dagger(\mathbf{x})]\hat{\sigma}_r\frac{1}{i\partial^+}\hat{\sigma}_s[U_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})-\eta(\mathbf{x})]\\&+\frac{1}{a^4g^2}\sum_{r\neq s}\{\mathrm{Tr}[U_r(\mathbf{x})U_s(\mathbf{x}+a\hat{\mathbf{r}})U_{-r}(\mathbf{x}+a\hat{\mathbf{r}}+a\hat{\mathbf{s}})U_{-s}(\mathbf{x}+a\hat{\mathbf{s}})-1]\}\\&+\frac{g^2}{2a^4}J^{+\alpha}_{LINK}\left(\frac{1}{\partial^+}\right)^2J^{+\alpha}_{LINK}+\frac{1}{2a^4}g^2J^{+\alpha}_q\left(\frac{1}{\partial^+}\right)^2J^{+\alpha}_q-\frac{g^2}{a^4}J^{+\alpha}_{LINK}\left(\frac{1}{\partial^+}\right)^2J^{+\alpha}_q.\end{aligned}\tag{2.11}$$

Here $\hat{\sigma}_1=\sigma_2,\ \hat{\sigma}_2=-\sigma_1$. In the two-component representation [10], the dynamical fermion field

$$\psi^+(x^-,x^\perp)=\begin{bmatrix}\frac{1}{a}\eta(x^-,x^\perp)\\0\end{bmatrix},\tag{2.12}$$

where η is the dimensionless two component lattice fermion field.

2. Effective Hamiltonian

Because of the nonlinear constraints $U^\dagger U=1$, $\det U=1$, it is highly nontrivial to quantize the system. Hence Bardeen and Pearson [1] and Bardeen, Pearson, and Rabinovici [2] proposed to replace the nonlinear variables U by linear variables M where M belongs to $GL(N,\mathcal{C})$, i.e., we replace $(1/g)U_r(\mathbf{x})\rightarrow M_r(\mathbf{x})$. Once we replace U by M, many more terms are allowed in the Hamiltonian. Thus one needs to add an effective potential V_{eff} to the Lagrangian density

$$\begin{aligned}V_{eff}=&-\frac{\mu^2}{a^2}\mathrm{Tr}(M^\dagger M)+\lambda_1\mathrm{Tr}[(M^\dagger M)^2]\\&+\lambda_2[\det M+\mathrm{H.c.}]+\cdots.\end{aligned}\tag{2.13}$$

Thus, the effective Hamiltonian for QCD on the transverse lattice becomes

$$\begin{aligned}P^-_{fb}=&P^-_{ffree}+P^-_V+P^-_{fhf}+P^-_{hf}+P^-_{chnf}\\&+P^-_{qqc}+P^-_{ggc}+P^-_{qgc}+P^-_p.\end{aligned}\tag{2.14}$$

The free fermion part is

$$P^-_{ffree}=\int dx^-\sum_{\mathbf{x}}\left(m^2+\frac{2}{a^2}\right)\eta^\dagger(\mathbf{x})\frac{1}{i\partial^+}\eta(\mathbf{x}).\tag{2.15}$$

The effective potential part is

$$\begin{aligned}P^-_V=&\int dx^-a^2\sum_{\mathbf{x}}\left(\frac{\mu^2}{a^2}\mathrm{Tr}(M^\dagger M)-\lambda_1\mathrm{Tr}[(M^\dagger M)^2]\right.\\&\left.-\lambda_2[\det M+\mathrm{H.c.}]+\cdots\right).\end{aligned}\tag{2.16}$$

The free helicity-flip part is

$$P^-_{fhf}=2im\int dx^-\sum_{\mathbf{x}}\sum_s\eta^\dagger(\mathbf{x})\hat{\sigma}_s\frac{1}{a}\frac{1}{\partial^+}\eta(\mathbf{x}).\tag{2.17}$$

Helicity flip associated with the fermion hop is

$$\begin{aligned}P^-_{hf}=&-img\int dx^-\sum_{\mathbf{x}}\sum_s\eta^\dagger(\mathbf{x})\\&\times\hat{\sigma}_s\frac{1}{a}\frac{1}{\partial^+}[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})]\\&-img\int dx^-\sum_{\mathbf{x}}\sum_r[\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})M_r^\dagger(\mathbf{x})]\\&\times\hat{\sigma}_r\frac{1}{a}\frac{1}{\partial^+}\eta(\mathbf{x}).\end{aligned}\tag{2.18}$$

Canonical helicity nonflip terms are

$$P^-_{chnf} = -\frac{g}{a^4}\int dx^- a^2 \sum_{\mathbf{x}} \sum_{rs} [\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})M_r^\dagger(\mathbf{x})]\hat{\sigma}_r \frac{1}{i\partial^+}\hat{\sigma}_s[\eta(\mathbf{x})]$$
$$-\frac{g}{a^4}\int dx^- a^2 \sum_{\mathbf{x}} \sum_{rs} [\eta^\dagger(\mathbf{x})]\hat{\sigma}_r \frac{1}{i\partial^+}\hat{\sigma}_s[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})]$$
$$-\frac{g^2}{a^4}\int dx^- a^2 \sum_{\mathbf{x}} \sum_{rs} [\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})M_r^\dagger(\mathbf{x})]\hat{\sigma}_r \frac{1}{i\partial^+}\hat{\sigma}_s[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})]. \quad (2.19)$$

The four-fermion instantaneous term is

$$P^-_{qqc} = -2\frac{g^2}{a^2}\int dx^- \sum_{\mathbf{x}} \eta^\dagger(\mathbf{x})T^a\eta(\mathbf{x})\frac{1}{(\partial^+)^2}\eta^\dagger(\mathbf{x})T^a\eta(\mathbf{x}). \quad (2.20)$$

The four link instantaneous term is

$$P^-_{ggc} = -\frac{1}{2}\frac{g^2}{a^2}\int dx^- \sum_{\mathbf{x}} J^{+a}_{LINK}(\mathbf{x})\frac{1}{(\partial^+)^2}J^{+a}_{LINK}(\mathbf{x}). \quad (2.21)$$

The fermion-link instantaneous term is

$$P^-_{qgc} = 2\frac{g^2}{a^2}\int dx^- \sum_{\mathbf{x}} J^{+a}_{LINK}(\mathbf{x})\frac{1}{(\partial^+)^2}\eta^\dagger(\mathbf{x})T^a\eta(\mathbf{x}). \quad (2.22)$$

The plaquette term is

$$P^-_p = -\frac{g^2}{a^4}\int dx^- a^2 \sum_{\mathbf{x}} \sum_{r\neq s} \{\mathrm{Tr}[M_r(\mathbf{x})M_s(\mathbf{x}+a\hat{\mathbf{r}})$$
$$\times M_{-r}(\mathbf{x}+a\hat{\mathbf{r}}+a\hat{\mathbf{s}})M_{-s}(\mathbf{x}+a\hat{\mathbf{s}})-1]\}. \quad (2.23)$$

Here

$$J^{+\alpha}_{LINK}(\mathbf{x}) = \sum_r \mathrm{Tr}\{T^\alpha[M_r(\mathbf{x})i\overleftrightarrow{\partial}^+ M_r^\dagger(\mathbf{x})$$
$$+M_r^\dagger(\mathbf{x}-a\hat{\mathbf{r}})i\overleftrightarrow{\partial}^+ M_r(\mathbf{x}-a\hat{\mathbf{r}})]\}. \quad (2.24)$$

3. Violations of hypercubic symmetry

The canonical helicity nonflip interactions given in Eq. (2.19) for $r\neq s$ break the hypercubic symmetry on the transverse lattice. For interacting theory this is also true for the Hamiltonian with a symmetric derivative. In the free field limit they do not survive for the Hamiltonian with a symmetric derivative but for a forward-backward derivative they survive. In that case, in the free field limit they reduce to

$$\frac{1}{a^2}\int dx^- \sum_{\mathbf{x}} \sum_{r\neq s} \Bigg[\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})\hat{\sigma}_r\hat{\sigma}_s\frac{1}{\partial^+}\eta(\mathbf{x})$$
$$+\eta^\dagger(\mathbf{x})\hat{\sigma}_r\hat{\sigma}_s\frac{1}{\partial^+}\eta(\mathbf{x}+a\hat{\mathbf{s}})$$
$$-\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})\hat{\sigma}_r\hat{\sigma}_s\frac{1}{\partial^+}\eta(\mathbf{x}+a\hat{\mathbf{s}})\Bigg]. \quad (2.25)$$

Going to the transverse momentum space via

$$\eta(x^-,x^\perp) = \int d^2k^\perp e^{ik^\perp\cdot x^\perp}\phi_{k^\perp}(x^-) \quad (2.26)$$

we get

$$-\frac{2}{a^2}\int dx^- \int d^2k^\perp \phi^\dagger_{k^\perp}(x^-)\sigma_3\frac{1}{i\partial^+}\phi_{k^\perp}(x^-)$$
$$\times[\sin(k_y a)-\sin(k_x a)+\sin(k_x a-k_y a)]. \quad (2.27)$$

Thus the violations of hypercubic symmetry are of the order of the lattice spacing a. The sign in front of this term changes if we switch the forward and backward derivatives.

In our numerical studies presented in Ref. [8] and in this work, we have set the coefficients of the hypercubic symmetry violating terms to zero.

B. Canonical transverse lattice QCD with the Wilson term

When one uses symmetric derivatives for the fermion fields, doublers arise as a result of the decoupling of even and odd lattice sites. To remove the doublers one may use the Wilson fermions [7] or the Kogut-Susskind fermions [8,11]. In this section, the details of the structure of the Hamiltonian resulting from the modification of the Wilson term are presented.

1. Constraint equation

The symmetric derivative is defined by

$$D_r\psi^\pm(\mathbf{x}) = \frac{1}{2a}[U_r(\mathbf{x})\psi^\pm(\mathbf{x}+a\hat{\mathbf{r}})-U_{-r}(\mathbf{x})\psi^\pm(\mathbf{x}-a\hat{\mathbf{r}})]. \quad (2.28)$$

Again we make the replacement $(1/g)U_r(\mathbf{x}) = M_r(\mathbf{x})$. Anticipating doublers, we can add a "Wilson term":

$$\delta\mathcal{L}=\frac{\kappa}{a}\bar{\psi}(\mathbf{x})[gM_r(\mathbf{x})\psi(\mathbf{x}+a\hat{\mathbf{r}}) -2\psi(\mathbf{x})+gM_{-r}(\mathbf{x})\psi(\mathbf{x}-a\hat{\mathbf{r}})], \tag{2.29}$$

where κ is the dimensionless Wilson parameter. Explicitly, in terms of the dynamical field ψ^+ and the constrained ψ^-

$$\delta\mathcal{L}=\frac{\kappa}{a}\psi^{-\dagger}(\mathbf{x})\gamma^0[gM_r(\mathbf{x})\psi^+(\mathbf{x}+a\hat{\mathbf{r}})-2\psi^+(\mathbf{x}) +gM_{-r}(\mathbf{x})\psi^+(\mathbf{x}-a\hat{\mathbf{r}})] +\frac{\kappa}{a}\psi^{+\dagger}(\mathbf{x})\gamma^0[gM_r(\mathbf{x})\psi^-(\mathbf{x}+a\hat{\mathbf{r}})-2\psi^-(\mathbf{x}) +gM_{-r}(\mathbf{x})\psi^-(\mathbf{x}-a\hat{\mathbf{r}})]. \tag{2.30}$$

The constraint equation for ψ^- in the presence of the Wilson term is

$$i\partial^+\psi^-(\mathbf{x})=m\gamma^o\psi^+(\mathbf{x})+i\frac{\alpha_r}{2a}[gM_r(\mathbf{x})\psi^+(\mathbf{x}+a\hat{\mathbf{r}}) -gM_{-r}(\mathbf{x})\psi^+(\mathbf{x}-a\hat{\mathbf{r}})] -\frac{\kappa}{a}\gamma^0[gM_r(\mathbf{x})\psi^+(\mathbf{x}+a\hat{\mathbf{r}})-2\psi^+(\mathbf{x}) +gM_{-r}(\mathbf{x})\psi^+(\mathbf{x}-a\hat{\mathbf{r}})]. \tag{2.31}$$

2. Hamiltonian: Symmetric derivatives and the Wilson term

After a great deal of algebra, we arrive at the Hamiltonian,

$$P^-=P^-_{ffree}+P^-_V+P^-_{hf}+P^-_{whf}+P^-_{chnf}+P^-_{wnf1}+P^-_{wnf2} +P^-_{qqc}+P^-_{ggc}+P^-_{qgc}+P^-_p. \tag{2.32}$$

The free fermion part is

$$P^-_{ffree}=\int dx^-a^2\sum_{\mathbf{x}}\frac{1}{a^2}\left(m+4\frac{\kappa}{a}\right)^2\eta^\dagger(x^-,\mathbf{x})\frac{1}{i\partial^+}\eta(x^-,\mathbf{x}). \tag{2.33}$$

The helicity flip part is

$$P^-_{hf}=-g\int dx^-\sum_{\mathbf{x}}\Bigg\{\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\eta^\dagger(\mathbf{x})\sum_r\hat{\sigma}_r\frac{1}{i\partial^+}[M_r(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{r}})-M_{-r}(\mathbf{x})\eta(\mathbf{x}-a\hat{\mathbf{r}})] -\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sum_r[\eta^\dagger(\mathbf{x}-a\hat{\mathbf{r}})\hat{\sigma}_rM_r(\mathbf{x}-a\hat{\mathbf{r}})-\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})\hat{\sigma}_rM_{-r}(\mathbf{x}+a\hat{\mathbf{r}})]\frac{1}{i\partial^+}\eta(\mathbf{x})\Bigg\}. \tag{2.34}$$

The Wilson term induced helicity flip part

$$P^-_{whf}=g^2\int dx^-\sum_{\mathbf{x}}\Bigg\{\frac{\kappa}{a}\frac{1}{2a}\sum_r\sum_s[\eta^\dagger(\mathbf{x}-a\hat{\mathbf{r}})M_r(\mathbf{x}-a\hat{\mathbf{r}})+\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})M_{-r}(\mathbf{x}+a\hat{\mathbf{r}})] \times\frac{1}{i\partial^+}\hat{\sigma}_s[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})-M_{-s}(\mathbf{x})\eta(\mathbf{x}-a\hat{\mathbf{s}})] -\frac{\kappa}{a}\frac{1}{2a}\sum_r\sum_s[\eta^\dagger(\mathbf{x}-a\hat{\mathbf{r}})\hat{\sigma}_rM_r(\mathbf{x}-a\hat{\mathbf{r}})-\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})\hat{\sigma}_rM_{-r}(\mathbf{x}+a\hat{\mathbf{r}})] \times\frac{1}{i\partial^+}[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})+M_{-s}(\mathbf{x})\eta(\mathbf{x}-a\hat{\mathbf{s}})]\Bigg\}. \tag{2.35}$$

The canonical helicity nonflip term arising from the fermion constraint is

$$P^-_{chnf}=-g^2\int dx^-\sum_{\mathbf{x}}\frac{1}{4a^2}\sum_r\sum_s[\eta^\dagger(\mathbf{x}-a\hat{\mathbf{r}})\hat{\sigma}_rM_r(\mathbf{x}-a\hat{\mathbf{r}})-\eta^\dagger(\mathbf{x}+a\hat{\mathbf{r}})\hat{\sigma}_rM_{-r}(\mathbf{x}+a\hat{\mathbf{r}})] \times\frac{1}{i\partial^+}\hat{\sigma}_s[M_s(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{s}})-M_{-s}(\mathbf{x})\eta(\mathbf{x}-a\hat{\mathbf{s}})]. \tag{2.36}$$

The Wilson term induced helicity nonflip terms are

$$P^{-}_{wnf1} = -g \int dx^{-} \sum_{\mathbf{x}} \left\{ \left(m + 4\frac{\kappa}{a} \right) \frac{\kappa}{a} \eta^{\dagger}(\mathbf{x}) \frac{1}{i\partial^{+}} \sum_{r} \left[M_{r}(\mathbf{x}) \eta(\mathbf{x}+a\hat{\mathbf{r}}) + M_{-r}(\mathbf{x}) \eta(\mathbf{x}-a\hat{\mathbf{r}}) \right] + \left(m + 4\frac{\kappa}{a} \right) \frac{\kappa}{a} \sum_{r} \left[\eta^{\dagger}(\mathbf{x}-a\hat{\mathbf{r}}) M_{r}(\mathbf{x}-a\hat{\mathbf{r}}) + \eta^{\dagger}(\mathbf{x}+a\hat{\mathbf{r}}) M_{-r}(\mathbf{x}+a\hat{\mathbf{r}}) \right] \frac{1}{i\partial^{+}} \eta(\mathbf{x}) \right\} \tag{2.37}$$

and

$$P^{-}_{wnf2} = -g^{2} \int dx^{-} \sum_{\mathbf{x}} \frac{\kappa^{2}}{a^{2}} \sum_{r} \sum_{s} \left[\eta^{\dagger}(\mathbf{x}-a\hat{\mathbf{r}}) M_{r}(\mathbf{x}-a\hat{\mathbf{r}}) + \eta^{\dagger}(\mathbf{x}+a\hat{\mathbf{r}}) M_{-r}(\mathbf{x}+a\hat{\mathbf{r}}) \right] \times \frac{1}{i\partial^{+}} \left[M_{s}(\mathbf{x}) \eta(\mathbf{x}+a\hat{\mathbf{s}}) + M_{-s}(\mathbf{x}) \eta(\mathbf{x}-a\hat{\mathbf{s}}) \right]. \tag{2.38}$$

Comparing the Hamiltonians with (a) the forward-backward derivative and (b) the symmetric derivative with the Wilson term, we notice that the only differences are in the particle number changing interactions, namely, helicity flip and helicity nonflip terms.

III. ONE LINK APPROXIMATION

A. Relevant interactions

In the one link approximation, for either Hamiltonian, the four link instantaneous term and the plaquette term do not contribute and only the link mass term of the effective potential contributes. Further, in the case of the forward-backward Hamiltonian, the helicity nonflip part proportional to g^2 does not contribute. For the Wilson term modified Hamiltonian, the Wilson term induced helicity flip part P^{-}_{whf}, the canonical helicity nonflip term P^{-}_{cnhf}, and the term proportional to κ^2 in the Wilson term induced helicity nonflip part do not contribute. Thus in the case of the Wilson term modified Hamiltonian the entire fermion hopping with no helicity flip arises from the Wilson term.

B. Comparison with one gluon exchange in the continuum

It is interesting to compare the one link approximation on the transverse lattice with the one gluon exchange approximation in the continuum. In the latter, a major source of singularity is the $k^{\perp}/k^{+}$ term in the quark-gluon vertex where $k^{\perp}$ (k^{+}) is the gluon transverse (longitudinal) momentum. This originates from the $A^{-}J^{+}_{q}$ interaction term in the Hamiltonian via the $(1/\partial^{+})\partial^{\perp}\cdot A^{\perp}$ contribution to the constrained field A^{-}. This term gives rise to quadratic ultraviolet divergence in the transverse plane accompanied by linear divergence in the longitudinal direction in fermion self-energy. On the transverse lattice, $\partial^{+}A^{-} \propto (1/\partial^{+})J^{+}_{LINK}$ so that $A^{-}J^{+}_{q} \rightarrow J^{+}_{LINK}[1/(\partial^{+})^{2}]J^{+}_{q}$. Thus a term which gives rise to severe divergence structure in the continuum gets buried in the fermion-link instantaneous interaction term which gives rise to a term in the gauge boson fermion vertex in the continuum in Abelian theory. In the non-Abelian gauge theory this gives rise to a term in the quark-gluon vertex and also to the instantaneous quark-gluon interaction in the continuum.

The transfer of the troublesome term from the quark-gluon vertex in the continuum theory to the quark-link instantaneous interaction term in the lattice theory has an interesting consequence. In the continuum theory, the addition of a gluon mass term by hand spoils the cancellation of the light front singularity between one gluon exchange and the instantaneous four-fermion interaction. On the transverse lattice this cancellation is absent anyway with or without a link mass term.

C. Longitudinal dynamics and effects of transverse hopping

We first consider the dynamics in the absence of any link. In this case, fermions cannot hop, and at each transverse location we have $(1+1)$-dimensional light front QCD which reduces to the 't Hooft model in the large N_c limit. In this case quarks and antiquarks at the same transverse position interact via the spin independent instantaneous interaction which, in the nonrelativistic limit, reduces to the linear potential in the longitudinal direction. The only parameters in the theory are the dimensionless fermion mass $m_f = am$ and the gauge coupling g. The spectrum consists of a ground state and a tower of excited states corresponding to the excitations of the $q\bar{q}$ pair.

Next consider the inclusion of the $q\bar{q}$ link states. There are four independent amplitudes corresponding to whether the quark is left, right, above, or below the antiquark. With nonzero mass of the link, these states lie above the ground state of a pure quark-antiquark system. Further, the q, $\bar{q}$, and link (which are frozen at their transverse positions) undergo fermion-link instantaneous interactions in the longitudinal direction which further increases the mass of the $q\bar{q}$ link states. Now the quark or antiquark can hop via helicity flip or helicity nonflip. Here we find a major difference between the Hamiltonians resulting from a forward-backward derivative and a symmetric derivative. Let us first consider the helicity flip hopping term in the forward-backward case

$$P_{hf}^{-}=-img\int dx^{-}\sum_{\mathbf{x}}\sum_{r}\left[\eta^{\dagger}(\mathbf{x})\hat{\sigma}_r\frac{1}{a}\frac{1}{\partial^{+}}\eta(\mathbf{x}+a\hat{\mathbf{r}})+\eta^{\dagger}(\mathbf{x}+a\hat{\mathbf{r}})M_r^{\dagger}(\mathbf{x})\hat{\sigma}_r\frac{1}{a}\frac{1}{\partial^{+}}\eta(\mathbf{x})\right]. \quad (3.1)$$

If we consider the transition from a two particle to a three particle state by a quark hop, then the first term in Eq. (3.1) corresponds to $|2\rangle\rightarrow|3a\rangle$ and the second term corresponds to $|2\rangle\rightarrow|3b\rangle$. The helicity flip term in the symmetric derivative case, after making some shifts in lattice points, can be written as

$$P_{hf}^{-}=-g\left(m+4\frac{\kappa}{a}\right)\times\frac{1}{2a}\int dx^{-}\sum_{\mathbf{x}}\sum_{r}\left[\left\{\eta^{\dagger}(\mathbf{x})\hat{\sigma}_r\frac{1}{i\partial^{+}}M_r(\mathbf{x})\eta(\mathbf{x}+a\hat{\mathbf{r}})-\eta^{\dagger}(\mathbf{x})\hat{\sigma}_rM_r(\mathbf{x})\frac{1}{i\partial^{+}}\eta(\mathbf{x}+a\hat{\mathbf{r}})\right\}-\left\{\eta^{\dagger}(\mathbf{x})\hat{\sigma}_r\frac{1}{i\partial^{+}}M_{-r}(\mathbf{x})\eta(\mathbf{x}-a\hat{\mathbf{r}})-\eta^{\dagger}(\mathbf{x})\hat{\sigma}_rM_{-r}(\mathbf{x})\frac{1}{i\partial^{+}}\eta(\mathbf{x}-a\hat{\mathbf{r}})\right\}\right]. \quad (3.2)$$

For the Hamiltonian with symmetric derivative, a quark or antiquark hopping accompanied by helicity flip has opposite signs for forward and backward hops. On the other hand, hopping accompanied by helicity nonflip have the same signs. As a result, there is no interference between helicity flip and helicity nonflip interactions [7]. In the case of the Hamiltonian with the forward-backward derivative, quark, or antiquark hopping accompanied by helicity flip has the same sign for forward and backward hops. As a consequence the helicity nonflip hop can interfere with the helicity flip hop. This has immediate consequences for the spectrum. In the case with a symmetric derivative, in the lowest order perturbation theory, the helicity zero states mix with each other which causes a splitting in their eigenvalues resulting in the singlet state lower than the triplet state. On the other hand, helicity plus or minus one states do not mix with each other or with helicity zero states, resulting in a twofold degeneracy. In the case with forward and backward derivatives all helicity states mix with each other, resulting in the complete absence of degeneracy.

IV. SINGULARITIES, DIVERGENCE, AND COUNTERTERMS

Since the transverse lattice serves as an ultraviolet regulator, we need to worry about only light front longitudinal momentum singularities.

A. Tree level

We take all the terms in the Hamiltonian to be normal ordered. At tree level this leaves us with singular factors of the form $1/(k)^2$ in the normal ordered four fermion and fermion link instantaneous interactions. The singularities are removed by adding the counterterms used in the previous work [4] on transverse lattice. The explicit forms of the counterterms are given in Appendix C in the appropriate places.

B. Self-energy corrections

In the one link approximation, a quark can make a forward (backward) hop followed by a backward (forward) hop resulting in self-energy corrections. In a single hop, helicity flip or nonflip can occur. In the case of symmetric derivatives, helicity flip cannot interfere with helicity nonflip, and as a consequence, self-energy corrections are diagonal in helicity space. In the case of forward and backward derivatives, the interference is nonzero, resulting in self-energy corrections both diagonal and off-diagonal in the helicity space. Similar self-energy corrections are generated for an antiquark also. These self-energy corrections contain a logarithmic light front infrared divergence which must be removed by counterterms. In Appendix E we present the explicit form of counterterms in the two cases separately. In previous works on one link approximation [4,5,7], these counterterms were not implemented.

V. NUMERICAL RESULTS

We diagonalize the dimensionless matrix a^2P^{-}. We further divide the matrix elements by g^2C_f which is the strength of the matrix elements for four fermion and fermion-link instantaneous interactions. Now, we define the constant G with a dimension of mass by $G^2=(g^2/a^2)C_f$. DLCQ yields M^2/G^2.

The dimensionless couplings are introduced [4] as follows: fermion mass $m_f=m/G$, link mass $\mu_b=\mu/G$, particle number conserving helicity flip coupling $m_f/(aG)=m_fC_1$, particle number nonconserving helicity flip $\sqrt{N}gm_f/(aG)=m_fC_2$, and particle number nonconserving helicity nonflip $\sqrt{N}g/(a^2G^2)=C_3$. In the case of the Wilson term modified Hamiltonian, we have the fermion mass term $m_f=(m+4\kappa/a)/G$, helicity-flip coupling $\sqrt{N}gmf/(2aG)=m_f\tilde{C}_2$, and helicity nonflip coupling $\sqrt{N}gm_f\kappa/(aG)=m_f\tilde{C}_3$.

All the results presented here were obtained on a small cluster of computers using the many fermion dynamics (MFD) code [12] that implements the Lanczos diagonalization method in parallel environment. For low K values, the results were checked against an independent code running on a single processor.

A. Cancellation of divergences

As we already mentioned, we encounter $1/(k^{+})^2$ singularities with instantaneous four fermion and instantaneous fermion-link interactions which give rise to linear divergences. We remove the divergences by adding appropriately chosen counterterms. We have numerically checked the removal of a linear divergence by counterterms in DLCQ. First we consider only $q\bar{q}$ states with instantaneous interaction.

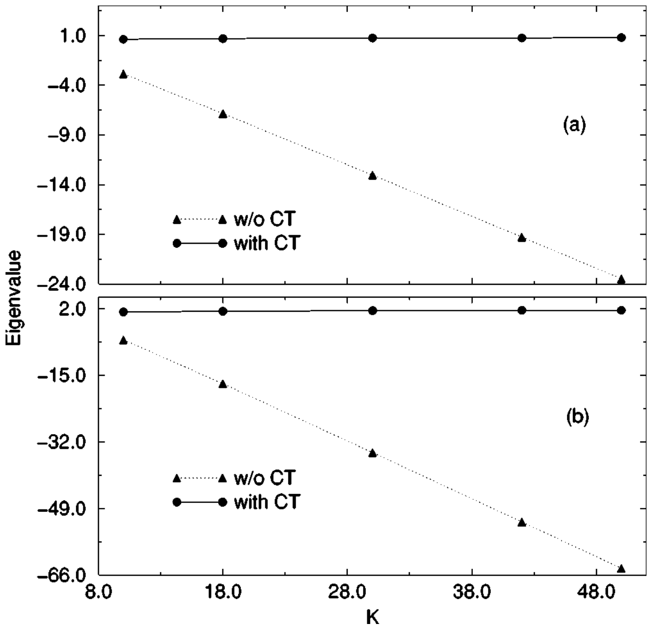

FIG. 1. Effect of the counterterm on the ground state eigenvalue. (a) With and without the counterterm in the $q\bar{q}$ sector for $m_f = 0.3$. (b) With and without the counterterm in the $q\bar{q}$ link sector for $m_f = 0.3$ and $\mu_b = 0.2$.

We study the ground state eigenvalue as a function of K with and without the counterterm. Results are presented in Fig. 1(a). Next we consider only $q\bar{q}$ link states with fermion-link instantaneous interaction with and without the counterterms. The behavior of ground state eigenvalue as a function of K is presented in Fig. 1(b). In both cases, it is evident that the counterterms are efficient in removing the divergence.

B. $q\bar{q}$ at the same transverse location

Next we study the spectrum of the Hamiltonian in the absence of any links. Since in this case the Hamiltonian depends only on the dimensionless ratio m_f/g we fix $g=1$ and vary m_f to study the spectra. The Hamiltonian matrix is diagonalized for various values of K. The convergence of the ground state eigenvalue as a function of K is presented in Table I. The ground state wave function squared as a function of the longitudinal momentum fraction x is plotted in Fig. 2. The convergence of the wave function has a very different behavior as a function of fermion mass m_f. As can be seen from this figure, the convergence in K is from above for heavy m_f and from below for light m_f. As a consequence the wave function is almost independent of K when m_f is of order g.

TABLE I. Ground state eigenvalue (in units of G^2) for $q\bar{q}$ sitting at the same transverse location.

K	Eigenvalue ($\mathcal{M}^2$)		
	$m_f = 0.3$	$m_f = 0.9$	$m_f = 3.0$
10	0.620	4.547	39.233
18	0.693	4.664	39.861
30	0.745	4.724	40.053
50	0.788	4.762	40.163
78	0.819	4.783	40.220
98	0.832	4.791	40.241
$K\rightarrow\infty$	0.869	4.820	40.285

C. Results of the one link approximation

We encountered logarithmic infrared divergences due to self-energy corrections and, in Appendix E, we discuss the associated counterterms. In Fig. 3 we show the effect of self-energy counterterms on the ground state energy in the two Hamiltonian cases we studied.

The quark distribution function for the ground state and the fifth state for the set of parameters $m_f = 0.3$, $\mu_b = 0.2$, $C_2 = 0.4$, $C_3 = 0.01$, and $K = 30$ is presented in Fig. 4. In this figure we also present separately the contribution from two particle and three particle states. As expected, the contribution from the three particle state peaks at smaller x compared to the two particle state. The exact location of this peak depends on the link mass. The convergence of the lowest four eigenvalues with K for the Hamiltonian with forward-backward and symmetric lattice derivatives is shown in Table II for $m_f = 0.3$, $\mu_b = 0.2$. We also show the results extrapolated to $K\rightarrow\infty$.

It is interesting to see the effect of the fermion-link instantaneous interaction on the low-lying eigenvalues. In its absence, there is no confining interaction in the longitudinal direction in the $q\bar{q}$ link sector. Furthermore, the mass of the lowest state in this sector corresponds to the threshold mass in this sector. Since its mass is lowered, it mixes more strongly with the $q\bar{q}$ sector in the ground state. The fifth state now corresponds to an almost free $q\bar{q}$ link state with an infinitesimal $q\bar{q}$ component as shown in Fig. 5.

VI. SUMMARY, DISCUSSION, AND CONCLUSIONS

In this work we have performed an investigation of $q\bar{q}$ states using two different light front Hamiltonians in the one link approximation. The Hamiltonians correspond to two different ways of formulating fermions on the transverse lattice, namely, (a) forward and backward derivatives for ψ^+ and ψ^-, respectively, or vice versa and (b) symmetric derivatives for both ψ^+ and ψ^-. In the latter, fermion doubling is present which is removed by the addition of the Wilson term. In this case there is no interference between helicity flip hop and helicity nonflip hop and, as a result, the $q\bar{q}$ component of the ground state wave function, which has helicity plus or minus one, is degenerate. In the former case, interference between helicity flip and helicity nonflip leads to the absence of degeneracy in the low-lying spectra. One can recover approximate degeneracy of helicity plus or minus one components only by keeping the strength of the helicity nonflip hopping very small. In the case of forward and backward derivatives, terms are also present which violate hypercubic symmetry on the transverse lattice. They become irrelevant in the continuum limit when the linear variables M are re-

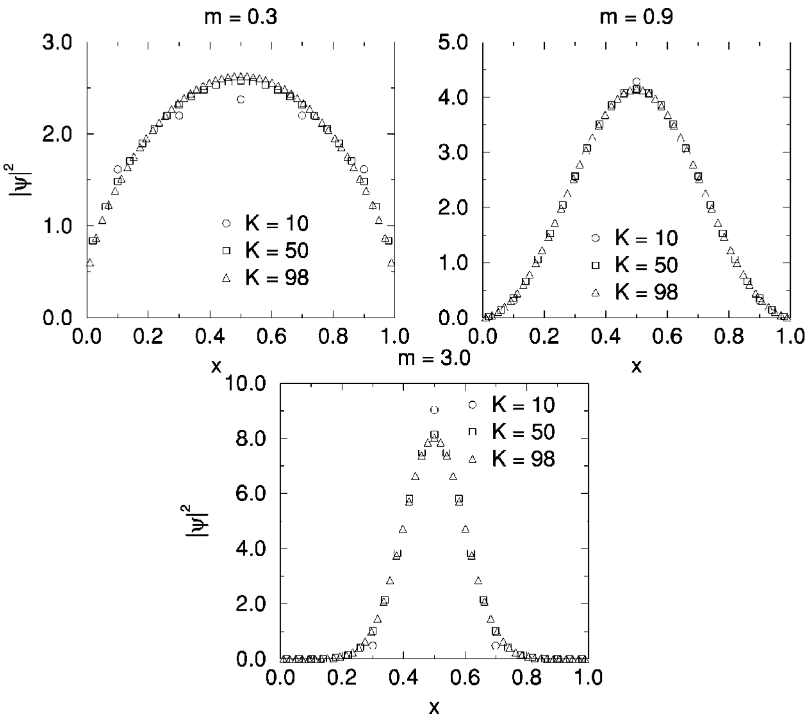

FIG. 2. Quark distribution function $|\psi(x)|^2$ of the ground state in the $q\bar{q}$ approximation for three choices of quark masses with the coupling constant $g=1.0$.

placed by nonlinear variables U. We have removed them entirely from the Hamiltonian in the present investigation.

Since the one link approximation is very crude, we have not attempted a detailed fit to low-lying states in the meson sector. Instead, we have explored the effects of various coupling strengths on the low-lying spectra and associated wave functions. In our work, longitudinal dynamics is handled by DLCQ. We have performed a detailed study of various convergence issues in DLCQ using a wide range of K values.

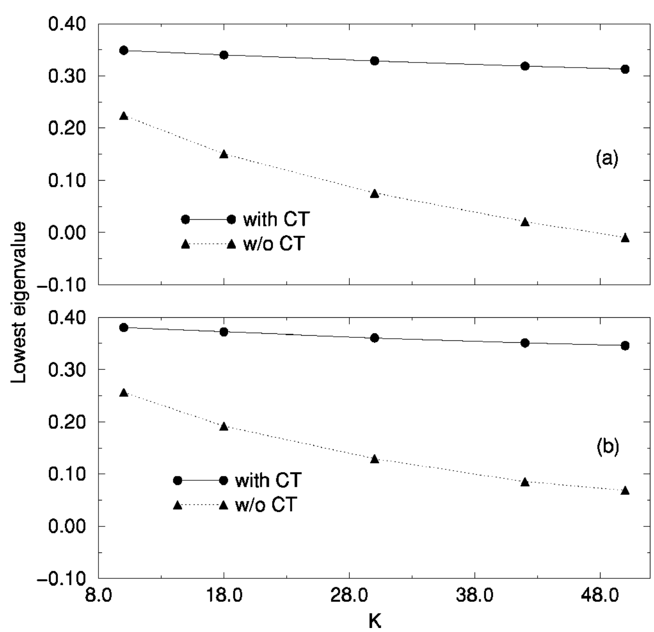

FIG. 3. Effect of self-energy counterterms on the ground state eigenvalue in the case of (a) the symmetric derivative with $\tilde{C}_2=0.4$, $\tilde{C}_3=0.1$ and (b) the forward-backward derivative with $C_2=0.4$, $C_3=0.01$. $m_f=0.3$, $\mu_b=0.2$ for both cases.

We summarize our results as follows. We have shown the effectiveness of appropriate counterterms in the $q\bar{q}$ and $q\bar{q}$ link sector to regulate the instantaneous fermion and fermion-link interactions, respectively. We have also checked the cancellation of logarithmic divergences due to self-energy effects. In the limit where fermions are frozen on the transverse lattice but undergo instantaneous longitudinal interaction, we have studied the convergence of ground state wave functions with respect to K for three typical values of the fermion mass. We have studied how the presence or absence of the fermion-link instantaneous interaction in the $q\bar{q}$ link sector affects the wave function of low-lying states. We have also studied the consequences of the interference of helicity flip and helicity nonflip hopping in the Hamiltonian with forward-backward derivatives. This interference is absent in the symmetric derivative case.

For future studies we would like to address the problem of mesons containing one light and one heavy quark in the context of heavy quark effective theory on the transverse lattice. A systematic study of the effects of sea quarks and additional links on the meson observables also need to be undertaken. A major unsettled issue in the transverse lattice formulation is the continuum limit of the theory when nonlinear link variables are replaced by link variables. It will be interesting to investigate the light front quantization problem with nonlinear constraints. In this respect the study of the nonlinear sigma model on the light front appears worthwhile.

ACKNOWLEDGMENTS

We thank Asit K. De for many helpful discussions. This work is supported in part by the Indo-U.S. Collaboration project jointly funded by the U.S. National Science Founda-

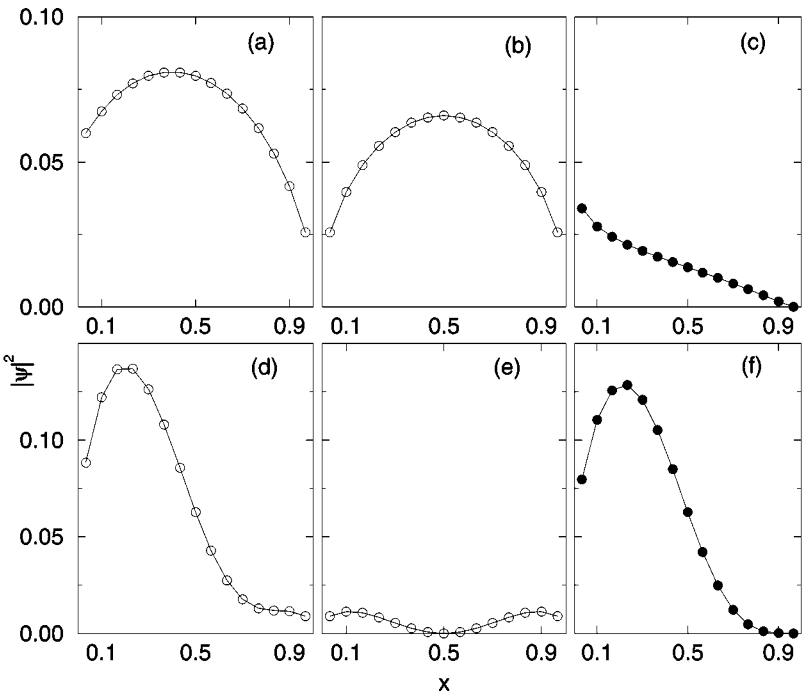

FIG. 4. (a) Quark distribution function $|\psi(x)|^2$ of the ground state in the one link approximation, (b) the $q\overline{q}$ contribution to the ground state, and (c) the $q\overline{q}$ link contribution to the ground state. (d) The quark distribution function $|\psi(x)|^2$ of the fifth eigenstate in the one link approximation, (e) the $q\overline{q}$ contribution to the fifth eigenstate, and (f) the $q\overline{q}$ link contribution to the fifth eigenstate. The parameters are $m_f=0.3$, $\mu_b=0.2$, $C_2=0.4$, $C_3=0.01$, and $K=30$.

tion (INT0137066) and the Department of Science and Technology, India [DST/INT/US (NSF-RP075)/2001]. This work is also supported in part by the U.S. Department of Energy, Grant No. DE-FG02-87ER40371, Division of Nuclear Physics and a project under Department of Atomic Energy, India.

APPENDIX A: STRUCTURE OF TERMS IN DLCQ

We use DLCQ for the longitudinal dimension ($-L \leqslant x^- \leqslant +L$) and implement an antiperiodic boundary condition for the two component fermion field,

$$\eta_c(x^-,\mathbf{x}) = \frac{1}{\sqrt{2L}} \sum_{\lambda} \chi_\lambda \sum_{m=1,3,5,\dots} [b_c(m,\mathbf{x},\lambda) e^{-i\pi m x^-/(2L)} + d_c^\dagger(m,\mathbf{x},-\lambda) e^{i\pi m x^-/(2L)}] \tag{A1}$$

with

$$\{b_c(m,\mathbf{x},\lambda), b_c^\dagger(m',\mathbf{x}',\lambda')\} = \{d_c(m,\mathbf{x},\lambda), d_c^\dagger(m',\mathbf{x}',\lambda')\} = \delta_{mm'}\delta_{\mathbf{x},\mathbf{x}'}\delta_{c,c'}\delta_{\lambda,\lambda'}. \tag{A2}$$

The link field has a periodic boundary condition (with the omission of the zero momentum mode),

$$M_{rpq}(x^-,\mathbf{x}) = \frac{1}{\sqrt{4\pi}} \sum_{m=1,2,3,\dots} \frac{1}{\sqrt{m}} [B_{-rpq}(m,\mathbf{x}+a\hat{\mathbf{r}}) \times e^{-i\pi m x^-/L} + B_{rpq}^\dagger(m,\mathbf{x}) e^{i\pi m x^-/L}] \tag{A3}$$

with

$$[B_{rpq}(m,\mathbf{x}), B_{r'Tts}^\dagger(m',\mathbf{x}')] = \delta_{mm'}\delta_{\mathbf{x},\mathbf{x}'}\delta_{r,r'}\delta_{ps}\delta_{qt}. \tag{A4}$$

The Hamiltonian $P^- = (L/\pi)H$. In the following section we give the explicit structure of terms in the Hamiltonian in the

TABLE II. Lowest four eigenvalues (in units of G^2) in one link approximation

K	Forward-backward ($C_2=0.01$, $C_3=0.4$)				Symmetric ($\tilde{C}_2=0.1$, $\tilde{C}_3=0.4$)			
	$\mathcal{M}_1^2$	$\mathcal{M}_2^2$	$\mathcal{M}_3^2$	$\mathcal{M}_4^2$	$\mathcal{M}_1^2$	$\mathcal{M}_2^2$	$\mathcal{M}_3^2$	$\mathcal{M}_4^2$
10	0.38041	0.4800	0.4899	0.5996	0.3486	0.4507	0.4507	0.5980
18	0.3722	0.4968	0.5110	0.6447	0.3402	0.4673	0.4673	0.6409
30	0.3606	0.5027	0.5210	0.6680	0.3288	0.4702	0.4702	0.6620
42	0.3511	0.5029	0.5240	0.6765	0.3189	0.4677	0.4677	0.6682
50	0.3457	0.5019	0.5246	0.6790	0.3130	0.4651	0.4651	0.6693
$K\to\infty$	0.3243	0.5022	0.5313	0.6979	0.2913	0.4589	0.4589	0.6837

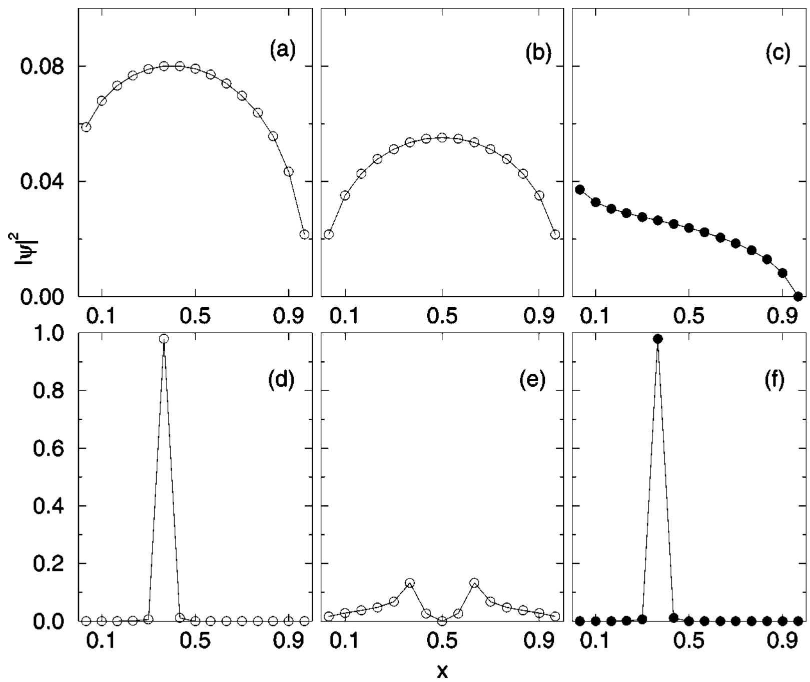

FIG. 5. Without the fermion-link instantaneous interaction: (a) The quark distribution function $|\psi(x)|^2$ of the ground state in the one link approximation, (b) the $q\bar{q}$ contribution to the ground state, and (c) the $q\bar{q}$ link contribution to the ground state. (d) The quark distribution function $|\psi(x)|^2$ of the fifth eigenstate in the one link approximation, (e) the $q\bar{q}$ contribution to the fifth eigenstate multiplied by 10^4, and (f) the $q\bar{q}$ link contribution to the fifth eigenstate. Parameters are the same as in Fig. 4.

forward-backward case in DLCQ restricting our discussion to those relevant for the one link approximation.

1. Mass terms

The mass terms are

$$H_{ffree}=m^2\sum_{\mathbf{x}}\sum_{c}\sum_{\lambda}\sum_{n}\frac{1}{n}[b_c^\dagger(n,\mathbf{x},\lambda)b_c(n,\mathbf{x},\lambda)+d_c^\dagger(n,\mathbf{x},\lambda)d_c(n,\mathbf{x},\lambda)], \tag{A5}$$

$$H_{LINKfree}=\frac{\mu^2}{2}\sum_{\mathbf{x}}\sum_{\hat{\mathbf{r}}}\sum_{n}\frac{1}{n}[B_r^\dagger(m,\mathbf{x})B_r(m,\mathbf{x})+B_{-r}^\dagger(m,\mathbf{x}+a\hat{\mathbf{r}})B_{-r}(m,\mathbf{x}+a\hat{\mathbf{r}})]. \tag{A6}$$

2. Four fermion instantaneous term

The four fermion instantaneous term which gives rise to a linear potential in the color singlet state is

$$2\frac{g^2}{\pi a^2}\sum_{cc'c''c'''}\sum_{\lambda\lambda'\lambda''\lambda'''}\sum_{\mathbf{x}}\delta_{\lambda\lambda'}\delta_{\lambda''\lambda'''}\times\sum_{m_1m_2m_3m_4}b_c^\dagger(m_1,\mathbf{x},\lambda)d_{c'''}^\dagger(m_4,\mathbf{x},-\lambda''')\times b_{c'}(m_2,\mathbf{x},\lambda')d_{c''}(m_3,\mathbf{x},\lambda''')\times\frac{1}{(m_3-m_4)^2}\delta_{m_1+m_4,m_2+m_3}. \tag{A7}$$

3. Helicity flip terms

The particle number conserving terms are

$$\frac{mg}{a}\sum_{r}\sum_{\mathbf{x}}\sum_{\lambda_1,\lambda_2}\chi_{\lambda_1}^\dagger\hat{\sigma}_r\chi_{\lambda_2}\times\sum_{m_1}\frac{1}{m_1}[b_c^\dagger(m_1,\mathbf{x},\lambda_1)b_c(m_1,\mathbf{x},\lambda_2)+d_c^\dagger(m_1,\mathbf{x},-\lambda_2)d_c(m_1,\mathbf{x},-\lambda_2)]. \tag{A8}$$

For the particle number nonconserving terms, a typical term is

$$\frac{mg}{a}\frac{1}{\sqrt{4\pi}}\sum_{r}\sum_{\mathbf{x}}\sum_{\lambda_1,\lambda_2}\chi_{\lambda_1}^\dagger\hat{\sigma}_r\chi_{\lambda_2}\times\sum_{m_1m_2m_3}\frac{1}{\sqrt{m_3}}\frac{1}{2m_3+m_2}\delta_{m_1-m_2,2m_3}b_c^\dagger(m_1,\mathbf{x},\lambda_1)\times B_{-rcc'}(m_3,\mathbf{x}+a\hat{\mathbf{r}})b_{c'}(m_2,\mathbf{x}+a\hat{\mathbf{r}},\lambda_2). \tag{A9}$$

4. Helicity nonflip terms

For two operators we have

$$\frac{2}{a^2}\sum_{\mathbf{x}}\sum_{\lambda}\sum_{n}\frac{1}{n}[b_c^\dagger(n,\mathbf{x},\lambda)b_c(n,\mathbf{x},\lambda)+d_c^\dagger(n,\mathbf{x},\lambda)d_c(n,\mathbf{x},\lambda)]. \tag{A10}$$

For three operators, a typical term is

$$-g\frac{1}{a^2}\frac{1}{\sqrt{4\pi}}\sum_{r}\sum_{\mathbf{x}}\sum_{\lambda}\sum_{m_1m_2m_3}\frac{1}{\sqrt{m_3}}\frac{1}{2m_3+m_2}\times\delta_{m_1-m_2,2m_3}b_c^\dagger(m_1,\mathbf{x},\lambda)\times B_{-rcc'}(m_3,\mathbf{x}+a\hat{\mathbf{r}})b_{c'}(m_2,\mathbf{x}+a\hat{\mathbf{r}},\lambda). \tag{A11}$$

5. Fermion-link instantaneous term

A typical term is

$$2\frac{g^2}{4\pi}\frac{1}{a^2}\sum_{\mathbf{x}}\sum_{r}\sum_{cc'c''}\sum_{dd'}T^{\alpha}_{cc'}T^{\alpha}_{dd'}\sum_{m_1m_2m_3m_4}\frac{1}{\sqrt{m_3}}\frac{1}{\sqrt{m_4}}$$
$$\times b_d^{\dagger}(m_1,\mathbf{x},\lambda_1)b_{d'}(m_2,\mathbf{x},\lambda_2)$$
$$\times B_{-rc'c''}(m_3,\mathbf{x}+a\hat{\mathbf{r}})B^{\dagger}_{-rc''c}(m_4,\mathbf{x}+a\hat{\mathbf{r}})$$
$$(-)(m_3+m_4)/(m_1-m_2)^2\delta_{m_1-m_2,2m_3-2m_4}. \quad \text{(A12)}$$

APPENDIX B: STATES IN DLCQ

We will consider states of zero transverse momentum. In the one-link approximation, the gauge invariant states are $q\bar{q}$ states

$$|2\rangle=\frac{1}{\sqrt{N}}\frac{1}{\sqrt{V}}\sum_{d}\sum_{\mathbf{y}(q)}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{y}(q),\mathbf{y}(\bar{q})}b_d^{\dagger}(n_1,\mathbf{y}(q),\sigma_1)$$
$$\times d_d^{\dagger}(n_2,\mathbf{y}(\bar{q}),\sigma_2)|0\rangle \quad \text{(B1)}$$

and the $q\bar{q}$ link states

$$|3a\rangle=\frac{1}{N}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2}}\sum_{dd'}\sum_{s}\sum_{\mathbf{y}(q)}\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{y}(l)}\delta_{\mathbf{y}(l),\mathbf{y}(q)}\delta_{\mathbf{y}(q),\mathbf{y}(\bar{q})-a\hat{\mathbf{s}}}$$
$$\times b_d^{\dagger}(n_1,\mathbf{y}(q),\sigma_1)B^{\dagger}_{sdd'}(n_3,\mathbf{y}(l))d^{\dagger}_{d'}(n_2,\mathbf{y}(\bar{q}),\sigma_2)|0\rangle$$

and

$$|3b\rangle=\frac{1}{N}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2}}\sum_{dd'}\sum_{s}\sum_{\mathbf{y}(q)}\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{y}(l)}\delta_{\mathbf{y}(l),\mathbf{y}(q)}\delta_{\mathbf{y}(q),\mathbf{y}(\bar{q})+a\hat{\mathbf{s}}}$$
$$\times b_d^{\dagger}(n_1,\mathbf{y}(q),\sigma_1)B^{\dagger}_{-sdd'}(n_3,\mathbf{y}(l))$$
$$\times d^{\dagger}_{d'}(n_2,\mathbf{y}(\bar{q}),\sigma_2)|0\rangle. \quad \text{(B2)}$$

We shall consider transition from these initial states to the following final states: The $q\bar{q}$ state

$$\langle 2'|=\frac{1}{\sqrt{N}}\frac{1}{\sqrt{V}}\sum_{e}\sum_{\mathbf{z}(q)}\sum_{\mathbf{z}(\bar{q})}\delta_{\mathbf{z}(q),\mathbf{z}(\bar{q})}\langle 0|d_e(n_2',\mathbf{z}(\bar{q}),\sigma_2')$$
$$\times b_e(n_1',\mathbf{z}(q),\sigma_1') \quad \text{(B3)}$$

and the $q\bar{q}$ link states

$$\langle 3a'|=\frac{1}{N}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2}}\sum_{ee'}\sum_{t}\sum_{\mathbf{z}(q)}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{z}(l)}\delta_{\mathbf{z}(l),\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{z}(\bar{q})-a\hat{\mathbf{t}}}$$
$$\times\langle 0|d_e(n_2',\mathbf{z}(\bar{q}),\sigma_2')B_{tee'}(n_3',\mathbf{z}(l))b_{e'}(n_1',\mathbf{z}(q),\sigma_1') \quad \text{(B4)}$$

and

$$\langle 3b'|=\frac{1}{N}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2}}\sum_{ee'}\sum_{t}\sum_{\mathbf{z}(q)}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{z}(l)}\delta_{\mathbf{z}(l),\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{z}(\bar{q})+a\hat{\mathbf{t}}}$$
$$\times\langle 0|d_e(n_2',\mathbf{z}(\bar{q}),\sigma_2')B_{-tee'}(n_3',\mathbf{z}(l))$$
$$\times b_{e'}(n_1',\mathbf{z}(q),\sigma_1'). \quad \text{(B5)}$$

APPENDIX C: FORWARD-BACKWARD DERIVATIVES: MATRIX ELEMENTS IN DLCQ

1. Transitions from the two particle state

a. To the two particle state

Let us consider transitions to the two particle state. We have, from the free particle term,

$$\langle 2'|H_{f\,free}|2\rangle=m^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathcal{N}_2, \quad \text{(C1)}$$

where

$$\mathcal{N}_2=\delta_{n_1,n_1'}\delta_{\sigma_1,\sigma_1'}\delta_{n_2,n_2'}\delta_{\sigma_2,\sigma_2'}. \quad \text{(C2)}$$

From the four fermion instantaneous term we get

$$\langle 2'|H_{qqc}|2\rangle=-2\frac{g^2}{\pi a^2}C_f\delta_{n_1+n_2,n_1'+n_2'}$$
$$\times\frac{1}{(n_1-n_1')^2}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'} \quad \text{(C3)}$$

where $C_f=(N^2-1)/2N$.

To implement the regulator prescription for $1/(k^+)^2$, we add the counterterm matrix elements

$$\langle 2'|H_{CT}|2\rangle=2\frac{g^2}{\pi a^2}C_f\delta_{n_1+n_2,n_1'+n_2'}$$
$$\times\sum_{n_{loop}=1}^{K}\frac{1}{(n_1-n_{loop})^2}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}. \quad \text{(C4)}$$

Here the term $n_{loop}=n_1$ is dropped from the sum.

From the helicity flip term we get

$$\langle 2'|H_{hf1}|2\rangle=-2\frac{1}{a}\sum_{s}\left[\frac{m}{n_1}\chi^{\dagger}_{\sigma_1'}\hat{\sigma}_s\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\right.$$
$$\left.+\frac{m}{n_2}\chi^{\dagger}_{-\sigma_2}\hat{\sigma}_s\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\right]\mathcal{N}_{hf} \quad \text{(C5)}$$

with

$$\mathcal{N}_{hf}=\delta_{n_1,n_1'}\delta_{n_2,n_2'}. \quad \text{(C6)}$$

From the helicity nonflip term we get

$$\langle 2'|H_{hnf}(1)|2\rangle=2\frac{1}{a^2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathcal{N}_2. \quad \text{(C7)}$$

b. To the three particle state

Next we consider the transitions to the three particle states.

To the state $|3a\rangle$. From the helicity flip term we get

$$\begin{aligned}\langle 3a'|H_{hf2}|2\rangle &= \frac{mg}{a}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{\sigma_1'}\hat{\sigma}_t\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\\ &\times\delta_{n_2,n_2'}\frac{\delta_{n_1'+2n_3',n_1}}{n_1'}\frac{1}{\sqrt{n_3'}}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{t}}}\\ &+\frac{mg}{a}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{-\sigma_2}\hat{\sigma}_t\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\\ &\times\delta_{n_1,n_1'}\frac{\delta_{n_2'+2n_3',n_2}}{n_2}\frac{1}{\sqrt{n_3'}}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{t}}a}.\end{aligned}\quad \text{(C8)}$$

From the helicity nonflip term we get

$$\begin{aligned}\langle 3a'|H_{hnf}(2)|2\rangle &= -g\frac{1}{a^2}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\delta_{n_2,n_2'}\\ &\times\frac{\delta_{n_1'+2n_3',n_1}}{n_1'}\frac{1}{\sqrt{n_3'}}\sum_t\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{t}}}\\ &-g\frac{1}{a^2}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\delta_{n_1,n_1'}\\ &\times\frac{\delta_{n_2'+2n_3',n_2}}{n_2}\frac{1}{\sqrt{n_3'}}\sum_t\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{t}}a}.\end{aligned}\quad \text{(C9)}$$

To the state $|3b\rangle$. From the helicity flip term we get

$$\begin{aligned}\langle 3b'|H_{hf2}|2\rangle &= \frac{mg}{a}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{\sigma_1'}\hat{\sigma}_t\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\\ &\times\delta_{n_2,n_2'}\frac{\delta_{n_1'+2n_3',n_1}}{n_1}\frac{1}{\sqrt{n_3'}}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{t}}}\\ &+\frac{mg}{a}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{-\sigma_2}\hat{\sigma}_t\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\\ &\times\delta_{n_1,n_1'}\frac{\delta_{n_2'+2n_3',n_2}}{n_2'}\frac{1}{\sqrt{n_3'}}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{t}}a}.\end{aligned}\quad \text{(C10)}$$

From helicity nonflip term we get

$$\begin{aligned}\langle 3b'|H_{hnf}(3)|2\rangle &= -g\frac{1}{a^2}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\delta_{n_2,n_2'}\\ &\times\frac{\delta_{n_1'+2n_3',n_1}}{n_1}\frac{1}{\sqrt{n_3'}}\sum_t\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{t}}}\\ &-g\frac{1}{a^2}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\delta_{n_1,n_1'}\\ &\times\frac{\delta_{n_2'+2n_3',n_2}}{n_2'}\frac{1}{\sqrt{n_3'}}\sum_t\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{t}}a}.\end{aligned}\quad \text{(C11)}$$

2. Transitions from the three particle ($q\bar{q}$ link) state $|3a\rangle$

a. To the three particle state

From the free particle term, we get

$$\langle 3a'|H_{free}|3a\rangle = \left[m^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)+\frac{1}{2}\mu^2\frac{1}{n_3}\right]\mathcal{N}_3 \quad \text{(C12)}$$

with

$$\mathcal{N}_3 = \delta_{n_1,n_1'}\delta_{n_2,n_2'}\delta_{n_3,n_3'}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}. \quad \text{(C13)}$$

The diagonal contribution from the four fermion instantaneous term to the three particle state vanishes due to the vanishing trace of the generators of $SU(N)$. The contribution from the fermion-link instantaneous term

$$\begin{aligned}\langle 3a'|H_{qgc}(1)|3a\rangle &= -\frac{g^2}{\pi}\frac{1}{a^2}C_f\delta_{n_1+2n_3,n_1'+2n_3'}\delta_{n_2,n_2'}\frac{1}{\sqrt{n_3}\sqrt{n_1-n_1'+2n_3}}\\ &\times\frac{(n_1-n_1'+4n_3)}{(n_1-n_1')^2}\frac{1}{\sqrt{2}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\\ &-\frac{g^2}{\pi}\frac{1}{a^2}C_f\delta_{n_2+2n_3,n_2'+2n_3'}\delta_{n_1,n_1'}\frac{1}{\sqrt{n_3}\sqrt{n_2-n_2'+2n_3}}\\ &\times\frac{(n_2-n_2'+4n_3)}{(n_2-n_2')^2}\frac{1}{\sqrt{2}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}.\end{aligned}\quad \text{(C14)}$$

Counterterm matrix elements in DLCQ to implement the regulated prescription for $1/(k^+)^2$

$$\langle 3a'|H_{CT}(1)|3a\rangle = \frac{g^2}{\pi}\frac{1}{a^2}C_f \delta_{n_1+2n_3,n_1'+2n_3'}\delta_{n_2,n_2'}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\Bigg[\sum_{n_{loop}=1}^{n_{1max}} \frac{1}{\sqrt{n_3}\sqrt{n_1-n_{loop}+2n_3}}\frac{(n_1-n_{loop}+4n_3)}{(n_1-n_{loop})^2}\frac{1}{\sqrt{2}} + \sum_{n_{loop}=1}^{n_{2max}} \frac{1}{\sqrt{n_3}\sqrt{n_2-n_{loop}+2n_3}}\frac{(n_2-n_{loop}+4n_3)}{(n_2-n_{loop})^2}\frac{1}{\sqrt{2}}\Bigg], \quad \text{(C15)}$$

where $n_{1max}<n_1+2n_3$ and $n_{2max}<n_2+2n_3$.

The contribution from the helicity flip term that conserves the particle number is

$$\langle 3a'|H_{hf}(1)|3a\rangle = -2\frac{m}{a}\delta_{n_1,n_1'}\delta_{n_2,n_2'}\delta_{n_3,n_3'}\Bigg[\frac{1}{n_1}\sum_r \chi^{\dagger}_{\sigma_1'}\hat{\sigma}_r\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'} + \frac{1}{n_2}\sum_r \chi^{\dagger}_{-\sigma_2}\hat{\sigma}_r\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\Bigg]. \quad \text{(C16)}$$

The contribution from the helicity nonflip term that conserves the particle number is

$$\langle 3a'|H_{hnf}(1)|3a\rangle = \frac{2}{a^2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathcal{N}_3. \quad \text{(C17)}$$

b. To the two particle state

From the helicity flip term we get

$$\langle 2'|H_{hf2}|3a\rangle = \frac{mg}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_s \chi^{\dagger}_{\sigma_1'}\hat{\sigma}_s\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'} \times \delta_{n_2,n_2'}\frac{\delta_{n_1',n_1+2n_3}}{n_1}\frac{1}{\sqrt{n_3}}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{s}}} + \frac{mg}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_s \chi^{\dagger}_{-\sigma_2}\hat{\sigma}_s\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'} \times \delta_{n_1,n_1'}\frac{\delta_{n_2',n_2+2n_3}}{n_2'}\frac{1}{\sqrt{n_3}}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{s}}a}. \quad \text{(C18)}$$

From the helicity nonflip term we get

$$\langle 2'|H_{hnf}|3a\rangle = -g\frac{1}{a^2}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\delta_{n_2,n_2'} \times \frac{\delta_{n_1',n_1+2n_3}}{n_1}\frac{1}{\sqrt{n_3}}\sum_s\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{s}}} - g\frac{1}{a^2}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\delta_{n_1,n_1'} \times \frac{\delta_{n_2',n_2+2n_3}}{n_2'}\frac{1}{\sqrt{n_3}}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{s}a}. \quad \text{(C19)}$$

3. Transitions from the three particle ($q\bar{q}$ link) state $|3b\rangle$

a. To the three particle state

From the free particle term, we get

$$\langle 3b'|H_{free}|3b\rangle = \left[m^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)+\frac{1}{2}\mu^2\frac{1}{n_3}\right]\mathcal{N}_3 \quad \text{(C20)}$$

with

$$\mathcal{N}_3 = \delta_{n_1,n_1'}\delta_{n_2,n_2'}\delta_{n_3,n_3'}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}. \quad \text{(C21)}$$

The diagonal contribution from the four fermion instantaneous term to the three particle state vanishes due to the vanishing trace of the generators of $SU(N)$.

The contribution from the fermion-link instantaneous term is

$$\langle 3b'|H_{qgc}(1)|3b\rangle = -\frac{g^2}{\pi}\frac{1}{a^2}C_f\delta_{n_1+2n_3,n_1'+2n_3'}\delta_{n_2,n_2'}\frac{1}{\sqrt{n_3}\sqrt{n_1-n_1'+2n_3}} \times \frac{(n_1-n_1'+4n_3)}{(n_1-n_1')^2}\frac{1}{\sqrt{2}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'} - \frac{g^2}{\pi}\frac{1}{a^2}C_f\delta_{n_2+2n_3,n_2'+2n_3'}\delta_{n_1,n_1'}\frac{1}{\sqrt{n_3}\sqrt{n_2-n_2'+2n_3}} \times \frac{(n_2-n_2'+4n_3)}{(n_2-n_2')^2}\frac{1}{\sqrt{2}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}. \quad \text{(C22)}$$

Here also we have the counterterm matrix elements given in Eq. (C15).

The contribution from the helicity flip term that conserves the particle number is

$$\langle 3b'|H_{hf}(1)|3b\rangle = -2\frac{m}{a}\delta_{n_1,n_1'}\delta_{n_2,n_2'}\delta_{n_3,n_3'}\left[\frac{1}{n_1}\sum_r \chi^\dagger_{\sigma_1'}\hat{\sigma}_r\chi_{\sigma_1}\delta_{\sigma_2\sigma_2'} + \frac{1}{n_2}\sum_r \chi^\dagger_{-\sigma_2}\hat{\sigma}_r\chi_{-\sigma_2'}\delta_{\sigma_1\sigma_1'}\right]. \tag{C23}$$

The contribution from the helicity nonflip term that conserves the particle number is

$$\langle 3b'|H_{hnf}(1)|3b\rangle = \frac{2}{a^2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathcal{N}_3. \tag{C24}$$

b. To the two particle state

From the helicity flip term we get

$$\begin{aligned}\langle 2'|H_{hf2}|3b\rangle &= \frac{mg}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_s \chi^\dagger_{\sigma_1'}\hat{\sigma}_s\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'} \times\delta_{n_2,n_2'}\frac{\delta_{n_1',n_1+2n_3}}{n_1'}\frac{1}{\sqrt{n_3}}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{s}}}\\ &+\frac{mg}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_s \chi^\dagger_{-\sigma_2}\hat{\sigma}_s\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'} \times\delta_{n_1,n_1'}\frac{\delta_{n_2',2n_3+n_2}}{n_2}\frac{1}{\sqrt{n_3}}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{s}}a}.\end{aligned} \tag{C25}$$

From the helicity nonflip term we get

$$\begin{aligned}\langle 2'|H_{hnf}|3b\rangle &= -g\frac{1}{a^2}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'} \times\delta_{n_2,n_2'}\frac{\delta_{n_1',n_1+2n_3}}{n_1'}\frac{1}{\sqrt{n_3}}\sum_s\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{s}}}\\ &-g\frac{1}{a^2}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\delta_{n_1,n_1'} \times\frac{\delta_{n_2',n_2+2n_3}}{n_2}\frac{1}{\sqrt{n_3}}\sum_s\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(q)+a\hat{\mathbf{s}}}.\end{aligned} \tag{C26}$$

APPENDIX D: SYMMETRIC DERIVATIVES AND THE WILSON TERM: MATRIX ELEMENTS IN DLCQ

In this section we list only those matrix elements that differ from the forward-backward case.

1. Transitions from the two particle state

a. To the state $|3a\rangle$

For the helicity flip we have

$$\begin{aligned}\langle 3a'|P^-_{whf}|2\rangle &= \left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{\sigma_1'}\hat{\sigma}_t\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\sum_{\mathbf{y}(q)}\sum_{\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{t}}}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_1}-\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1'+2n_3',n_1}\\ &+\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_t \chi^\dagger_{-\sigma_2}\hat{\sigma}_t\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{z}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{t}}a}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_2'}-\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2'+2n_3',n_2}.\end{aligned} \tag{D1}$$

For the helicity nonflip we have

$$\begin{aligned}\langle 3a'|P^-_{wnf1}|2\rangle &= -\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_t\sum_{\mathbf{y}(q)}\sum_{\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{t}}}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_1}+\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1'+2n_3',n_1}\\ &-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_t\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{z}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{t}}a}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_2'}+\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2'+2n_3',n_2}.\end{aligned} \tag{D2}$$

b. To the state $|3b\rangle$

For the helicity flip we have

$$\begin{aligned}\langle 3b'|P^{-}_{whf}|2\rangle=&\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{t}\chi^{\dagger}_{\sigma_1'}\hat{\sigma}_t\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\sum_{\mathbf{y}(q)}\sum_{\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{t}}}\frac{1}{\sqrt{n_3'}}\left(-\frac{1}{n_1}+\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1'+2n_3',n_1}\\&+\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{t}\chi^{\dagger}_{-\sigma_2}\hat{\sigma}_t\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{z}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{t}}a}\frac{1}{\sqrt{n_3'}}\left(-\frac{1}{n_2'}+\frac{1}{n_2}\right)\\&\times\delta_{n_1,n_1'}\delta_{n_2+2n_3,n_2'}.\end{aligned}\tag{D3}$$

For the helicity nonflip we have

$$\begin{aligned}\langle 3a'|P^{-}_{wnf1}|2\rangle=&-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{t}\sum_{\mathbf{y}(q)}\sum_{\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{t}}}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_1}+\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1'+2n_3',n_1}\\&-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\sum_{t}\sum_{\mathbf{y}(\bar{q})}\sum_{\mathbf{z}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{t}}a}\frac{1}{\sqrt{n_3'}}\left(\frac{1}{n_2'}+\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2'+2n_3',n_2}.\end{aligned}\tag{D4}$$

2. Transitions from the three particle state $|3a\rangle$ to the two particle state

For the helicity flip we have

$$\begin{aligned}\langle 2'|P^{-}_{whf}|3a\rangle=&\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{s}\chi^{\dagger}_{\sigma_1'}\hat{\sigma}_s\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\sum_{\mathbf{z}(q)}\sum_{\mathbf{z}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{s}}}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_1'}-\frac{1}{n_1}\right)\delta_{n_2,n_2'}\delta_{n_1+2n_3,n_1'}\\&+\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{s}\chi^{\dagger}_{-\sigma_2}\hat{\sigma}_s\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})-\hat{\mathbf{s}}a}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_2}-\frac{1}{n_2'}\right)\delta_{n_1',n_1}\delta_{n_2'+2n_3',n_2}.\end{aligned}\tag{D5}$$

For the helicity nonflip we have

$$\begin{aligned}\langle 2'|P^{-}_{wnf1}|3a\rangle=&-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{s}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)+a\hat{\mathbf{s}}}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_1}+\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1+2n_3,n_1'}\\&-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{s}}a}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_2'}+\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2+2n_3,n_2'}.\end{aligned}\tag{D6}$$

3. Transitions from the three particle state $|3b\rangle$ to the two particle state

For the helicity flip we have

$$\begin{aligned}\langle 2'|P^{-}_{whf}|3b\rangle=&\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{s}\chi^{\dagger}_{\sigma_1'}\hat{\sigma}_s\chi_{\sigma_1}\delta_{\sigma_2,\sigma_2'}\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{s}}}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_1}-\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1+2n_3,n_1'}\\&+\left(m+4\frac{\kappa}{a}\right)\frac{1}{2a}\sqrt{N}\frac{1}{V}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{4\pi}}\sum_{s}\chi^{\dagger}_{-\sigma_2}\hat{\sigma}_s\chi_{-\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{s}}a}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_2'}-\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2+2n_3,n_2'}.\end{aligned}\tag{D7}$$

For the helicity nonflip we have

$$\langle 2'|P^-_{whf}|3b\rangle = -\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\,\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\,\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_s\sum_{\mathbf{z}(q)}\sum_{\mathbf{y}(q)}\delta_{\mathbf{z}(q),\mathbf{y}(q)-a\hat{\mathbf{s}}}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_1}+\frac{1}{n_1'}\right)\delta_{n_2,n_2'}\delta_{n_1+2n_3,n_1'}$$
$$-\left(m+4\frac{\kappa}{a}\right)\frac{\kappa}{a}\sqrt{N}\frac{1}{V}\,\frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{4\pi}}\,\delta_{\sigma_2,\sigma_2'}\delta_{\sigma_1,\sigma_1'}\sum_s\sum_{\mathbf{z}(\bar{q})}\sum_{\mathbf{y}(\bar{q})}\delta_{\mathbf{z}(\bar{q}),\mathbf{y}(\bar{q})+\hat{\mathbf{s}}\,a}\frac{1}{\sqrt{n_3}}\left(\frac{1}{n_2'}+\frac{1}{n_2}\right)\delta_{n_1,n_1'}\delta_{n_2+2n_3,n_2'}. \tag{D8}$$

APPENDIX E: SELF-ENERGY COUNTERTERMS

In this appendix we list the self-energy counterterms.

1. Symmetric derivatives case

The counterterm for self-energy for a quark or an antiquark with longitudinal momentum n_1 due to double helicity flip hops is

$$CT_1 = \frac{2}{n_1}\sum_{n_1'=1}^{n_1}\frac{1}{n_1'}\frac{(n_1-n_1')^2}{\mu^2 n_1 n_1' + m^2(n_1-n_1')^2}. \tag{E1}$$

The counterterm for self-energy for a quark or an antiquark with longitudinal momentum n_1 due to double helicity nonflip hops is

$$CT_2 = \frac{2}{n_1}\sum_{n_1'=1}^{n_1}\frac{1}{n_1'}\frac{(n_1+n_1')^2}{\mu^2 n_1 n_1' + m^2(n_1-n_1')^2}. \tag{E2}$$

2. Forward and backward derivative case

In this case we have three types of contributions: (1) the helicity flip acting twice, (2) the helicity nonflip acting twice, and (3) the interference of helicity flip and helicity nonflip hops. The first two are diagonal in helicity space but the last one is off-diagonal in helicity space.

The transition from state $|2\rangle$ to state $|3a\rangle$ and back due to a quark hop gives rise to longitudinal infrared divergence. In this case the counterterm due to double helicity flip is

$$CT_3 = 2\sum_{n_1'=1}^{n_1}\frac{1}{n_1'}\frac{n_1}{\mu^2 n_1 n_1' + m^2(n_1-n_1')^2}. \tag{E3}$$

The counterterm due to double helicity nonflip is the same without the factor of 2. The transition from state $|2\rangle$ to state $|3b\rangle$ and back due to a quark hop does not give rise to longitudinal infrared divergence. Similarly the transition from state $|2\rangle$ to state $|3a\rangle$ and back due to an antiquark hop does not give rise to longitudinal infrared divergence. The transition from state $|2\rangle$ to state $|3b\rangle$ and back due to an antiquark hop gives rise to longitudinal infrared divergence which requires counterterms, the explicit forms of which are the same as in the quark case for the transition from $|2\rangle$ to state $|3a\rangle$. Lastly we consider counterterms for self-energy contributions arising from the interference of helicity flip and helicity nonflip hopping. The counterterms have the same structure as in the case of helicity nonflip transitions accompanied by the following extra factors. Since we have two possibilities, namely helicity flip followed by helicity nonflip and vice versa, and these two contributions are the same, we get a factor of two. We also get a factor $\chi^\dagger_{s'}\hat{\sigma}^\perp\chi_s$ where s (s') is the initial (final) helicity and $\hat{\sigma}^1=\sigma^2$, $\hat{\sigma}^2=-\sigma^1$.

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