B-meson distribution amplitude in QCD

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The *B*-meson distribution amplitude is calculated using QCD sum rules. In particular we obtain an estimate for the integral relevant to exclusive *B* decays $\lambda_B = 460 \pm 110$ MeV at the scale of 1 GeV. A simple QCD-motivated parametrization of the distribution amplitude is suggested.

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I. INTRODUCTION

The *B*-meson distribution amplitude was introduced in [1] as the direct analogue of light-cone distribution amplitudes of light mesons [2–4] in an attempt to describe generic exclusive *B* decays by the contribution of the hard gluon exchange. Since then, considerable effort has been invested in understanding the QCD dynamics of heavy meson decays in the heavy quark limit. The radiative decay $B \to \gamma e \nu$ provides one with the simplest example of such processes [5]. This form factor can be calculated in terms of the *B*-meson distribution amplitude to one-loop accuracy [6] and arguments have been given that the corresponding factorization formula is valid to all orders in the strong coupling [7]. Similar QCD factorization formulas have also been proposed for the related processes $B \to \gamma\gamma$ and $B \to \gamma \ell^+ \ell^-$ [8,9]. For weak decays involving energetic light hadrons in the final states the QCD factorization is more complex since one must isolate the end-point soft contributions in terms of additive contributions. This is a hot topic (see e.g. [10]) and the results have been encouraging although, as has been repeatedly pointed out [11,12], the $1/m_b$ corrections to heavy-to-light exclusive decays are most likely large and require quantitative treatment.

The *B*-meson distribution amplitude plays the central role in all known factorization formulas, but, surprisingly, received relatively little attention in the past. In the present work we use QCD sum rules to present a realistic model for the *B*-meson distribution amplitude, consistent with all QCD constraints. We also take this opportunity to clarify its theoretical status about which there has been some confusion. The approach that we take in this paper is a modification of the classic work by Chernyak and Zhitnitsky on the QCD sum rule analysis of the distribution amplitudes of light mesons [13]. In particular we argue that the relevant matrix element of a bilocal quark-antiquark operator can be calculated by the QCD-corrected expansion at *imaginary* light-cone distances. We find that the nonperturbative corrections remain under control and present quantitative estimates for the distribution amplitude and its first inverse moment which enters decay form factors at the tree level. Our results can be considered as an extension of an earlier QCD sum rule calculation by Grozin and Neubert [14] (see also [15]) with the main difference being that we calculate next-to-leading order (NLO) radiative corrections to the sum rule. This is important since the true analytic structure of the distribution amplitude is seen only at this level.

The presentation is organized as follows. Section II is introductory and contains the necessary definitions. A simple model for the distribution is obtained in Sec. III using QCD perturbation theory and duality. The complete sum rule is constructed in Sec. IV where we discuss the structure of nonperturbative corrections. This section also contains our main results, the summary and conclusions.

II. DEFINITIONS

Following Ref. [16] we define the *B*-meson distribution amplitude as the renormalized matrix element of the bilocal operator built of an effective heavy quark field $h_v(0)$ and a light antiquark $\bar{q}(tn)$ at a lightlike separation:

$$\langle 0|[\bar{q}(tn)\not{n}[tn,0]\Gamma h_v(0)]_R|\bar{B}(v)\rangle = -\frac{i}{2}F(\mu)\,\mathrm{Tr}[\gamma_5\not{n}\Gamma P_+]\Phi_+(t,\mu) \qquad (1)$$

with

$$[tn,0] \equiv \mathrm{P}\exp\left[ig\int_0^1 du\, n_\mu A^\mu(utn)\right]. \qquad (2)$$

Here v_μ is the heavy quark velocity, n_μ is the lightlike vector, $n^2=0$, such that $n\cdot v=1$, $P_+=\frac{1}{2}(1+\not{v})$ is the projector on upper components of the heavy quark spinor, Γ stands for an arbitrary Dirac structure, $|\bar{B}(v)\rangle$ is the $\bar{B}$-meson state in the heavy quark effective theory (HQET) and $F(\mu)$ is the decay constant in HQET, which is related to the physical *B*-meson decay constant to one-loop accuracy as

$$f_B\sqrt{m_B} = F(\mu)\left[1+\frac{C_F\alpha_s}{4\pi}\left(3\ln\frac{m_b}{\mu}-2\right)+\ \ldots\right]. \qquad (3)$$

The notation $[\dots]_R$ in Eq. (1) stands for the renormalization in a minimal subtraction– (MS-)like scheme and μ here and below refers to the modified minimal subtraction ($\overline{\mathrm{MS}}$) normalization scale.

The invariant function $\Phi_+(t,\mu)$ where t is a real number defines what is usually called the leading twist B-meson distribution amplitude in position space, in contrast to the amplitude $\Phi_-(t,\mu)$ which involves a different light-cone projector ($\bar{\not n}$ instead of $\not n$) in between the quarks; here $\bar{n}^2=0,\bar{n}\cdot n=2$. This name is not exact since the translation symmetry of the theory is broken by the presence of the effective heavy quark field and hence neither geometrical nor collinear twist is defined. In the present paper we consider only the distribution $\Phi_+(t,\mu)$ and its Fourier transform

$$\phi_+(k,\mu)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt e^{ikt}\Phi_+(t-i0,\mu),$$

$$\Phi_+(t,\mu)=\int_0^\infty dk e^{-ikt}\phi_+(k,\mu), \tag{4}$$

where in the first equation the integration contour goes below the singularities of $\Phi_+(t,\mu)$ that are located in the upper half plane.

The scale dependence of the distribution amplitude is driven by the renormalization of the corresponding nonlocal operator

$$O_+(t)=\bar{q}(tn)\not n[tn,0]\Gamma h_v(0).$$

The corresponding Z factor was computed in [16] to one-loop order. In momentum space, the result reads

$$O_+^{\mathrm{ren}}(k,\mu)=\int dk' Z_+(k,k';\mu)O_+^{\mathrm{bare}}(k'), \tag{5}$$

where

$$Z_+(k,k';\mu)=\delta(k-k')+\frac{\alpha_s C_F}{4\pi}Z_+^{(1)}(k,k';\mu)+\cdots,$$

$$Z_+^{(1)}(k,k';\mu)=\left(\frac{4}{\hat{\epsilon}^2}+\frac{4}{\hat{\epsilon}}\ln\frac{\mu}{k}-\frac{5}{\hat{\epsilon}}\right)\delta(k-k') -\frac{4}{\hat{\epsilon}}\left[\frac{k}{k'}\frac{\theta(k'-k)}{k'-k}+\frac{\theta(k-k')}{k-k'}\right]_+ \tag{6}$$

with $d=4-\epsilon$ and the standard notation $2/\hat{\epsilon}=2/\epsilon-\gamma_E+\ln 4\pi$. Here $[\dots]_+$ is the usual "plus" distribution.

In order to understand the meaning of this result it is instructive to consider operator renormalization in position space. The one-loop expression corresponding to Eq. (6) is

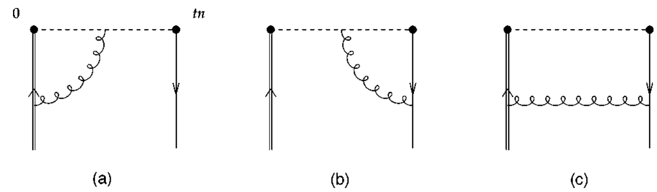

FIG. 1. One-loop renormalization of the nonlocal light-cone operator built of one light and one effective heavy quark field (double line). The dashed line indicates the gluon Wilson line insertion in between the quark fields.

$$O_+^{\mathrm{ren}}(t,\mu)=O_+^{\mathrm{bare}}(t)+\frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2}+\frac{4}{\hat{\epsilon}}\ln(it\mu)\right)O_+^{\mathrm{bare}}(t) -\frac{4}{\hat{\epsilon}}\int_0^1 du\frac{u}{1-u}[O_+^{\mathrm{bare}}(ut)-O_+^{\mathrm{bare}}(t)] -\frac{1}{\hat{\epsilon}}O_+^{\mathrm{bare}}(t)\right\}, \tag{7}$$

where the first two terms in curly brackets correspond to vertex-type corrections shown in Fig. 1a and Fig. 1b, respectively (in Feynman gauge), and the third term takes into account the quark field renormalization: $q^{\mathrm{ren}}=Z_q^{-1/2}q^{\mathrm{bare}}$ and $h_v^{\mathrm{ren}}=Z_h^{-1/2}h_v^{\mathrm{bare}}$ with $Z_q=1-(2/\hat{\epsilon})(\alpha_s C_F/4\pi)$, $Z_h=1+(4/\hat{\epsilon})(\alpha_s C_F/4\pi)$ [17]. The exchange diagram in Fig. 1c is UV finite and does not contribute [14,16].

Note the following property: renormalization of the nonlocal light-cone operator $O_+(t)$ (7) is quasilocal: it only gets mixed with itself and with operators with smaller light-cone separation. In fact the heavy quark vertex correction in Fig. 1a corresponds to a multiplicative (cusp) renormalization in coordinate space [18] while the light quark vertex correction is identical to the similar contribution to the light-quark–antiquark nonlocal operators [19]. For light quarks, this property of quasilocality guarantees existence of the Wilson short distance operator product expansion (OPE) since it implies that the "size" of the operator is not altered by the renormalization. In the present case, however, the local OPE does not exist because of the term $\ln(it\mu)$ which is nonanalytic at $t\to 0$. It is easy to see that this contribution arizes from the term $\sim(\mu t)^\epsilon/\epsilon^2$ in the dimensionally regularized diagram in Fig. 1a so that the answer for this diagram depends on the order of limits $t\to 0$ and $4-d=\epsilon\to 0$. We conclude that renormalization of the nonlocal light-cone operator built of one light and one effective heavy quark field does not commute with the short-distance expansion. In particular

$$[\bar{q}(tn)\not n[tn,0]\Gamma h_v(0)]_R\neq\sum_{p=0}^\infty\frac{t^p}{p!}[\bar{q}(0)(\overleftarrow{D}\cdot n)^p h_v(0)]_R, \tag{8}$$

and the equality does not hold even as an asymptotic expansion. As a consequence, non-negative moments of the B-meson distribution amplitude $\int dk k^p\phi_+(k,\mu)$ for $p=0,1,2,\dots$ are not related to matrix elements of local operators and in fact do not exist: It is easy to see that the logarithmic singularity of the amplitude in position space $\Phi_+(t,\mu)\sim\ln(it)$ for $t\to 0$ implies that the Fourier integral (4) is logarithmically divergent at $k\to\infty$, that is $\phi_+(k,\mu)\sim 1/k$ for $k\gg\mu$, in agreement with the analysis in [16]. The analysis of moments $\int dk k^p \phi_+(k,\mu)$ in [14,20] tacitly assumes a different definition of the B-meson distribution amplitude, such that the nonlocal light-cone operator is defined as the generating function for renormalized local operators on the right-hand side (RHS) of Eq. (8). This implies e.g. that power divergences are subtracted. This is a different object which, most likely, does not satisfy any simple renormalization group equation and has no obvious relation to exclusive B decays.

III. SUM RULES: PERTURBATION THEORY

The aim of the present study is to suggest a realistic model of the B-meson distribution amplitude that would be consistent with all QCD constraints. To this end we evaluate the necessary B-meson matrix elements using the standard QCD sum rule approach [21]. In this section we set up the framework and present intermediate results that include only perturbation theory contributions and the assumption of duality. The complete treatment including nonperturbative corrections is presented in the next section.

To derive the sum rules we consider the following correlation functions in HQET:

$$i\int d^4x e^{-i\omega(vx)}\langle 0|\mathrm{T}\{\bar{q}(0)\Gamma_1 h_v(0)\bar{h}_v(x)\Gamma_2 q(x)\}|0\rangle = -\frac{1}{2}\mathrm{Tr}[\Gamma_1 P_+\Gamma_2]\Pi(\omega) \tag{9}$$

and

$$i\int d^4x e^{-i\omega(vx)}\langle 0|\mathrm{T}\{\bar{q}(tn)\not{n}\Gamma_1[tn,0]h_v(0)\bar{h}_v(x)\Gamma_2 q(x)\}|0\rangle = -\frac{1}{2}\mathrm{Tr}[\not{n}\Gamma_1 P_+\Gamma_2]T(t,\omega). \tag{10}$$

The correlation function $\Pi(\omega)$ has a pole at $\omega=\bar{\Lambda}$ where $\bar{\Lambda}=m_B-m_b$ is the usual HQET parameter, and the residue at this pole is proportional to the HQET decay constant $F(\mu)$:

$$\Pi(\omega)=\frac{1}{2}F^2(\mu)\frac{1}{\bar{\Lambda}-\omega}+\text{higher resonances and continuum.} \tag{11}$$

Similarly,

$$T(t,\omega)=\frac{1}{2}F^2(\mu)\frac{1}{\bar{\Lambda}-\omega}\int_0^\infty dk e^{-ikt}\phi_+(k,\mu)+\ \dots. \tag{12}$$

On the other hand, both correlation functions can be calculated in QCD at negative values of ω of the order of 1 GeV in perturbation theory and taking into account nonperturbative effects induced by vacuum condensates [21]. Matching the two representations one obtains a sum rule. There are two technical details: First, one makes an assumption that contributions of the continuum and of higher resonances can be taken into account by the restriction to the so-called duality region $0<s<\omega_0$ in the dispersion representation for the correlation functions, e.g.

$$\Pi(w)=\int_0^\infty \frac{ds}{s-\omega}\rho_\Pi(s)\to\int_0^{\omega_0}\frac{ds}{s-\omega}\rho_\Pi(s), \tag{13}$$

where $\rho_\Pi(s)$ is the corresponding spectral density. The numerical value for the parameter ω_0 (called the continuum threshold) is usually taken to be in the interval 0.8–1.0 GeV [17,22–24]. Second, one makes the so-called Borel transformation

$$\int_0^{\omega_0}\frac{ds}{s-\omega}\rho_\Pi(s)\to\int_0^{\omega_0}ds e^{-s/M}\rho_\Pi(s) \tag{14}$$

introducing the variable M (Borel parameter) instead of the energy ω in order to suppress higher-order nonperturbative corrections and minimize the dependence on the continuum model. The resulting sum rule for the correlation function $\Pi(\omega)$ is well known [22,23,25]:

$$\begin{aligned}\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}=&\frac{N_c}{2\pi^2}\int_0^{\omega_0}ds s^2 e^{-s/M}\\&\times\left[1+\frac{\alpha_s}{\pi}\left(\frac{17}{3}+\frac{4\pi^2}{9}-2\ln\frac{2s}{\mu}\right)\right]\\&-\frac{1}{2}\langle\bar{q}q\rangle\left[1+\frac{2\alpha_s}{\pi}-\frac{m_0^2}{16M^2}\right].\end{aligned} \tag{15}$$

Here $\alpha_s=\alpha_s(\mu)$, $\langle\bar{q}q\rangle\simeq-(240\text{ MeV})^3$ is the quark condensate and m_0^2 is the ratio of the quark-gluon and quark condensates $m_0^2=\langle\bar{q}g(\sigma G)q\rangle/\langle\bar{q}q\rangle\simeq 0.8\text{ GeV}^2$. The given numbers correspond to the renormalization scale $\mu=1$ GeV. With the choice $\bar{\Lambda}=0.4$–0.5 GeV and $\omega_0=0.8$–1.0 GeV the sum rule in Eq. (15) is satisfied for a wide range of values of the Borel parameter 0.3 GeV$<M<\infty$ and is used [22,23] to determine the B-meson decay constant $F(\mu)$ in the heavy quark limit. In the numerical estimates in this paper we will take the "window" 0.3 GeV $<M<$0.6 GeV in which the matching is done [24] and use the value $\alpha_s(1\text{ GeV})=0.5$ ($\Lambda_{\mathrm{QCD}}^{(3)\mathrm{NLO}}\simeq 360$ MeV) which is consistent with the world average.

Our task in this work is to derive the similar sum rule for the correlation function $T(t,\omega)$ defined in Eq. (10). The perturbative contributions are shown in Fig. 2. The corresponding sum rule reads, so far without nonperturbative corrections:

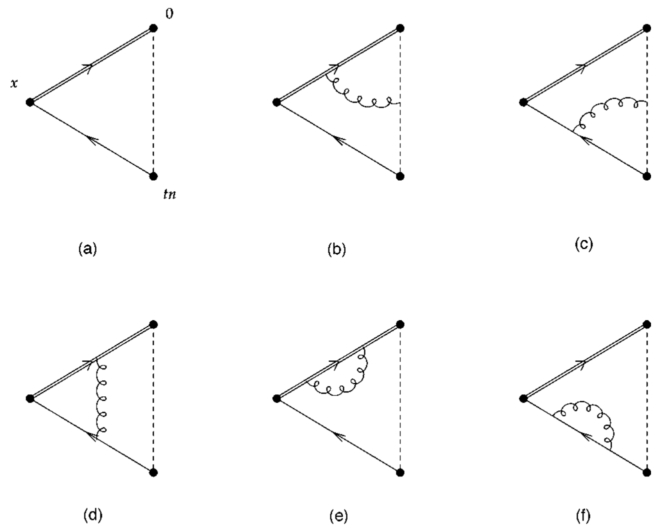

FIG. 2. Correlation function (10) in QCD perturbation theory to first order.

$$\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}\phi_+(k,\mu)$$
$$=k\,\theta(2\omega_0-k)\left[\int_{k/2}^{\omega_0}ds\,e^{-s/M}\rho_<(s,k,\mu)+\int_0^{k/2}ds\,e^{-s/M}\rho_>(s,k,\mu)\right]$$
$$+k\,\theta(k-2\omega_0)\int_0^{\omega_0}ds\,e^{-s/M}\rho_>(s,k,\mu) \qquad (16)$$

where

$$\rho_<(s,k,\mu)=\frac{N_c}{4\pi^2}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[\frac{7}{2}+\frac{7\pi^2}{24}-\ln^2\frac{k}{\mu}-\frac{5}{2}\ln(x-1)-(x-1)\ln(x-1)-\frac{1}{2}\ln^2(x-1)-2\ln\frac{k}{\mu}[1+\ln(x-1)]+x\ln x+\mathrm{Li}_2\left(\frac{1}{1-x}\right)\right]\right\},$$

$$\rho_>(s,k,\mu)=\frac{\alpha_s C_F N_c}{8\pi^3}\left[-x+\ln(1-x)-2(1-x)\ln(1-x)+2\ln^2(1-x)+2\ln\frac{k}{\mu}[x+\ln(1-x)]\right]. \qquad (17)$$

Here $\mathrm{Li}_2(x)$ is the Euler dilogarithm function and we used the shorthand notation $x=2s/k$.

Neglecting α_s corrections for a moment, one gets the simple expression

$$\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}\phi_+(k,\mu)=\frac{N_c}{4\pi^2}\theta(2\omega_0-k)k\int_{k/2}^{\omega_0}ds\,e^{-s/M}. \qquad (18)$$

In the local duality limit $M\to\infty$ using the sum rule expression for $F(\mu)$ [Eq. (15)] with the same accuracy, $1/2F^2(\mu)\simeq(N_c/6\pi^2)\omega_0^3$, one obtains

$$\phi_+(k)^{\mathrm{LD}}=\frac{3}{4\omega_0^3}\theta(2\omega_0-k)k(2\omega_0-k) \qquad (19)$$

which resembles the asymptotic light-cone distribution amplitude of light mesons if rewritten in terms of the scaling variable $\xi=k/(2\omega_0)$. For finite values of the Borel parameter M the B-meson distribution amplitude gets skewed toward smaller values of the momentum but qualitatively remains the same; see Fig. 3. Note that it has finite support $k<2\omega_0$ and can be interpreted as the probability amplitude to find the light quark (on shell) in the B meson with momentum k.

Beyond the Born approximation this simple parton-model interpretation is lost since the distribution amplitude develops a high-momentum "tail" with $k>2\omega_0$ and in this region cannot be thought of as a probability amplitude for the two-particle state on the mass shell. The $\mathcal{O}(\alpha_s)$ radiative correction turns out to be very large ($\sim$100% of the Born term) but cancels to a large extent against the similar large radiative correction to $F(\mu)$ [22,25].

The numerical results for two values of the Borel parameter $M=0.3$ GeV and $M=0.6$ GeV are shown in Fig. 3. We choose $\omega_0=1$ GeV for this plot and substitute the coupling $F^2(\mu)$ appearing on the LHS of the sum rule (16) by the sum rule (15) to the same accuracy, i.e. neglecting nonperturbative corrections. In this way the dependence on $\bar{\Lambda}$ cancels out and the sensitivity to other parameters (ω_0 and M) is strongly reduced. Indeed, it is seen in Fig. 3 that dependence on the Borel parameter is rather mild. Note that for large k the distribution amplitude becomes negative. The asymptotic behavior is

$$\phi_+(k)\sim k \quad \text{for} \quad k\to 0,$$
$$\phi_+(k)\sim-\frac{1}{k}\ln(k/\mu) \quad \text{for} \quad k\gg\mu, \qquad (20)$$

in agreement with [16]. Also the scale dependence of the distribution amplitude extracted from the sum rule (16) agrees with [16]. All results are shown for $\mu=1$ GeV.

Of particular interest for the QCD description of B decays is the value of the first negative moment

$$\lambda_B^{-1}(\mu)=\int_0^\infty\frac{dk}{k}\phi_+(k,\mu). \qquad (21)$$

We obtain from the sum rules

$$\lambda_B^{-1}=1.49-1.83\ \mathrm{GeV}^{-1} \quad \text{for} \quad \omega_0=1.0\ \mathrm{GeV},$$

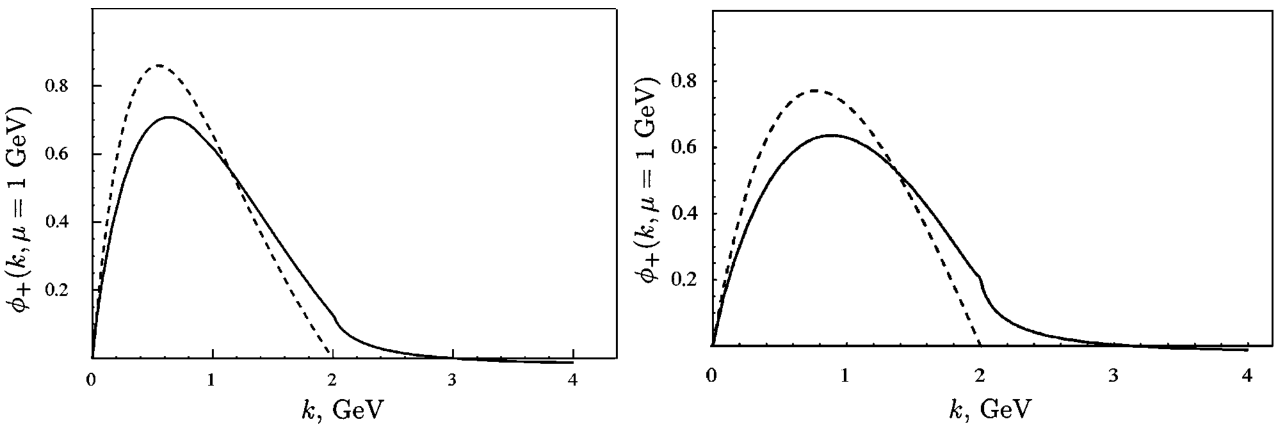

FIG. 3. *B*-meson distribution amplitude $\phi_+(k,\mu=1\text{ GeV})$ calculated from the sum rule (16) in QCD perturbation theory to leading order (dashed curves) and next-to-leading order (solid curves) for the continuum threshold $\omega_0=1$ GeV and two values of the Borel parameter $M=0.3$ GeV (left panel) and $M=0.6$ GeV (right panel). The value of the decay constant $F(\mu)$ appearing on the LHS of the sum rule is substituted by the corresponding sum rule (15) with the appropriate accuracy (LO or NLO) and neglecting the condensate contributions.

$$\lambda_B^{-1}=1.79-2.08\ \text{GeV}^{-1}\quad\text{for}\ \omega_0=0.8\ \text{GeV},\tag{22}$$

where the lower value corresponds to $M=0.6$ GeV and the higher one to $M=0.3$ GeV for each choice of ω_0. Notice that λ_B^{-1} decreases as M increases and in the local duality limit we obtain

$$\lambda_B^{-1}(\mu=2\omega_0)^{\rm LD}=\frac{3}{2\omega_0}\left[1-\frac{\alpha_s(2\omega_0)}{\pi}\left(\frac{5}{3}+\frac{5\pi^2}{36}\right)\right],\tag{23}$$

where it is taken into account that in the limit $M\to\infty$ the sum rules effectively become normalized at the scale $\mu=2\omega_0$ because of subtraction of the continuum from the running coupling; cf. [26]. To avoid misunderstanding we recall that all results of this section correspond to the sum rules in QCD perturbation theory and the given numbers will be superseded by those in the next section where we consider the nonperturbative corrections.

IV. SUM RULES: NONPERTURBATIVE CORRECTIONS

The primary source of nonperturbative corrections to the sum rules in HQET is provided by the quark condensate. The corresponding diagrams (leading and next-to-leading order) are shown in Fig. 4. The leading-order contribution in Fig. 4a is simply

$$T^{\langle\bar qq\rangle}(t,\omega)=\frac{\langle\bar qq\rangle}{2\omega}.\tag{24}$$

It does not depend on the quark-antiquark separation and gives rise to the δ-function type contribution to the RHS of the sum rule in Eq. (16):

$$\ldots-\frac{1}{2}\langle\bar qq\rangle\,\delta(k).\tag{25}$$

Condensates of higher dimension produce even more singular terms; the expansion goes in derivatives of the δ function at $k=0$. This is a well-known problem which is familiar

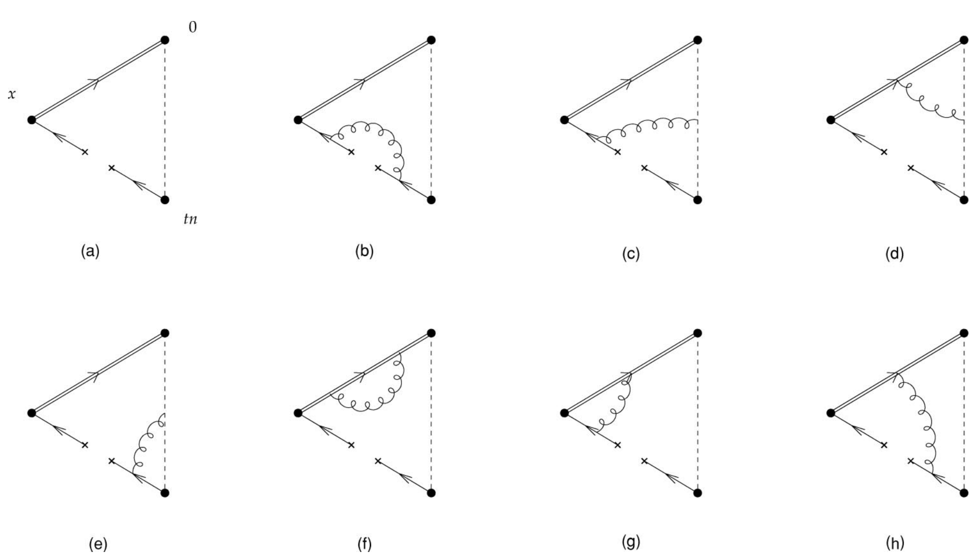

FIG. 4. Quark condensate contribution to the correlation function (10).

from the QCD sum rule studies of light-cone distribution amplitudes of light mesons [13] and nucleon parton distributions [27,28]: The short-distance OPE which is the the basis of the Shifman-Vainstein-Zakharov approach is inadequate for a calculation of distribution functions point by point in the momentum fraction space. As a consequence, QCD sum rules cannot be used for a direct calculation of distribution amplitudes (unless they are supplemented by additional assumptions) but rather provide constraints which have to be implemented within QCD-motivated parameterizations (models) of the distributions, consistent with perturbative QCD. For a model to be self-consistent, there are three conditions:

The end-point behavior of the distributions has to be consistent with QCD.

The model has to be closed under the QCD evolution, i.e. calculation of the scale dependence has to be possible and not involve nonperturbative parameters other than those specified by the model at the reference scale.

The model has to involve a minimum possible number of nonperturbative parameters.

The Chernyak-Zhitnitsky models of light-cone distribution amplitudes of light mesons give the classical example of such an approach. In this case one expands the distribution amplitude in a series over orthogonal polynomials, e.g. for the pion

$$\phi_\pi(\xi,\mu)=6\xi(1-\xi)\sum_{p=0,2,\dots}^{\infty}\varphi_p(\mu)C_p^{3/2}(2\xi-1), \tag{26}$$

so that coefficients in this expansion correspond to (Gegenbauer) moments of $\phi_\pi(\xi)$, and defines a *model* by truncating this expansion at a certain $p=p_{\rm max}$. The first $p_{\rm max}$ coefficients are then estimated using QCD sum rules. (In practice one takes $p_{\rm max}=2$ since estimates of higher-order coefficients turn out to be unreliable.) The model satisfies all the above criteria since the correct end-point behavior is built in by construction and higher-order coefficients can get mixed only with lower-order coefficients but not vice versa; it follows that the set of coefficients $\{\varphi_0,\varphi_2,\dots,\varphi_{k_{\rm max}}\}$ is closed under renormalization [29] and the distribution amplitude $\phi_\pi(\xi,\mu)$ can be calculated at arbitrary scale from its model at $\mu=\mu_0$. It indeed involves a minimum number of parameters, each of which has a clear meaning in QCD as the matrix element of a certain local operator and can eventually be calculated e.g. on the lattice.

In contrast to Eq. (26), the *B*-meson distribution amplitude cannot be written as a *sum* of independent terms that have autonomous QCD evolution but rather is given by the *integral* in the complex moment plane [16]. This feature reminds one of the evolution of parton distributions in the deep-inelastic inclusive lepton-hadron scattering, but in contrast to the latter case one cannot obtain complex moments of the *B*-meson distribution amplitude by analytic continuation from the set of real integers [as we mentioned in Sec. II, every non-negative moment of $\phi_+(k,\mu)$ diverges]. As a result, one necessarily has a continuous rather than discrete set of nonperturbative parameters.

One option [14,28,30] is to parametrize the *B*-meson distribution amplitude by the matrix element of the bilocal operator in Eq. (1) at imaginary light-cone separation

$$t=-i\tau,\quad \varphi_+(\tau,\mu)=\Phi_+(-i\tau,\mu). \tag{27}$$

Obviously

$$\varphi_+(\tau,\mu)=\int_0^\infty dk e^{-\tau k}\phi_+(k,\mu) \tag{28}$$

and the parameter λ_B is given by the simple integral

$$\lambda_B^{-1}(\mu)=\int_0^\infty d\tau\varphi_+(\tau,\mu). \tag{29}$$

The purpose of going over to imaginary light-cone times (distances) is similar to that of the usual Wick rotation: In this way the oscillating exponents corresponding to the light-cone time dependence of intermediate states propagating along the light cone are converted to falling exponents suppressed by the energy of the state, where the light-cone quantization is implied. Simultaneously, the normalization scale μ acquires the physical meaning of the cutoff in energy of the intermediate states. Note that the renormalization of $\varphi_+(\tau,\mu)$ involves only the distribution at smaller light-cone separations [cf. Eq. (7)]. This implies that knowledge of $\varphi_+(\tau,\mu)$ at small distances up to $\tau<\tau_{\rm max}$ is sufficient to calculate its scale dependence in the same distance range, in agreement with the self-consistency criterion formulated above.

On the other hand, it is easy to understand that the function $\varphi_+(\tau,\mu)$ can be calculated at small τ using OPE; expansion in vacuum condensates of increasing dimension corresponds to the Laurent expansion of $\varphi_+(\tau,\mu)$ in powers of τ, which is modified by calculable perturbative corrections. The condensate expansion seems to be under control up to distances of order $\tau\sim1$ GeV^{-1} (that is, of order 0.2 fm). At larger distances the OPE diverges and one has to either truncate $\varphi_+(\tau,\mu)$ at a certain $\tau_{\rm max}$ or rely on a certain model for the large τ behavior. Provided that the nonperturbative corrections decrease sufficiently fast for large τ one can hope that the model assumptions do not lead to a large uncertainty in the overall result.

To illustrate this construction, we have calculated the quark condensate contribution including the α_s correction (see Fig. 4), the contribution of the gluon condensate (Fig. 5), and the contribution of the mixed condensate $\langle\bar q\sigma gGq\rangle\simeq m_0^2\langle\bar qq\rangle$ which is obtained as the expansion of the diagram in Fig. 4a in the background gluon field. The resulting sum rule in which we have also included the perturbative contribution of Fig. 2 reads

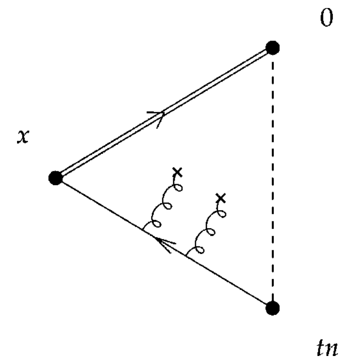

FIG. 5. Gluon condensate contribution to the correlation function (10). Only this diagram contributes in the Fock-Schwinger gauge.

$$
\begin{aligned}
&\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}\varphi_+(\tau,\mu)\\
&\quad=\int_0^{\omega_0}ds\,e^{-s/M}\rho_{\rm pert}(s,\tau,\mu)\\
&\qquad-\frac{1}{2}\langle\bar{q}q\rangle\left\{1+\frac{\alpha_sC_F}{2\pi}\left[3-\frac{5\pi^2}{24}-\ln^2(\tau\mu e^{\gamma_E})\right.\right.\\
&\qquad\left.\left.-\ln(\tau\mu e^{\gamma_E})-\ln(1+2\tau M)-{\rm Li}_2(-2\tau M)\right]\right\}\\
&\qquad+\frac{1}{48}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{M\tau^2}{(1+2\tau M)^2}+\frac{1}{32}\frac{m_0^2}{M^2}\langle\bar{q}q\rangle\\
&\qquad\times(1+2\tau M),
\end{aligned}
\tag{30}
$$

where the perturbative spectral density can be read off Eq. (16). The contribution of the gluon condensate turns out to be very small and will be neglected in what follows. We further note that the sum of the diagram in Fig. 4d and one-half of the heavy quark self-energy correction in Fig. 4f defines the universal renormalization factor of the Wilson line built of the lightlike segment of length $-i\tau$ and the timelike segment of length $-i/M$. It can be shown that the corresponding contributions $\sim(\alpha_sC_F)^n$ exponentiate to all orders [31] and produce a Sudakov-like exponential suppression factor

$$
\begin{aligned}
S(\tau,M,\mu)=\exp&\left\{-\frac{\alpha_sC_F}{2\pi}\left[\ln^2(\tau\mu e^{\gamma_E})+\frac{5\pi^2}{24}-1-\ln\frac{\mu e^{\gamma_E}}{2M}\right.\right.\\
&\left.\left.+{\rm Li}_2(-2\tau M)\right]\right\}
\end{aligned}
\tag{31}
$$

which is the same for the quark and the quark-gluon condensate. Note that ${\rm Li}_2(-2\tau M)\sim-\frac{1}{2}\ln^2(2\tau M)$ for $\tau\gg 1/M$. We end up with an improved sum rule

$$
\begin{aligned}
&\frac{1}{2}F^2(\mu)e^{-\bar{\Lambda}/M}\varphi_+(\tau,\mu)\\
&\quad=\int_0^{\omega_0}ds\,e^{-s/M}\rho_{\rm pert}(s,\tau,\mu)-\frac{1}{2}\langle\bar{q}q\rangle S(\tau,M,\mu)\\
&\qquad\times\left\{1+\frac{\alpha_sC_F}{2\pi}\left[2-\ln(\tau\mu e^{\gamma_E})-\ln\frac{\mu e^{\gamma_E}}{2M}-\ln(1+2\tau M)\right]\right.\\
&\qquad\left.-\frac{1}{16}\frac{m_0^2}{M^2}(1+2\tau M)\right\},
\end{aligned}
\tag{32}
$$

in which the double-logarithmic corrections to the quark and quark-gluon chiral condensates are resummed. We do not attempt a similar resummation in the perturbative contribution since its effect is negligible compared to the $1/\tau^2$ falloff inherited from the Born term.

The perturbative and the nonperturbative contributions to the sum rule result (32) for $\varphi_+(\tau,\mu)$ are shown separately as a function of distance τ in Fig. 6 ($\omega_0=1$ GeV) and Fig. 7 ($\omega_0=0.8$ GeV) for two different values of the Borel parameter $M=0.3$ GeV and $M=0.6$ GeV. Note that at small distances the nonperturbative corrections are significantly smaller than the perturbative contribution. The nonperturbative correction turns to zero at a certain value of τ as a result

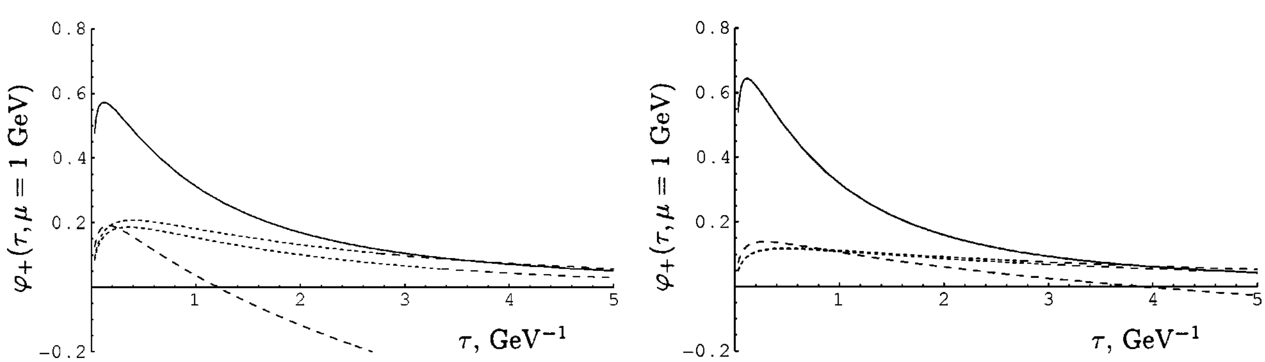

FIG. 6. Perturbative (solid curves) and nonperturbative (long dashes) contributions to the B-meson distribution amplitude $\varphi_+(\tau,\mu=1\text{ GeV})$ calculated from the sum rule (32) to NLO accuracy. In addition, the nonlocal condensate models (35), (36) of resummed nonperturbative contributions to the sum rule, cf. Eq. (37), are shown by short dashes. The continuum threshold is chosen to be $\omega_0=1$ GeV and two values of the Borel parameter are used: $M=0.3$ GeV (left panel) and $M=0.6$ GeV (right panel). The value of the decay constant $F(\mu)$ appearing on the LHS of Eq. (32) is substituted by the corresponding sum rule (15).

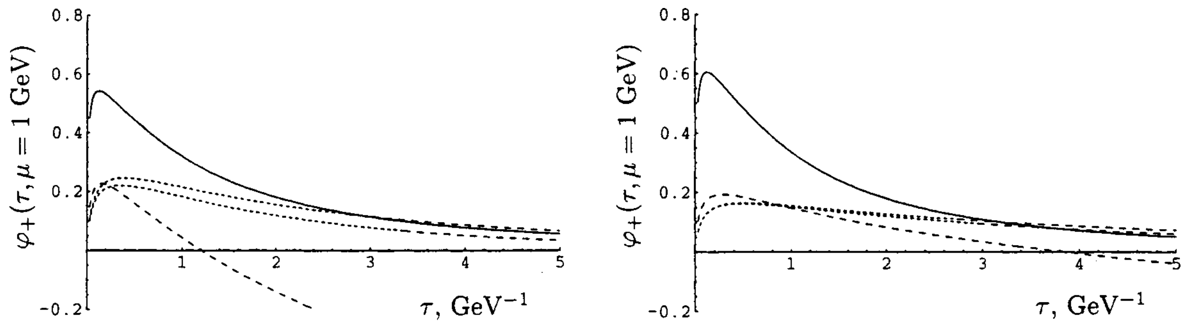

FIG. 7. Same as in Fig. 6, with a different value of the continuum threshold $\omega_0=0.8$ GeV.

of the cancellation between the quark condensate contribution ($\sim$ const) and that of the mixed condensate ($\sim\tau$). This cancellation of course cannot be taken seriously and indicates only that the OPE breaks down since the hierarchy of contributions is lost. We conclude that the classical QCD sum rule is valid only up to light-cone distances of order 1–3 GeV^{-1}, depending on the value of the Borel parameter. A rough estimate for the nonperturbative contribution to λ_B^{-1} in the strict OPE-based approach is, therefore, given by the integral over the region of small τ where the correction is still positive, that is up to the crossing point with the zero axis. In order to get an estimate of a possible nonperturbative contribution from large distances we use the concept of a nonlocal quark condensate introduced in [32] and later used rather extensively in QCD sum rule calculations of the distribution amplitudes of light mesons by the Dubna group [33,34]. The same approach was taken up in [14].

The nonlocal quark condensate presents a model for a partial resummation of the OPE to all orders in terms of the vacuum expectation value of the single nonlocal operator

$$\langle 0|\bar{q}(x)[x,0]q(0)|0\rangle=\langle\bar{q}q\rangle\int_0^\infty d\nu e^{\nu x^2/4}f(\nu). \tag{33}$$

The first two moments of $f(\nu)$ are fixed by the OPE:

$$\int_0^\infty d\nu f(\nu)=1,$$

$$\int_0^\infty d\nu\nu f(\nu)=\frac{1}{4}m_0^2, \tag{34}$$

and in addition one requires that the correlation function (33) decreases exponentially at large Euclidian separations $x^2\to-\infty$. The two simplest choices are

$$\text{model I:}\quad f(\nu)=\delta(\nu-m_0^2/4)\quad [32] \tag{35}$$

corresponding to the Gaussian large-distance behavior $\sim\exp[-|x^2|m_0^2/16]$ and

$$\text{model II:}\quad f(\nu)=\frac{\lambda^{p-2}}{\Gamma(p-2)}\nu^{1-p}e^{-\lambda/\nu},$$

$$p=3+\frac{4\lambda}{m_0^2}\quad [28] \tag{36}$$

corresponding to $\langle 0|\bar{q}(x)q(0)|0\rangle\sim\exp[-\lambda\sqrt{-x^2}]$. Here λ is a parameter with the physical meaning of the vacuum quark correlation length. In this work we take $\lambda=400$ MeV as a representative number; cf. [34,35]. We will see that sensitivity of the sum rules to the shape of $f(\nu)$ is in fact small; the major shortcoming of this approach is rather that other condensates (e.g. the nonlocal quark-antiquark-gluon condensate) are not included and there exists no parameter that would justify such a truncation.

Nonlocality of the quark condensate is easy to implement within our sum rules and it amounts to a simple substitution in Eq. (32) (cf. [14]):

$$-\frac{1}{2}\langle\bar{q}q\rangle S(\tau,M,\mu)\left\{1+\mathcal{O}(\alpha_s)-\frac{m_0^2}{16M^2}(1+2\tau M)\right\}$$
$$\to-\frac{1}{2}\langle\bar{q}q\rangle S(\tau,M,\mu)\int_0^\infty d\nu f(\nu)e^{-\nu(1+2\tau M)/(4M^2)}. \tag{37}$$

One sees that an unphysical rise of power corrections for large τ is replaced by an exponential falloff so that with this substitution the sum rule (32) can be extended to arbitrarily large light-cone distances. Note that the mixed condensate contribution is now included as a part of the nonlocal condensate and we are forced to neglect the α_s correction to the (local) quark condensate in Eq. (32) apart from the universal Sudakov exponent. The size of this correction does not exceed 10–15% for τ in the few GeV^{-1} region so that its sacrifice is numerically not important. In the same region, the Sudakov perturbative suppression appears to be comparable to the nonperturbative suppression incorporated in the nonlocal condensates which is due to a finite vacuum quarks correlation length. The results are shown in Fig. 6 and Fig. 7 by short dashes; the lower and the higher of the curves correspond to the choices in Eq. (35) and Eq. (36), respectively. The corresponding results for λ_B^{-1} are, for $\mu=1$ GeV,

$$\omega_0=1\ \text{GeV},\quad M=0.6\ \text{GeV}:\quad \lambda_B^{-1}=1.23+\begin{cases}0.26\\0.60\\0.83\end{cases}=1.95\pm0.23\ \text{GeV}^{-1},$$

$$\omega_0=1\ \text{GeV},\quad M=0.3\ \text{GeV}:\quad \lambda_B^{-1}=1.32+\begin{cases}0.13\\0.54\\0.88\end{cases}=2.03\pm0.29\ \text{GeV}^{-1},$$

$$\omega_0=0.8\ \text{GeV},\quad M=0.6\ \text{GeV}:\quad \lambda_B^{-1}=1.36+\begin{cases}0.35\\0.84\\1.16\end{cases}=2.36\pm0.33\ \text{GeV}^{-1},$$

$$\omega_0=0.8\ \text{GeV},\quad M=0.3\ \text{GeV}:\quad \lambda_B^{-1}=1.39+\begin{cases}0.15\\0.64\\1.05\end{cases}=2.24\pm0.35\ \text{GeV}^{-1}, \tag{38}$$

where the first number gives the perturbative contribution [the difference from Eq. (22) is due to the different value used for $F(\mu)$] and the three numbers under the brace correspond to three different estimates for the nonperturbative contribution: (1) quark and mixed condensate contribution to Eq. (32) restricted to the positivity region, (2) nonlocal condensate model I [Eq. (35)] and (3) nonlocal condensate model II [Eq. (36)]. The first (upper) number should be considered as an estimate of the nonperturbative correction from below while the difference between the two lower ones characterizes the uncertainty in the choice of the parametrization of the nonlocal condensate. We take the average between the two models as our central value, and one half of the difference between this central value and the first (upper) number, coming from local OPE, as an estimate of the overall uncertainty of the result. In other words, we ascribe 50% uncertainty to the extrapolation of the nonperturbative contribution to large distances as suggested by the nonlocal condensate model, which is rather conservative; cf. [33]. From the numbers in Eq. (38) we obtain the final result

$$\lambda_B^{-1}(\mu=1\ \text{GeV})=2.15\pm0.5\ \text{GeV}^{-1} \tag{39}$$

or

$$\lambda_B(\mu=1\ \text{GeV})=460\pm110\ \text{MeV}. \tag{40}$$

Several other models of the *B* meson distribution amplitude have been proposed in the literature. Our value in Eq. (40) can be compared with

$$\lambda_B=350\pm150\ \text{MeV}\ [6,36],$$

$$\lambda_B=\bar{\Lambda}\sim400\text{–}500\ \text{MeV}\ [20],$$

$$\lambda_B=473\ \text{MeV}\ [37],$$

$$\lambda_B\simeq600\ \text{MeV}\ [15]. \tag{41}$$

One observes a good overall agreement which is gratifying given the very different approaches involved.

As follows from Eqs. (5) and (6), the scale dependence of λ_B also involves the first logarithmic moment of the distribution amplitude [12,16]

$$\lambda_B^{-1}(\mu)=\left[1+\frac{\alpha_s C_F}{2\pi}\ln\frac{\mu}{\mu_0}\right]\lambda_B^{-1}(\mu_0) -\frac{\alpha_s C_F}{\pi}\ln\frac{\mu}{\mu_0}\int_0^\infty\frac{dk}{k}\ln\frac{\mu_0}{k}\,\phi_+(k,\mu_0), \tag{42}$$

where $(\alpha_s C_F/\pi)\ln(\mu/\mu_0)<1$. We define

$$\sigma_B(\mu)=\lambda_B(\mu)\int_0^\infty\frac{dk}{k}\ln\frac{\mu}{k}\,\phi_+(k,\mu) \tag{43}$$

$$=\lambda_B(\mu)\int_0^\infty d\tau\ln(\tau\mu e^{\gamma_E})\,\varphi_+(\tau,\mu)$$

and calculate $\sigma_B(1\ \text{GeV})$ from the QCD sum rule (32) repeating the same procedure as explained above for λ_B. Without going into details we simply quote the final result:

$$\sigma_B(\mu=1\ \text{GeV})=1.4\pm0.4. \tag{44}$$

Note that $\sigma_B(\mu)$ defines the average value of $\ln\mu/k$ in the integral for the first inverse moment λ_B^{-1}, so that the number in Eq. (44) implies that main contribution to λ_B^{-1} comes from momenta ~250 MeV. With this value for σ_B, the two contributions $\mathcal{O}(\alpha_s)$ in Eq. (42) tend to cancel each other to a large extent, so that the remaining scale dependence of λ_B^{-1} is weak.

A simple model of the distribution amplitude $\phi_+(k,\mu)$ with given values of the parameters λ_B and σ_B and correct asymptotic behavior can be chosen as

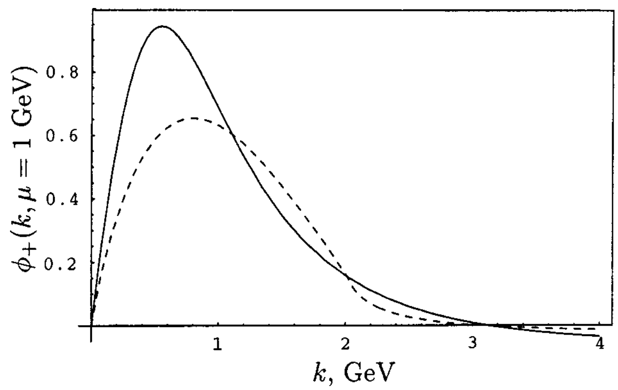

FIG. 8. A QCD model for the B-meson distribution amplitude (45) (solid curve) compared with the perturbative sum rule prediction (16) with $M=0.45$ GeV, $\omega_0=1$ GeV (dashed curve).

$$\phi_+(k,\mu=1\ \ \mathrm{GeV})=\frac{4\lambda_B^{-1}}{\pi}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2(\sigma_B-1)}{\pi^2}\ln k\right] \tag{45}$$

(k in units of GeV). Using the values of λ_B and σ_B in Eqs. (39),(44) one obtains the distribution shown in Fig. 8 by the solid curve. For comparison, we also show in this plot a typical distribution amplitude obtained from QCD sum rules in perturbation theory (16) in Sec. III; cf. Fig. 3. The effect of nonperturbative corrections is to shift the distribution toward softer momenta, which is natural. One minor drawback of such a parametrization is that the set of parameters λ_B and σ_B is not closed under renormalization. In view of a very limited range of scales that are interesting for B-decay phenomenology this seems to be not a problem, however.

To summarize, in this paper we have derived QCD sum rules for the B-meson distribution amplitude (1) and, in particular, obtained an estimate of its first inverse moment λ_B^{-1} (39) and the parameter σ_B (43) that characterizes the shape of the distribution; see Eq. (44). A simple model is suggested [Eq. (45)] that incorporates all existing constraints. We believe that our estimates are interesting for the studies of the heavy quark limit in exclusive B decays and can be used in a broad context. Concrete applications go beyond the aim of this work.

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[1] A. Szczepaniak, E.M. Henley, and S.J. Brodsky, Phys. Lett. B **243**, 287 (1990).
[2] V.L. Chernyak and A.R. Zhitnitsky, JETP Lett. **25**, 510 (1977); Yad. Fiz. **31**, 1053 (1980); V.L. Chernyak, V.G. Serbo, and A.R. Zhitnitsky, JETP Lett. **26**, 594 (1977); Sov. J. Nucl. Phys. **31**, 552 (1980).
[3] G.P. Lepage and S.J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. Lett. **43**, 545 (1979); **43**, 1625(E) (1979); Phys. Rev. D **22**, 2157 (1980); S.J. Brodsky, G.P. Lepage, and A.A. Zaidi, *ibid.* **23**, 1152 (1981).
[4] A.V. Efremov and A.V. Radyushkin, Phys. Lett. **94B**, 245 (1980); Teor. Mat. Fiz. **42**, 147 (1980).
[5] G.P. Korchemsky, D. Pirjol, and T.M. Yan, Phys. Rev. D **61**, 114510 (2000).
[6] S. Descotes-Genon and C.T. Sachrajda, Nucl. Phys. **B650**, 356 (2003).
[7] E. Lunghi, D. Pirjol, and D. Wyler, Nucl. Phys. **B649**, 349 (2003); S.W. Bosch, R.J. Hill, B.O. Lange, and M. Neubert, Phys. Rev. D **67**, 094014 (2003).
[8] S.W. Bosch and G. Buchalla, J. High Energy Phys. **08**, 054 (2002).
[9] S. Descotes-Genon and C.T. Sachrajda, Phys. Lett. B **557**, 213 (2003).
[10] E. Bagan, P. Ball, and V.M. Braun, Phys. Lett. B **417**, 154 (1998); J. Charles, A. Le Yaouanc, L. Oliver, O. Pene, and J.C. Raynal, Phys. Rev. D **60**, 014001 (1999); M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2001); M. Beneke, A.P. Chapovsky, M. Diehl, and T. Feldmann, *ibid.* **B643**, 431 (2002); C.W. Bauer, D. Pirjol, and I.W. Stewart, Phys. Rev. D **67**, 071502(R) (2003); M. Beneke and T. Feldmann, hep-ph/0308303.
[11] A. Ali, V.M. Braun, and H. Simma, Z. Phys. C **63**, 437 (1994).
[12] P. Ball, hep-ph/0308249.
[13] V.L. Chernyak and A.R. Zhitnitsky, Nucl. Phys. **B201**, 492 (1982); Phys. Rep. **112**, 173 (1984).
[14] A.G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
[15] P. Ball and E. Kou, J. High Energy Phys. **04**, 029 (2003).
[16] B.O. Lange and M. Neubert, Phys. Rev. Lett. **91**, 102001 (2003).
[17] M. Neubert, Phys. Rep. **245**, 259 (1994).
[18] I.A. Korchemskaya and G.P. Korchemsky, Phys. Lett. B **287**, 169 (1992); G.P. Korchemsky and G. Marchesini, Nucl. Phys. **B406**, 225 (1993).
[19] I.I. Balitsky and V.M. Braun, Nucl. Phys. **B311**, 541 (1989).
[20] H. Kawamura, J. Kodaira, C.F. Qiao, and K. Tanaka, Phys. Lett. B **523**, 111 (2001); **536**, 344(E) (2002).
[21] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).
[22] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **278**, 457 (1992).
[23] M. Neubert, Phys. Rev. D **45**, 2451 (1992).
[24] P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994).
[25] D.J. Broadhurst and A.G. Grozin, Phys. Lett. B **274**, 421 (1992).
[26] I.I. Balitsky, A.V. Kolesnichenko, and A.V. Yung, Yad. Fiz. **41**, 282 (1985).
[27] V.M. Belyaev and B.L. Ioffe, Nucl. Phys. **B310**, 548 (1988); Int. J. Mod. Phys. A **6**, 1533 (1991); B.L. Ioffe and A. Khod-

jamirian, Phys. Rev. D **51**, 3373 (1995).

[28] V.M. Braun, P. Gornicki, and L. Mankiewicz, Phys. Rev. D **51**, 6036 (1995).

[29] The coefficients in the Gegenbauer expansion are renormalized multiplicatively to leading order because of conformal symmetry; this property is, however, not essential for our argument.

[30] A.G. Grozin and G.P. Korchemsky, Phys. Rev. D **53**, 1378 (1996).

[31] J.G.M. Gatheral, Nucl. Phys. **B133**, 90 (1983); J. Frenkel and J.C. Taylor, *ibid.* **B246**, 231 (1984).

[32] S.V. Mikhailov and A.V. Radyushkin, Pisma Zh. Éksp. Teor. Fiz. **43**, 551 (1986) [JETP Lett. **43**, 712 (1986)]; Yad. Fiz. **49**, 794 (1988) [Sov. J. Nucl. Phys. **49**, 494 (1989)].

[33] A.P. Bakulev and A.V. Radyushkin, Phys. Lett. B **271**, 223 (1991); S.V. Mikhailov and A.V. Radyushkin, Phys. Rev. D **45**, 1754 (1992); A.P. Bakulev and S.V. Mikhailov, Z. Phys. C **68**, 451 (1995); Phys. Lett. B **436**, 351 (1998).

[34] A.P. Bakulev and S.V. Mikhailov, Phys. Rev. D **65**, 114511 (2002).

[35] A.V. Radyushkin, Phys. Lett. B **271**, 218 (1991).

[36] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).

[37] T. Kurimoto, H.n. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2002).