Multiparton interactions and hadron structure

W. D. Walker
Physics Department, Duke University, Durham, North Carolina 27708, USA
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We have studied the charged multiplicity distributions arising from p-p and p-$\overline{p}$ collisions over the range of center of mass energies from 30 to 1800 GeV. We find that a portion of each distribution does obey Koba-Nielsen-Olesen (KNO) scaling. Those parts of the distributions that do not scale are the result of collisions which are initiated by multiparton collisions. Results from experiment E735 show not only collisions initiated by double but also good evidence for triple parton-parton collisions. These multi-parton collisions seem to account for essentially all the increase in the nonsingle diffractive inelastic cross section in this energy domain. The threshold for multiparton collisions seems to be associated with the advent of sea quarks in the structure function of the proton. We have developed a means of extrapolating these results to higher energies. The results indicate that small Feynman x partons seem to become more dominant as the energy is increased. It seems very likely that the sea quarks are important at larger impact parameters than the valence quarks. This means that the gluons and sea quarks are increasingly important in describing the collisions as the energy grows. The threshold for multiparton collisions seems to be associated with the advent of the sea quarks in the reactions.

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I. INTRODUCTION

The theories and ideas concerning multiple particle production go back to the late 1930s with a significant interlude at Fermi's statistical theory of particle production [1]. The theories progress through fireballs [2], strings [3], and quark-gluon plasma [4,5]. We have continued the analysis of multiplicity distributions that was described in two previous papers [6,7]. The data come from the collider experiments at CERN-UA5 [8] and the Fermilab experiment E735 [9]. It had been found that distributions observed at the CERN Intersecting Storage Rings (ISR) and fixed target energies obey Koba-Nielsen-Olesen (KNO) scaling [10]. It was discovered in the UA5 experiments that KNO scaling seems to break down by c.m. energies ($\sqrt{s}$) of 200 GeV. This breakdown becomes more severe as the bombarding energy is increased. The apparent breakdown of KNO scaling produced a lot of activity in attempts to understand the reason for the breakdown [11]. Different types of distributions were proposed. In particular, fits were made to the distribution using a negative binomial distribution which proposes that there are $\langle k\rangle$ sources for the $\langle n\rangle$ particles observed. We have discovered that a part of each multiplicity distribution does in fact KNO scale with respect to the average multiplicity for events initiated by single parton-parton collisions. The average multiplicities we observe for the single parton-parton initiated collisions are readily obtained from the UA5 and E735 multiplicity distributions as we will explain below. The quantity we denote by $\langle n_1\rangle$ agrees quite well with the $\langle n_{soft}\rangle$ of Giovannini and Ugoccioni [12]. It is the average multiplicity produced in single parton-parton collisions. Our terminology is quite different from theirs in that we do not believe that all the increase in p_T with multiplicity is the result of minijets but is the result of the production of a larger fraction of more massive particles (k's for example) [13].

II. DATA

We will now present the data and their interpretation. We show in Fig. 1 the data on which our results are based. These are the multiplicity distributions from the experiment UA5 and the Tevatron experiment E735. These are the full phase-space multiplicity distributions. The E735 results have been extrapolated to the full phase space using the same computer code as was used for experiment UA5 [8]. There is one energy where the two experiments overlap, namely at $\sqrt{s}$

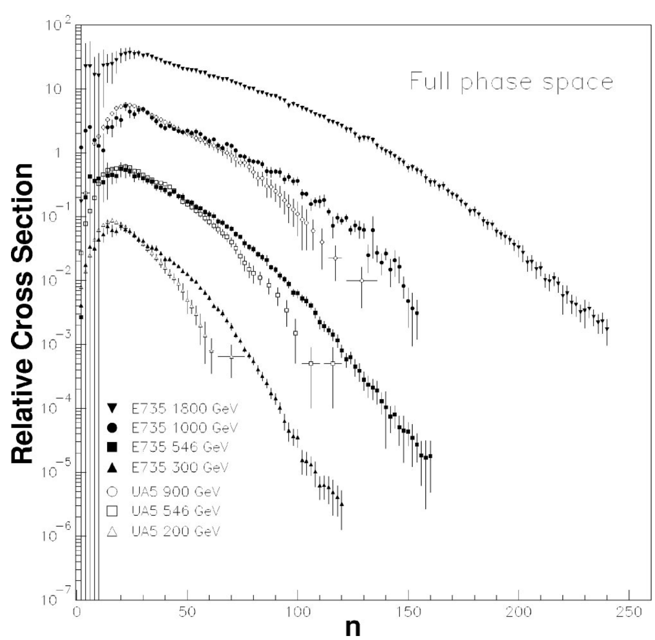

FIG. 1. The multiplicity distributions as measured by experiments UA5 and E735 at various collider energies. Data from the two experiments which were taken at the same or nearly the same energy have been normalized to each other over a range of multiplicities just past the peaks of the distributions.

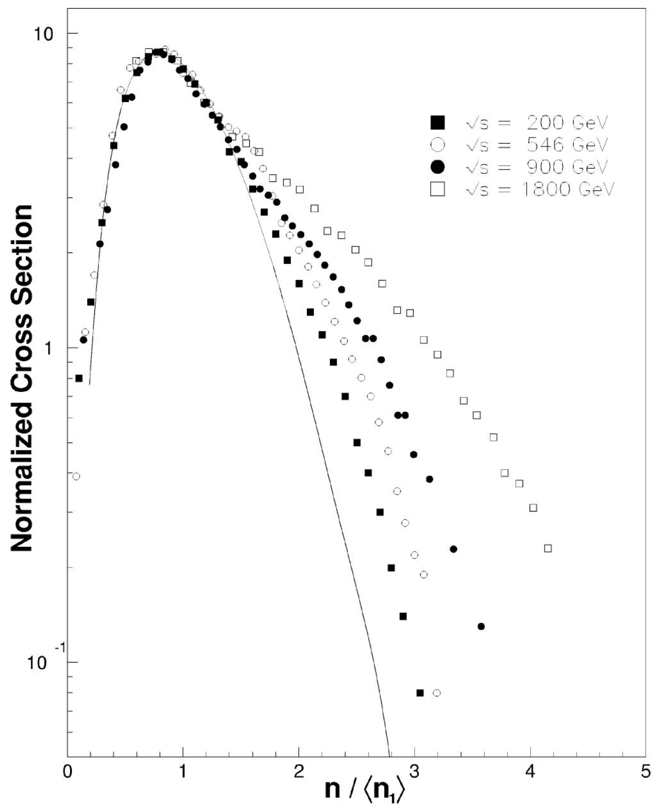

FIG. 2. A comparison of multiplicity distributions at different collider energies. The distributions have been normalized at the maximum value of $d\sigma/dx$ where $x=n/\langle n_1\rangle$. $\langle n_1\rangle$ is the average multiplicity for a single parton-parton collision as obtained from the solid curve which is the KNO distribution from the ISR data. The actual cross section $d\sigma/dx$ may be obtained by integrating the overall curves as presented and equating the result to the measured value of σ_{NSD} to determine the scale constant.

$=546$ GeV. It can be seen that the UA5 data seem to fall below the E735 data at high multiplicities and the E735 data are less accurate at low multiplicities. Experiment E735 is certainly statistically more reliable at the higher multiplicities since the experiment was designed to detect and study events of high multiplicity and consequently there were many more data at high multiplicities. Otherwise the two experiments agree rather well over a large range of multiplicities.

We show in Fig. 2 the collider data at four different energies superimposed in one plot. We have plotted the partial cross sections as a function of $x=n/\langle n_1\rangle$ where $\langle n_1\rangle$ is the average number of particles initiated in a single parton-parton collision (i.e. that part of the distribution that does KNO scale). The solid curve is the distribution of multiplicities as found by fitting the ISR data using a polynomial in the quantity x. A useful aspect of this curve is that the multiplicity at the distribution maximum n_{max} is simply related to $\langle n\rangle$ as $\langle n\rangle=1.25n_{max}$ at each energy. This curve is used to find the quantity $\langle n_1\rangle$ at each energy. We note that the quantity $d\sigma/dx$ is nearly energy independent up to a bit past $x=1$ for the distributions shown. The different data sets are normalized to the same value of $d\sigma/dx$ at $n=n_{max}$. We will later

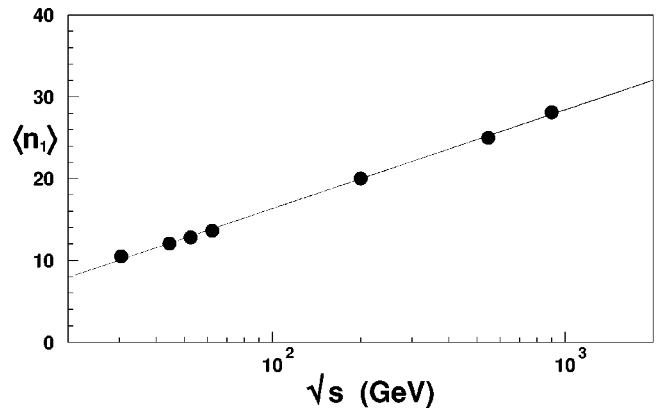

FIG. 3. The average multiplicity, $\langle n_1\rangle$, obtained from the multiplicity distributions shown in Fig. 1, as explained in the text.

show that the cross section for single parton-parton interactions does indeed seem to be nearly energy independent. This is not terribly surprising since the parton-parton energies which give rise to the single parton-parton collisions vary widely at a given bombarding energy. Thus we have found that KNO scaling seems to hold up to a value of $n\approx1.2\langle n_1\rangle$. We show in Fig. 3 the plot of $\langle n_1\rangle$ versus $\log(\sqrt{s})$. The quantity $\langle n_1\rangle=12.06\log(\sqrt{s}/4.27)$ over a wide range of energies. The line shown is the result of a χ^2 fit of the ISR, UA5 and E735 data.

To further analyze the breakdown of KNO scaling at high multiplicities we subtract the multiplicity distribution from the distribution for collisions initiated by single parton-parton collisions from the observed distributions. The result is the set of distributions shown in Fig. 4. These distributions show clear peaks close to $x=2$. We believe that the events contributing to these distributions shown are mainly the result of double parton-parton collisions. The poorest data set is that at $\sqrt{s}=900$ GeV. The UA5 data fail at high multiplicities and have been corrected by using the comparison between the UA5 and the E735 data at $\sqrt{s}=546$ GeV. We have not put error bars on the points in these distributions since they would make the distributions more difficult to read. We can estimate the errors mainly on the basis of the errors as calculated in the UA5 reports. We find that the KNO distributions from the lower energies are known to better precision. The poorest data from a statistical point of view are the data from $\sqrt{s}=200$ GeV which have errors ranging from 15% to 20%. The most accurate data are those from $\sqrt{s}=546$ GeV and 1800 GeV and these data are accurate to about 10%. It is very difficult to estimate the errors at $\sqrt{s}=900$ GeV since the curve is a mixture of data from E735 and UA5 which were taken at slightly different energies.

Next we discuss the energetics of the interactions in order to understand better the results shown in Fig. 4. The threshold for double parton-parton collisions is at a center of mass energy between 100 and 200 GeV. The other result is that both the parton-parton collisions seem to give rise to nearly equal multiplicities which would indicate that the amount of energy involved in each of the parton-parton collisions is comparable. We know of no simple explanation of this result. To get an estimate of the amount of energy involved we use

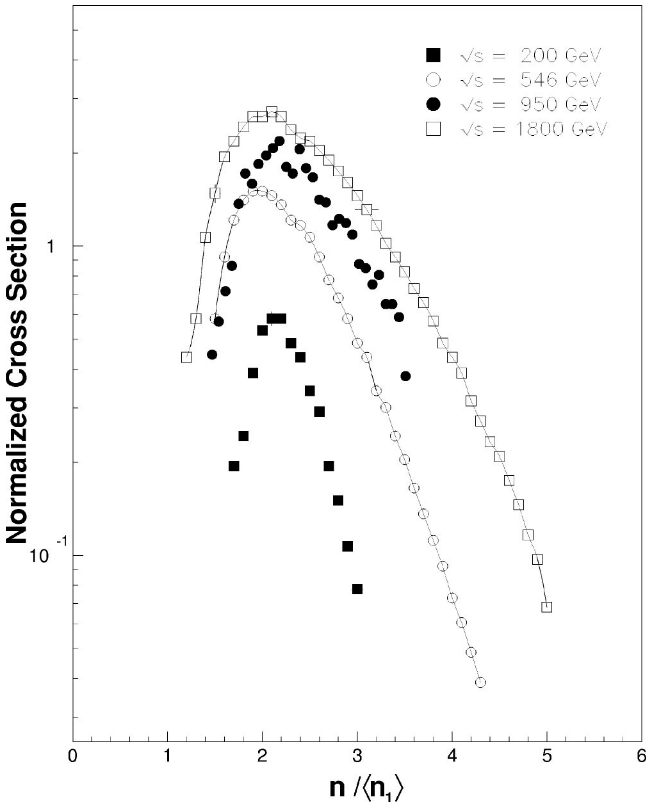

FIG. 4. The non-KNO scaling part of the multiplicity distribution. These distributions were obtained by subtracting the KNO distribution from the experimentally determined distributions.

the results on multiplicities produced in electron-positron annihilations. We assume that the amount of energy expended in producing a given charged multiplicity in a parton-parton collision is the same as in the e^+-e^- annihilation process which produces the same multiplicity. The strong correlation between multiplicity from e^+-e^- annihilation and hadronic production has been known for over 20 years [14]. This fact has also recently been observed in multihadron collisions at the Relativistic Heavy Ion Collider (RHIC) [15]. In a previous paper we found good agreement in comparing annihilation data with hadronic interactions at fixed target energies [16]. Using a plot of average multiplicity versus $\sqrt{s}$ and comparing this with our expression for $\langle n_1 \rangle$ we find that the threshold for multiparton collisions should be close to a $\sqrt{s}$ of 120 GeV.

We use the formula

$$\langle n \rangle = 0.084[\alpha(\sqrt{s})]^{0.49}\exp\{2.27/[\alpha(\sqrt{s})]^{0.5}\}$$

derived by Webber [17] from QCD considerations to estimate the multiplicity of charged particles from e^+-e^- annihilation for energies above the LEP energies. The factor 0.084 is chosen to agree with existing data. Using s' for e^+-e^- and s for p-$\bar{p}$ we define the quantity $y = \sqrt{s'}/\sqrt{s}$. We show in Table I $\langle n_1 \rangle$ and the corresponding experimentally determined values along with the quantity y. As the energy increases the value of y decreases. This will mean that the sea quarks and gluons will become increasingly important in determining the characteristics of the interactions as the energy is increased. It is also obvious from an examination of the table that the threshold for double parton-parton interactions should be slightly above a $\sqrt{s}$ of 120 GeV. We also note that the threshold for 3 parton-parton collisions should occur slightly below $\sqrt{s} = 546$ GeV. We conclude that the subtracted distributions shown in Fig. 4 are probably purely double parton-parton collisions up through $\sqrt{s} = 546$ GeV. Above that energy, the subtracted distribution is a mixture of double and perhaps triple or more parton-parton collisions. An examination of Fig. 4 shows that the subtracted distributions become broader as the energy is increased. We use the subtracted distribution at $\sqrt{s} = 546$ GeV to estimate the number of triple collisions at the higher energies. We use the same method as before, namely we subtract the high x part of the double collision distribution at $\sqrt{s} = 546$ GeV from the subtracted distributions obtained at 900 and 1800 Gev. The results of this subtraction are shown in Fig. 5 for $\sqrt{s} = 1800$ GeV. We also can estimate the number of triple collisions by using a convolution of $d\sigma_1/dx$ to estimate the shape of $d\sigma_2/dx$. The resulting shape agrees with $d\sigma_2/dx$ very well on the high x side for $\sqrt{s} = 546$ GeV.

Using the σ_{NSD} [18–21] we determine the cross sections σ_2 and σ_3 which are shown in Fig. 6. The triple parton-parton cross section probably contains some more highly multiple parton-parton collisions. We note that σ_2 agrees very well with that measured by CDF [22] at $\sqrt{s} = 1800$ GeV. The two experiments used entirely different methods to determine the cross section.

III. EXTRAPOLATION TO LHC ENERGIES

We show in Fig. 7 a plot of $\langle n \rangle$ versus $\sqrt{s}$. The dependence deduced on the basis of this plot is $\langle n \rangle = 3.25(\sqrt{s})^{0.35}$. We can estimate the width of the multiplicity

TABLE I. Cross section summary.

$\sqrt{s}$ in GeV	$\langle n_1 \rangle$	$y = \sqrt{s'}/\sqrt{s}$	σ_{NSD} in mb	σ_1 in mb	$\sigma_2 + \sigma_3$ in mb
62	13.8	0.58	31.3 ± 1	31.3 ± 1	0.0
200	19.85	0.423	33.8 ± 1.5	32.2 ± 1.5	1.66 ± 0.3
546	25.0	0.273	39.3 ± 3	32.8 ± 2	7.0 ± 0.6
900	27.6	0.21	42.7 ± 3.3	31.1 ± 3	11.6 ± 1.5
1800	31.1	0.145	48.5 ± 3.7	31.3 ± 2.5	17.1 ± 1.5
14000	41.6	0.04	64.2 ± 6	32.4 ± 3.0	≈ 30

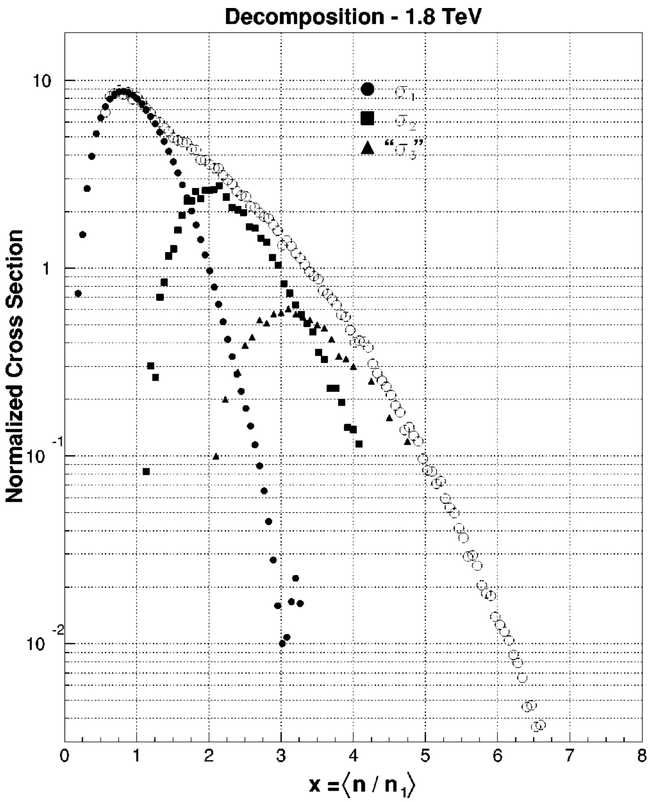

FIG. 5. The decomposition of the multiplicity distribution at $\sqrt{s}=1800$ GeV. The multiplicity distributions for 1, 2 and 3 parton-parton collisions are shown.

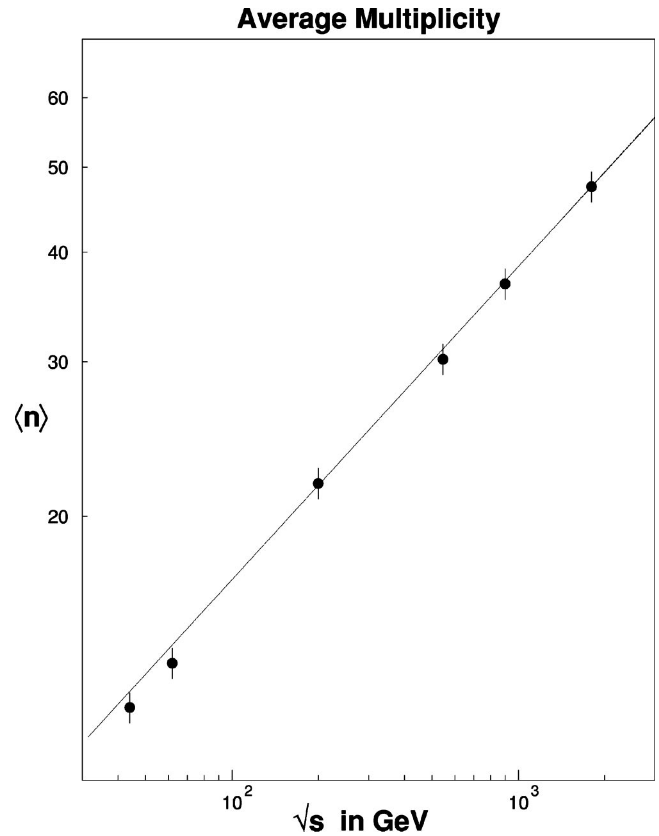

FIG. 7. A plot of the average charged multiplicity as a function of $\sqrt{s}$.

distribution on the basis of similar plots of the widths of the multiplicity distributions at various fractions z of the peak value of the distribution. The width of the distribution $\Delta(z)=B(z)(\sqrt{s})^{\alpha}$. For small values of $1/z$ the $B(z)$ are small and α is of the order of 0.25, and when the fraction z is small then $B(z)$ is large and α is ≈ 0.4. We show in Fig. 8 a set of the curves used to extrapolate the widths from lower energies to the Large Hadron Collider (LHC) energy. Using these extrapolations we constructed the multiplicity distribution shown in Fig. 9. We perform the same subtractions that we used for the lower energy distributions. The decomposition of this distribution gives a σ_3 which extends out to values of x of the order of 4 or 5. Thus it indicates that there are cases which are possibly the result of 4 or 5 parton-parton collisions in a single nucleon-nucleon interaction. The multiplicity distribution gives a $\langle n\rangle$ which is fairly close to the value of $\langle n\rangle$ calculated from the expression given above. The agreement is to within slightly more than 2%. It is very difficult if not impossible to estimate the accuracy of such extrapolations.

Using the cross section estimates of Block *et al.* [21] we have a total cross section of 108 ± 5 mb and an elastic cross

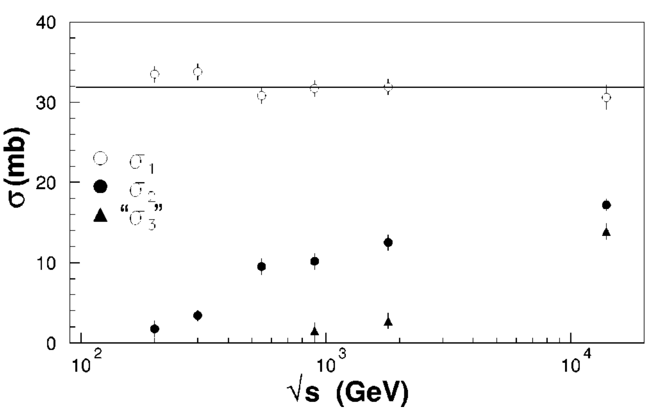

FIG. 6. A plot of the partial cross sections for 1, 2 and 3 parton-parton collisions. These cross sections can be deduced from the σ_{NSD} by integrating the curves shown in Fig. 2.

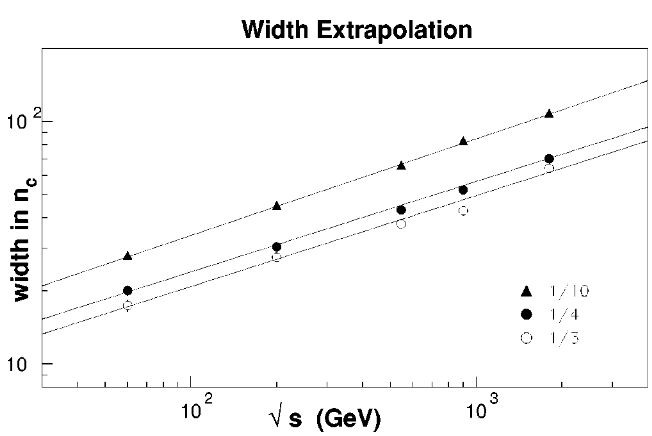

FIG. 8. The width of the multiplicity distributions for different fractions of the peak value of the distributions.

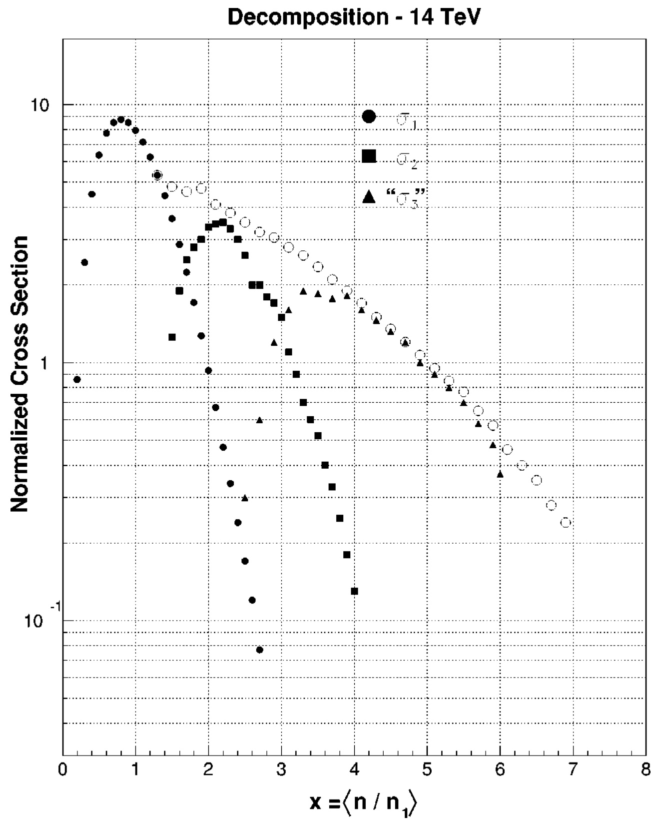

FIG. 9. The multiplicity distribution at 14 TeV obtained by extrapolation. It is obviously very difficult to estimate the precision of this extrapolation.

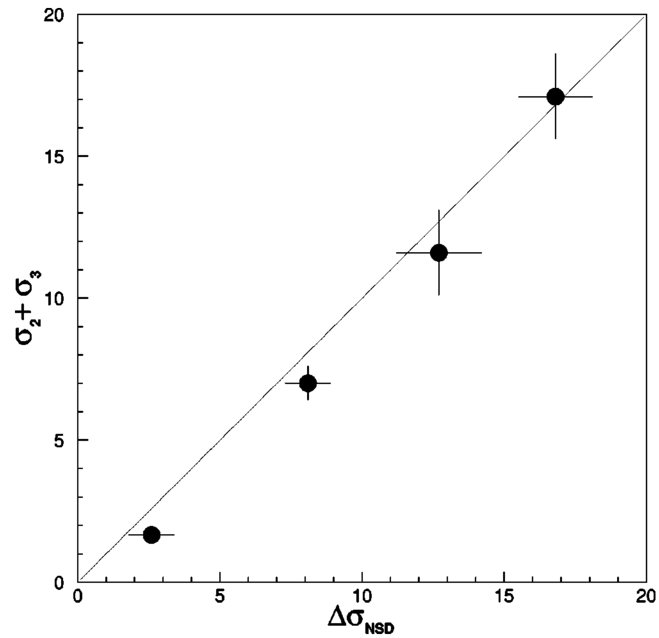

FIG. 10. The increase in the multiparton cross section is plotted against the increase in σ_{NSD}. We have used the fit to the cross sections given in the *Review of Particle Physics*.

section of 30.6 mb. Extrapolating the single diffractive cross sections from the lower energies [22] yields a single diffractive cross section of 10 ± 2 mb. We believe these values are internally consistent and yield a σ_1 of 32 ± 3 mb.

An examination of the quantity $\sigma_2+\sigma_3$ as a function of $\sqrt{s}$ in Table I shows that the increase in σ_{NSD} is well accounted for by the multiparton processes. We show in Fig. 10 a plot of the increase in the sum of $\sigma_2+\sigma_3$ versus $\Delta\sigma_{NSD}$. This clearly demonstrates that the increase in the cross section is produced by the multiparton collisions. There is also an interesting question as to why the threshold for double parton-parton interactions should be in the neighborhood of 120 GeV. We relate this to the fact that the sea quark distribution starts contributing to the structure function of the proton [23] at Feynman x of less than 0.35 to 0.40. The quantity y is related to the Feynman x in the following way. It is easy to show that $s_{effective}=sx_1x_2$ where x_1 and x_2 are the Feynman x values for the colliding partons. We relate $s_{effective}$ to the energy required in e^+-e^- annihilation. Thus the quantity y is closely related to $\langle\sqrt{x_1x_2}\rangle$ (the quantity y has a value of 0.42 at $\sqrt{s}=200$ GeV). Examination of Table I shows that the multiparton collisions have a threshold above 120 GeV. We believe that the advent of interactions with the sea quarks gives rise to the threshold for double parton-parton collisions. It also seems likely that the sea quarks are spread outside the valence quark domain. This fits in with the fact that the cross section increase is well accounted for by multiparton events. There is a useful summary of results on a picture of the nucleon from data on p-$\overline{p}$ total and elastic cross sections [24,25]. According to the discussion in the paper by Bourrely *et al.* [25] there seems to be an opaque core surrounded by a less opaque part at larger impact parameters. We would identify the transparent rim with the sea quark contribution to the cross section. We show in Fig. 10 a plot of $\sigma_2+\sigma_3$ versus $\Delta\sigma_{NSD}$ which plots the data shown in Table I and shows that the cross section increase is well accounted for by the multiparton interactions. We have not attempted to fit these data with any particular model of particle production. The dual parton model (DPM) [26] would probably predict a rapidity distribution different from that produced by parton-parton collisions. A single parton-parton collision will produce a single accumulation of particles in either the forward or backward hemisphere. The DPM tends to produce accumulations in both hemispheres since the two diquarks carry most of the momentum and end up in their respective hemispheres. A sample of single parton interactions at 1800 GeV would make possible a decision as to the accuracy of the DPM.

IV. CONCLUSIONS

(1) A portion of each multiplicity distribution does obey KNO scaling.

(2) Multiple parton-parton collisions become increasingly important as the collision energy is increased.

(3) The increase in the NSD cross section with energy is well accounted for by multiparton interactions.

(4) The data suggest that triple or more parton-parton col-

lisions are present at $\sqrt{s}=1800$ GeV and at higher energies.

(5) We suggest that the sea quarks are distributed over a larger volume in the nucleon than are the valence quarks.

(6) Further study of events of this type should allow one to measure a higher order structure function, namely the probability $P(x_1,x_2)$ which is the probability of finding two partons with Feynman x_1 and x_2 respectively.

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