Kinematical bound in asymptotically translationally invariant space-times

Tetsuya Shiromizu,[1,2] Daisuke Ida,[1] and Shinya Tomizawa[1]
[1]*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*
[2]*Advanced Research Institute for Science and Engineering, Waseda University, Tokyo 169-8555, Japan*
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We present positive energy theorems in asymptotically translationally invariant space-times which can be applicable to black strings and charged branes. We also address the bound property of the tension and charge of branes.

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I. INTRODUCTION

Asymptotic flatness is a useful working assumption in studying four-dimensional general relativity, in particular in the field of black hole physics. Of course, this assumption is well justified for the gravitational phenomena of isolated systems within the Hubble horizon of our Universe. Within the framework of string theory, on the other hand, we have to take account of nonasymptotically flat space-times, because the vacuum of the theory is considered to be four-dimensional space-time times compact extra dimensions at low energy to realize our apparently four-dimensional universe. For example, we need more insight into the black strings of branes, which are nonasymptotically flat solutions typically arising in supergravity theories, to extract some information on quantum gravity or the unified theory of interactions.

The stability of the Schwarzschild space-time is one of fundamental properties of black holes in asymptotically flat space-time. However, this is not the case for the black strings or branes; namely, they are unstable under the linear perturbations of sufficiently long wave length along the brane [1]. We have no definitive answer concerning the end point of this Gregory-Laflamme instability, but there are several possibilities; the final state might be naked singularities joining array of black holes, or an inhomogeneous black string or brane [2], and there also is a possibility that there is no equilibrium state. Since the subject concerns nonasymptotically flat inhomogeneous space-time, the analysis will be quite difficult. We would ultimately need a dynamical analysis directly solving the Einstein equation [3,4]. However, it might be also useful to have a kinematical bound irrelevant for the details of the underlying theory for such nonasymptotically flat space-times. Such a kinematical bound might be also useful to restrict the form of the metric like the uniqueness theorem in asymptotically flat space-times [5,6].

In this Brief Report, utilizing the spinorial approach, we present bound theorems (positive mass theorem, Bogomol'nyi-Prasad-Sommerfield (BPS) bound, positive tension theorem) in asymptotically translationally invariant space-times. Recently Traschen discussed the positive mass theorem in such space-times without horizon and gauge fields [7]. In this Brief Report we will extend Traschen's work to cases with horizon in higher dimensions, which is relevant for black string or brane space-times, and include the gauge fields in four dimensions.

The rest of the present Brief Report is composed of two main parts. In Sec. II, we present the positive mass theorem in higher dimensions with the horizon. Then we prove the positive energy and tension theorems for charged branes in Sec. III. Finally we give a discussion in Sec. IV. In the Appendix we give formulas for the calculation of the boundary term at horizon.

II. ASYMPTOTICALLY TRANSLATIONALLY INVARIANT SPACE-TIMES

First of all, we must specify asymptotically translationally invariant space-times. The metric of full space-times is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{1}$$

Let $n \propto \partial_t$ and $z \propto \partial_{x^1}$ to be timelike and spacelike unit vector fields such that $n^\mu z_\mu = 0$. In addition, $\hat{r} \propto \partial_{x^2}$ to be spacelike perpendicular unit normal vector fields to n and z, $\hat{r}^\mu n_\mu = \hat{r}^\mu z_\mu = 0$. We assume that z becomes proportional to the asymptotically translational Killing vector toward the infinity directed to $\hat{r}$. x^μ, x^i, x^I, x^A and x^a spans the full space-times $\mathcal{M}$, $(n-1)$-dimensional spacelike hypersurface V_0 normal to n, the $(n-1)$-dimensional timelike hypersurface V_1 normal to z, $(n-2)$-dimensional spacelike submanifold V_{01} orthogonal to n and z, and the $(n-3)$-dimensional spacelike submanifold V_{012} orthogonal to n, z, and $\hat{r}$. Each induced metricies can be written as

$$q_{ij} dx^i dx^j = (g_{\mu\nu} + n_\mu n_\nu) dx^\mu dx^\nu, \tag{2}$$

$$h_{IJ} dx^I dx^J = (g_{\mu\nu} - z_\mu z_\nu) dx^\mu dx^\nu, \tag{3}$$

$$p_{AB} dx^A dx^B = (g_{\mu\nu} + n_\mu n_\nu - z_\mu z_\nu) dx^\mu dx^\nu, \tag{4}$$

$$s_{ab} dx^a dx^b = (g_{\mu\nu} + n_\mu n_\nu - z_\mu z_\nu - \hat{r}_\mu \hat{r}_\nu) dx^\mu dx^\nu. \tag{5}$$

Then $\mu,\nu = 0,1,2,\dots,n$, $i = 1,2,\dots,n$, $I = 0,2,3,\dots,n$, $A = 2,3,\dots,n$, and $a = 3,4,\dots,n$ (see Fig. 1).

We assume that the submanifold V_{01} is a $(n-2)$-dimensional asymptotically Euclid space.

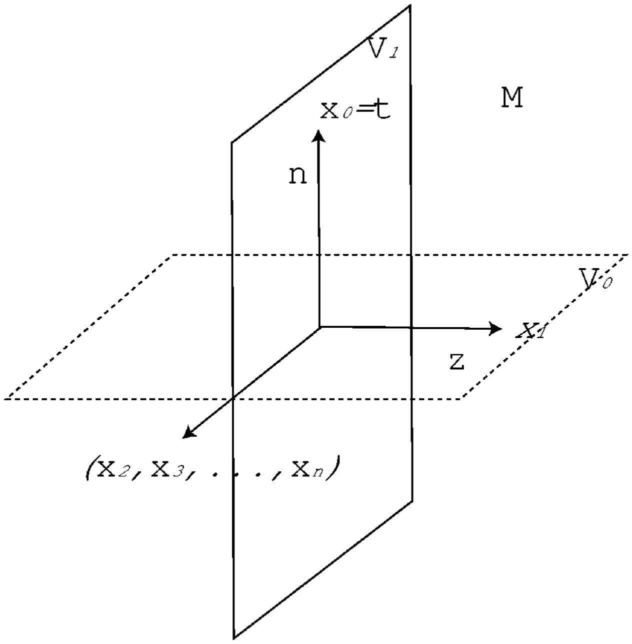

FIG. 1. Full space-time $\mathcal{M}$ can be foliated by spacelike hypersurfaces V_0 normal to the timelike vector field $n \propto \partial_t$ and timelike hypersurfaces V_1 normal to the spacelike vector field $z \propto \partial_{x^1}$. We can define coordinate $\{x^i\} = (x_1, x_2, \ldots, x_n)$ in V_0, $\{x^I\} = (x_0, x_2, \ldots, x_n)$ in V_1 and $\{x^A\} = (x_2, \ldots, x_n)$ in $(n-2)$-dimensional spacelike surface V_{01} normal to both vector fields n and z. Furthermore, we set coordinate $\{x^a\} = (x_3, \ldots, x_n)$ in the $(n-3)$-dimensional spacelike submanifold V_{012} normal to n, z and $\hat{r} \propto \partial_{x^2}$.

III. POSITIVE MASS THEOREM FOR BLACK STRING

In this section, we present the positive energy theorem in asymptotically translationally invariant space-time with horizon. See Ref. [7] for cases without a horizon. If one thinks of the gravitational energy as evaluated in slices which have appropriate asymptotic boundaries and a regular center, it is not necessary to take the event horizon as the boundary term. However, the proof independent of the inner structure of the horizon is useful.

Let us consider a spinor ϵ satisfying a Dirac-type equation [7]

$$\gamma^A \nabla_A \epsilon = 0. \tag{6}$$

Note that we usually suppose $\gamma^i \nabla_i \epsilon = 0$ for the spinor to prove the original positive energy theorem [8]. In asymptotically translationally invariant space-times, it is likely that the existence of solutions to Eq. (6), which approaches a constant spinor ϵ_0, is guaranteed rather than the solution to $\gamma^i \nabla_i \epsilon = 0$. This is because the space spanned by coordinate $\{x^A\}$ is asymptotically flat and we can expect almost the same proof of the existence of solutions as that in asymptotically flat space-times.

Let us define the Nester tensor $E_{\mu\nu}$ by

$$E^{\mu\nu} = \tfrac{1}{2} (\bar{\epsilon} \gamma^{\mu\nu\alpha} \nabla_\alpha \epsilon + \text{c.c.}); \tag{7}$$

we obtain the formula

$$\nabla_\mu E^{\mu\nu} = \tfrac{1}{2} G^\nu_\mu \xi^\mu + \overline{\nabla_\mu \epsilon} \gamma^{\mu\nu\alpha} \nabla_\alpha \epsilon, \tag{8}$$

where $\bar{\epsilon} = \epsilon^\dagger \gamma^{\hat{0}}$. According to Ref. [8], a surface integral of Nester tensor at spatial infinity over V_{02} gives the ADM energy-momentum vector P^μ, that is,

$$-P^\mu \xi_\mu = \frac{1}{16\pi} \int_{V_{02}^\infty} E^{\mu\nu} dS_{\mu\nu}. \tag{9}$$

Integrating Eq. (8) over spacelike manifold V_{02} and using Stokes's theorem and Eq. (6), we obtain the formula

$$\int_{V_{02}^\infty} dS_{\hat{0}\hat{2}} E^{\hat{0}\hat{2}} - \int_{V_{02}^{\mathrm{H}}} dS_{\hat{0}\hat{2}} E^{\hat{0}\hat{2}} = \int dV_0 (8\pi T^\mu_\nu \xi^\nu n_\mu + 2|\nabla_A \epsilon|^2), \tag{10}$$

where $\xi^\mu = -\bar{\epsilon} \gamma^\mu \epsilon$. Following the proof in Ref. [8], we require that spinor ϵ approaches a constant spinor ϵ_0 at infinity V_{02}^∞. In the above we used the Einstein equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$. The first and second terms on the left-hand side are boundary terms at infinity and the horizon. The first term gives us the gravitational energy. Thus, what we must focus on is the boundary term at the horizon. This is a nontrivial issue and the point here. We modify the proof in asymptotically flat space-times with the horizon [9]. The detail of the computation is described in the Appendix. As a result, it becomes

$$\begin{aligned}\int_{V_{02}^H} dS_{\hat{0}\hat{2}} E^{\hat{0}\hat{2}} &= \frac{1}{2} \int dS_{\hat{0}\hat{2}} [\epsilon^\dagger (\nabla_{\hat{2}} - \gamma^{\hat{2}} \gamma^{\hat{1}} \nabla_{\hat{1}}) \epsilon + \text{c.c.}] \\ &= \tfrac{1}{2} \int dS_{\hat{0}\hat{2}} \epsilon^\dagger [-\tfrac{1}{2} (K - K_{\hat{2}\hat{2}} + k) \gamma^{\hat{2}} \gamma^{\hat{0}} \epsilon \\ &\quad - \gamma^{\hat{2}} \gamma^{\hat{1}} D_{\hat{1}} \epsilon - \gamma^{\hat{2}} \gamma^a d_a \epsilon + \tfrac{1}{2} K_{\hat{a}\hat{2}} \gamma^{\hat{a}} \gamma^{\hat{0}} \epsilon] + \text{c.c.} \\ &= \tfrac{1}{2} \int dS_{\hat{0}\hat{2}} \epsilon^\dagger [-\tfrac{1}{2} (K - K_{\hat{2}\hat{2}} + k) \gamma^{\hat{2}} \gamma^{\hat{0}} \epsilon + \psi] \\ &\quad + \text{c.c.}\end{aligned} \tag{11}$$

where

$$\psi = -\gamma^{\hat{2}} \gamma^{\hat{1}} D_{\hat{1}} \epsilon - \gamma^{\hat{2}} \gamma^a d_a \epsilon + \tfrac{1}{2} K_{\hat{a}\hat{2}} \gamma^{\hat{a}} \gamma^{\hat{0}} \epsilon. \tag{12}$$

D_i, $\mathcal{D}_A$ and d_a are covariant derivative with respect to q_{ij}, p_{AB} and s_{ab}, respectively. K_{ij} and k_{ab} are defined by $K_{ij} = q_i^k \nabla_k n_j$ and $k_{ab} = s_a^c \mathcal{D}_c \hat{r}_b$, respectively.

At the horizon we impose

$$\gamma^{\hat{2}} \gamma^{\hat{0}} \epsilon = \epsilon \tag{13}$$

and use $\theta_+ \propto K - K_{\hat{2}\hat{2}} + k = 0$ at the apparent horizon. θ_+ is the expansion of the outgoing null geodesic congruence. Then we can see the boundary term at the horizon vanishes. We used the fact that ψ anti-commutes with $\gamma^{\hat{2}} \gamma^{\hat{0}}$ and then the contribution of ψ to Eq. (11) disappears. Finally

$$E_{\rm ADM}=\frac{1}{8\pi|\epsilon_0|^2}\int_{V_{02}^{\infty}}dS_{\hat{0}\hat{2}}E^{\hat{0}\hat{2}}$$
$$=\frac{1}{8\pi|\epsilon_0|^2}\int dV_0(8\pi T^{\mu}_{\nu}\xi^{\nu}n_{\mu}+2|\nabla_A\epsilon|^2). \tag{14}$$

Together with the dominant energy condition, we can see that $E_{\rm ADM}$ is positive definite.

Let us discuss the $M=0$ cases. In this case,

$$\nabla_{\hat{A}}\epsilon=0 \tag{15}$$

and then

$$^{(n)}R_{\hat{A}\hat{B}\mu\nu}\gamma^{\mu\nu}\epsilon=0. \tag{16}$$

From the above we see that

$$^{(n)}R_{\mu\nu\alpha\beta}=0. \tag{17}$$

This means that the space-time with zero energy is flat. Even for asymptotically translationally invariant space-times, the ground state is flat space-time.

IV. BOUND THEOREMS FOR CHARGED BRANES IN FOUR DIMENSIONS

A. Positive energy theorem for charged brane

In this subsection, we extend Traschen's study to cases with gauge field in four dimensions. It is easy to extend to higher dimensions following Ref. [10]. For this we define the following covariant tensor motivated by N=2 supergravity [11]:

$$\hat{\nabla}_{\mu}\epsilon=\nabla_{\mu}\epsilon+\frac{i}{4}F_{\alpha\beta}\gamma^{\alpha\beta}\gamma_{\mu}\epsilon. \tag{18}$$

Let us consider a spinor ϵ satisfying

$$\gamma^A\hat{\nabla}_A\epsilon=0. \tag{19}$$

The Nester tensor is defined by

$$\hat{E}^{\mu\nu}=\tfrac{1}{2}(\overline{\epsilon}\gamma^{\mu\nu\alpha}\hat{\nabla}_{\alpha}\epsilon+\text{c.c.})=E^{\mu\nu}-i\overline{\epsilon}(F^{\mu\nu}-\gamma_5\tilde{F}^{\mu\nu})\epsilon \tag{20}$$

and we obtain the following formula:

$$\nabla_{\mu}\hat{E}^{\mu\nu}=\tfrac{1}{2}G^{\nu}{}_{\mu}\xi^{\mu}+\overline{\hat{\nabla}_{\mu}\epsilon}\gamma^{\mu\nu\alpha}\hat{\nabla}_{\alpha}\epsilon-i\overline{\epsilon}(\nabla_{\mu}F^{\mu\nu}-\gamma_5\nabla_{\nu}\tilde{F}^{\mu\nu})\epsilon$$
$$+4\pi\overline{\epsilon}T^{\mu\nu}(F)\gamma_{\nu}\epsilon \tag{21}$$

where $\tilde{F}^{\mu\nu}=(1/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ and

$$T_{\mu\nu}(F)=(4\pi)^{-1}(F_{\mu}{}^{\alpha}F_{\nu\alpha}-\tfrac{1}{4}g_{\mu\nu}F^2). \tag{22}$$

Integrating over the spacelike hypersurface, the result is

$$8\pi\epsilon_0^{\dagger}[E_{\rm ADM}-i\gamma^{\hat{0}}(Q_{\rm e}-\gamma_5Q_{\rm m})]\epsilon_0$$
$$=\int_{V_{02}^{\infty}}dS_{\mu\nu}\hat{E}^{\mu\nu}$$
$$=\int_{V_0}d\Sigma[G^{\mu}_{\nu}\xi^{\nu}n_{\mu}+2|\hat{\nabla}_A\epsilon|^2-2i\overline{\epsilon}(j^{\mu}_{\rm e}-\gamma_5j^{\mu}_{\rm m})\epsilon n_{\mu}$$
$$-8\pi T_{\mu\nu}(F)\xi^{\mu}n^{\nu}]$$
$$=\int_{V_0}d\Sigma[8\pi T^{\mu}_{\nu}\xi^{\nu}n_{\mu}+2|\hat{\nabla}_A\epsilon|^2-2i\overline{\epsilon}(j^{\mu}_{\rm e}-\gamma_5j^{\mu}_{\rm m})\epsilon n_{\mu}], \tag{23}$$

where

$$Q_{\rm e}=\frac{1}{8\pi}\int dS_{\mu\nu}F^{\mu\nu},\quad Q_{\rm m}=\frac{1}{8\pi}\int dS_{\mu\nu}\tilde{F}^{\mu\nu} \tag{24}$$

$$j^{\mu}_{\rm e}=\nabla_{\nu}F^{\nu\mu},\quad j^{\mu}_{\rm m}=\nabla_{\nu}\tilde{F}^{\nu\mu}. \tag{25}$$

From first line to the second one, we used the Einstein equation $G_{\mu\nu}=8\pi(T_{\mu\nu}(F)+T_{\mu\nu})$. Using the above a dominant energy condition, we can obtain the BPS bound

$$E_{\rm ADM}\geqslant\sqrt{Q_{\rm e}^2+Q_{\rm m}^2}. \tag{26}$$

As the inequality is saturated, $\hat{\nabla}_{\hat{A}}\epsilon=0$ holds. In general, $\hat{\nabla}_{\hat{0}}\epsilon\neq0$ and $\hat{\nabla}_{\hat{1}}\epsilon\neq0$. Since $\hat{\nabla}_{\hat{A}}\epsilon$ can be regarded as a infinitesimal local supersymmetric transformation of the gravitino, it is well-known fact that a part of supersymmetry is broken.

We note that the current BPS bound theorem is slightly different from that given in Ref. [10]. Therein the term corresponding to $\nabla_A\epsilon$ in Eqs. (19) and (23) is $\nabla_i\epsilon$.

B. Positive tension theorem for charged branes

Let discuss the issue of the positive tension theorem [7] or BPS bound [13]. As discussed in Ref. [7], we can expect the tension of a brane is a conserved charge associated with an asymptotic spatial translational Killing vector parallel to the brane, as just the ADM energy is one associated with an asymptotic time translational Killing vector. In analogy with the construction of positive energy theorem, the Nester tensor is defined by

$$\hat{B}^{\mu\nu}=\tfrac{1}{2}(\tilde{\epsilon}\gamma^{\mu\nu\alpha}\hat{\nabla}_{\alpha}\epsilon+\text{c.c.})$$
$$=\tfrac{1}{2}(\tilde{\epsilon}\gamma^{\mu\nu\alpha}\nabla_{\alpha}\epsilon+\text{c.c.})-\tfrac{1}{2}[i\tilde{\epsilon}(F^{\mu\nu}-\gamma_5\tilde{F}^{\mu\nu})\epsilon+\text{c.c.}]$$
$$=\tfrac{1}{2}(\tilde{\epsilon}\gamma^{\mu\nu\alpha}\nabla_{\alpha}\epsilon+\text{c.c.})=B^{\mu\nu} \tag{27}$$

where $\tilde{\epsilon}=\epsilon^{\dagger}\gamma^{\hat{1}}$. The integration over time is taken to be finite interval Δt. We should note that time direction in the construction of the previous theorem is replaced with x^1 direction. In a way similar to that in Sec. III, we can easily show

$$8\pi\mu|\epsilon_0|^2=\frac{1}{\Delta t}\int_{V_1}dtdS_{\hat{A}}B^{\hat{A}\hat{2}}$$

$$=\frac{1}{\Delta t}\int dV_1(2|\nabla_A\epsilon|^2-8\pi T^{\rm tot}_{\mu\hat{1}}\tilde{\xi}^\mu) \qquad (28)$$

where $\tilde{\xi}^\mu=\tilde{\epsilon}\gamma^\mu\epsilon$ and $T^{\rm tot}_{\mu\nu}=T_{\mu\nu}(F)+T_{\mu\nu}$. We followed the Traschen's definition of the tension. See Refs. [7,12,13] for the issue of the definition.

Note that the gauge field does not contribute to the tension. Thus the BPS bound cannot be proven although it has been argued in Ref. [13]. To prove that in general cases, we must improve the proof nontrivially.

V. SUMMARY

In this paper we proved several bound theorems in asymptotically translationally invariant space-times. More precisely we could prove the positive energy theorem for space-times with an event horizon such as black strings. We also proved a positive energy and tension theorem for charged brane configurations. For a current definition of the tension, the gauge field does not contribute to the tension.

The positive energy theorem for black string space-times might be used to gain insight into the issue of the final fate. We might be able to prove a sort of uniqueness theorem using the positive energy theorem. Indeed, in asymptotically flat space-times, the uniqueness theorem for static black holes can be proved in this line [6].

APPENDIX: BOUNDARY TERM AT THE HORIZON

Here we present some useful formulas. Using the Dirac-Witten equation, $\nabla_{\hat{2}}\epsilon$, which appeared as the first term in the integrand of the right-hand side in the first line of Eq. (11), can be written as

$$\begin{aligned}\nabla_{\hat{2}}\epsilon &= -\gamma^{\hat{2}}\gamma^{\hat{a}}\nabla_{\hat{a}}\epsilon\\ &= -\gamma^{\hat{2}}\gamma^{\hat{a}}(D_{\hat{a}}\epsilon+\tfrac{1}{2}K_{\hat{a}\hat{i}}\gamma^{\hat{i}}\gamma^{\hat{0}}\epsilon)\\ &= -\gamma^{\hat{2}}\gamma^{\hat{a}}(\mathcal{D}_{\hat{a}}\epsilon+\tfrac{1}{2}J_{\hat{a}\hat{A}}\gamma^{\hat{A}}\gamma^{\hat{1}}\epsilon+\tfrac{1}{2}K_{\hat{a}\hat{i}}\gamma^{\hat{i}}\gamma^{\hat{0}}\epsilon)\\ &= -\gamma^{\hat{2}}\gamma^{\hat{a}}(d_{\hat{a}}\epsilon+\tfrac{1}{2}k_{\hat{a}\hat{b}}\gamma^{\hat{b}}\gamma^{\hat{2}}\epsilon+\tfrac{1}{2}J_{\hat{a}\hat{A}}\gamma^{\hat{A}}\gamma^{\hat{1}}\epsilon\\ &\quad+\tfrac{1}{2}K_{\hat{a}\hat{i}}\gamma^{\hat{i}}\gamma^{\hat{0}}\epsilon)\\ &= -\gamma^{\hat{2}}\gamma^{\hat{a}}d_{\hat{a}}\epsilon-\tfrac{1}{2}k\epsilon-\tfrac{1}{2}J_{\hat{a}\hat{A}}\gamma^{\hat{2}}\gamma_{\hat{a}}\gamma^{\hat{A}}\gamma^{\hat{1}}\epsilon\\ &\quad-\tfrac{1}{2}K_{\hat{a}\hat{i}}\gamma^{\hat{2}}\gamma^{\hat{a}}\gamma^{\hat{i}}\gamma^{\hat{0}}\epsilon \qquad (A1)\end{aligned}$$

where J_{AB} is defined by $J_{AB}=p_A^C D_C z_B$.

Let us define a scalar field ϕ by

$$\phi:=\epsilon^\dagger J_{\hat{a}\hat{A}}\gamma^{\hat{2}}\gamma^{\hat{a}}\gamma^{\hat{A}}\gamma^{\hat{1}}\epsilon=-\epsilon^\dagger J_{\hat{a}\hat{2}}\gamma^{\hat{2}}\gamma^{\hat{a}}\gamma^{\hat{1}}\epsilon+\epsilon^\dagger J^{\hat{a}}_{\hat{a}}\gamma^{\hat{2}}\gamma^{\hat{1}}\epsilon. \qquad (A2)$$

It is easy to see that ϕ is pure imaginal, $\phi^*=-\phi$. Then

$$\mathrm{Re}(\epsilon^\dagger\nabla_{\hat{2}}\epsilon)=-\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{a}}d_{\hat{a}}\epsilon-\tfrac{1}{2}k|\epsilon|^2+\tfrac{1}{2}K_{\hat{a}\hat{2}}\epsilon^\dagger\gamma^{\hat{a}}\gamma^{\hat{0}}\epsilon$$
$$-\tfrac{1}{2}K^{\hat{a}}_{\hat{a}}\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{0}}\epsilon. \qquad (A3)$$

In the same way, we obtain the following formula for the second term of the integrand in the right-hand side in the first line of Eq. (11):

$$\mathrm{Re}(\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{1}}\nabla_{\hat{1}}\epsilon)=\mathrm{Re}[\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{1}}(D_{\hat{1}}\epsilon+\tfrac{1}{2}K_{\hat{1}\hat{i}}\gamma^{\hat{i}}\gamma^{\hat{0}}\epsilon)]$$
$$=\mathrm{Re}[\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{1}}D_{\hat{1}}\epsilon]+\tfrac{1}{2}K_{\hat{1}\hat{1}}\mathrm{Re}[\epsilon^\dagger\gamma^{\hat{2}}\gamma^{\hat{0}}\epsilon]. \qquad (A4)$$

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