Spinning pulsating string solitons in $AdS_5\times S^5$

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We point out the existence of some simple string solitons in $AdS_5\times S^5$, which at the same time are spinning in AdS_5 and pulsating in S^5, or vice versa. This introduces an additional arbitrary constant into the scaling relations between energy and spin or *R* charge. The arbitrary constant is not an angular momentum, but can be related to the amplitude of the pulsation. We discuss the solutions in detail and consider the scaling relations. Pulsating multispin or multi-*R*-charge solutions can also be constructed.

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I. INTRODUCTION

Recent progress in understanding the conjectured duality [1–3] between superstring theory on $AdS_5\times S^5$ and $\mathcal{N}=4$ SU(N) super Yang-Mills theory in Minkowski space is based on scaling relations between energy and angular momentum for straight spinning strings [4–16]. Circular pulsating strings have also been analyzed in some detail [4,10]. Even more recently, multispin and multi-*R*-charge solutions were constructed [17–19] and classified [20,21].

The geometry of anti–de Sitter space profoundly alters the properties of strings, as compared to Minkowski space. For instance, for a straight spinning string in anti–de Sitter space [22], the energy scales with the angular momentum, while in Minkowski space it scales with the square root of the angular momentum. Similarly, for a circular pulsating string in anti–de Sitter space [23], the energy scales with the square of the amplitude, while in Minkowski space it scales with the amplitude (the amplitude is of course not a coordinate invariant, but the maximal circumference is).

Comparison with Yang-Mills computations (see for instance [24–28]) is based on detailed analysis of the subleading terms in the scaling relations. It is therefore of impotance to search for general families of strings for which it is still possible to obtain analytical results. In the present paper, we explicitly construct new families of string solitons in $AdS_5\times S^5$. Our strings are straight and spinning in one direction but circular and pulsating in another, and with a nontrivial coupling between the two. Pulsating multispin solutions are also constructed. In each case, we obtain the explicit solutions in terms of elliptic functions, analyze the scaling relations in various limits, and compare with previously known results. We note in passing that our solutions fall outside the classification of [29,30].

The paper is organized as follows: In Sec. II, we set our notation and conventions, and derive and analyze the solutions which are spinning in AdS_5 but pulsating in S^5. Section III is devoted to the "opposite" situation, i.e., pulsating in AdS_5 but spinning in S^5. In Sec. IV, we take the simplest multispin solution [17] and generalize it to be pulsating in S^5. Finally in Sec. V, we present our conclusions and we suggest some further investigations in these directions.

II. SPINNING IN AdS_5 BUT PULSATING IN S^5

We take the $AdS_5\times S^5$ line element in the form

$$ds^2=-(1+H^2r^2)dt^2+\frac{dr^2}{1+H^2r^2}+r^2(d\beta^2+\sin^2\beta d\phi^2+\cos^2\beta d\tilde{\phi}^2)+H^{-2}(d\theta^2+\sin^2\theta d\psi^2+\cos^2\theta(d\psi_1^2+\sin^2\psi_1 d\psi_2^2+\cos^2\psi_1 d\psi_3^2)) \tag{2.1}$$

where H^{-1} is the scale of AdS_5 and the radius of S^5. The 't Hooft coupling in this notation is $\lambda=(H^2\alpha')^{-2}$.

The string which is straight and spinning in AdS_5 but circular and pulsating in S^5, is obtained by the ansatz

$$t=t(\tau),\ r=r(\sigma),\ \beta=\pi/2,\ \phi=\omega\tau,\ \theta=\theta(\tau),\ \psi=\sigma \tag{2.2}$$

with the remaining coordinates being arbitrary constants.

The *t* equation is solved by $t=c_0\tau$, and then the *r* and θ equations become

$$r''-\frac{H^2r'^2r}{1+H^2r^2}-H^2c_0^2(1+H^2r^2)r+\omega^2(1+H^2r^2)r=0 \tag{2.3}$$

$$\ddot{\theta}+\sin\theta\cos\theta=0 \tag{2.4}$$

while the nontrivial conformal gauge constraint is

$$\frac{H^2r'^2}{1+H^2r^2}+\dot{\theta}^2-H^2c_0^2(1+H^2r^2)+H^2r^2\omega^2+\sin^2\theta=0. \tag{2.5}$$

Notice that Eq. (2.5) involves τ and σ derivatives. However, everything is consistently solved by

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$$r(\sigma)=\frac{c_0\cos\alpha_0}{\sqrt{\omega^2-H^2c_0^2}}\times\mathrm{cn}\left(\sqrt{\omega^2-H^2c_0^2\sin^2\alpha_0}\,\sigma\,\middle|\,\frac{H^2c_0^2\cos^2\alpha_0}{\omega^2-H^2c_0^2\sin^2\alpha_0}\right) \tag{2.6}$$

$$\sin\theta(\tau)=\begin{cases} Hc_0\sin\alpha_0\,\mathrm{sn}(\tau|H^2c_0^2\sin^2\alpha_0), & Hc_0\sin\alpha_0<1\\ \mathrm{sn}(Hc_0\sin\alpha_0\tau|1/(H^2c_0^2\sin^2\alpha_0)), & Hc_0\sin\alpha_0>1\end{cases} \tag{2.7}$$

where α_0 is an integration constant. In the first case of Eq. (2.7) the string oscillates around one of the poles, while in the second case it oscillates between the two poles. In the limiting case, $Hc_0\sin\alpha_0=1$, it oscillates between a pole and the equator. To ensure that $r(\sigma)$ is periodic, we have the condition

$$2\pi\sqrt{\omega^2-H^2c_0^2\sin^2\alpha_0}=4K\left(\frac{H^2c_0^2\cos^2\alpha_0}{\omega^2-H^2c_0^2\sin^2\alpha_0}\right) \tag{2.8}$$

where K is a complete elliptic integral.

The solution (2.6), (2.7) is parametrized by (c_0,ω,α_0), of which one is fixed by Eq. (2.8). It is convenient to trade the two remaining parameters for

$$A\equiv Hc_0\sin\alpha_0 \tag{2.9}$$

$$B\equiv\frac{Hc_0\cos\alpha_0}{\sqrt{\omega^2-H^2c_0^2}}. \tag{2.10}$$

A is the amplitude of $\sin\theta(\tau)$, while B is the extension of $Hr(\sigma)$. Actually, A can only be interpreted as the amplitude as long as $Hc_0\sin\alpha_0\leqslant 1$, but since the solution for $\theta(\tau)$ is defined for any $Hc_0\sin\alpha_0$, we also define the amplitude for any $Hc_0\sin\alpha_0$. Now Eqs. (2.8)–(2.10) lead to

$$\tan\alpha_0=\frac{\pi A\sqrt{1+B^2}}{2BK\left(\frac{B^2}{1+B^2}\right)} \tag{2.11}$$

It is straightforward to compute the conserved energy and spin (the R charge is zero). In the present parametrization, they are

$$E=\frac{2\sqrt{1+B^2}}{\pi H\alpha'}\sqrt{B^2+\frac{\pi^2A^2(1+B^2)}{4K^2\left(\frac{B^2}{1+B^2}\right)}}\;E\left(\frac{B^2}{1+B^2}\right) \tag{2.12}$$

$$S=\frac{2\sqrt{1+B^2}}{\pi H^2\alpha'}\sqrt{1+B^2+\frac{\pi^2A^2(1+B^2)}{4K^2\left(\frac{B^2}{1+B^2}\right)}}\times\left(E\left(\frac{B^2}{1+B^2}\right)-\frac{1}{1+B^2}K\left(\frac{B^2}{1+B^2}\right)\right) \tag{2.13}$$

Now we can consider the short strings in AdS^5 corresponding to $B\ll 1$. We get

$$E\approx\frac{1}{H\alpha'}\sqrt{B^2+A^2} \tag{2.14}$$

$$S\approx\frac{1}{2H^2\alpha'}B^2\sqrt{1+A^2} \tag{2.15}$$

such that

$$\alpha' E^2\approx\frac{A^2}{H^2\alpha'}+\frac{2S}{\sqrt{1+A^2}}. \tag{2.16}$$

For $A\approx 0$, corresponding to small oscillations near one of the poles of S^5, we get

$$\alpha' E^2\approx 2S \tag{2.17}$$

as in Minkowski space. For $A\approx 1$, corresponding to oscillations between a pole and the equator of S^5, we get

$$E/H\approx\frac{1}{H^2\alpha'}+\frac{S}{\sqrt{2}} \tag{2.18}$$

while for $A\gg 1$, corresponding to high frequency oscillations between the poles of S^5, we get

$$E/H\approx\frac{A}{H^2\alpha'}+\frac{S}{A^2} \tag{2.19}$$

and the energy is now completely dominated by the contribution from the oscillations.

For $B\gg 1$, corresponding to long strings in AdS_5, we get

$$E\approx\frac{2}{\pi H\alpha'}\left(B^2+\frac{1}{2}\log B\right)\sqrt{1+\frac{\pi^2A^2}{4\log^2B}} \tag{2.20}$$

$$S\approx\frac{2}{\pi H^2\alpha'}\left(B^2-\frac{1}{2}\log B\right)\sqrt{1+\frac{\pi^2A^2}{4\log^2B}} \tag{2.21}$$

such that

$$E/H-S\approx\frac{1}{\pi H^2\alpha'}\sqrt{4\log^2B+\pi^2A^2} \tag{2.22}$$

where $B=B(S,A)$ is the solution of

$$S\approx\frac{2B^2}{\pi H^2\alpha'}\sqrt{1+\frac{\pi^2A^2}{4\log^2B}}. \tag{2.23}$$

If the logarithm dominates over A in Eq. (2.23), we have

$$B^2\approx\frac{\pi H^2\alpha' S}{2} \tag{2.24}$$

such that

$$E/H-S\approx\frac{1}{\pi H^2\alpha'}\log\left(\frac{\pi H^2\alpha' S}{2}\right)+\frac{\pi A^2}{2H^2\alpha'\log\left(\frac{\pi H^2\alpha' S}{2}\right)}. \tag{2.25}$$

This formula is valid for arbitrary A, provided that $A\ll\log(H^2\alpha' S)$, in particular, it holds for $A\approx 0$ and $A\approx 1$. If A dominates over the logarithm in Eq. (2.23), we have instead

$$\frac{B^2}{\log B}\approx\frac{H^2\alpha' S}{A} \tag{2.26}$$

such that

$$E/H-S\approx\frac{A}{H^2\alpha'}+\frac{1}{2\pi^2AH^2\alpha'}\log^2\frac{A}{H^2\alpha' S} \tag{2.27}$$

which holds for $A\gg\log(H^2\alpha' S)$.

It is interesting that Eqs. (2.25) and (2.27) are formally quite similar to the results obtained by Russo [7] if we replace the amplitude A with the shifted R charge $H^2\alpha' J+2/\pi$. But the physics is of course completely different here.

III. SPINNING IN S^5 BUT PULSATING IN AdS_5

We now consider a string which is spinning in S^5 but circular and pulsating in AdS_5. It is obtained by the ansatz

$$t=t(\tau),\ r=r(\tau),\ \beta=\pi/2,\ \phi=\sigma,\ \theta=\theta(\sigma),\ \psi=\nu\tau \tag{3.1}$$

with the remaining coordinates being arbitrary constants.

The t equation can be integrated to

$$\dot{t}=\frac{c_0}{1+H^2r^2} \tag{3.2}$$

and then the r and θ equations become

$$\ddot{r}-\frac{H^2\dot{r}^2r}{1+H^2r^2}+\frac{H^2c_0^2r}{1+H^2r^2}+(1+H^2r^2)r=0 \tag{3.3}$$

$$\theta''+\nu^2\sin\theta\cos\theta=0 \tag{3.4}$$

while the nontrivial conformal gauge constraint is

$$\frac{H^2\dot{r}^2}{1+H^2r^2}+\theta'^2-\frac{H^2c_0^2}{1+H^2r^2}+H^2r^2+\nu^2\sin^2\theta=0. \tag{3.5}$$

Again, the constraint mixes τ and σ derivatives, but the solution is easily obtained as

$$r(\tau)=\frac{1}{H\sqrt{2}}\Big(\sqrt{(1+H^2c_0^2\sin^2\alpha_0)^2+4H^2c_0^2\cos^2\alpha_0}-(1+H^2c_0^2\sin^2\alpha_0)\Big)^{1/2}\mathrm{cn}\big([(1+H^2c_0^2\sin^2\alpha_0)^2+4H^2c_0^2\cos^2\alpha_0]^{1/4}\tau\big|m\big) \tag{3.6}$$

$$\sin\theta(\sigma)=\frac{Hc_0\sin\alpha_0}{\nu}\mathrm{sn}\left(\nu\sigma\,\middle|\,\frac{H^2c_0^2\sin^2\alpha_0}{\nu^2}\right) \tag{3.7}$$

where the elliptic parameter of the r expression is

$$m=\frac{1}{2}-\frac{1+H^2c_0^2\sin^2\alpha_0}{2\sqrt{(1+H^2c_0^2\sin^2\alpha_0)^2+4H^2c_0^2\cos^2\alpha_0}}. \tag{3.8}$$

Equation (3.2) can now be integrated in terms of an elliptic integral of the third kind, but we shall not need the explicit expression. The solution (3.7) is valid for $Hc_0\sin\alpha_0\leq\nu$. There is another solution for $Hc_0\sin\alpha_0>\nu$ but it is believed to be unstable [4] so we shall not consider it here. To ensure periodicity of $\theta(\sigma)$, we have the condition

$$2\pi\nu=4K\left(\frac{H^2c_0^2\sin^2\alpha_0}{\nu^2}\right) \tag{3.9}$$

As in the previous section, it is convenient to trade the two remaining parameters (say c_0 and α_0) for two new ones

$$A=\frac{1}{\sqrt{2}}\Big(\sqrt{(1+H^2c_0^2\sin^2\alpha_0)^2+4H^2c_0^2\cos^2\alpha_0}-(1+H^2c_0^2\sin^2\alpha_0)\Big)^{1/2} \tag{3.10}$$

$$B=\frac{Hc_0\sin\alpha_0}{\nu} \tag{3.11}$$

such that A is the amplitude of $Hr(\tau)$ while B is the extension of $\sin\theta(\sigma)$. As noted in the Introduction, the amplitude is coordinate dependent. A coordinate invariant measure of the oscillations is given by the maximal circumference. This is precisely our A, up to a factor of 2π.

A straightforward computation gives the energy and R charge in this parametrization (the spin is zero)

$$E=\frac{1}{H\alpha'}\sqrt{A^2\left(A^2+1+\frac{4B^2}{\pi^2}K^2(B^2)\right)+\frac{4B^2}{\pi^2}K^2(B^2)} \tag{3.12}$$

$$J=\frac{2}{\pi H^2\alpha'}(K(B^2)-E(B^2)). \tag{3.13}$$

First consider short strings in S^5 corresponding to $B \ll 1$

$$E \approx \frac{1}{H\alpha'}\sqrt{A^2(A^2+1+B^2)+B^2} \tag{3.14}$$

$$J \approx \frac{B^2}{2H^2\alpha'} \tag{3.15}$$

such that

$$E \approx \frac{1}{H\alpha'}\sqrt{A^2(A^2+1+2H^2\alpha' J)+2H^2\alpha' J}. \tag{3.16}$$

For $A \approx 0$ we get

$$\alpha' E^2 \approx 2J \tag{3.17}$$

which is like in Minkowski space. For $A \gg 1$ we get

$$E/H \approx \frac{A^2}{H^2\alpha'} + \frac{1}{2H^2\alpha'} + J \tag{3.18}$$

such that the energy is completely dominated by the pulsation, $E \sim A^2$.

Now consider long strings in S^5 ("long" meaning extending almost down to the equator) corresponding to $B \approx 1$

$$E \approx \frac{1}{H\alpha'}\sqrt{A^2\left(A^2+1+\frac{1}{\pi^2}\log^2\frac{16}{1-B^2}\right)+\frac{1}{\pi^2}\log^2\frac{16}{1-B^2}} \tag{3.19}$$

$$J \approx \frac{2}{\pi H^2\alpha'}\left(\frac{1}{2}\log\frac{16}{1-B^2}-1\right) \tag{3.20}$$

such that

$$E \approx \frac{1}{H\alpha'}\sqrt{A^2\left(A^2+1+\frac{1}{\pi^2}(\pi H^2\alpha' J+2)^2\right)+\frac{1}{\pi^2}(\pi H^2\alpha' J+2)^2}. \tag{3.21}$$

For $A \approx 0$ we get

$$E/H - J \approx \frac{2}{\pi H^2\alpha'} + \frac{JA^2}{2}. \tag{3.22}$$

For $A \gg 1$ we must distinguish between different cases. If $A \gg H^2\alpha' J$ we get

$$E/H \approx \frac{A^2}{H^2\alpha'} + \frac{1}{2}H^2\alpha' J^2. \tag{3.23}$$

If $A \ll H^2\alpha' J$ we get

$$E/H \approx AJ + \frac{A^3}{2H^4\alpha'^2 J}. \tag{3.24}$$

The scaling relations obtained here are, to our knowledge, completely new. They generalize the ones of Gubser, Klebanov and Polyakov [4] and supplement the ones of Russo [7].

IV. PULSATING MULTISPIN SOLUTIONS

Multi spin and multi-R-charge solutions were recently obtained by Tseytlin and Frolov [17–19]. Such solutions can easily be combined with pulsation. Here we take for simplicity the simplest 2 spin solution in AdS_5 and couple it with pulsation in S^5. The ansatz is

$$t = c_0\tau, \; r = r_0 = \text{const}, \; \beta = \sigma, \; \phi = \omega\tau,$$

$$\tilde{\phi} = \omega\tau, \; \theta = \theta(\tau), \; \psi = \sigma \tag{4.1}$$

with the remaining coordinates being constants. This is a circular string in AdS_5 spinning in two different directions. It is also a circle in S^5, but pulsating there.

The r and θ equations become

$$\omega^2 = 1 + H^2 c_0^2 \tag{4.2}$$

$$\ddot{\theta} + \sin\theta\cos\theta = 0 \tag{4.3}$$

while the nontrivial conformal gauge constraint is

$$\dot{\theta}^2 + \sin^2\theta - H^2c_0^2(1+H^2r_0^2) + H^2r_0^2(1+\omega^2) = 0. \tag{4.4}$$

The θ equation and constraint are solved by

$$\sin\theta(\tau) = \begin{cases} A\,\mathrm{sn}(\tau|A^2), & A<1 \\ \mathrm{sn}(A\tau|1/A^2), & A>1 \end{cases} \tag{4.5}$$

where

$$A = H\sqrt{c_0^2 - 2r_0^2} \tag{4.6}$$

with the same interpretation as in Sec. II. The energy E and the 2 spins $S_1=S_2\equiv S$ are easily computed

$$E=\frac{(1+H^2r_0^2)\sqrt{2H^2r_0^2+A^2}}{H\alpha'} \tag{4.7}$$

$$S=\frac{r_0^2}{2\alpha'}\sqrt{2r_0^2H^2+A^2+1}. \tag{4.8}$$

For short strings (say $Hr_0\ll 1$) we get

$$S\approx\frac{r_0^2}{2\alpha'}\sqrt{A^2+1}\left(1+\frac{H^2r_0^2}{A^2+1}\right) \tag{4.9}$$

such that

$$H^2r_0^2\approx\frac{2H^2\alpha' S}{\sqrt{A^2+1}}-\frac{4H^4\alpha'^2S^2}{(A^2+1)^2} \tag{4.10}$$

which inserted into E gives

$$E(S,A)\approx\frac{1}{H\alpha'}\left(1+\frac{2H^2\alpha' S}{\sqrt{A^2+1}}-\frac{4H^4\alpha'^2S^2}{(A^2+1)^2}\right)\times\sqrt{\frac{4H^2\alpha' S}{\sqrt{A^2+1}}-\frac{8H^4\alpha'^2S^2}{(A^2+1)^2}+A^2}. \tag{4.11}$$

For $A=0$ we get

$$E\approx\frac{2\sqrt{S}}{\sqrt{\alpha'}}(1+H^2\alpha' S) \tag{4.12}$$

which to leading order is just the Minkowski result $\alpha' E^2=2(2S)$. For $A=1$ we get

$$E\approx\frac{1}{H\alpha'}(1+2^{3/2}H^2\alpha' S) \tag{4.13}$$

and for $A\gg 1$

$$E\approx\frac{1}{H\alpha'}(A+2H^2\alpha' S), \tag{4.14}$$

For long strings (say $Hr_0\gg 1$) we get

$$E/H-2S\approx\frac{\sqrt{2H^2r_0^2+A^2}}{H^2\alpha'}-\frac{H^2r_0^2}{2H^2\alpha'\sqrt{2H^2r_0^2+A^2}}. \tag{4.15}$$

Now we have to distinguish between different cases. If $Hr_0\gg A$ we get from Eq. (4.8)

$$S\approx\frac{Hr_0^3}{\sqrt{2}\alpha'}\left(1+\frac{A^2+1}{4H^2r_0^2}\right) \tag{4.16}$$

such that

$$Hr_0\approx(\sqrt{2}H^2\alpha' S)^{1/3}-\frac{A^2+1}{12(\sqrt{2}H^2\alpha' S)^{1/3}} \tag{4.17}$$

which when inserted into Eq. (4.15) gives

$$E/H-2S\approx\frac{3(\sqrt{2}H^2\alpha' S)^{1/3}}{2^{3/2}H^2\alpha'}+\frac{4A^2-1}{2^{7/2}H^2\alpha'(\sqrt{2}H^2\alpha' S)^{1/3}}. \tag{4.18}$$

This result is valid for $H^2\alpha' S\gg\{1, A^3\}$, and therefore holds in particular for $A=0$ and $A=1$. Notice also that the pulsation only gives a contribution to the nonleading terms, in this limit. On the other hand, if $Hr_0\ll A$ we get from Eq. (4.8)

$$S\approx\frac{Ar_0^2}{2\alpha'}\left(1+\frac{r_0^2H^2}{A^2}\right) \tag{4.19}$$

such that

$$H^2r_0^2\approx\frac{2H^2S\alpha'}{A}-\frac{4S^2\alpha'^2H^4}{A^4} \tag{4.20}$$

and insertion into Eq. (4.15) gives

$$E/H-2S\approx\frac{A}{H^2\alpha'}+\frac{S}{A^2} \tag{4.21}$$

which holds for $1\ll H^2\alpha' S\ll A^3$. Thus, in this limit, the pulsation completely changes the scaling relation.

All our results reduce for $A=0$ to those obtained in [17], but otherwise they are quite different. It is straightforward also to generalize the multi-R-charge solutions of [17] to include pulsation in AdS_5, but we shall not go into the details here.

We end this section with some comments about stability. Contrary to circular pulsating strings, circular spinning strings tend to be unstable. In particular for very large angular momentum. In the case of the 2-spin solution in AdS_5, corresponding to the solution considered here in the limit $A=0$, it was shown explicitly [17] that it is stable for $S\leq 3.97\sqrt{\lambda}$, with $\sqrt{\lambda}\gg 1$ in the semiclassical approximation. Thus it is stable for large angular momentum, as long as it is not "very large."

Simply by continuity, our pulsating spinning 2-spin solutions will also be stable for small A (say $A<1$, at least). What happens for large A, i.e., whether the coupling to pulsation more generally leads to stabilization or the contrary, is an open problem. A generic stability analysis is most easily performed using the world-sheet covariant approach developed in [31,32]. Unfortunately, it turns out to be extremely complicated for the solutions considered in this section, because of the nontrivial time dependent $\theta(\tau)$. As a result, we get timedependent coefficients in the 8 coupled equations for the physical perturbations, which should be compared with the case considered in [17] where the coefficients were constant in the 3 equations for the physical perturbations. At the moment, we have no solution to this problem.

V. CONCLUDING REMARKS

In conclusion, we have found several new relatively simple families of string solitons in $AdS_5 \times S^5$. They generalize some of the previously known solitons in the sense that they combine spin and pulsation in a nontrivial way. For each family, we analyzed in detail the scaling relations between energy and angular momentum. The scaling relations reduce in a certain limit ($A=0$) to previously known relations, but are otherwise quite different.

It would be interesting to find the Yang-Mills operators corresponding to the spinning pulsating strings constructed here. It seems, however, to be a highly nontrivial problem due to the coupling between spin and pulsation. The Yang-Mills operators for spinning strings were given in [4], and those for pulsating strings were suggested in [10], but it is not clear how to combine them.

Another direction which could be interesting to pursue, is to consider linearized perturbations around these solutions. Classically, this could reveal how the pulsation affects the stability properties of spinning strings, a question which could be important especially for the multispin solutions of [17]. At the quantum level, the perturbations could give a contribution to the scaling relations, similarly to the results obtained in [5,6].

These problems are currently under investigation.

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