Resultant parameters of effective theory

A. Vereshagin[1,2,*] and V. Vereshagin[2,†]
[1]*University of Bergen, Allegt. 55, N5007 Bergen, Norway*
[2]*Theoretical Physics Department, Institute of Physics, St. Petersburg State University, St. Petersburg, Petrodvoretz 198504, Russia*
(Received 25 July 2003; published 21 January 2004)

This is the fourth paper in a series devoted to a systematic study of the problem of a mathematically correct formulation of the rules needed to manage an effective field theory. Here we consider the problem of constructing the full set of essential parameters in the case of the most general effective scattering theory containing no massless particles with spin $J>1/2$. We perform a detailed classification of combinations of the Hamiltonian coupling constants and select those which appear in the expressions for renormalized S-matrix elements at a given loop order.

DOI: 10.1103/PhysRevD.69.025002 PACS number(s): 11.10.Ef, 02.30.Fn, 11.55.Hx, 12.40.Yx

I. INTRODUCTION

At first glance, the concept of an effective field theory (first formulated in [1]) looks too general to be of practical use in computing the characteristics of hadron scattering processes. In all the known cases of its application (see, e.g., [1–3]), the authors, in fact, mostly rely on the philosophy rather than on a certain computational scheme accounting for specific features of effective theories. The point is that such a scheme has not yet been developed, and many questions still require answers. At the same time, the importance of the subject is beyond question because, if constructed, such a scheme could provide us with a tool allowing us to manage the conventionally nonrenormalizable theories (see [4]).

In our previous publications ([5–8]) it was shown that, under certain conditions, it is possible to derive quite reasonable results already from the analysis of the lowest order amplitudes computed with the help of the most general effective Hamiltonian constructed from local fields describing free particles with arbitrary spins and masses. In those papers, however, many important issues concerning the details of our approach have not been explained. In particular, the solution to the problem of parametrization of scattering amplitudes was declared without proof. In this paper we discuss this issue in detail. We introduce the notion of minimal parametrization and show that the set of minimal parameters is quite sufficient for fixing the analytic form of an arbitrary complex graph. Moreover, it happens to be possible to single out those combinations of minimal parameters which are needed to fix the form of the amplitude of a given process at arbitrary high loop order. The latter combinations are called the resultant parameters. At last, we briefly discuss the problem of ordering of infinite sums of graphs describing the tree level amplitudes of binary processes (this can be generalized for more involved cases) and outline a way to construct the essential parameters—the only ones which require formulating the renormalization prescriptions.

This paper should not be considered self-contained. The reason is that it happened to be necessary to tie together several fundamental concepts that were earlier discussed in the literature. To make the reading easier it would be useful if the reader recalls first the contents of the following issues.

The concept of quasiparticles in nonrelativistic scattering problem: see [9].

The general scheme of constructing the intrinsically quantum relativistic scattering theory, canonical formalism, essential and redundant parameters, the notion of asymptotic safety: see [4] and Chaps. 2–7 in the monograph [10].

The notion of generalized renormalizability, the effective theory concept: see [1,3], and Chap. 12.3 in [10].

The problem of canonical quantization of theories with too many field derivatives: see [11,12].

Unstable particles and unitarity: see [13].

The general philosophy and methods of constructing reasonable perturbation series: [14].

Besides, it is implied that the reader is familiar with the method of Cauchy forms allowing to present the polynomially bounded meromorphic function in the form of uniformly converging series of pole contributions. The case of one variable is described in, e.g., [15,16]. The generalization for several variables is discussed in [6,8]. The main line of our approach is briefly (perhaps, too briefly) discussed in [8].

II. PRELIMINARIES

First of all we need to specify the precise meaning of the term *effective theory*. It is often understood as just a theory describing physics below some scale Λ (see, e.g., [3]). In fact, this definition tacitly implies that the corresponding perturbation series loses its meaning at energy $E\sim\Lambda$ where a kind of new physics comes into play. We would like to stress that we consider here just an opposite case. It is assumed that the discussed below general effective theory does not contain any kind of a latent inner cutoff. Owing to this, we use the term effective theory in its original meaning defined in [1]. Namely, *we call a theory as effective if the corresponding quantum Hamiltonian (in the interaction picture) takes a form of the formal infinite series containing all the local terms consistent with a given symmetry requirements.*

In this paper we are interested in consideration of the general features of effective theories. Because of this reason we do not imply the presence of any other symmetry but

*Email address: alexand@fi.uib.no
†Email address: vvv@av2467.spb.edu

 ©2004 The American Physical Society

Lorentz invariance. The problem of accounting for the requirements of dynamical (nonlinear) symmetries is briefly discussed below.

It is necessary to stress here that the given above definition is only meaningful if the quantum interaction Hamiltonian can be constructed by hands without any referring to the corresponding classical Lagrangian. This means that the canonical quantization scheme (based on the Lagrangian) cannot be considered as the basis for constructing the quantum effective theory because the most general form of classical Lagrangian must contain the terms with arbitrary high powers (and orders) of the time derivatives; in such a situation the canonical quantization looks impracticable. Because of this reason we rely upon the alternative—intrinsically quantum—scheme of constructing an effective theory. In this scheme, developed by Weinberg in the series of papers [17],[1] the structure of the Fock space of asymptotic states is postulated, and field operators are constructed in accordance with symmetry properties of those states. The Hamiltonian is also postulated as the interaction picture operator only depending on those fields and their derivatives. The *S*-matrix elements are computed with the help of Dyson's formula

$$S_{fi}=\langle f|T_W \exp\left\{-i\int H_{\rm int}\,dx\right\}|i\rangle, \qquad (1)$$

where the symbol T_W stands for Wick's T-product.[2] The noncovariant terms in the Hamiltonian and in propagators (see [10] and the Refs. quoted therein) should be neglected—in the case of effective theory this does not introduce any uncertainty because, by construction, the Hamiltonian contains *all* the terms consistent with Lorentz symmetry. This means that the total effect of noncovariant terms might, at most, result in a renormalization of some coupling constants.

Thus we see that Weinberg's scheme happens well suited for constructing the effective field theory Hamiltonian. However, there is one problem revealing itself when this scheme is used to describe the hadron dynamics. The point is that in this scheme the Hamiltonian contains those and only those field operators which correspond to the states of stable particles. *Weinberg's scheme is adapted to describe the scattering processes with true stable particles solely in terms of the corresponding creation and annihilation operators; the possibility to describe the physics of resonances in the framework of this scheme looks questionable.* Fortunately, this problem is quite solvable. The results obtained in [13] show that, in the case when the Hamiltonian contains the fields of unstable particles,[3] the formal construction (1) remains applicable. In this case it defines the *S*-matrix as the unitary operator on the space of stable particle states. The fields of unstable particles do not create true asymptotic states; they can be treated as the fields describing resonances. In our next paper we will have to say more about this scenario. For the present we just shut our eyes to the existence of the problem of interpretation and consider in this paper the most general effective Hamiltonians constructed from the infinite set of fields corresponding to free particles with arbitrary spins and masses.

Another problem connected with Weinberg's scheme is that of nonlinearly realized (dynamical) symmetries.[4] It is extremely difficult (if ever possible) to formulate the conditions providing a guarantee of the desired dynamical symmetry properties of amplitudes resulting from the effective quantum Hamiltonian written in the interaction picture. Surely, this difficulty is explained by the fact that neither free nor interaction Hamiltonian by itself commutes with the dynamical symmetry generators. The solution (at least, partial) to this problem can be obtained from the results of [1]. In that paper it is shown that, leaning upon the canonical quantization scheme and using the minimal invariant Lagrangian (that containing the minimal number of field derivatives required by symmetry), it is possible to calculate the lowest order terms in series expansion (in small momenta) of the amplitude describing a process with Goldstone bosons. This means that the dynamical symmetry requirements can be formulated—at least, in lowest orders—directly in terms of amplitudes; one has no necessity in formulating them on the Hamiltonian level.[5] In turn, this means that, in order to account for the dynamical symmetry requirements in the framework of effective theory, one needs to compute amplitudes of the processes involving Goldstone bosons and then compare the results with those obtained from the canonically quantized invariant classical Lagrangian of the lowest order. This very approach has been used in [5] and [6] to derive the restrictions imposed by Chiral $SU_2\times SU_2$ symmetry on the structure of meson resonance spectrum. The answer to the question on how to write down the restrictions imposed by a certain kind of dynamical symmetry on the higher order amplitudes still remains unclear. In this paper we do not discuss this point.

One note is in order. In what follows we assume that the effective theory under consideration does not contain massless particles of higher spin $J>1/2$. This is just a technical assumption, but at the moment we do not know how to avoid it.

III. CLASSIFICATION OF THE PARAMETERS

The effective Hamiltonian contains all the types of local terms consistent with Lorentz symmetry. For example, along with the simple interaction term ϕ^4, it contains also the terms of the form $\phi^2\partial_\mu\phi\partial^\mu\phi$, $\phi^2\partial_{\mu\nu}\phi\partial^{\mu\nu}\phi$, $\phi\partial_\mu\phi\partial_\nu\phi\partial^{\mu\nu}\phi$, ϕ^5, and so on. This means that many Hamiltonian coupling constants contribute to the same kinematical structure in the amplitude of a given process (say, to the term $\sim s^2$ in the tree-level amplitude of the process 2

[1]See also Chaps. 2–5 of the monograph [10].

[2]It is explicitly covariant—see, e.g., [12].

[3]Those with masses large enough to make it possible for the decay into lighter particles.

[4]In the case of linear (algebraic) symmetry there is no problem at all.

[5]An example is provided by famous Low's theorems in QED.

FIG. 1. Examples of 1- and 2-loop order vertices.

$\rightarrow 2$). Hence, to perform the renormalization program, one needs first to solve the problem of classification of couplings in order to avoid attracting unnecessary (dependent) counterterm vertices. Another reason, explaining why the solution to this *problem of couplings* might be extremely useful, is the following. As known (see, e.g., [10,18]), the most difficult problem connected with renormalization of effective theories (which are renormalizable by the very construction) is the necessity to formulate an infinite number of renormalization prescriptions needed to fix the finite parts of counterterms. This looks impracticable until one finds a regularity effectively reducing the number of independent prescriptions. It seems quite natural to look for the mathematical expression of such a regularity in terms of *independent* parameters appearing in a theory.

In as much as we are only interested in describing the scattering processes,[6] it looks reasonable to work in terms of the parameters appearing in *S*-matrix elements. Those parameters are the functions of Hamiltonian coupling constants. Clearly, the Green functions of a theory depend on the same parameters as the *S*-matrix elements do; but, in addition, they may depend on the orthogonal combinations only contributing off the mass shell. Hence it makes sense to classify the parameters as *essential* and *redundant* ones (see Chap. 7.7 of Ref. [10]). We follow the general line of this classification but we find it necessary to make more precise definitions of the terms.

First, we work with the quantum Hamiltonian in the interaction picture. In contrast, the definitions in [10] refer to the Lagrangian coupling constants. As we have already noted, in the case of effective theory the simple connection between the canonical Lagrangian and quantum Hamiltonian approaches is lost and there is no real possibility to express the Hamiltonian parameters in terms of the Lagrangian ones. Second, in contrast with [10], we classify the parameters appearing in the expressions for *S*-matrix elements of a given loop order, not only those in the Hamiltonian. The reason for elaborating the more detailed classification of the effective theory parameters is that the form of dependence of matrix elements on the Hamiltonian coupling constants depends of the loop order in question. Hence it looks quite natural to elaborate a classification of parameters appearing in amplitudes of effective theory at a given order of loop expansion. As shown below, it happens possible to point out the set of independent[7] parameters (combinations of coupling constants) quite sufficient to describe all the *S*-matrix elements of a given order.

Because of all these reasons we use the following definitions. The independent combinations of Hamiltonian coupling constants needed to fix the kinematical structure of all the renormalized *N*-loop Green functions of a given effective theory we call as just the *N*th level parameters. We separate them into two groups. The first group only contains those combinations of the parameters which do not appear in the expressions for renormalized *S*-matrix elements. We call these combinations as the *redundant parameters of the Nth level*. All the other independent combinations we collect in the second group and call them as the *resultant parameters of the Nth level*. The term *essential* is reserved for those combinations of resultant parameters which appear in the *well-defined* (converging) series presenting the amplitudes of a given loop order in certain kinematical domains.

The similar classification applies also for the parameters appearing in the expression for *pointlike vertex of the Nth loop order*: that, containing self-closed lines—bubbles or/and tadpoles—which, in turn, may have complex multiloop inner structure (see Fig. 1), the total number of loops being *N*. The presence of an arbitrary number of such bubbles (or/and tadpoles) does not change analytic structure of the vertex,[8] it only may change the numerical coefficients in the corresponding polynomials (series).

Sometimes, it is convenient to classify in the same way the parameters appearing in the Hamiltonian. In this case we use the term Hamiltonian level parameters (effective, minimal, nonminimal).

Clearly, the full set of the parameters needed to describe the amplitude of a given process (at a given order of loop expansion) is exhausted by the power series expansion coefficients around an arbitrary nonsingular point in the space of corresponding kinematical variables. The problem is that the full collection of such sets necessarily contains dependent parameters because general principles (causality, crossing, etc.) impose certain limitations on its structure. This is the reason why we work with pointlike vertices of different loop orders and classify the coefficients appearing in corresponding analytical expressions.

For the following it is also useful to introduce the notion of the *effective vertex*. Let us consider a formal sum of all the Hamiltonian monomials constructed from a given set of *n* fields and differing from one another by the total number and/or positions of differential operators ∂_μ (for example, $\phi^2\partial_\mu\phi\partial^\mu\phi, \phi^2\partial_{\mu\nu}\phi\partial^{\mu\nu}\phi, \phi\partial_\mu\phi\partial_\nu\phi\partial^{\mu\nu}\phi, \ldots$). Each one of these monomials corresponds to an individual vertex (polynomial in kinematical variables) in the system of Feynman rules. It happens convenient to consider the infinite sum of all such vertices. It takes a form of infinite formal series in powers of variables. We call this series as the effective vertex of the Hamiltonian order. The Hamiltonian can be rewritten in the form of an infinite sum of effective vertices, the single items differing from one another by the number or/and by

[6]In other words, we are interested in constructing the *effective scattering theory*.

[7]As long as coupling constants in the effective Hamiltonian are considered independent.

[8]It is just a polynomial or power series in kinematical variables.

quantum numbers of field operators. Hence, the full sum of Feynman graphs (of a given loop order) describing the amplitude of a given process can be always presented as a sum of graphs written in terms of effective vertices of the Hamiltonian order. In what follows we imply tacitly that this is done. The problem of convergence of formal infinite sums will be discussed in Sec. VI.

The notion of the effective vertex of Nth loop order is introduced in much the same way—this point is considered in more detail in Sec. IV. The coefficients appearing in corresponding series we call as the Nth *level effective parameters*.

By construction, the effective theory Hamiltonian contains an infinite number of coupling constants, only a part of them (or, better, their combinations) contributing to S-matrix elements. We do not need to compute all the Green functions of a theory because we are only interested in the amplitudes of various scattering processes. This means that for our purpose it is quite sufficient to consider theories only renormalizable in the sector of essential parameters. The divergences in Green functions unrelated to S-matrix elements[9] will never bother us.

Thus we need to select the set of essential parameters. This cannot be done through just a classification of coupling constants appearing in the Hamiltonian. The reason is that, except for a few trivial cases, both essential and redundant parameters are very complicated functions of the Hamiltonian coupling constants G_i. Suppose for a moment that all such functions are constructed and classified as essential (E_i, $i=1,2,\ldots$) and redundant (R_i, $i=1,2,\ldots$) parameters. This would provide us with an infinite system of algebraic equations of the form

$$E_i=E_i(G_1,\ldots),\quad R_j=R_j(G_1,\ldots)\quad (i,j=1,2,\ldots). \tag{2}$$

Resolving this system with respect to G_i, one obtains

$$G_i=G_i(E_1,\ldots,R_1,\ldots)\quad (i=1,2,\ldots). \tag{3}$$

Hence, when dealing with S-matrix elements, one can assign to R_i whatever values are convenient for computations. In particular, there is no necessity in formulating the renormalization prescriptions fixing the finite parts of redundant counterterms. In turn, this is especially useful if we are interested in finding a regularity allowing to put in order the infinite system of normalization prescriptions needed to compute amplitudes of various scattering processes in the framework of effective theory.

Thus we see that it would be extremely useful if we find a way to write down the explicit form of the relations (2). We do not know how to solve this problem in general. Instead, one can try to find a *perturbative* solution providing the required relations at every fixed order of loop expansion. To realize this idea, one needs to perform certain reconstruction (reparametrization) of the initial Dyson series. Below we describe a special kind of parametrization which serves this purpose.

We imply tacitly that there exists a regularization consistent with all the desired symmetries. This suggestion looks harmless if the Euclidean version of a theory is considered. However, if one works in Minkowski space (as we do), it seems much less trivial. Nevertheless, we believe that it is true.

This classification happens especially convenient when one is only interested in computing the renormalized S-matrix elements in the effective theory framework. As far as we know, the special role of essential parameters has been first stressed in [4].

The logical line of subsequent consideration is the following. We start from the basic effective Hamiltonian written in terms of the physical masses and physical couplings[10] (plus counterterms). Next, we note that some combinations of G_i (their forms depend on the loop order in question) certainly contribute to measurable quantities and thus could be considered, at least, as building blocks for essential parameters. Then we prove that it is always possible to rewrite the expression for arbitrary graph of a given loop order in such a way that the renormalized S-matrix only depends on those latter combinations called below as *minimal parameters*. Further, we show that the full sum of graphs of the same loop order can be rewritten in a form quite similar to that constructed on the previous step for an individual graph with a fixed set of internal lines. The parameters appearing on this stage are called the *resultant parameters*. At last, we direct the way allowing to construct all the essential parameters as certain infinite sums of the resultant ones. This result shows that, when dealing with effective scattering theory, it is always possible (at least, in principle) to make use of the scheme of renormalized perturbation theory only appropriate in the sector of essential parameters.

IV. MINIMAL PARAMETERS

The immediate task we are going to solve is to prove the following statement. *The full set of the essential parameters of effective theory is constructed solely from those combinations of the Hamiltonian coupling constants (including masses) which are needed to fix the independent on-shell kinematic structures appearing in the expressions for effective vertices (of different orders) multiplied by the relevant wave functions*. In fact, this statement is almost trivial but its precise meaning deserves comments. This section is devoted to the preliminary consideration needed for better understanding of the proof given in Sec. V.

The proof consists of two steps. First, we show that all

[9]An excellent example of such divergences is provided by the Standard Electroweak Model in the unitary gauge.

[10]When speaking about the mass of unstable particle, it is more appropriate to use the term renormalized. In fact, this implies using the renormalized perturbation theory with the conventional OMS (on-mass-shell) normalization conditions (see, e.g., [18–22]). The quotation marks are used to stress that only certain combinations of the Hamiltonian parameters present measurable quantities.

invariant scalar structures (form factors), describing a given vertex in arbitrary *S*-matrix graph, can be reduced to a simpler form (called minimal). Second, we show that it is always possible to reduce the full set of tensor structures needed to fix the form of this vertex to a subset only containing a part of them (also called minimal). The corresponding procedure—called below as the graph reduction—eliminates certain part of the parameters which we call as nonminimal. When applied to a given graph, it results in the sum of two items. The first one is just the initial graph written in terms of new—minimal—vertices (of different orders) completely described by the relevant minimal parameters. The second item does not contribute to the renormalized *S*-matrix under the condition that the normalization point is taken on mass shell.

We would like to note that the reduction procedure is only needed to prove the completeness of the full set of minimal parameters. We do not imply its using in practical calculations, which can be started already in terms of minimal parameters.

It is a point here to stress the difference between two terms often used throughout the paper. The term *on-shell graph* means that the graph in question (say, pointlike vertex) is computed at all external momenta on the mass shell. The term *S-matrix graph* (or, the same, *amplitude graph*) means that the on-shell graph is dotted by the relevant wave functions. The difference between the corresponding expressions manifests itself in the case when particles with spin $J\neq 0$ are considered.

Now we need to explain the precise meaning of the term *minimal* (minimal vertex, minimal propagator). The reason why we use one more special term in addition to those defined above (essential, redundant) is explained by the following circumstance. The difference between the essential and redundant parameters manifests itself when one considers the structure of the amplitude of a given scattering process. This amplitude results from contributions of many different *S*-matrix graphs of a given loop order. Thus the essential parameters of a given level happen constructed from the Hamiltonian coupling constants describing the vertices with different numbers of field operators. This language is not suitable for discussing the problems of renormalization. That is why we need the more detailed classification of various combinations of the Hamiltonian coupling constants appearing in the process of calculation of a given graph.

Consider an effective vertex $V_{\dots}(p_1,\dots,p_n)$ (the ellipses stand for Lorentz indices) with n lines carrying the momenta $(p_1,p_2,\dots,p_n)$ only restricted by the conservation law. As explained in the preceding section, this vertex corresponds to an infinite sum of monomials in the Hamiltonian. Each monomial is constructed from fields and their derivatives, the total number of field operators being n. The explicit expression for this vertex reads

$$V_{\dots}(p_1,\dots,p_n)=\sum_{a=1}^{M+N} T^{(a)}_{\dots}F_a, \tag{4}$$

where $T^{(a)}_{\dots}$ stand for whatever independent tensor structures needed (their total number is denoted as $M+N$) and F_a—for the corresponding scalar form factors (formal power series in invariant kinematical variables).

It is pertinent to remind that the expression (4) is equally applicable in the case if we consider the pointlike vertex of the Lth order. In accordance with the definition given in Sec. III, the corresponding coefficients of formal power series for F_a are called as the Lth level parameters.

Further, choose a set of independent scalar variables (their total number is $4n-10$) as follows:

$$[\pi_1,\dots,\pi_n;\nu_1,\dots,\nu_{3n-10}]. \tag{5}$$

Here

$$\pi_i\equiv p_i^2-m_i^2,$$

and ν_r stand for the rest (arbitrarily choosen[11]) independent linear combinations of scalar products

$$\nu_r\equiv\sum_{i,j=1}^{n} s^r_{ij}(p_i\cdot p_j),\quad (r=1,\dots,3n-10) \tag{6}$$

with numerical coefficients s^r_{ij}.

It is always possible to rewrite F_a as follows:

$$\begin{aligned}F_a(\pi_1,\dots,\pi_n;\nu_1,\dots,\nu_{3n-10})&\\ =F_a(\pi_1,\dots,\pi_{i-1},0,\pi_{i+1},\dots,\pi_n;\nu_1,\dots,\nu_{3n-10})&\\ +\pi_iP_a(\dots).&\end{aligned}$$

Thus the vertex under consideration takes a form of a sum of two items:

$$V_{\dots}(p_1,\dots,p_n)=\sum_a T^{(a)}_{\dots}[F^{(i)}_a+\pi_iP_a]. \tag{7}$$

The scalar functions $F^{(i)}_a$ appearing in the first term are called minimal with respect to the ith line. They do not change their form when this line is put on its mass shell. The second—nonminimal—term vanishes in this case.

We call the propagator as minimal if its numerator is just a spin sum written in a covariant form and considered as a function of four independent variables p_μ. The nonminimal propagator differs from the minimal one by nonpole terms.[12] In what follows we imply using the minimal propagators. This does not reduce the generality of our analysis because nonpole terms result in precisely the same effect as that caused by nonminimal parameters. This will become more clear after reading the next section. Besides, as shown in [5] and [6], in practical calculations in the framework of the Cauchy form techniques one only needs to know the residues of propagators.

Next, let us consider the tensor structures $T^{(a)}_{\dots}$ occurring in (4). They may contain the factors (γ matrices, tensors $g_{\mu\nu}$

[11]The problem of appropriate choice of those variables will be discussed in more detail in a separate publication.

[12]It is this point where our suggestion on the absence of massless particles of higher spin becomes important. Note also, that the definition of the minimal propagator given above is not quite exact, though sufficient for current consideration; we will discuss it in subsequent papers.

and $\varepsilon_{\alpha\beta\gamma\delta}$, momentum p_i^μ) resulting in constants when the line in question is put on the mass shell and dotted by the corresponding wave function. We call such factors as nonminimal (with respect to a given line). For example, if the line under consideration corresponds to a vector particle (with momentum p_i), every tensor structure containing p_i^μ is classified as nonminimal.

The full set of independent tensor structures $T^{(a)}_{\dots}$ can be separated into two groups as follows:[13]

$$T^{(a)}_{\dots}=\{T^{(1,i)}_{\dots},\dots,T^{(M_i,i)}_{\dots};R^{(1,i)}_{\dots},\dots,R^{(N_i,i)}_{\dots}\}. \quad (8)$$

Here, the first group

$$T^{(k,i)}_{\dots} \quad (k=1,\dots,M_i;\ \ 1\leqslant i\leqslant n) \quad (9)$$

does not contain any nonminimal (with respect to ith line) structures, while the second one

$$R^{(k,i)}_{\dots} \quad (k=1,\dots,N_i;\ \ 1\leqslant i\leqslant n) \quad (10)$$

consists of all such structures. The structures from the first group are called minimal (with respect to the given line). The meaning of this separation is explained by the fact that, when dotted by the relevant propagator or wave function, the nonminimal structures result in the same terms as the minimal ones or/and in terms proportional to π_i. In other words, the effect of nonminimal tensor structures is quite similar to that of nonminimal parameters appearing in scalar form factors.

By way of illustration, let us consider the case of nonminimal structure containing the factor p_μ corresponding to a vector particle (with 4-momentum p). If the line in question is external, this structure does not contribute to *S*-matrix due to the transversality of the vector particle wave function. In the case of internal line this factor is multiplied by the vector particle propagator. The resulting expression does not contain a pole:

$$p_\mu\frac{-g^\mu{}_\nu+p^\mu p_\nu/M^2}{p^2-M^2}=\frac{1}{M^2}p_\nu.$$

This means that inside a graph the nonminimal structure plays a role of pole killer.

Note, that nonminimal structures never survive as independent items in the expressions for scattering amplitudes.

The vertex is called as minimal if it is minimal with respect to all its lines and the corresponding expression does not contain any nonminimal tensor structures. The algebraic form of Lorentz invariant expression for minimal vertex does not change its appearance when the momenta are considered on the mass shell $\pi_i=0$ $(i=1,\dots,n)$.

The explicit form of the minimal vertex differs from that of nonminimal one by the items proportional to $(p_i^2-m_i^2)$ or/and by those proportional to nonminimal (at least, with respect to one of the lines) tensor structures.[14] Inside a graph such terms work as pole killers. This very property provides a basis for the statement formulated in the beginning of this section.

The instructive example where the difference between minimal and nonminimal elements (vertices and propagators) manifests itself explicitly (and happens important) is provided by the conventionally used propagator and interaction Hamiltonian of the spin-3/2 Rarita–Schwinger field (see, e.g., [23] and [24] and references therein). This field corresponds, in particular, to the well established resonance $\Delta(1232)$ playing an important role in low energy pion-nucleon processes. Because of this reason this field is often used in various Lagrangian models. The problem appears when different authors use different forms of the interaction term and (or) propagator, this difference sometimes leading to contradictive results.[15] The most popular forms of those elements used in the literature differ from one another by the terms resulting in a pole killer. This difference produces an additional (smooth) contribution to the amplitude which, in turn, changes the results of data fitting. This is just an artifact of Rarita–Schwinger formalism.[16] Surely, the pole term is the same in both cases, the residue being just a spin sum. The so-called off-shell couplings turn out to be redundant (see [23] and [26]).

V. THE PROOF OF THE STATEMENT

To prove the statement formulated in the beginning of the preceding section, it is sufficient to show that *an arbitrary S-matrix graph can be rewritten in the form only constructed from the minimal vertices of different orders plus the terms which do not contribute to the renormalized S-matrix*.

The proof is straightforward. Consider an arbitrary complex graph[17] (amputated Green function) constructed in accordance with Feynman rules derived from the effective theory Hamiltonian.[18] Further, consider the inner line q connecting the vertices $V_1{}^{\mu\dots}(p_1,\dots,p_n,q)$ and $V_2{}^{\nu\dots}(k_1,\dots,k_m,q)$ (see Fig. 2). We do not make any suggestions about the other lines: a part of them may be taken external while the rest may be taken internal.

First, let us consider the case when this line is the only one connecting the vertices in question and the propagator contains nonminimal terms

[13]The numbers M_i and N_i depend on spin of the line in question; the total number M_i+N_i of tensor structures only depends on the vertex type.

[14]This is also true with respect to the vertices containing self-closed lines (bubbles or tadpoles). As explained in Sec. III, we classify such vertices as pointlike.

[15]For the references and discussion see, e.g., [25] and [26].

[16]This would not occur if Weinberg's formalism [17] for the spin-J field were used.

[17]Regularization is tacitly implied.

[18]It is important that we consider a graph constructed from a fixed set of effective vertices of the Hamiltonian order; no summation over the different types of inner lines as well as over different types of the effective vertices is implied on this stage.

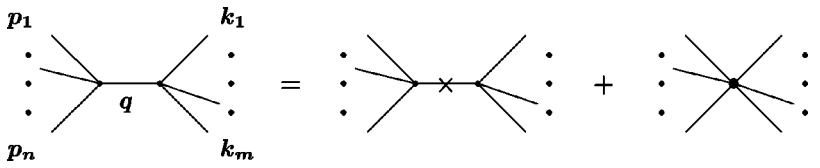

FIG. 2. Line reduction procedure.

$$\mathcal{P}^{\cdots}_{,,,}(q)=\frac{1}{q^2-m^2}\left(\Pi^{\cdots}_{,,,}(q)+\underbrace{(q^2-m^2)\phi^{\cdots}_{,,,}(q)}_{\text{nonminimal terms}}\right).$$

Here $\phi^{\cdots}_{,,,}(q)$ is some nonsingular tensor and $\Pi^{\cdots}_{,,,}(q)$ is just a spin sum written in a covariant form and understood as a function of four *independent* components of momentum. Besides, let us write down the vertices in the form (7) explicitly showing the presence of nonminimal terms in scalar form factors:

$$V^{\cdots}_{1,2}(\ldots,q)=\sum_a T^{(a)\cdots}[F^{1,2}_a+(q^2-m^2)P^{1,2}_a]$$

(in what follows we omit tensor indices).

It is easy to understand that nonminimal terms just kill the denominator of the propagator and thus result in a new quasivertex with $(n+m)$ lines $(p_1,\ldots,p_n,k_1,\ldots,k_m)$. In other words, one can represent (rewrite) the graph in the following way [below $\delta(\cdots)$ denotes the momentum conservation delta function needed for each vertex]:

$$\begin{aligned}&\cdots\int dq\ \delta(\cdots)\delta(\cdots)V_1\mathcal{P}V_2\\&=\cdots\int dq\ \delta(\cdots)\delta(\cdots)\left(\sum_a T^{(a)}F^1_a\right)\\&\quad\times\frac{\Pi(q)}{q^2-m^2}\left(\sum_b T^{(b)}F^2_b\right)\\&\quad+\cdots\underbrace{\delta\left(\sum p_i-\sum k_i\right)\int dq\ \delta\left(\sum p_i-q\right)}_{\text{new vertex}}\cdots,\end{aligned}$$

where ellipses before the integral stand for the rest part of the graph. Besides, the minimal elements of V_1 and V_2 transform the line q into a new one (minimal, labeled by a cross). Thus, the initial graph gets transformed into two new ones. The first graph has the same structure as the initial one except that the forms of the vertices V_1 and V_2 have been changed—the terms proportional to q^2-m^2 disappeared and the minimal propagator appeared in place of the nonminimal one. The second graph has quite a different structure: the new pointlike quasivertex with $(n+m)$ lines has appeared in place of two original ones—V_1 and V_2. This quasivertex does not follow from the Feynman rules based on the effective Hamiltonian. Nevertheless, it has precisely the same analytic structure as that of "true" vertex with the same number of lines. The only difference is that the crossing symmetry properties may be broken if the initial graph was not properly symmetrized with respect to the lines under consideration. Clearly, this difficulty would never appear if—in place of single graph—we consider the symmetric sum of all its topological copies. Below we imply that this is the case. This means that the effect of nonminimal terms results in a symmetric sum of corresponding quasivertices. We call this sum the *secondary vertex of order zero* (or, the same, tree order secondary vertex). Recall, that we are dealing with an effective theory, hence all possible vertices are already included. Thus our procedure (later on we call it the reduction of a given line) only leads to a renormalization of the parameters fixing the form of the Hamiltonian order effective vertex with $(n+m)$ lines.

The case when there are two lines (q_1 and q_2) connecting the vertices under consideration can be analyzed precisely in the same way as above. The result is illustrated in Fig. 3 (for simplicity, here V_1 and V_2 are taken to be four-vertices).

So, in this case the reduction of both lines results in a sum of two kinds of graphs (see Fig. 3).

(1) The same graph as the initial one but with two crossed (minimal) lines in place of two original ones.
(2) Three graphs with pointlike vertices dotted by the factors stemming from crossed or uncrossed self-closed lines and from the effect of pole killers.[19] Purely for the sake of uniformity, one can further rewrite the graph with uncrossed bubble as a sum of two items: the same graph as the initial one but with the crossed bubble in place of uncrossed one plus the reminder caused by the effect of relevant pole killers. We would like to stress that each one of these pointlike graphs should be considered as the 1-loop order graph whether or not the bubble is drawn explicitly (see the last of graphs shown in Fig. 3).

Proceeding in the same way one can realize that, in the case when there are l lines connecting V_1 and V_2, the reduction procedure creates the same two vertices with l minimal lines in place of the original ones. Besides, it creates a set of vertices with more (also minimal) external lines and $n<l$ bubbles (visible or/and invisible) some of which, in turn, may present a complex loop structure. It is important that these new vertices possess the pointlike kinematical structure.

In order to preserve the loop counting rules, we use special terms for the sum of secondary quasivertices resulting from the reduction of one of l lines connecting two vertices under consideration. This sum can be considered as a single *secondary vertex of the* $(l-1)$*th order*. In general, the pointlike vertex with several bubbles (tadpoles), having in total

[19]When the self-closed line corresponds to a particle with spin $J\neq 0$ these factors may result in additional reparametrization.

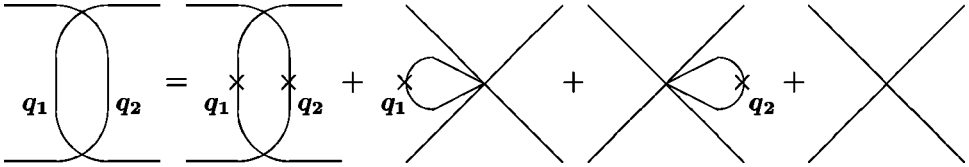

FIG. 3. Example of reduction of two adjacent lines.

L-loops, is called the secondary vertex of the Lth order. For example, the sum of three pointlike graphs depicted in Fig. 3 is defined as a single secondary vertex of the first order.

When continued for all internal lines of a given graph, the reduction procedure results in a sum of graphs constructed from minimal propagators and pointlike vertices (with different number of bubbles) in which all the lines are minimal except those happened to be external in the initial graph. But each one of these latter lines may be internal in the case if a given vertex appears also in the inner part of the graph. To avoid inconsistency, let us present the vertices connected with external lines in the form (7):

$$V_{\dots}(p_1,\dots,p_n)=\sum_a T^{(a)}_{\dots}[F^{(i)}_a+\pi_i P_a].$$

This results in a sum of graphs which can be divided into two groups. The first group consists of all graphs constructed solely in terms of minimal propagators and minimal vertices of different orders. In contrast, every graph from the second group contains at least one vertex of the type P_a connected with one of the external lines (say, ith) and dotted by the factor π_i corresponding to this line.

Graphs from the second group (let us call them nonminimal) do not contribute to the amplitude of the process under consideration. Nevertheless, they cannot be simply neglected. The point is that those graphs might result in nontrivial contributions of two different kinds. First, they contribute to the amplitudes (of the same loop order as that in question) corresponding to the processes involving more particles. Second, they can contribute to the values of renormalization constants.

The contribution of nonminimal graphs (with a given number of external lines) to the amplitudes of the processes involving more particles can be rewritten in terms of minimal parameters precisely in the same way as above. In what follows we tacitly imply that this is done with respect to all S-matrix graphs of the loop order L under consideration.

As to the influence of nonminimal graphs on the values of renormalization constants, it happens irrelevant if we are only interested in the corresponding S-matrix elements of a given order and, in addition, rely on the conventional OMS renormalization scheme (see, e.g., [20] and [21]). In this case one can simply forget about this group of graphs because the only quantities depending on their parameters are the wave function renormalization constants[20] which, in turn, are just redundant parameters having no influence on renormalized S-matrix elements of the order in question. In the opposite case, when one needs to calculate the amplitudes of the loop order $(L+1)$, the Lth order nonminimal graphs cannot be neglected. Instead, they must be taken into account when constructing the next order graphs which, in turn, should be further subjected to the reduction procedure. The important point is that, after this is done, the parameters appearing in nonminimal graphs of the Lth loop order will happen absorbed into the structure of minimal parameters describing the vertices of the order $(L+1)$, this being also true with respect to Lth order nonminimal counterterms. From this note it follows the important conclusion: *to obtain finite results for S-matrix elements in the framework of effective theory, one has no need in formulating the normalization conditions fixing the finite parts of nonminimal counterterms*.

Now, the first step is done. We have shown, that it is always possible to pick out certain group of parameters which do not produce the kinematically independent contributions to renormalized amplitudes at a given order of loop expansion. So, from this point we can consider the scalar form factors F_a being minimal with respect to each line. This, in turn, means that they only depend on kinematical variables (6), the dependence on π_i may be dropped.

We would like to stress once more that the above analysis is only true in the framework of OMS renormalization scheme: the renormalization point must be taken on mass shell. It is this condition which allowed us to consider both external and internal lines on the same footing. In turn, this means that for unstable particles the Hamiltonian mass parameters may happen only indirectly connected with pole positions of the corresponding full propagators (see [20] and [21]).

Thus, in order to calculate the amplitude of a given scattering process up to a given order of loop expansion, one only needs to formulate the normalization prescriptions for the remaining group of parameters. However, as yet this cannot be done in terms of measurable quantities because this latter group still contains the redundant combinations. To reveal them we need to consider the influence of nonminimal tensor structures.

Let us rewrite each of the vertices V_1, V_2 as follows:

$$V_{\dots}(p_1,\dots,p_n)=\sum_{a=1}^{M} T^{(a)}_{\dots}F^t_a+\sum_{a=1}^{N} R^{(a)}_{\dots}F^r_a. \quad (11)$$

The first sum in (11) contains all the independent minimal (with respect to each of the lines) tensor structures $T^{(a)}_{\dots}$, while the second one contains all the other independent structures (nonminimal, at least, with respect to one of the lines). This means that every coefficient of the polynomials (series)

$$F^t_a(\nu_1,\dots,\nu_{3n-10})$$

[20]This relates to the case of self-energy graphs.

presents a measurable quantity.[21] This is so just because each one of those coefficients results in the individual kinematical structure in the amplitude.

Hence we conclude that all the combinations of coupling constants appearing (as expansion coefficients) in the invariant form factors F_a^t should be classified as building blocks for the essential parameters.

Now, let us consider the parameters from the second group, namely, those appearing in the form factors $F_a^r(\nu_1, \ldots, \nu_{3n-10})$ describing the contributions of nonminimal tensor structures $R^{(a)}_{\ldots}$. Below it is shown that the effect produced by this group is reduced to just a renormalization of the minimal parameters [those appearing in "minimal" form factors $F_a^t(\nu_1, \ldots, \nu_{3n-10})$].

For simplicity, we only consider here the case of structures of the bosonic type. The generalization for fermions is straightforward. To describe the fields with spin $J \neq 0$ we use the conventional Rarita–Schwinger formalism [27] and rely upon the method of contracted projecting operators (see, e.g., [17,28,29]). The corresponding wave functions $\epsilon_{\mu_1 \ldots \mu_j}(j,q)$ (here q stands for momentum and j for polarization) possess symmetry, tracelessness and transversality properties.

First, consider the case when one of the lines of the vertex (say, V_1) is external and corresponds to a particle with spin $J \neq 0$ and momentum p. We are only interested in nonminimal tensor structures, hence the relevant expression necessarily contains the terms of the form

$$p_{\mu_1} \ldots p_{\mu_J}, \quad g_{\mu_1\mu_2} p_{\mu_3}, \ldots, p_{\mu_J}, \quad \ldots .$$

The corresponding amplitude graph equals zero.

Now, consider the case when this line is internal. Keeping in mind that the numerator of the propagator is just a spin sum (written in covariant form and considered as a function of four independent components of momentum), it is easy to understand that nonminimal tensor structures result in polynomial contributions. This follows from the fact that in this case the residue equals zero due to the properties of spin sums. The symmetry, tracelessness and transversality[22] properties (only valid on the mass shell) play precisely the same role as pole killers discussed above. Thus we conclude that the only effect produced by nonminimal tensor structures is reduced to a renormalization of the coefficients in invariant form factors F_a^t. This may result in reappearing of the variables π_i but now we know how to manage this problem: it is sufficient to repeat the reduction procedure once more.

Thus it is shown that, at every fixed order L of loop expansion, the contribution of an arbitrary graph to the amplitude of a given process can be rewritten solely in terms of minimal parameters of the Lth and lower levels. Hence, all the essential parameters of the Lth level are constructed solely from those minimal parameters. Note, that no distinction between the basic and counterterm vertices has been made in the course of our analysis.

In particular, this means that, when calculating the amplitude of pion–nucleon scattering in a framework of effective theory, one can use the nonchiral interaction Hamiltonian $\overline{N}\gamma_5 N\pi$: this does not necessarily lead to a contradiction with chiral invariance.

We will say that the amplitude graph[23] of a given (true) loop order L is presented in the *minimal (or, unitary) parametrization*, if it is rewritten in terms of minimal propagators and minimal vertices of different orders $l \leqslant L$. The graph constructed solely from minimal elements we call as the *minimal graph*.

As follows from the above analysis, the reduction procedure transforms a given L-loop graph, constructed in accordance with conventional Feynman rules, into a sum of minimal graphs of different topological structure plus the sum of graphs with at least one nonminimal external line. When drawing the minimal graphs, it is convenient to supply every vertex V_i with the special index l_i showing its order. The value $l=0$ should be assigned to all the initial Hamiltonian vertices as well as to the secondary vertices of the tree level.[24] Under this condition, the true loop order L of the minimal graph with $L_{\min}$ loops and p vertices $V_1, \ldots, V_p$ of orders $l_1, \ldots, l_p$ equals

$$L = L_{\min} + \sum_{i=1}^{p} l_i .$$

The corresponding counterterm vertex V_c should be supplied with the index $l_c = L$. The important point is that, *as far as we consider all the Hamiltonian couplings as independent constants, the minimal parameters describing vertices of different orders are also independent*. This statement can be easily proved by induction.

The special convenience of dealing with minimal parametrization becomes clear from the following note. In the cases of customary finite-component renormalizable theories (as well as their infinite-component vector copies) one needs to formulate as many normalization prescriptions as there are coupling constants (including masses) in the basic Hamiltonian. This is so because of two reasons. First, in those cases we are interested in complete renormalizability of a theory; this means that we have to fix the finite parts of all the counterterms including those needed to renormalize the off-shell Green functions. Second, in conventional renormalizable theories every coupling constant presents an essential

[21] Strictly speaking, this is not quite true. It would be better to say that those coefficients contribute to measurable quantities. The point is that it is impossible to measure the contribution of the individual vertex—only a sum of all the relevant graphs of a given order presents the measurable quantity. We will come back to this point below.

[22] Plus γ-transversality in the case of fermion fields.

[23] Recall that the proper symmetrization with respect to all the lines of identical particles is tacitly implied.

[24] It should be kept in mind that there is no difference between the Hamiltonian and tree levels in the case of triple vertices.

parameter.[25] The situation looks much more complicated in effective theories. In this case there are certain combinations of the Hamiltonian parameters which do not contribute to renormalized *S*-matrix and, hence, cannot be related to any observable. In fact, these—redundant—combinations are not needed at all if we are only interested in describing scattering processes. The reason why the minimal parametrization happens most suitable in the case of effective scattering theory, is that it provides us with the (infinite) set of constants needed to construct the full set of the essential parameters directly connected with observable quantities. The structure of this connection is discussed in more detail in Sec. VII. However, before discussing this structure we need to introduce the notion of resultant parameters.

VI. RESULTANT PARAMETERS

In the preceding section we have considered an individual amplitude graph (more precisely, a symmetric sum) with a given number of external lines and certain fixed set of inner vertices. However, this graph (sum) only presents a part of the Lth order contribution to the amplitude describing the process under consideration. To obtain the net result, one needs to make four steps more.

(1) First, it is necessary to carry out the reduction of all the graphs (of the order L) with the same set of external lines but with different structure of the set of vertices (different numbers of legs), no summation over the kinds of internal lines (virtual particles) being implied on this step.
(2) Second, it is necessary to sum over all possible kinds of inner lines in every graph considered above.
(3) Third, it is necessary to sum up all the expressions obtained on the previous steps.
(4) Fourth, it is necessary to take account of contributions due to counterterm vertices of the Lth loop order.

The same should be done with respect to all the amplitude graphs with different numbers (and types) of external lines. It is easy to understand that this program results in a set of graphs constructed solely from minimal propagators and minimal effective vertices of various loop orders $l \leqslant L$ with different numbers and types of legs. Every such (lth order) vertex $V^{l}_{\dots}(p_1, \dots, p_n)$ with certain set of n legs takes the following typical form:

$$V^{(l)}_{\dots}(p_1, \dots, p_n) = \sum_{a=1}^{M} T^{(a)}_{\dots} V^{(l)}_{a}(\nu_1, \dots, \nu_{3n-10}), \tag{12}$$

where M is the number of relevant minimal tensor structures $T^{(a)}_{\dots}$ and $V^{(l)}_{a}$ stands for the infinite formal series

$$V^{(l)}_{a}(\nu_1, \dots, \nu_t) = \sum_{k_1, \dots, k_t = 0}^{\infty} V^{(a,l)}_{k_1 \cdots k_t} \nu_1^{k_1} \cdots \nu_t^{k_t}, \qquad t \equiv (3n-10) \tag{13}$$

in powers of kinematical variables (6).

Clearly, the general form of *minimal* counterterm vertices of the loop order L under consideration looks precisely like that of (12) and (13). The corresponding coefficients can be considered as the pieces of those appearing in the expression (13) for the highest order $l=L$ minimal effective vertex—there is no necessity in writing them down as special items. In turn, this means that, until we fix the set of normalization prescriptions for minimal vertices, all the coefficients $V^{(a,L)}_{k_1 \cdots k_t}$ in (13) should be taken as free parameters. We call them *the Lth level resultant parameters* (which are minimal by the very construction). The only limitations for their values follow from the requirement of finiteness of the Lth loop order amplitudes and the formal restrictions imposed by crossing and Bose (Fermi) symmetry (until we fix the renormalization prescriptions).

The important feature of the set of resultant parameters with $l=0,1,\dots,L$ is that this set is *full* and *closed*. It is full because no other parameters are needed to compute all the *S*-matrix elements of the Lth order. It is closed in the sense that taking account of graphs with $l>L$ loops leaves the lower level ($l \leqslant L$) parameters unchanged.

According to the results of Sec. V, there is no need in formulating normalization prescriptions adjusting finite parts of the coefficients at nonminimal counterterm vertices. This means that, except the infinite parts needed to remove divergences in subgraphs of the next loop order, those coefficients can be chosen in a way most suitable for subsequent calculations. In turn, this means that the full set of normalization conditions, needed to fix the physical content of effective scattering theory, is not larger than the set of corresponding resultant parameters.

Starting from this point we consider all the infinite renormalizations done. Let us now briefly discuss the problems of convergence. In fact, there are two problems closely connected with one another. The first one is the problem of convergence of numerical series constructed from the minimal parameters. Every coefficient in the form (13) for the resultant vertex presents an infinite sum of the parameters describing individual secondary vertices. Since no one of those latter parameters presents a measurable quantity, we do not think that the problem of convergence of their infinite sums should be taken too seriously.

Another problem is that of convergence of formal power series presenting the resultant effective vertices. Let us first discuss the case of tree-level resultant vertices with $n=4$ lines (recall that, irrespectively to a level, the resultant triple vertices are just constants). Each one of the corresponding resultant parameters (see Fig. 4) presents a sum of two items. The first item is just the relevant minimal parameter appearing in the effective 4-vertex of the Hamiltonian level. The second item stems from the reduction of graphs with resonance exchanges in *s*-, *t*-, and *u*-channels. It presents an in-

[25] Recall, that the gauge fixing parameter in gauge theories appears in the framework of Lagrangian formalism.

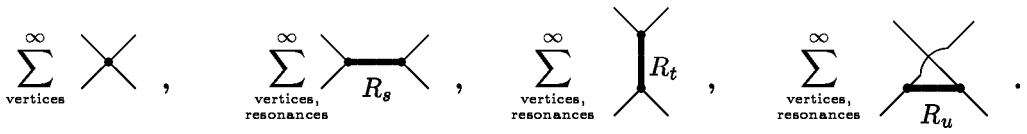

FIG. 4. Formal sum of graphs describing the tree-level amplitude of the process $2\to 2$ before reduction of the resonance lines. R_s, R_t, and R_u stand for all possible resonances in the *s*-, *t*-, and *u*-channels, respectively. The effective triple vertices contain both minimal and nonminimal (with respect to inner line) parameters.

finite sum of products of the Hamiltonian triple coupling constants, at least one of which being nonminimal with respect to inner line. All the minimal triple couplings of the Hamiltonian level are contained in the triple vertices describing the pole parts of resonance contributions.

The resultant effective 4-vertex does not present an independent element of Feynman rules: every time when it appears as a part of a larger graph, one also has to take account of contributions due to the resonance exchange graphs shown in Fig. 4. This note allows one to conclude that it makes no sense to discuss the convergence of infinite series (13) for the resultant 4-vertex: only the full sum of tree-level graphs under consideration must possess the desired convergency property. This means that in the full sum of graphs (of a given loop order), presenting an amplitude under consideration, we expect mutual cancellations among various unwanted contributions which might occur in every individual item.

Clearly, this argumentation equally applies to arbitrary effective resultant vertex with $n>4$ lines as well as to the case of higher loop order vertices. Thus it may happen that the resultant parameters describing the vertices with different numbers of legs are not completely independent. Indeed, as argued in [8] (the detailed analysis will be published elsewhere), the requirements of convergence, crossing symmetry and polynomial boundedness lead to highly nontrivial relations connecting the resultant parameters of the vertices differing from one another by the number of legs.

VII. THE ESSENTIAL PARAMETERS

In this section we just give an idea on how to construct the essential parameters from the resultant ones. The detailed analysis would require too much space; it will be published elsewhere. A preliminary discussion can be found in [8].

By way of illustration, let us consider the tree-level amplitude describing a scattering process $2\to 2$. For the following, it is convenient to consider in parallel three different pairs of independent kinematical variables

$$[x,\nu_x] \quad (x=s,t,u).$$

Here s,t,u stand for the conventional Mandelstam variables, and

$$\nu_s\equiv(u-t), \quad \nu_t\equiv(s-u), \quad \nu_u\equiv(t-s). \tag{14}$$

From (14) it follows that

$$u=\tfrac{1}{2}(2\sigma-s+\nu_s), \quad t=\tfrac{1}{2}(2\sigma-s-\nu_s), \tag{15}$$

$$s=\tfrac{1}{2}(2\sigma-t+\nu_t), \quad u=\tfrac{1}{2}(2\sigma-t-\nu_t), \tag{16}$$

$$t=\tfrac{1}{2}(2\sigma-u+\nu_u), \quad s=\tfrac{1}{2}(2\sigma-u-\nu_u), \tag{17}$$

where $2\sigma\equiv(m_1+m_2+m_3+m_4)$ and $m_i (i=1,\ldots,4)$ are the external particle masses.

The tree-level amplitude of the process under consideration is a sum of four items each of which, in turn, presents an infinite sum of contributions stemming either from the effective 4-vertex or from graphs with resonance exchanges (see Fig. 4). In particular, the first term is an infinite sum of items each of which originates from the corresponding Hamiltonian monomial constructed from four field operators or/and their derivatives. It takes a form of (formal) infinite power series in two independent kinematical variables. All the coefficients appearing in this series are constructed from the corresponding minimal parameters of the Hamiltonian level.

As to the triple vertices appearing in graphs with resonance exchanges, they contain both minimal and nonminimal (with respect to inner lines) parameters of the Hamiltonian level. The nonminimal parameters do not contribute to the pole parts of graphs: as shown in Sec. IV, they only contribute to smooth (analytic) part. In contrast, all the minimal parameters contribute to the values of residues at corresponding poles. This means that, after the reduction of inner lines, the amplitude can be presented in one of three equivalent forms only differing from one another by the choice of variables

$$M(s,\nu_s)=\sum_{i,j=0}^{\infty} A_{ij}^{(s)} s^i {\nu_s}^j+\sum_{R_s}\frac{N_s^{(s)}(\nu_s)}{s-M_R^2}+\sum_{R_t}\frac{N_t^{(s)}(s)}{\nu_s-(\theta_t-s)}+\sum_{R_u}\frac{N_u^{(s)}(s)}{\nu_s+(\theta_u-s)}, \tag{18}$$

$$M(t,\nu_t)=\sum_{i,j=0}^{\infty} A_{ij}^{(t)} t^i {\nu_t}^j+\sum_{R_s}\frac{N_s^{(t)}(t)}{\nu_t+(\theta_s-t)}+\sum_{R_t}\frac{N_t^{(t)}(\nu_t)}{t-M_R^2}+\sum_{R_u}\frac{N_u^{(t)}(t)}{\nu_t-(\theta_u-t)}, \tag{19}$$

$$M(u,\nu_u)=\sum_{i,j=0}^{\infty} A_{ij}^{(u)} u^i {\nu_u}^j+\sum_{R_s}\frac{N_s^{(u)}(u)}{\nu_u-(\theta_s-u)}+\sum_{R_t}\frac{N_t^{(u)}(u)}{\nu_u+(\theta_t-u)}+\sum_{R_u}\frac{N_u^{(u)}(\nu_u)}{u-M_R^2}. \tag{20}$$

Here

$$\theta_x \equiv (2\sigma - M_{Rx}^2) \quad (x=s,t,u); \tag{21}$$

the relations (15)–(17) have been used to rewrite denominators in terms of relevant pairs of variables.

No one of the formal series (18)–(20) makes sense until we fix the order of summation and point out the areas where we would like to assign meaning to those series. As argued in [6] (see also [8]), it is natural to consider every series written in terms of the pair $[x,\nu_x]$ in the corresponding thin three-dimensional band (layer)

$$B_x: \quad \{x \in \mathbf{R}, \quad \nu_x \in \mathbf{C}; \quad x \in (-\epsilon,\epsilon)\} \quad (x=s,t,u).$$

By condition, the thickness 2ϵ of the layer B_x should be taken sufficiently small such that $\epsilon < \min\{M_{Rx}^2\}$. This means that those items which contain fixed (independent of ν_x) poles in x do not result in singular contributions in B_x. Hence, in B_x the expression for the amplitude can be rewritten as formal sum of contributions due to sliding (depending on x) poles in ν_x plus the term which is formally regular in both variables. For example, in B_u we have

$$M(u,\nu_u) = \sum_{i,j=0}^{\infty} M_{ij}^{(u)} u^i \nu_u{}^j + \sum_{R_s} \frac{N_s^{(u)}(u)}{\nu_u - (\theta_s - u)} + \sum_{R_t} \frac{N_t^{(u)}(u)}{\nu_u + (\theta_t - u)}, \quad ([u,\nu_u] \in B_u). \tag{22}$$

The corresponding formal expressions for the amplitude in B_s and B_t can be rewritten precisely in the same way. We would like to stress that every coefficient in (22) is constructed from the tree level resultant parameters.

The special convenience of the form (22) is explained by the following reason. At every fixed $u \in B_u$ this form can be treated as a uniformly converging series presenting a meromorphic function of one complex variable ν_u and one real parameter u. The possibility of such interpretation is provided by the general theorem (due to Mittag–Leffler) known from complex analysis (see, e.g., [15] and [16]). To make use of this theorem in its constructive form, one has to impose certain limitations on the values of resultant parameters. Besides, in order to provide a guarantee that the amplitude possesses desired properties of crossing symmetry, one needs to consider in parallel three different forms of the type (22) (in B_s, B_t, and B_u). In the domains of mutual intersections

$$D_s \equiv B_t \cap B_u, \quad D_t \equiv B_u \cap B_s, \quad D_u \equiv B_s \cap B_t,$$

the corresponding forms must identically coincide in pairs. This requirement leads to additional limitations[26] strongly restricting the allowed values of resultant parameters. Those limitations take a form of an infinite system of algebraic equations connecting different parameters among themselves and, hence, reducing the number of *independent* parameters needed to fix a particular effective scattering theory. The full set of independent combinations of resultant parameters can be considered as the set of true essential parameters which require formulating the renormalization prescriptions.

VIII. CONCLUSION

The main result of the above analysis can be formulated as follows. *To describe the scattering processes in the framework of an effective field theory one has no need in fixing the detailed structure of particle interactions off the mass shell. All the information needed to fix the numerical values of S-matrix elements at a given loop order L is contained in the values of the resultant parameters of Lth and lower levels.* This result coincides with that obtained by Weinberg, Scadron, and Wright in series of papers [9] on nonrelativistic scattering theory.

The central idea of our work is that the number of *independent* normalization prescriptions needed to fix the physical content of an effective scattering theory is much less than the total number of resultant parameters. As explained in Sec. VII, certain natural consistency requirements lead to an infinite number of constraints strongly restricting the allowed physical values of those parameters. This point will be discussed in detail in the next paper.

ACKNOWLEDGMENTS

We are grateful to K. Semenov-Tian-Shanski, A. Vasiliev, M. Vyazovski, V. Cheianov, H. Nielsen and J. Schechter for stimulating discussions. The work was supported in part by INTAS (project 587, 2000), RFBR (Grant No. 01-02-17152) and by Ministry of Education of Russia (Grant No. E00-3.3-208). The work by A. Vereshagin was supported by Meltzers Høyskolefond (Studentprosjektstipend, 2002).

[26] Called in [5–8] as bootstrap equations.

[1] S. Weinberg, Physica A **96A**, 327 (1979).
[2] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
[3] H. Georgi, Annu. Rev. Nucl. Part. Sci. **43**, 209 (1993).
[4] S. Weinberg, in *General Relativity–An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).
[5] V. Vereshagin, Phys. Rev. D **55**, 5349 (1997).
[6] A. Vereshagin and V. Vereshagin, Phys. Rev. D **59**, 016002 (1999).
[7] A. Vereshagin, πN Newsletter **16**, 426 (2002).
[8] A. Vereshagin, V. Vereshagin, and K. Semenov-Tian-Shanski, Zap. Nauchn. Semin. POMI **291**, 78 (2002) (in Russian) [hep-th/0303242, the English translation to appear soon in J. Math. Sci. (N.Y.) (2003)].
[9] S. Weinberg, Phys. Rev. **130**, 776 (1963); **131**, 440 (1963);

133, B232 (1964); M. Scadron and S. Weinberg, *ibid.* **133**, B1589 (1964); M. Scadron, S. Weinberg, and J. Wright, *ibid.* **135**, B202 (1964).
[10] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 2000), Vols. 1–3.
[11] A. Salam and J. Strathdee, Phys. Rev. D **2**, 2869 (1970).
[12] A. N. Vasiliev, *Functional Methods in Quantum Field Theory and Statistical Physics* (Gordon and Breach, Amsterdam, 1998).
[13] M. Veltman, Physica (Amsterdam) **29**, 186 (1963).
[14] N. N. Bogoliubov and Ju. A. Mitropolski, *Asymptotic Methods in Nonlinear Oscillation Theory* (Nauka, Moscow, 1974) (Russian edition).
[15] B. V. Shabat, *An Introduction to Complex Analysis* (Nauka, Moscow, 1969) (in Russian).
[16] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists* (Academic, New York, 1995).
[17] S. Weinberg, Phys. Rev. **133**, B1318 (1964); **134**, B882 (1964); **135**, B1049 (1964); **138**, B988 (1965); **181**, 1893 (1969).
[18] J. C. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984).
[19] M. E. Peskin and D. V. Schroeder, *Quantum Field Theory* (Addison-Wesley, New York, 1997).
[20] M. L. Nekrasov, Phys. Lett. B **531**, 225 (2002); Eur. Phys. J. C **19**, 441 (2001).
[21] B. A. Kniehl and A. Sirlin, Phys. Lett. B **530**, 129 (2002).
[22] F. V. Tkachov, Phys. Part. Nucl. **25**, 649 (1994).
[23] Hua-Bin Tang and P. Ellis, Phys. Lett. B **387**, 9 (1996).
[24] P. Ellis and Hua-Bin Tang, Phys. Rev. C **56**, 3363 (1997).
[25] V. V. Vereshagin, K. N. Mukhin, and O. O. Patarakin, Phys. Usp. **43**, 315 (2000).
[26] V. V. Vereshagin *et al.*, Nucl. Phys. **A592**, 413 (1995).
[27] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[28] M. D. Scadron, Phys. Rev. **165**, 1640 (1968).
[29] V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973).