On the status of highly entropic objects

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It has been proposed that the entropy of any object must satisfy fundamental (holographic or Bekenstein) bounds set by the object's size and perhaps its energy. However, most discussions of these bounds have ignored the possibility that objects violating the putative bounds could themselves become important components of Hawking radiation. We show that this possibility cannot *a priori* be neglected in existing derivations of the bounds. Thus this effect could potentially invalidate these derivations; but it might also lead to observational evidence for the bounds themselves.

I. INTRODUCTION

The laws of thermodynamics and the concepts of entropy (S) and energy (E) express fundamental aspects of physics. In the conventional understanding, these quantities are related to each other and to the size of an object only through the first law of thermodynamics $dE = TdS - PdV$. However, there have been intriguing suggestions (see e.g. [1–5]) that more fundamental laws (e.g., quantum gravity and/or string theory effects) should change this picture. In particular, the entropy of any object might be bounded by some function of its size, typically characterized by a length scale R or an enclosing area A, and perhaps its energy E. Suggestions of this form include Bekenstein's proposed bound [1],

$$S < \alpha RE/\hbar c, \qquad (1.1)$$

and the so-called holographic bound [2,3],

$$S < Ac^3/4\hbar G. \qquad (1.2)$$

Here we have displayed the fundamental constants explicitly, but below we use geometric units with $k_B = \hbar = c = G = 1$.[1] The original version [1] of Bekenstein's bound has $\alpha = 2\pi$, while some subsequent discussions (e.g. [6]) weaken the bound somewhat, enlarging α by a factor of order ten.

Arguments in favor of these bounds [1–3,6,7] typically suggest that inserting or transforming bound-violating objects into black holes leads to contradictions with the second law of thermodynamics. Many counterarguments have been given and the subject remains in a state of controversy. The original argument [1] for Eq. (1.1) involved slowly lowering a "box" toward a black hole and then, at some point, letting it fall freely through the horizon. Counterarguments appealing to a buoyant force exerted on the box by the "thermal atmosphere" of the black hole were given by Unruh and Wald in [8]. The question was reconsidered recently in [9] in the context of a resolution of the "self-accelerating box paradox." Under plausible assumptions as to the treatment of certain boundary effects, it was shown in [9] that a box violating Eq. (1.1) would make a notable contribution to the thermal atmosphere of the very black hole with which it was supposed to violate the second law. This contribution might be negligible far from the black hole, but would become important in the region near the horizon from which the box was to be dropped. This opens the door to new effects which might provide loopholes in the original argument of [1]. A few such effects were discussed in [9] and similar effects will be described below.

However, other arguments for a version of Eq. (1.1) (with α somewhat greater than 2π) have been made in which one releases the object to fall into the black hole from far away [6]. When applied to such processes, the comments of [9] suggest that thermally produced copies of bound-violating objects would be relevant even far away from the black hole. In other words, despite the very low Hawking temperature of any macroscopic black hole, they suggest that objects violating this version of Eq. (1.1) would be Hawking radiated at a significant rate by the particular black hole used in the argument. We explicitly verify this suggestion below, noting that the situation far from the black hole is under much better control than that studied in [9].

We then note that these considerations generalize to any setting (e.g. those of [10] and [1,6]) in which the absorption of a "highly entropic object" by a black hole would, in the absence of further entropy generation, lead to a violation of the second law. Thus, such highly entropic objects and their kin will be important components of the black hole's thermal atmosphere so that further processes will indeed occur. We show that similar comments apply to the holographic bound (1.2). Finally, we suggest how this same effect could lead to *observational* evidence in favor of both Eqs. (1.1) and (1.2) in certain regimes, independently of whether, as a matter of principle, these bounds necessarily hold in all possible hypothetical worlds.

We remark here that the majority of the thought experiments we consider herein (as well as those considered in [9]) involve only semiclassical processes which are quasistation-

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[1]In many ways, the choice $8\pi G = 1$ is more natural than $G = 1$. With this choice of units, the horizon entropy becomes $2\pi A$ and Eq. (1.2) reads $S < 2\pi A$.

ary *for the black hole*, that is to say processes in which the black hole may be treated classically such that its mass changes only incrementally. (An important exception is the gedanken experiment for deriving the holographic bound in [3].) In this context, a very general argument presented in [11,12] establishes[2] that no violation of the GSL can occur if the matter outside the black hole is correctly described by some quantum field theory. From this point of view any attempt to decrease the total entropy of a black hole by inserting highly entropic objects is doomed in advance to fail, and the only question is how this failure will work itself out in the given case. Thus, a proof that one could decrease the entropy with the aid of a certain type of highly entropic object would amount to a proof that no such object could exist in any self-consistent quantum field theory (which extended to curved spacetime). Conversely, if one could imagine a quantum field theory in which such an object definitely could exist, then one would be guaranteed that the theory would provide for some effect to protect the GSL from violation, when such objects were made to interact with black holes in the above semiclassical setting. To a certain extent, the remainder of this paper is just a more detailed working out of this implication.

II. HIGHLY ENTROPIC OBJECTS AT EQUILIBRIUM

It is well established [14–19] that the radiation surrounding a black hole of temperature T_{BH} is thermal in the sense that, *in equilibrium*, it is described by an ensemble of the form $e^{-\beta H}$, where $\beta = 1/T_{BH}$. When a black hole radiates into empty space and the thermal ensemble would be dominated by weakly interacting particles, the Hawking radiation is just the outgoing component of the radiation described by this ensemble.

The point stressed in [9] is that, according to statistical mechanics, the probability to find a particular macrostate in a thermal ensemble is not $e^{-\beta E}$ but $e^{-\beta F}$, where $F = E - T_{BH}S$ is its free energy[3] at the temperature T_{BH} corresponding to the black hole. In converting this into an emission rate, the only other relevant factor is a "gray body factor" that enters the absorption cross section σ for our object. (The absorption and emission rates are related by the assumption of "detailed balance." We assume that this assumption is valid for our objects.[4])

A. The Bekenstein bound

Let us now recall the setting for the argument of [6] in favor of Eq. (1.1). One considers an object of size R, energy

E, and entropy S which falls into a Schwarzschild black hole of size $R_{BH} = 2\zeta R$ from a distance $d \gg R_{BH}$. The parameter ζ is taken to be large enough that the object readily falls into the black hole without being torn apart. In other words, we engineer the situation so that the black hole is, at least classically, a perfect absorber of such objects. It is also assumed that the Hawking radiation emitted during the infall of our object is dominated by the familiar massless fields, in which case it is a small enough effect so as not to significantly impede the fall of our object. Consideration of the second law [6] then leads to the bound

$$S < 8\pi\nu\zeta RE, \tag{2.1}$$

where ν is a numerical factor in the range $\nu = 1.35–1.64$. Here the energy E has been assumed to be much smaller than the mass M_{BH} of the black hole and, up to the factor ν, the above bound is obtained by considering the entropy increment of the black hole, $dS_{BH} = dE_{BH}/T_{BH}$. Notice here that the black hole was assumed not to readily emit copies of our object as part of its Hawking radiation.

Suppose, now, that a "highly entropic object" does exist with $S > 8\pi\nu\zeta RE$. The arguments of [9] suggest that this large entropy will induce such objects to be emitted copiously by the black hole, and it is clear that no violation of the second law will result if the net flux of such objects vanishes or is directed outward from the black hole. To see whether this is indeed the case, let us compute the free energy of our object at the black hole temperature $T_{BH} = (4\pi R_{BH})^{-1} = (8\pi\zeta R)^{-1}$:

$$F = E - \frac{S}{4\pi R_{BH}} < E - \nu E < 0. \tag{2.2}$$

Now if we assume that no objects are present, then we find $F = E - TS = 0 - 0 = 0$. By Eq. (2.2), objects violating Eq. (1.1) have significantly lower free energy than this, and are therefore more likely to exist than not in a state of thermal equilibrium at T_{BH}. (In fact, the most likely macrostate is one that is so full of such objects that new ones cannot be squeezed in at the same low free energy.) Consequently, it is unjustified to assume that such objects are unlikely to be radiated by the black hole, during the course of one of the putatively entropy-violating processes under consideration. (On the other hand, we cannot simply assert that they must be radiated in great numbers, because the nature of the equilibrium state does not in itself determine what happens away from equilibrium.)

Consideration of simple models elucidates the ways in which this loophole might play itself out. Suppose for example that our object's free energy were independent of the number of such objects already present.[5] Then the putative thermal ensemble would be unstable, as adding an additional

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[2]Assuming that certain *a priori* divergent quantities can be handled appropriately.

[3]Since $e^{-F/T} = e^{-E/T}e^S$, the free energy includes the effect of collecting e^S microstates into a single macrostate.

[4]If the object can be described as a field quantum and the black hole metric treated as fixed, then one just has potential scattering, for which detailed balance can be derived in the usual manner. More generally, one might appeal to some version of time reversal invariance, or better *CPT* invariance, but the status of the latter is not settled within quantum gravity.

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[5]This supposition is instructive but not realistic. Note in particular that free bosons do *not* fall into this category, as a thermal ensemble of any number N of free boson fields exists at any temperature T. The free boson case is quite interesting and will be studied in detail in [20].

such object would lower the free energy, no matter how many were already present. (Hence, strictly speaking, there could be no state of equilibrium at all, much as with the super-radiant modes in the case of a rotating black hole.) Thus, we would expect the Hawking radiation to contain so many of our objects that the usual semiclassical approximation would fail and the black hole would quickly decay.

As a second example, suppose that our objects can be modeled by weakly interacting fermions. Then all macrostates with $F < 0$ will be occupied, although states with sufficiently high kinetic energy will remain empty. If the parameters are chosen correctly, the rate of Hawking radiation can remain low enough that the semiclassical approximation remains valid and the black hole does exist as a meta-stable state. However, since the object we wish to drop is by construction in a state with insufficient kinetic energy to satisfy $S < 8\pi\nu\zeta RE$, it represents an ingoing state with $F < 0$. Thus, a corresponding outgoing state is occupied with high probability and the black hole will very likely emit such an object during the time that our ingoing object is being absorbed. In fact, it is very likely to emit a large number of such objects in various directions.

In the third instructive case we suppose that the thermal atmosphere of the black hole blocks the passage of our highly-entropic objects so that energetic objects cannot stream freely outward from the black hole. Let us assume it also blocks the passage of the *CPT* conjugate objects, since these will carry equal entropy. This case might arise because the thermal atmosphere already contains many densely packed copies of our object, or it might arise because our objects are excluded by interactions with some other component of the atmosphere. Note that due to detailed balance (or *CPT* invariance) this atmosphere will also obstruct us from dropping in a new object from far away. Our new object will bounce off the thermal atmosphere or be otherwise prevented from entering the black hole to the same extent that an outgoing such object emitted by the black hole will fail to escape. Thus, again it is plausible that the black hole is very likely to emit at least one such object before we manage to send a new one into the black hole.

In each case we find, with high plausibility, that the Hawking radiation adds at least as much entropy to the Universe as is removed when our object falls through the horizon. Note that none of the caveats from [9] apply here: the relevant region is far from the black hole so that it is large and homogeneous and no boundary effects should be important.

B. A generalization

Since the end result did not rely on particular properties of Schwarzschild black holes, one might expect that our argument can be formulated much more generally. To see that this is the case, let us proceed along the lines of [11]. Consider then any process in which a given object with entropy S is destroyed, giving its energy E to a black hole. Note that this includes both processes of the original form [1] as well as the more recent [6]. As above, we suppose that this represents a small change, with E being small in comparison to

the total energy of the black hole. The change in the total entropy of the universe is at least

$$\Delta S_{total} \geqslant \Delta S_{BH} - S. \qquad (2.3)$$

But using the first law of thermodynamics for the black hole, this is just

$$\Delta S_{total} \geqslant \frac{E}{T_{BH}} - S = \frac{F}{T_{BH}}, \qquad (2.4)$$

where F is the free energy of the object at the Hawking temperature T_{BH}. In particular, since $T_{BH} > 0$, the sign of ΔS_{total} must match that of F. One concludes that the absorption of an object by a black hole can violate the second law only if $F < 0$, in which case any of the mechanisms from Sec. II A may come into play to prevent the process from occurring. Note that only the first law (energy conservation) has been assumed in our argument and that no special properties of black holes have been used; the argument would proceed as well if one replaced the black hole by any object at the same temperature. (However, in places we did assume that emission and absorption rates could be analyzed as if the black hole were in equilibrium with its surroundings, unlike in the more general treatment of [11,12].)

C. The holographic bound

Let us now consider the holographic bound (1.2). Suppose in particular that we have a (spherical, uncharged) object with $S \geqslant A/4$ and consider a Schwarzschild black hole of equal area $A = 4\pi R_{BH}^2$. Since our highly entropic object is not itself a black hole, its energy E must be less than the mass M_{BH} of the black hole. The arguments of [2,3] now ask us to consider what happens if we drop our highly entropic object into a black hole of mass M_{BH} or otherwise transform it into a black hole of this mass. For arguments which drop the object into a preexisting black hole, one typically[6] assumes $E \ll M_{BH}$, but this is not the case for all the arguments.

Based on effects like those described in [13] (Sec. II) and in [9] one may speculate that some Hawking-like process forbids this transformation. More specifically the suggestion is that if the transformation does proceed at first, then the resulting black hole state will be a mere "thermal fluctuation" that lasts for no more than a time of order R_{BH}. In order to assess this suggestion, let us suppose for the moment that $E \ll M_{BH}$ so that the emission rate of such "highly entropic objects" from a black hole of mass M_{BH} can be analyzed as in Secs. II A and II B. Then the free energy of our highly entropic object at the Hawking temperature $T_{BH} = (4\pi R_{BH})^{-1}$ of the black hole is

$$F = E - T_{BH} S < E - \frac{A/4}{4\pi R_{BH}} = E - M_{BH}/2 < 0. \qquad (2.5)$$

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[6]See, e.g., the weakly gravitating case described in [7].

Thus, we again see that our object is likely to be emitted readily in Hawking radiation.

In the case where E and M_{BH} are comparable, the emission of our object will react back significantly on the black hole itself. In this case it no longer seems possible to analyze the emission rate by comparison with a state of thermal equilibrium in a fixed black hole background. [Indeed a canonical ensemble at fixed temperature seems inappropriate, and one would have to replace it by a microcanonical ensemble for the system of radiation plus black hole(s).] Therefore, we will fall back on a more general, but less compelling type of argument which we could also have used above, but did not since the equilibrium alternative was available.

Instead of reasoning from detailed balance and equilibrium abundances, we could have just assumed that our object was emitted as if it were a field quantum of a massive free field (as in the original calculations of Hawking radiation). This yields an emission rate, which, if we ignore the prefactor, takes the Boltzmannian form, $\exp(-E/T_{BH})$. This can also be written as $\exp(\Delta S_{BH})$, where ΔS_{BH} (a negative number) is the entropy lost by the black hole in emitting the object of energy E, and we are still assuming that $E \ll M_{BH}$. If we now assume further that this rate applies equally to each *microstate* of our object, then the total emission rate for the macrostate of the object acquires a factor of $\exp(S)$, whence the overall rate (still neglecting "prefactors") takes on the "naive thermodynamic value" of $\exp(\Delta S_{BH}+S)$. This coincides with the form utilized above, $\exp(-E/T_{BH}+S)$.

Now this form of the argument has the weakness that the assumptions going into it seem to be under poorer control and less convincing than those going into our equilibrium analysis above. However, unlike the latter, the present analysis carries over to the case where E is comparable to M_{BH}, at least in the sense that, according to Refs. [22,23], the emission rate for a microstate retains a factor proportional to $\exp(\Delta S_{BH})$. If we accept this, then the rest of the argument is just as before: The formation of a black hole from our object could violate the second law only if $S>S_{BH}$. But since $\Delta S_{BH}>-S_{BH}$, one finds $\Delta S_{total}=\Delta S_{BH}+S>0$ for the corresponding emission process. Since this implies an exponentially large emission rate for our object, we conclude that the combined process of collapse and emission would actually result in a net *increase* in the total entropy.

III. DISCUSSION

The proposals (1.1) and (1.2) for fundamental entropy bounds would forbid the existence of objects with extremely high entropy. In fact, we have seen that the entropy of the putatively forbidden objects is so high that they (or even more entropic objects) would be an important component of Hawking radiation even for large black holes where the temperature is low. This expands an interesting loophole in existing arguments for such fundamental bounds. In a more special context where such a violation arises from a large number of light fields, a very similar loophole was discussed in [13]. To quote from that reference, " . . . the bound should be necessary in order for black holes to be stable or meta-

stable states, but should not be needed for the validity of the GSL [generalized second law]." We found evidence for such an assertion in two different regimes. In the first, where the energy E of the putative highly entropic object (HEO) is much less than that of the black hole, one knows on general grounds that the GSL cannot be violated in any semiclassical process with a quasistationary black hole [11,12]. Therefore, if one imagines a HEO which would lead to a violation, then the conclusion must be either that the HEO cannot actually exist (compare how self-accelerating boxes were excluded in [9]) or that some effect has been overlooked which would avoid the violation in another way; we presented evidence that emission of HEO's by the black hole is such an effect. In the second regime, where E is comparable to M_{BH}, things are much less clear cut, but a similar argument can be made if one accepts the conclusions of [22,23] concerning the emission of such objects.

One might think that such highly entropic objects are in any case experimentally excluded due to our excellent understanding of high temperature thermal states produced in the laboratory. However, states under experimental control are produced by interactions with normal matter. As a result, they place only loose constraints on highly entropic objects made from unknown fundamental fields which might interact extremely weakly with those of the standard model. One may imagine such objects as being made from exotic dark matter or other "hidden-sector" fields. One might also imagine that, even if made of standard model fields, some dynamical effect might cause these objects to come into equilibrium only after a cosmologically long timescale.

Since objects violating Eqs. (1.1) and (1.2) can be abundant in Hawking radiation, it is interesting to speculate that the production of highly entropic objects could lead to observable rates of mass loss from known black holes. For example, let us consider the case where such objects pass unimpeded through the thermal atmosphere of the black hole but where semiclassical black holes nevertheless exist as metastable states. A good model for this case is the scenario of weakly interacting fermions discussed above. Then if the putative bounds are violated by a factor of order 1, our objects have negative free energy even when their kinetic energies are relativistic. The Hawking radiation may then be modeled as a "fluid" of such objects[7] which flows outward from the black hole with density ρ and speed $v \sim c$. The black hole loses mass at a rate of $\dot{M}=4\pi\rho R_{BH}^2 c$. On the other hand, we have observed various black holes for some time and thus have at least rough bounds on the rate at which they lose mass. Consideration of a black hole of a few solar masses whose mass remains roughly constant over a period of ten years would rule out the existence of such a fluid with $\rho \gtrsim 5\times 10^4$ kg/m^3, while similar observations of a 10^6 solar mass black hole would exclude a corresponding fluid with $\rho \gtrsim 0.2$ kg/m^3. One of course obtains much stronger limits if the accepted age of such objects is used as the relevant ti-

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[7]Similar fluid pictures were described in [21] as candidate descriptions of Hawking radiation at temperatures high enough to create hadrons.

mescale. The detailed modeling of similar scenarios may provide fertile ground for future investigations.

We conclude with a hand waving argument that also allows one to set observational limits on certain highly entropic objects. An enthusiastic seminar speaker can probably wave his or her hand with an acceleration exceeding 10^4 cm/s^2. If massive objects were present in the thermal radiation associated with this acceleration then, unless these objects were transparent to human hands, one would bump into them and the vacuum would not feel empty. Consider, for example, an object of mass ~ 1 gram and size ~ 1 cm. In order that such an object not impede our waving hand, its entropy cannot exceed 10^{54}. This is tighter than the holographic bound by about ten orders of magnitude, though much looser than the Bekenstein bound. It is even possible that one can extend this hand waving argument to rule out certain objects at zero temperature, but we leave that discussion for another place.

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