

New mechanism for generating density perturbations from inflation

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We propose a new mechanism to generate density perturbations in inflationary models. Spatial fluctuations in the decay rate of the inflaton field to ordinary matter lead to fluctuations in the reheating temperature. We argue that in most realistic models of inflation the coupling of the inflaton to normal matter is determined by the vacuum expectation values of fields in the theory. If those fields are light during inflation (this is a generic situation in the minimal models of supersymmetric inflation) they will fluctuate leading to density perturbations through the proposed mechanism. We show that these fluctuations could easily dominate over the ones generated through the standard mechanism. The new scenario has several consequences for inflation model building and observations. The proposed mechanism allows us to generate the observed level of density perturbations with a much lower scale of inflation and thus generically predicts a smaller level of gravitational waves. The relation between the slope of the spectrum of the produced density perturbations and the potential of the inflaton field is different from the standard relations obtained in the context of slow roll inflation. Because the field responsible for the fluctuations is not the inflaton, it can have significantly larger self-couplings and thus density perturbations could be non-Gaussian. The non-Gaussianity can be large enough to be detectable by CMB and large scale structure observations.

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I. BASIC MECHANISM

In the standard picture [1], the observed density perturbations are produced as follows. As the inflaton field ϕ rolls down its potential it is effectively massless, so it fluctuates up and down. These fluctuations are different in different regions, resulting effectively in inflation lasting different amounts of time in different places. At the end of inflation, the inflaton oscillates about the minimum of its potential and decays, reheating the universe. As a result of the fluctuations each region of the Universe goes through the same history but at slightly different times leading to adiabatic density perturbations.

Our idea is different. To reheat the universe the inflaton has to couple to the ordinary particles through a coupling schematically given by

$$\lambda \phi q \bar{q}, \quad (1)$$

where q stands for the ordinary particles (e.g., quarks, leptons and their scalar superpartners), and λ is the coupling strength, which is assumed to be *constant* in the standard picture. For simplicity we assume that the density perturbations produced at the inflationary stage are negligible. So inflation ends at the same time everywhere in the Universe. The density fluctuations are created during the reheating process, because of fluctuations in the coupling “constant” λ . Since λ controls the efficiency by which the energy stored in

the ϕ -condensate gets converted into radiation, the fluctuations in λ translate into the fluctuations in the final reheating temperature. Note that in our scenario different regions go through slightly different histories, as the couplings are different. Thermal equilibrium ensures that we get adiabatic fluctuations.

The key point of our approach is to notice that λ is not a constant, and can fluctuate in space. The reason why λ could fluctuate is that in supersymmetric theories as well as in the theories inspired by superstrings, the effective couplings are not constants, but rather functions of the scalar fields in the theory. In the early universe, these scalar fields can assume different values in different regions. We shall come back to this issue below.

Assuming that λ is a stochastic variable, the new mechanism works as follows. The decay rate of the inflaton field is

$$\Gamma \sim \lambda^2 m \quad (2)$$

where m is the inflaton mass at its minimum. We shall assume that Γ is less than the Hubble parameter during inflation. If this were not the case and $\Gamma \gg H$ then the reheating would be instantaneous and all the energy stored in the inflaton would immediately be converted into radiation. In that case the reheating temperature would be independent of Γ .

When $\Gamma < H$ during inflation, the reheating temperature is roughly

$$T_R \sim \sqrt{\Gamma M_{PL}} \sim \lambda \sqrt{m M_{PL}}. \quad (3)$$

If λ fluctuates, the corresponding fluctuations in the reheating temperature are

$$\frac{\delta T_R}{T_R} \sim \frac{\delta \Gamma}{\Gamma} \sim \frac{\delta \lambda}{\lambda}. \quad (4)$$

The fluctuations are completely determined by the fluctuations of Γ not of ϕ .

In order to understand in more detail how our mechanism works we will consider a toy model (see also Ref. [12]). We write down the full set of perturbation equations in a later section. Let us follow the cosmological evolution in two different domains of the Universe in which λ takes two different values equal to λ_1 and λ_2 respectively. For definiteness we shall assume that $\lambda_1 > \lambda_2$. According to our assumption, right after inflation the energy densities in the two domains are equal, and evolve as nonrelativistic matter,

$$\epsilon_{1matter} = \epsilon_{2matter} \sim \frac{1}{a^3} \quad (5)$$

where a is the scale factor. The inflaton decay in each domain takes place when

$$\Gamma = H \sim \frac{\sqrt{\epsilon}}{M_{PL}} \quad (6)$$

where H is a Hubble parameter. In the approximation of an instant decay (which we adopt for the moment) the reheating temperature is given by Eq. (3). Since $\lambda_1 > \lambda_2$, the condition (6) gets first satisfied in the λ_1 domain. Thus when

$$\Gamma_1 = \lambda_1^2 m \sim H \quad (7)$$

the energy in this domain gets converted into radiation:

$$\epsilon_{1rad} \sim T_{R1}^4 \sim \lambda_1^4 m^2 M_{PL}^2 \quad (8)$$

which subsequently scales as $1/a^4$. Right after this moment the energy densities in two domains are still equal. However, in the λ_1 domain it is stored in radiation, whereas in the λ_2 domain it is still stored in inflaton oscillations and evolves as nonrelativistic matter. From this moment on up until reheating in the λ_2 domain, the energy densities in the two domains evolve *differently*.

The λ_2 domain gets reheated when

$$\Gamma_2 = \lambda_2^2 m \sim H \quad (9)$$

and gets filled with radiation of an energy density

$$\epsilon_{2rad} \sim T_{R2}^4 \sim \lambda_2^4 m^2 M_{PL}^2. \quad (10)$$

For equal values of scale factors, the energy in the first domain is redshifted to

$$\epsilon_{1rad} \sim \left(\frac{\lambda_2}{\lambda_1}\right)^{4/3} \lambda_2^4 m^2 M_{PL}^2. \quad (11)$$

The resulting radiation energy densities in these domains are related as

$$\epsilon_{1rad} \sim \left(\frac{\lambda_2}{\lambda_1}\right)^{4/3} \epsilon_{2rad}. \quad (12)$$

The density perturbations are

$$\frac{\delta \epsilon_{rad}}{\epsilon_{rad}} \propto \frac{\delta \lambda}{\lambda} \propto \frac{\delta \Gamma}{\Gamma}. \quad (13)$$

Thus, during the reheating process, the fluctuations in λ are translated in density perturbations. The precise calculation of the above model (Sec. IV and the Appendix) actually gives $\delta \epsilon_{rad} / \epsilon_{rad} \approx 0.1 \delta \Gamma / \Gamma$.

II. THE ORIGIN OF THE FLUCTUATIONS IN THE DECAY RATE

Let us now discuss why the fluctuations in Γ could be generated in the first place. The reason why coupling ‘‘constants’’ can fluctuate, is that in unified theories, such as supersymmetric theories, the couplings are not constant but are determined by expectation values of fields in the theory. Many of these fields have very flat potentials and thus can strongly fluctuate during inflation.

In general we can think of $\lambda(S)$ as a function of a scalar field S . Let us assume that q -s are the ordinary fermions, and their supersymmetric partners. The coupling can be expanded in series of S

$$\lambda(S) = \lambda_0 \left(1 + \frac{S}{M} + \dots \right) \quad (14)$$

where M is some scale, which may be of order M_{PL} or smaller. Since we want to keep our discussion maximally general, we should not specify the value of M . Depending on the gauge symmetries of the theory the first (and some of the higher terms) of expansion may or may not be absent. We shall assume that S is a light field, with mass smaller than the Hubble parameter during inflation. Then S fluctuates during inflation, $\delta S \sim H$ and these fluctuations translate into fluctuations of λ , which after reheating translate into the density fluctuations.

We can define f to be the fraction of the coupling controlled by the fluctuating field. If the VEV of S during inflation is $\langle S \rangle$ then $f = \langle S \rangle / M$. In terms of f we can write, to first order in S/M ,

$$\frac{\delta \Gamma}{\Gamma} = 2f \frac{\delta S}{\langle S \rangle}. \quad (15)$$

The observed level of density perturbation implies that $\delta \Gamma / \Gamma \sim 10^{-5}$. The fluctuations of S are of order, $\delta S \sim H$, so the observed level of fluctuations can be the result of $\langle S \rangle \gg H$ or of $f \ll 1$.

The fluctuations of S will be Gaussian if $\langle S \rangle \gg H$ and non-Gaussian in the opposite limit. Thus depending on the VEV of S during inflation we can get fluctuations with a varying level of non-Gaussianities. Our mechanism for generating

non-Gaussianities is different from the multifield inflation models (eg. [2,3] and references therein).

Example

Let us consider a simple illustrative example: minimal supersymmetric standard model (MSSM) plus an inflaton ϕ . The chiral superfield content of the model consists of two Higgs doublets (we shall call them h, \bar{h}), quark and lepton superfields plus their antiparticle superfields (we shall denote them as q, q_c), and an inflaton superfield (ϕ). Following the usual practice, we shall assume that the inflaton is a singlet under the MSSM gauge group. The specific form of the inflaton potential is unimportant. All we assume is that it gives us a sufficient amount of inflation, for solving the flatness and horizon problems. We shall also assume that the density fluctuations created by the inflaton fluctuations are very small.

At the end of inflation, ϕ has to decay and reheat the Universe. Let us ask, how could ϕ decay into quarks and leptons. Since ϕ is a gauge singlet, it has three options for transmitting its energy into ordinary particles.

(1) ϕ can directly decay into the ordinary particles through the renormalizable interaction in the superpotential

$$\lambda_0 \phi h \bar{h}. \tag{16}$$

In fact due to the gauge symmetry, the only candidates for such a decay are the Higgs doublets.

(2) ϕ can also decay through non-renormalizable interactions in the superpotential

$$\phi \frac{q}{M} q q + \phi \frac{q_c}{M} q_c q_c + \phi \frac{h}{M} q q_c + \dots \tag{17}$$

where M is some mass scale. This scale can be $\sim M_{PL}$ or may be a lower scale coming from integrating out some heavy particles below M_{PL} . The precise origin of the coupling is not important.¹ Analogous, couplings may be included in Kähler potential.

(3) Finally, ϕ can decay into some heavy exotic particles with MSSM gauge charges, which quickly rescatter and thermalize with the ordinary particles. In particular, the effective non-renormalizable couplings of the form (17) can result from integrating out such states.

Let us focus on the second possibility. If during inflation and reheating one of the sfermions participating in the coupling gets a VEV, then through Eq. (17) ϕ can experience a direct two-body decay into the q -quanta. The decay rate is regulated by an effective coupling

$$\lambda = \frac{\langle S \rangle}{M} \tag{18}$$

¹Some of the above interactions break baryon number, however they cannot lead to proton decay if ϕ has a zero VEV today. Breaking of baryon number conservation could be even welcome for generating baryon asymmetry during reheating. This issue will not be discussed here.

where $\langle S \rangle$ is the VEV of a scalar component of one of the q superfields. The properties of the density perturbations, such as Gaussianity will be determined by the balance between the different channels, and on the value of δS . Let us briefly discuss this issue.

In MSSM S corresponds to one of the many *flat directions* of the potential. These flat directions are flat only in the unbroken SUSY limit, but are lifted by SUSY breaking effects, and acquire curvature $\sim \text{TeV}$.

During inflation (or reheating) the inflationary energy density breaks supersymmetry and lifts the flat directions even more, provided $H > \text{TeV}$. So in general, the flat direction fields acquire masses $\sim H$ and may acquire big VEVs as well (see [4]).

The physical reason behind the $\sim H^2$ curvature of flat directions during inflation is very transparent. The point is that a positive vacuum energy density, that drives inflation, breaks supersymmetry spontaneously. This breaking is either F or a D -type, meaning that the vacuum energy density is dominated by either the F or the D terms. In case of, F -breaking, the breaking is universally transmitted to all the fields through the couplings in the Kähler potential, e.g., of the following form:

$$\int d^4x d^4\theta \frac{\phi^* \phi}{M^2} S^* S \tag{19}$$

where ϕ is the superfield with a nonzero F -term, and S is a flat direction field. Such couplings will be generated both by supergravity (in which case $M \sim M_{PL}$) and by gauge interactions [4] (in which case $M \ll M_{PL}$). For instance, in the case of a minimal Kähler, the gravity-mediated curvature of the flat directions is

$$m_{gravity}^2 = 3H^2 + \frac{|W|^2}{M_{PL}^4} \tag{20}$$

where W is a superpotential. However, in the MSSM model the flat directions are usually lifted also by gauge-mediated contribution. For instance, if the inflaton ϕ can couple to some gauge-charged fields (which is usually the case), the flat directions receive a two-loop gauge contribution to the masses [4]

$$m_{gauge}^2 = H^2 \left(\frac{\alpha}{4\pi} \right)^2 \left(\frac{M_{PL}}{|\phi|} \right)^2. \tag{21}$$

One-loop contributions are also possible. The over-all sign of the contribution and their balance is very model dependent. For instance, in D -term inflation [5] gravity-mediated curvature is $\ll H^2$, and the gauge-mediated contribution is dominant.

The bottomline of our discussion is that, generically, flat directions are lifted during inflation by gravitational and gauge effects. As a result their VEVs are shifted. The location of the minima, and effective masses in those minima are very model dependent and it is impossible to draw a general conclusion. On the other hand construction of a particular

model which would accommodate our needs is trivial. The condition under which the new mechanism of the density perturbations will dominate is that the mass of S during inflation be slightly below H (say $\sim 0.1H$ or so). Then H behaves as practically massless during inflation and we have $\delta S \sim H$.

Corrections of the form (20) would inevitably ruin inflation, and must be avoided. Inflationary scenarios that universally suppress the gravity-mediated curvature (20) of flat directions, e.g., such as the D -term inflation [5], are therefore highly motivated, as they protect the flatness of the inflaton potential from dangerous gravity-mediated corrections $\sim H^2$. In such scenarios our mechanism will generically be operative and contribute to the density perturbations.

Then depending on which is the dominant channel of the inflaton decay, we get different pictures in perturbation spectrum. If couplings like Eq. (17) are the dominant source of reheating, then the density perturbations are given by

$$\frac{\delta\epsilon}{\epsilon} \sim \frac{\delta S}{\langle S \rangle}. \quad (22)$$

In which case to get the correct level of density perturbations we have to demand that $\langle S \rangle \sim 10^5 H$, and resulting perturbations will be Gaussian, as in the case of an ordinary inflation.

Another possibility is when the channel (16) dominates. In this case

$$\frac{\delta\epsilon}{\epsilon} \sim \frac{\langle S \rangle \delta S}{\lambda_0^2 M^2} \sim \frac{\langle S \rangle H}{\lambda_0^2 M^2}. \quad (23)$$

Since now S can stay near its inflationary minimum, the perturbations can be non-Gaussian.

III. CONSEQUENCES FOR MODEL BUILDING

We have argued that our new mechanism for generating density perturbations creates fluctuations of order H/M where M is a scale that could be significantly lower than the Planck scale (the origin of M depends on the particular model, it could be related to the VEV of a field or with physics at scales intermediate between Hubble during inflation and the Planck scale). In other words our mechanism can create the observed density perturbations with a much lower scale of inflation than the standard scenario where density perturbations are given by $H/M_{PL}\sqrt{\epsilon}$, with $\epsilon = (M_{PL}V'/V)^2$.

If our mechanism is responsible for the observed perturbations then we predict a much lower background of gravitational waves than in standard scenarios. The amplitude of the gravity wave background is still given by H/M_{PL} and thus it is greatly suppressed relative to density perturbations, of order H/M .

The relation between the slope of the power spectrum of density perturbations and the characteristics of the inflaton potential is different in our scenario,

$$n-1 = \frac{d \ln H^2}{d \ln a} \quad \text{our scenario}$$

$$n-1 = \frac{d \ln H^2/\epsilon}{d \ln a} \quad \text{standard case.} \quad (24)$$

As a result, both the energy scale of inflation and the characteristics of the inflaton potential would be misinterpreted if our mechanism was operating but the observations were analyzed using the standard assumptions.

Moreover, there is a significant chance that the fluctuations generated by our mechanism could be at least slightly non-Gaussian. The fields that determine the couplings are not slow rolling during inflation and do not dominate the energy density. As a result, their potentials need not satisfy the stringent slow-roll conditions the inflaton needs to satisfy. These conditions imply that non-Gaussianities generated in standard one-field models of inflation are very small, probably unobservable with current techniques [6]. The fields determining the couplings could have for example larger self couplings (such would be, for instance, the quasiflat directions stabilized by quartic Yukawa couplings in the superpotential) leading to observable non-Gaussianities.

The slow roll condition on the k th derivative of the inflaton potential ($V^{(k)}$) is

$$\epsilon^{k/2-1} \frac{M_{PL}^k V^{(k)}}{V} < 1. \quad (25)$$

For $k=3$ for example, this leads to the constraint that $V^{(3)}/H < H/M_{PL}\epsilon^{1/2} \sim 10^{-5}$. In our scenario couplings much larger than this are allowed. The cubic coupling over H is roughly given by $m_s^2/\langle S \rangle H \sim H/\langle S \rangle$ so if $\langle S \rangle$ is not much larger than H significant non-Gaussianities are possible.

The bound on the three point function from WMAP can roughly be translated into $V^{(3)}/H < 10^{-3}$ [7]. On the other hand to match the correct level of density perturbations $f\delta S/\langle S \rangle \sim 10^{-4}$ (as we will show in the next section the solution of the full equations imply that the potential fluctuations are of order $1/9 \delta\Gamma/\Gamma$ so we set the combination $f\delta S/\langle S \rangle$ to 10^{-4} rather than 10^{-5}). This means that when $f \sim 0.1$ the non-Gaussianities saturate the WMAP bound. A more detailed analysis of the constraints set by WMAP is left for a future publication.

IV. PERTURBATION EQUATIONS

In this section we will explicitly write the relevant perturbation equations in the conformal gauge [8,9]. We will consider two fluids—radiation and the inflaton field which for simplicity we will treat as nonrelativistic matter. Our mechanism should also work for preheating [11], but here we only discuss the simpler case.

First we write the equations for the background evolution. The background metric is flat FRW universe, $ds^2 = a^2(d\eta^2 - dx^2)$, η is the conformal time, $a'/a = \mathcal{H}$ is the conformal Hubble constant, with $' \equiv d/d\eta$. The unperturbed Friedmann equations are

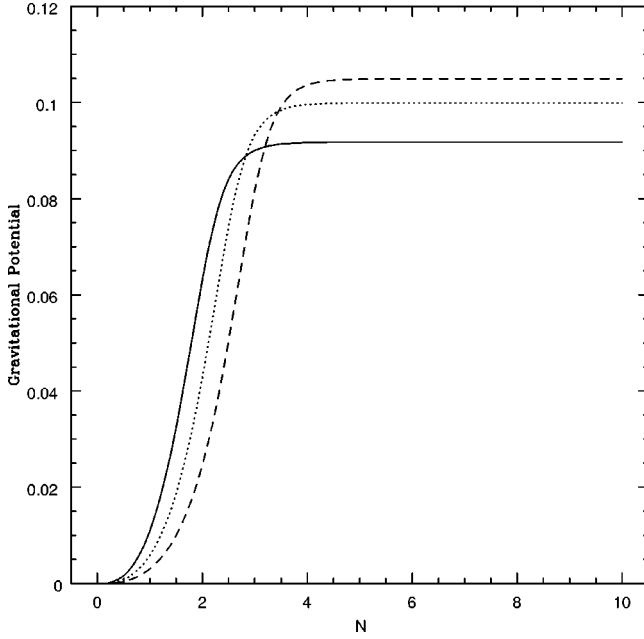


FIG. 1. Evolution of the gravitational potential in units of $\delta\Gamma/\Gamma$ as a function of $N=\ln a$. The curves correspond to $H_*/\Gamma = 5, 10, 20$ (solid, dotted, dashed).

$$\epsilon'_m = -3\mathcal{H}\epsilon_m - \Gamma a \epsilon_m, \quad (26)$$

$$\epsilon'_r = -4\mathcal{H}\epsilon_r + \Gamma a \epsilon_m, \quad (27)$$

$$\mathcal{H}^2 = (8/3)\pi G a^2 (\epsilon_r + \epsilon_m). \quad (28)$$

These describe a matter-dominated universe at small time and a radiation-dominated universe at large time. We solved them assuming that expansion rate was H_* and $x \equiv \epsilon_m/(\epsilon_m + \epsilon_r) = 1$ at the beginning. The fraction x remains around one until it suddenly drops to $x=0$ at the time when $H \sim \Gamma$. Before that time $H^2 \propto a^{-3}$ and after that $H^2 \propto a^{-4}$. Here $H \equiv \mathcal{H}/a$ is the standard Hubble parameter.

We now consider *super-horizon* perturbations induced by perturbations in the decay rate Γ . We use the conformal gauge with perturbed metric $ds^2 = a^2((1+2\Phi)d\eta^2 - (1-2\Phi)dx^2)$. The perturbed Einstein and matter equations describing forced *super-horizon* perturbations are

$$\mathcal{H}\Phi' + \mathcal{H}^2\Phi = -(4/3)\pi G a^2 (\epsilon_m \delta_m + \epsilon_r \delta_r), \quad (29)$$

$$\delta'_m = 3\Phi' - \Gamma a (\delta_\Gamma + \Phi), \quad (30)$$

$$\delta'_r = 4\Phi' + (\epsilon_m/\epsilon_r)\Gamma a (\delta_\Gamma + \Phi + \delta_m - \delta_r). \quad (31)$$

Here $\delta_m \equiv \delta\epsilon_m/\epsilon_m$, $\delta_r \equiv \delta\epsilon_r/\epsilon_r$, $\delta_\Gamma \equiv \delta\Gamma/\Gamma$.

We solved Eqs. (29)–(31) numerically. The solution for the gravitational potential is shown in Fig. 1. The gravitational potential directly gives the level of microwave background anisotropies which are simply proportional to Φ (see [8] for details on the calculation of CMB anisotropies in the conformal gauge). As we have estimated before, the potential fluctuations are of order $\delta\Gamma/\Gamma$. More specifically $\Phi \approx 1/9 \delta\Gamma/\Gamma$ with only a weak dependence of Γ/H_* , pro-

vided Γ/H_* is relatively small. In the limit $\Gamma/H_* \rightarrow 0$, one obtains (Appendix) the exact result $\Phi = (1/9)\delta\Gamma/\Gamma$.

V. CONCLUSIONS

We have proposed a new way in which density perturbations could be created during inflation. The density fluctuations produced by the new mechanism could easily dominate over the ones produced in the standard scenario. Our mechanism allows for the energy scale of inflation to be significantly lower than usually assumed. The fluctuations produced are also adiabatic and nearly scale invariant, although the relation of the spectral slope to the inflation potential is different from the usual. If the scale of inflation is lower, we generically predict a lower background of gravitational waves.

Our mechanism has no trouble generating non-Gaussianities that could be observed by future experiments. If such signal were to be found it would give additional clues as to the details of the inflationary model and how it fits in the general framework of particle physics.

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APPENDIX: ISOMORPHISM BETWEEN FREE AND INDUCED PERTURBATIONS. EXACT SOLUTION

Equations for free perturbations are obtained just by dropping the terms with δ_Γ from equations for induced perturbations (29)–(A1):

$$\mathcal{H}\Phi' + \mathcal{H}^2\Phi = -(4/3)\pi G a^2 (\epsilon_m \delta_m + \epsilon_r \delta_r), \quad (A1)$$

$$\delta'_m = 3\Phi' - \Gamma a \Phi, \quad (A2)$$

$$\delta'_r = 4\Phi' + (\epsilon_m/\epsilon_r)\Gamma a (\Phi + \delta_m - \delta_r). \quad (A3)$$

Let Φ , δ_m , δ_r be a solution of the free perturbation equations (A1)–(A3). Then $\Phi_1 = \Phi - \delta_\Gamma$, $\delta_{m1} = \delta_m + 2\delta_\Gamma$, $\delta_{r1} = \delta_r + 2\delta_\Gamma$ is a solution of the forced perturbation problem. We are interested in the purely forced mode, when $\Phi_1 = \delta_{m1} = \delta_{r1} = 0$ at $\eta=0$. The purely forced mode can be obtained from the free solution which satisfies $\Phi = \delta_\Gamma$, $\delta_m = -2\delta_\Gamma$, $\delta_r = -2\delta_\Gamma$ at $\eta=0$. This is just an adiabatic perturbation in the matter-dominated universe. That $\delta_r \neq 0$ at $\eta=0$ is irrelevant, because the radiation energy is small at small η .

We will show below that the free adiabatic (not growing for $\eta \rightarrow 0$) perturbation satisfies the well known matching condition [10]

$$\frac{\Phi(\infty)}{\Phi(0)} = \frac{1+p_m}{1+p_r} = \frac{10}{9}. \quad (\text{A4})$$

Here the indices p characterize the expansion rate of the universe, $a \propto t^p$. For matter domination $p_m=2/3$, and for radiation domination $p_r=1/2$.

The problem of calculating the forced metric perturbation is thus solved. The fluctuating reaction rate $\delta\Gamma$ induces a super-horizon potential perturbation

$$\Phi = \frac{1}{9} \frac{\delta\Gamma}{\Gamma}. \quad (\text{A5})$$

It remains to prove Eq. (A4). The free perturbation equations (A1)–(A3) can be written as

$$(3\epsilon_m + 4\epsilon_r)^2 \zeta' = \epsilon_m S, \quad (\text{A6})$$

$$S' = -\frac{9\epsilon_m + 10\epsilon_r}{2\epsilon_m + 2\epsilon_r} \mathcal{H} S, \quad (\text{A7})$$

where ζ (the Bardeen parameter) and S are

$$\zeta = \Phi - \frac{\epsilon_m \delta_m + \epsilon_r \delta_r}{3\epsilon_m + 4\epsilon_r}, \quad (\text{A8})$$

$$S = \mathcal{H}\epsilon_r(3\delta_r - 4\delta_m) + \Gamma a(\epsilon_m \delta_m + \epsilon_r \delta_r). \quad (\text{A9})$$

There are two scalar modes of free perturbations. The mode $S \neq 0$ is obtained from Eq. (A7). The other mode is $S=0$, $\zeta = \text{const}$. One can show that the latter mode describes adiabatic perturbations at $\eta \rightarrow 0$. We can therefore use $\zeta = \text{const}$ to prove the matching condition (A4).

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