# Radiation and evolution of a small relativistic dipole in QED

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We study in the quasiclassical approximation the radiation reaction and its influence on the space-time evolution for the small relativistic dipole moving in a constant external electromagnetic field in QED.

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## I. INTRODUCTION

The problem of the radiation loss (radiation reaction) of a particle moving in a given external field is a classical problem both in quantum and in classical physics that has always attracted a lot of attention. The classical physics studies are thoroughly reviewed in Refs. [1–3]. Since the creation of QED this problem was thoroughly studied on the quantum level by several groups of authors, and the results were reviewed in Refs. [4–10]. Two different lines of approach were developed: one based on the use of the exact wave functions in the external field [5,6,8], and the other based on the quasiclassical approach [7].

In particular, the classical results for the radiation reaction were extended to the quantum case, and it was shown that for ultrarelativistic particles such that $FE/m^3 \gg 1$ ($F$ is the field strength, $E$ the energy and $m$ the mass of the particle), the law of radiation reaction changes drastically compared to the classical case due to the strong recoil effects. Recently a new version of the quasiclassical approach based on the use of the quasiclassical Schrödinger wave functions was developed in Refs. [9,10] and references therein.

Although the theory of a particle in an external field seems to be thoroughly developed, there is still a lot of interest in the subject. The reasons, apart from the internal beauty of the subject, include a number of practical reasons. First, the external field is the simplest model of the media. Second, the QED results can be viewed as a starting point for the discussion of the propagation of the QCD particles in the media, this subject being extremely popular recently due to the recent interest in the quark-gluon plasma [11]. Next, it was realized that the space-time evolution of the point charge in the external field is closely connected with the fundamental properties of QED, leading to the concept of the semibare electron [12].

The above research was devoted, however, to the radiation reaction of the charged particle in the external field. Much less is known about the dipole propagation in the external field. The experimental research of fast $e^+$-$e^-$ pairs propagating in the media continues since the 1950s, including the famous experiments by Perkins [13] in 1957. The theoretical investigation of the fast $e^+$-$e^-$ pairs leading to the concept of charge transparency was started in Refs. [14,15]. Recently, there was renewed theoretical interest in the study of the relativistic dipole in QED. The reasons are both practical (explanation of the experimental data on $e^+$-$e^-$ pairs) and theoretical. In particular, it was realized that quantum effects play a much bigger role in the propagation of the dipole in the external field than that of the single particle, leading to the discovery of the quantum diffusion [16]. The essence of the latter phenomenom is the diffusion-type law of the fast dipole expansion in the weak external field due to the non-Coulombic quantum photon exchange between the components of the dipole. Thus it was realized that the study of the propagating QED dipole, and in particular of its space-time evolution, is important for the understanding of the fundamental properties of the QED, in light of Ref. [12]. Moreover, the study of the propagating dipole is extremely important due to its possible generalization to QCD, where the dipole, due to confinement, may be the basic degree of freedom [17–19]. This approach led to the discovery of color transparency phenomena in QCD [17]. Moreover, the QED dipole is identical to the QCD dipole connected to the deep inelastic scattering on the longitudinal virtual photons [20]. However, there is still very little knowledge about the properties of the propagating relativistic dipole (in particular relative to what we know about the propagation of the single charged particles).

The main goal of the present paper is to study the radiation reaction and in particular the pattern of the charge transparency and its influence on the evolution of the small ultrarelativistic dipole in the arbitrary strong external field in QED. In particular we shall be interested in the influence of the interference between the fields created by different components of the dipole on the radiation reaction. For simplicity we shall consider the case of the dipole containing two oppositely charged scalar particles of the same mass, moving in a constant external field whose direction is orthogonal to the direction of the motion of the center of mass of the dipole. We shall assume that two particles were created at the time $T=0$ in the same point of the space-time $\vec{r}(0)=\vec{0}$.

The main goal of this paper is to take into account the influence of the quantum effects on the radiation reaction of the dipole. We shall be able to take into account the quantum effects connected with the recoil. We will not be able to take into account the quantum effects connected with the quantum character of the motion of the dipole, in particular we shall not be able to take into account the spread of the dipole wave packets and the quantum diffusion. We will not take into account the spin of the particle, limiting ourselves to the scalar particle case. Throughout the paper we use the quasiclassical wave functions first derived in Refs. [9,10].

We shall see that there are three distinct time scales: $1/E$

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$\ll T \ll m/F$ (this time regime exists for the dipole such that the initial transverse motion of its components is nonrelativistic), $m/F \ll T \ll E/F$, $T \gg E/F$. For the first regime (we shall call it the very small dipole regime) the radiation reaction is strongly suppressed by interference. The interference also decreases a number of emitted photons. For the second regime the decrease in radiation reaction, relative to the sum of radiation reactions of two independent particles, depends on the Lorentz invariant parameter $\chi = FE/m^3$. For $\chi \ll 1$ the interference quickly decreases starting from $T \sim m/F$. For $\chi \gg 1$ the interference starts to decrease only starting from the larger time $T^* \sim (E/F^2)^{1/3}$. In the latter case the dipole affects especially change the photon spectrum. The relevant photons first are concentrated near the end point of the spectrum, and not in the middle, as for a single charged particle. The maximum of the radiation reaction spectral curve moves towards saturation at frequencies $\omega \sim 0.4E$.

The interference radically changes the frequency distribution of the number of radiated photons. Instead of an unbounded increase at small frequencies, it now goes to zero as $\omega \to 0$ and has a maximum at finite frequency of the order of the maximum of the radiation reaction.

Finally in the third regime, the interference does not influence the radiation reaction, but still cuts off the soft photons with $\omega \ll 1/T$. The photon distributions will have a finite maximum at $\omega \sim 2/T$.

Our results, derived in the approximation of the constant external field, can be translated to the model-independent language of the propagation of the dipole through the arbitrary external media. Indeed, the Lorentz-invariant parameter $\chi = FE/m^3$ is really a ratio of two parameters: the parameter $l_c = E/m^2$, which is (up to a numerical coefficient, unimportant here) a coherence length, and $l_F = m/F$, which is the field regeneration length (or time between successful interactions with the external field). Thus the parameter $\chi$ actually measures a number of collisions once the dipole propagates through the coherence length. In particular the color transparency and quantum diffusion considered in Refs. [16,17,19] correspond to the case $\chi \ll 1$. The regime of the very small dipole corresponds to the case $T \leq m/F$, i.e., in model-independent language, to the case when the propagation time is less than a time $T_F$ required to meet an external field photon. In other words, $l_F$ is analogous to the mean free path in the media language. Then it is clear that in this regime the radiation is always suppressed, independent of the parameter $\chi$. However, the later time evolution depends on the parameter $\chi$. If $\chi \ll 1$ (which corresponds to the case considered in Refs. [16,17,19]), the coherence length is much less than $l_F$, the radiation suppression ends, as we shall see, at $T \sim m/F$, and apart from the small time interval in the beginning $\sim E/m^2 \ll m/F$, one can use for the study of the radiation reaction and the spectra of the emitted photons a quasiclassical approximation. However, in the opposite case, $\chi \gg 1$, we have the situation of multiple collisions during the coherence length. In this case, we were able to develop a quasiclassical theory of the radiation emission taking recoil into account. Our results show the suppression of the radiation reaction and the photon emission up to the time $T^* \gg T_F$. This looks quite similar to the Landau-Pomeranchuk effect for the propagation of the fast particle in the media. There the effect also appears when the coherence length is much larger than the free mean path [9]. However, as we discuss below, the classical approximation may be not applicable to the situation when $l_F \ll l_c$ for the dipole. This is the case that occurs in statistical mechanics, when the coherence length is larger than free mean path. Then there is a number of the important effects that arise only beyond the quasiclassical approximation [21]. In this paper we shall only study what one obtains sticking to the quasiclassical approximation. The results may be considered as a starting point for future study.

The paper is organized in the following way. In Sec. II we shall consider the small classical dipole but will derive its radiation reaction using relativistic quantum mechanics, and check that the classical approach corresponds to the recoiless limit of quasiclassical theory. We shall review the results for a single particle, then consider the case of the radiation of the arbitrary dipole, and then derive the radiation reaction in the small and very small dipole limits. In Sec. III we shall briefly review the classical wave-function method of Refs. [9,10] and extend it to the case of the arbitrary dipole. Next we shall assume that the dipole is small (in the plane transverse to the direction of its center of mass motion) and derive the general formula for the radiation reaction of such a small dipole. In Secs. IV and V we shall use the above formulas to study the radiation reaction in two important limiting cases. In Sec. IV we shall study the frequency distribution of the radiation and the time dependence of the total radiated energy for the limit of very small times when the dipole's own field was not generated yet. We shall call this regime the very small dipole regime. This regime can be also characterized as the regime when the particle deflection angle due to the external field is less that the radiation angle. In Sec. V we shall consider the scale of times when the dipole is still small, but its field has already been generated. The particle deflection angle is much larger than the radiation angle. We shall study the frequency distribution of the photons and the radiation reaction also in this case. We shall see that the radiation reaction depends on the parameter $\chi = FE/m^3$. [Recall that for the single fast moving charged particle, the radiation reaction qualitatively depends on this parameter, which is Lorentz invariant: $\chi = \sqrt{(F_{\mu\nu}p^\nu)^2/m^6}$ [1].] In Sec. VI we shall study the total back force acting on the dipole for very small times and its influence on both the transverse and the longitudinal evolution of the dipole. In Sec. VII we shall make some qualitative comments on the influence of the quantum nature of the dipole motion on the radiation reaction, in particular on the possibility to go beyond the quasiclassical approximation. Our results, the directions for the future work, and possible implications for QCD will be summarized in the Conclusion.

## II. RADIATION REACTION OF THE FAST RELATIVISTIC DIPOLE

### A. Radiation of the single scalar particle

Let us start by briefly recalling the basic quasiclassical formalism for radiation of photons by relativistic charged particles without taking recoil into account [4]. The results are the same as those obtained by using classical electromagnetism theory [1,9], but we shall use from the beginning not the wave but the photon formalism. This will easily be extended in the next section to the case when we need to take recoil into account and the classical electromagnetism theory will be inapplicable.

The matrix element of the interaction between the electromagnetic field and the scalar particle is given by

$$S^{(1)}=-iq\int d^4x A_\mu(x)J^\mu(x), \tag{2.1}$$

where $J_\mu(x)$ is the current density operator in the external field,

$$J^\mu=\Phi^*(P^\mu\Phi)-(P^\mu\Phi^*)\Phi. \tag{2.2}$$

The operator $P^\mu$ is the generalized momentum operator in the external field. Consequently, the matrix element for the emission of the photon with the frequency $\omega$, wave vector $\vec{k}$, and polarization vector $\vec{e}$ is given by

$$M_{\rm fi}=-iq\int_0^T dt\int d^3\vec{r}\sqrt{\frac{2\pi}{\omega}}\frac{1}{\sqrt{E_iE_f}}\phi_f^*(\vec{r},t)(\vec{e}\cdot\hat{P})\times\exp[i(\omega t-\vec{k}\cdot\vec{r})]\phi_i(\vec{r},t). \tag{2.3}$$

Here $\phi_i$ is the initial and $\phi_f$ is the final state wave functions, normalized by the condition

$$\int d^3\vec{r}\,\phi^*(\vec{r})\phi(\vec{r})=1. \tag{2.4}$$

The operator $\vec{P}$ is

$$\vec{P}=\vec{p}-q\vec{A},$$

$\vec{p}=-\partial/\partial x_i$ is the momentum, $q$ is the charge of the particle, and $A(\vec{r},t)$ is the vector potential. $E_i$ and $E_f$ are the energies of the initial and the final states.

We shall use the quasiclassical wave functions of the scalar particle in the external field:

$$\phi(\vec{r},t)=\sqrt{\frac{D}{E_i-qA_0}}\exp\left(\frac{i}{\hbar}S(r,p,t)\right). \tag{2.5}$$

Here $S(r,p,t)$ is the action of the particle with the momentum $\vec{p}$ calculated along the classical trajectory of the particle passing through the point with the coordinate $\vec{r}$ at the time $t$ and having the momentum $\vec{p}$ at $t=0$. $D$ is the Van Vleck determinant:

$$D=\sqrt{\left\|\frac{\partial^2S(\vec{r},\vec{p})}{\partial\vec{r}\partial\vec{p}}\right\|}=\frac{1}{E}\sqrt{\delta(\overrightarrow{r-\vec{r}(t)})}. \tag{2.6}$$

The wave functions (2.5) cannot be substituted directly into the matrix element (2.3), due to the appearance of the quickly oscillating factors

$$\exp\{i[S(r,p_f,t)-S(r,p_i,t)]\}/\hbar$$

for $\hbar\to 0$. In order to avoid this difficulty we have to use the representation

$$\phi_i(\vec{r},t)=\int d^3\vec{p}\,\phi_p(\vec{r},t)S_{\rm pp_0}, \tag{2.7}$$

where $\phi_p$ is the quasiclassical wave function of the particle in the external field possessing at $t\to\infty$ the asymptotics

$$\phi_p(\vec{r},t)\to\frac{1}{\sqrt{2E_p}}\exp[i(\vec{p}\cdot\vec{r}-Et)].$$

$S_{\rm pp_0}$ is the scattering matrix of the particle in the external field considered. If we neglect the recoil and substitute the representation (2.7) for the final state wave function into the matrix element (2.3), we shall recover the classical amplitude for the radiation of the electromagnetic waves, and the classical expression for the energy loss during a time interval $T$ (see, e.g., Refs. [7,9] for details):

$$dW_{\rm cl}=\frac{2q^2}{\pi^2}(d^3k)\int_0^T\int_0^T dt\,dt'[\vec{e}\cdot\vec{v}(t)][\vec{e}^*\cdot\vec{v}(t')]\times\exp[i\omega(t-t')]-i\vec{k}\cdot[\vec{r}(t)-\vec{r}(t')]. \tag{2.8}$$

After averaging over the photon polarization vectors we obtain

$$dW_{\rm cl}=q^2\frac{1}{2\pi^2}d^3\vec{k}\int_0^T\int_0^T dt\,dt'([\vec{v}(t)\cdot\vec{v}(t')]-[\vec{n}\cdot\vec{v}(t)]\times[\vec{n}\cdot\vec{v}(t')])\exp[i\omega(t-t')-i\omega\vec{n}\cdot[\vec{r}(t)-\vec{r}(t')]], \tag{2.9}$$

where $\vec{k}=\omega\vec{n}$. Note that $\vec{n}\cdot\vec{v}(t)=\vec{v}\cdot\vec{\nabla}_{\vec{r}}=\partial/\partial t$. Hence the terms in the latter equation containing $\vec{n}$ in the preexponential factor can be integrated by parts:

$$dW_{\rm fi}=q^2\frac{1}{2\pi^2}d^3k\Bigg(\int_0^T\int_0^T dt\,dt'[\vec{v}(t)\cdot\vec{v}(t')-1]\exp[i\omega(t-t')-i\omega\vec{n}\cdot[\vec{r}(t)-\vec{r}(t')]]\Bigg)$$
$$+\frac{4}{\omega}\int_0^T\sin\{[\omega T+\vec{n}\cdot\vec{r}(T)]/2\}\cos\{\omega(T-2s)+\vec{n}\cdot[\vec{r}(T)-2\vec{r}(s)]\}/2$$
$$-\frac{2}{\omega^2}[1-\cos\omega T+\omega\vec{n}\cdot\vec{r}(T)]. \tag{2.10}$$

It is straightforward to see, however, that the last two lines in Eq. (2.10) correspond to terms decreasing or bounded with $T$, while the expression in the first line increases with $T$. Thus the two last lines can be omitted if we are interested in large time intervals $T\gg 1/\omega$. Indeed, we can integrate Eq. (2.10) over the photon direction $\vec{n}$ and obtain

$$dW_{\rm cl}=q^2\frac{2}{\pi}\omega\,d\omega\Bigg[\int_0^T\int_0^T dt\,dt'[\vec{v}(t)\cdot\vec{v}(t')-1]\cos(\omega(t-t'))\frac{\sin[\omega|\vec{r}(t)-\vec{r}(t')|]}{|\vec{r}(t)-\vec{r}(t')|}$$
$$+\frac{2}{\omega}\int_0^T\frac{\cos[\omega s-\omega r(s)]-\cos[\omega s+\omega r(s)]}{r(s)}$$
$$+\frac{2}{\omega}\int_0^T\frac{\cos[\omega(T-s)-\omega|\vec{r}(T)-\vec{r}(s)|]-\cos[\omega s+\omega|\vec{r}(T)-\vec{r}(s)|]}{|\vec{r}(T)-\vec{r}(s)|}$$
$$-\frac{2}{\omega^2}\Bigg(1-\frac{\sin\{\omega[T+r(T)]\}-\sin[\omega T-r(T)]}{r(T)}\Bigg)\Bigg]. \tag{2.11}$$

It is easy to see that the last three lines in Eq. (2.11) are suppressed similar to $1/(\omega T)$ relative to the double integral in the first line, and thus can safely discarded if we are interested in the frequencies and time intervals $\omega T\gg 1$. In order to know numerically how large these terms are, we shall, however, keep them.

Finally, since we are usually interested in the energy losses in units of time, we can differentiate Eq. (2.11) over time $T$ and obtain

$$\frac{dW_{\rm cl}}{dT}=q^2\frac{4}{\pi}\omega d\omega\Bigg[\int_0^T dt[\vec{v}(T)\cdot\vec{v}(t)-1]\cos(\omega(T-t))\frac{\sin[\omega|\vec{r}(T)-\vec{r}(t)|]}{|\vec{r}(T)-\vec{r}(t)|}$$
$$+\frac{2}{\omega}\frac{\cos[\omega T-\omega r(T)]-\cos[\omega T+\omega r(T)]}{r(T)}$$
$$+\frac{2}{\omega}\int_0^T\frac{d}{d/T}\frac{\cos[\omega s-\omega|\vec{r}(T)-\vec{r}(T-s)|]-\cos[\omega s+\omega|\vec{r}(T)-\vec{r}(T-s)|]}{|\vec{r}(T)-\vec{r}(T-s)|}$$
$$-\frac{d}{dT}\frac{2}{\omega^2}\Bigg(1-\frac{\sin\{\omega[T+r(T)]\}-\sin[\omega T-r(T)]}{r(T)}\Bigg)\Bigg]. \tag{2.12}$$

Below we shall use the first line in the latter equation and check that the last three lines can be neglected:

$$\frac{dW}{dT}=q^2\frac{4}{\pi}\omega d\omega\Bigg[\int_0^T dt[\vec{v}(T)\cdot\vec{v}(t)-1]$$
$$\times\cos(\omega(T-t))\frac{\sin[\omega|\vec{r}(T)-\vec{r}(t)|]}{|\vec{r}(T)-\vec{r}(t)|}\Bigg]. \tag{2.13}$$

The latter equation, if the limits of integration are infinite, can be easily brought into the standard form of the classical electromagnetic theory [4,9].

### B. Radiation of the relativistic dipole: General theory

Let us now consider the radiation reaction of the relativistic dipole in the case we can neglect recoil, i.e., $\omega\ll E$. For simplicity we consider the symmetric dipole, whose center of mass moves with the speed $v\sim c$ in the direction orthogonal to the direction of the constant external field, and which was created at time $T=0$ in the point $\vec{r}(0)=0$. We shall denote the components of the dipole as $P$ and $A$ (particle and antiparticle). Let us assume that the particles of the dipole have, after its creation, the same initial energy $E_i$, and the orthogonal component of the velocity $v_{0t}$. Note that if $u_{0t}$ is the velocity in the transverse plane in the center of mass (c.m.) reference frame, moving with the dipole, then $v_{0t} = (m/E)u_{0t}$, meaning that in any case $v_{0t} \leqslant m/E$. In our kinematics the two components of the dipole will have the same velocity component in the direction of c.m. motion and the opposite sign components in the transverse plane.

The amplitude of the radiation of the photon with polarization vector $\vec{e}$, wave vector $\vec{k}$, and frequency $\omega$ will be the difference (due to the different charges of the dipole components) of the amplitudes of the photon emission of the particle and the antiparticle components of the dipole. Using the equations of the preceding section it is straightforward to write

$$M_{\mathrm{fi}} = -i\sqrt{\frac{2\pi}{\omega}}\frac{1}{\sqrt{E_i E_f}}\int_0^T dt\{\vec{e}\cdot\vec{v}_P(t)\exp i[\omega t - \vec{k}\cdot\vec{r}_P(t)] - \vec{e}\cdot\vec{v}_A(t)\exp i[\omega t - \vec{k}\cdot\vec{r}_A(t)]\}. \tag{2.14}$$

Here $\vec{r}_P(t)$ and $\vec{r}_A(t)$ are the radius vectors of the particle and antiparticle components of the dipole. The energy radiation loss during the time from creation of the dipole at time $t=0$ until time $T$ with the photons radiated in the frequency range $d\omega$ and the solid angle range $do$ is

$$dW = \frac{q^2}{4\pi^2}\omega^2 do\, d\omega \int_0^T dt \int_0^T dt' \exp i\omega(t-t') \times\{\vec{e}\cdot\vec{v}_P(t)\exp i[\vec{k}\cdot\vec{r}_P(t)]\} - \vec{e}\cdot\vec{v}_A(t)\exp i[\vec{k}\cdot\vec{r}_A(t)] \times\{\vec{e}^{*}\cdot\vec{v}_P(t')\exp - i[\vec{k}\cdot\vec{r}_P(t')]\} - \vec{e}^{*}\cdot\vec{v}_A(t')\exp - i[\vec{k}\cdot\vec{r}_A(t')]. \tag{2.15}$$

Summing over the polarizations of the photon we obtain

$$\frac{dW}{d\omega} = \frac{q^2}{4\pi^2}\omega^2\int_0^T dt \int_0^T dt'\{\exp i[\omega(t-t')]\}(\{\vec{v}_P(t)\cdot\vec{v}_P(t') - [\vec{n}\cdot\vec{v}_P(t)][\vec{n}\cdot\vec{v}_P(t')]\}\exp i(\vec{k}\cdot[\vec{r}_P(t)-\vec{r}_P(t')]) + (P\leftrightarrow A) - (\vec{v}_P(t)\cdot\vec{v}_A(t') - [\vec{n}\cdot\vec{v}_P(t)][\vec{n}\cdot\vec{v}_A(t')]) \times \exp i\{\vec{k}\cdot[\vec{r}_P(t)-\vec{r}_A(t')]\} + (P\leftrightarrow A)). \tag{2.16}$$

Here $\vec{k}=\omega\vec{n}$. Using, as for the single particle, $\vec{n}\cdot\vec{v}(t) = \vec{v}\cdot\vec{\nabla}_{\vec{r}} = \partial/\partial t$, we can carry out the integration by parts and obtain:

$$\frac{dW}{d\omega\, do} = \frac{q^2}{4\pi^2}\omega\int_0^T dt \int_0^T dt' \exp i[\omega(t-t')][\vec{v}_P(t)\cdot\vec{v}_P(t') - 1]\exp i\vec{k}\cdot[\vec{r}_P(t)-\vec{r}_P(t')] + (P\leftrightarrow A) - [\vec{v}_P(t)\cdot\vec{v}_A(t') - 1]\exp i\vec{k}\cdot[\vec{r}_P(t)-\vec{r}_A(t')] + (P\leftrightarrow A) + \Delta W(\omega,T). \tag{2.17}$$

The latter equation gives us the formula for the radiation of the arbitrary relativistic dipole. Note that it is a sum of two terms that correspond to the radiation of the single particle and two terms that correspond to the interference between the particle and the antiparticle.

The term $\Delta W$ arises from the integration by parts (cf. the single particle) and is equal to

$$\Delta W(\omega,T) = do\frac{q^2}{4\pi^2}\omega^2\left(\frac{2}{\omega}\int_0^T \sin\{\omega(T-s) + \omega\vec{n}\cdot[\vec{r}_P(T) - \vec{r}_P(s)]\} + (P\leftrightarrow A)\right) - \frac{2}{\omega}\int_0^T ds(\sin\{\omega(T-s) + \omega\vec{n}\cdot[\vec{r}_P(T)-\vec{r}_A(s)]\} + \sin\{\omega(T-s) + \omega\vec{n}\cdot[\vec{r}_A(T)-\vec{r}_P(s)]\}) - (P\leftrightarrow A) - \frac{2}{\omega^2}\{1-\cos\omega\vec{n}\cdot[\vec{r}_P(T)-\vec{r}_A(T)]\}. \tag{2.18}$$

We can integrate over the angle variable $do$ and obtain

$$\frac{dW}{d\omega} = \frac{q^2}{\pi}\int_0^T dt \int_0^T dt' \cos[\omega(t-t')][\vec{v}_P(t)\cdot\vec{v}_P(t') - 1] \times\frac{\sin|\vec{r}_P(t)-\vec{r}_P(t')|}{|\vec{r}_P(t)-\vec{r}_P(t')|} + (P\leftrightarrow A) - [\vec{v}_P(t)\cdot\vec{v}_A(t') - 1] \times\frac{\sin|\vec{r}_P(t)-\vec{r}_A(t')|}{|\vec{r}_P(t)-\vec{r}_A(t')|} - (P\leftrightarrow A) + \Delta G(\omega,T). \tag{2.19}$$

Here the term $\Delta G$ corresponds to the integral of $\Delta W$:

$$\delta G=\frac{2q^2\omega}{\pi}\left[\int_0^T ds\left(\frac{\cos[\omega(T-s)-\omega|\vec r_P(T)-\vec r_P(s)|]}{\omega|\vec r_P(T)-\vec r_P(s)|}-\frac{\cos[\omega(T-s)+\omega|\vec r_P(T)-\vec r_P(s)|]}{\omega|\vec r_P(T)-\vec r_P(s)|}\right)\right]$$
$$+(A\leftrightarrow P)-\left(\frac{\cos[\omega(T-s)-\omega|\vec r_P(T)-\vec r_A(s)|]}{\omega|\vec r_P(T)-\vec r_A(s)|}-\frac{\cos[\omega(T-s)+\omega|\vec r_P(T)-\vec r_A(s)|]}{\omega|\vec r_P(T)-\vec r_A(s)|}\right)$$
$$-(A\leftrightarrow P)-\frac{1}{\omega}\left(1-\frac{\sin\omega|\vec r_P(T)-\vec r_A(s)|}{\omega|\vec r_P(T)-\vec r_A(s)|}\right). \tag{2.20}$$

In order to get the radiation reaction it is enough to differentiate the above equations over $T$. The latter equations describe the radiation of an arbitrary relativistic dipole, integrated over the angles, for the time interval $T$. Now we can move to our goal—to consider the case of the small relativistic dipole.

### C. Radiation of the small relativistic dipole

Consider the small quasiclassical relativistic dipole, i.e., $v\gg v_t$, where $v$ is the center of mass velocity and $v_t$ is the transverse component of the velocity ($v_t$ can be both relativistic and nonrelativistic). For sufficiently small times one can estimate

$$v_t(T)\sim v_{0t}+FT/E. \tag{2.21}$$

Here $F$ is the external field,

$$\vec F=\vec E+\vec v\times\vec H, \tag{2.22}$$

$\vec E$ is an electric field, and $\vec H$ is a magnetic field. Consequently, one considers a dipole as small if

$$FT\ll E. \tag{2.23}$$

For larger time scales,

$$FT\gg E,$$

the components of the dipole behave as independent particles and there is no interference. Let us study the interference pattern in the small dipole. Let us assume that the condition (2.23) is satisfied. Then the photons are radiated into the small cone around the $z$ axis (we choose the $z$ axis in the direction of the propagation of the dipole), of order $m/E$ at $T\sim 1/E_i$, where $E_i$ is the initial energy of each of the components of the dipole. Later the radiated photons are concentrated in two cones around the directions of the components of the dipole. It is clear that there exist, even if the condition (2.23) is satisfied, two distinct possibilities: the two radiation cones, generated by the dipole components, overlap, and that they stop to overlap. Since the cone angle for the ultrarelativistic particle is $\theta\sim m/E_i$, we see that the condition for overlapping is

$$v_{ot}+FT/E\leq m/E. \tag{2.24}$$

If we can neglect the initial transverse velocity, the latter condition becomes

$$T\leq m/F. \tag{2.25}$$

If

$$m/F\ll T\ll E/F$$

the dipole is still small, but the cones do not overlap, and the interference must decrease drastically. There is also the self-consistency condition: since $T\gg 1/E$, we must have

$$mE\gg F \tag{2.26}$$

for the possibility of considering the very small dipole, with overlapping cones, quasiclassicaly. We need the weaker condition

$$E^2\gg F, \tag{2.27}$$

for the possibility to consider the small quasiclassical dipole. If the latter conditions are not fulfilled, we must take into account the interference of the dressing by the external field and the generation of the self-field by bare particles. This is beyond the scope of this research (although it could be that our analysis is qualitatively true even in the latter case, since the self-dressing usually generates quickly oscillating terms that can be singled out).

We conclude that the classical dipole has two regimes: (1) a very small dipole, when the radiation cones of the particle and the antiparticle overlap strongly, $T\ll m/F$, and (2) a small dipole in the sense that it still moves along the $z$ axis, but the cones of the radiation do not overlap, and the interference decreases. Note that these two cases correspond to two possible relations between the depletion angle of the single charged particle in the external field and the radiation angle. The very small dipole corresponds to the case when the latter angle is much larger than the former and the small dipole corresponds to the case when the former is larger than the latter. Note also that for a relativistic dipole in the c.m. of the dipole, transverse motion means (then $v_t\sim m/E$) we have only the small dipole regime.

Suppose we have the very small dipole. Let us analyze the interference pattern. Consider the exponents in Eq. (2.17). The exponents in the terms that contain only the particle or only the antiparticle radiation are

$$\omega\{\cos\theta[z(T)-z(t)]+\sin\theta[y(T)-y(t)]\}.$$

Here $\theta$ is the angle between the photon wave vector and direction of the $z$ axis, and

$$z(T)-z(t)\sim v_z(T-t),\quad y(T)-y(t)\sim v_t(T-t).$$

It is clear that the corresponding integrals will be saturated by $t\sim T$, and the first term will be dominant since $\sin\theta\ll 1$ Consider now the exponents in the interference terms in Eq. (2.17). These exponents have the form for the chosen kinematics

$$\omega\{\cos\theta[z(T)-z(t)]+\sin\theta\sin(\phi)[y(t)+y(T)]\}.$$

Here $\phi$ is the azimutal angle. In the first approximation we can set $y(t)\sim y(T)=d(T)/2$ in the latter equation, and instead of integrating, substitute $\sin\theta$ by its characteristic value $m/E$. [$d(T)$ is the scale of the dipole, i.e., the separation between the charges, which in our kinematics is purely transverse.] Then the integral over the angle $\phi$ gives the Bessel function

$$\frac{1}{2\pi}\int_0^{2\pi}\exp[i\sin\phi\omega d(T)m/E]d\phi=J_0\left(\frac{\omega}{E}d(T)m\right). \tag{2.28}$$

With the same accuracy we can substitute $\cos\theta[z(t)-z(T)]$ with $\cos\theta[z(t)-z(T)]+\sin\theta\sin\phi[y(t)-y(T)]$, i.e., after taking into account the interference term, the exponent in the interference term will be the same as in the direct terms. Then for very small dipoles we can rewrite Eq. (2.17) as

$$\frac{dW}{d\omega dT}=\frac{4q^2}{\pi}\int_0^T dt\cos[\omega(T-t)][\vec{v}_P(T)\cdot\vec{v}_P(t)-1]\frac{\sin|\vec{r}_P(T)-\vec{r}_P(t)|}{|\vec{r}_P(T)-\vec{r}_P(t)|}\left[1-J_0\left(\frac{\omega}{E}md(T)\right)\right]. \tag{2.29}$$

In addition, there is contribution from the terms that correspond to integration by parts, where it is enough to do the same approximation:

$$\frac{dG}{dTd\omega}=\frac{q^2}{\pi}\frac{2}{\omega}\int_0^T\frac{d}{dT}\frac{\cos(\omega s)-\omega|\vec{r}(T)-\vec{r}(T-s)|-\cos[\omega s+\omega|\vec{r}(T)-\vec{r}(T-s)|]}{|\vec{r}(T)-\vec{r}(T-s)|}\left[1-J_0\left(\frac{\omega}{E}md(T)\right)\right]+\frac{d}{dT}\frac{2}{\omega^2}\frac{\sin\omega d(T)}{T}. \tag{2.30}$$

Here $\dot{\vec{d}}(s)$ is the time derivative of the dipole moment, i.e., the relative velocity of the particle and antiparticle:

$$\dot{\vec{d}}(s)=\frac{\partial[\vec{r}_P(s)-\vec{r}_A(s)]}{\partial s}. \tag{2.31}$$

The latter equation gives the radiation energy loss rate for the very small relativistic dipole between times 0 and $T$, emitted in the particular interval of photon frequencies.

Note that our interference analysis could be made in terms not of the characteristic radiation angles, but in terms of the longitudinal and transverse momenta. Our characteristic angles $m/E$ correspond to the characteristic transverse momentum of the emitted photons $q_t\sim m\omega/E$. In particular, if we consider photons whose energy is a finite part of $E$, the characteristic transverse momentum will be $q_t\sim m$.

Consider now the next regime, $E/F\gg T\gg m/F$. This is the case of the small, but not very small dipole. In this case we can still consider the trajectory of each of the particles as almost a straight line. We can follow the above derivation of the interference terms, but in this case, although still $\theta\ll 1$, we need to take as $\theta$ the angle $v_t/v\sim v_t\sim v_{0t}+FT/E$. We then get an equation similar to Eq. (2.29), but with a different argument for the Bessel function:

$$\frac{dW}{d\omega dT}=\frac{4q^2}{\pi}\int_0^T dt\cos[\omega(T-t)][\vec{v}_P(T)\cdot\vec{v}_P(t)-1]\times\frac{\sin|\vec{r}_P(T)-\vec{r}_P(t)|}{|\vec{r}_P(T)-\vec{r}_P(t)|}[1-J_0(\omega\theta(T)d(T))], \tag{2.32}$$

where

$$\theta(T)=v_{0t}+FT/E=v_y(T). \tag{2.33}$$

Since for the classical dipole $d(T)\sim v_y(T)T\sim v_{ot}T+FT^2/(2E)$, we see that interference is suppressed as

$$1-J_0\left(\frac{\omega}{E}[v_{0t}^2TE+3v_{ot}FT^2/2+F^2T^3/(2E^2)]\right).$$

Since $v_t\leq m/E$ (due to the relativistic law of the velocity summation) and $T\gg m/F$, the third term in the argument of the Bessel function will be dominant, i.e., the interference decreases as $J_0((\omega/E)(F^2T^3/E))$, and quickly becomes negligible. Finally note that for very small frequencies one always has interference.

Note also that the argument of the Bessel function can be represented as

$$xb(\tau),\quad b(\tau)=m\frac{dd^2(\tau)}{d\tau},$$

i.e., as a Lorentz invariant (see also the discussion below). Here $\tau$ is the proper time in the reference frame of the c.m. of the dipole.

Finally, since the integrands in Eqs. (2.29) and (2.32) are concentrated near $T=s$, we can expand them in a Taylor series near $s=T$. Consider first the difference $\vec{r}(T)-\vec{r}(s)$ in the argument of the exponents.

For a small dipole it is possible to use the approximations [9,10]

$$\vec{v}(T)=v(0)[1-v_t^2(T)/2v(0)^2]+\vec{v}_t(T) \tag{2.34}$$

and

$$\dot{\vec{v}}(T)_t=q\vec{F}/E. \tag{2.35}$$

In this approximation up to the terms of order $m/E$,

$$\dot{\vec{v}}(T)\sim\dot{\vec{v}}_t\sim q\vec{F}/E,$$

i.e., the vectors $\vec{v}$ and $d\vec{v}/dT$ are orthogonal. Also

$$\frac{d^2\vec{v}}{dT^2}=-\omega_0^2\vec{v}(T),$$

$$\omega_0=qF/E.$$

Then

$$\begin{aligned}|\vec{r}(T)-\vec{r}(s)|\\ &=\sqrt{v^2(T)(T-s)^2[1-\omega_0^2(T-s)^2]^2+\omega_0^2(T-s)^2/4}\\ &\sim v(T)(T-s)[1+\omega_0^2(T-s)^2/24].\end{aligned} \tag{2.36}$$

Then we have in the standard way (Ref. [9]):

$$\vec{v}_P(T)\vec{v}_P(t)-1=-[1-v^2(T)+(T-t)^2\omega_0^2/2]. \tag{2.37}$$

In the same approximation

$$\dot{\vec{d}}(T)\cdot\dot{\vec{d}}(s)=4[v_t(T)^2-(T-s)\omega_0 v_t(T)]. \tag{2.38}$$

Thus we have our final result for the radiation of the small dipole:

$$\begin{aligned}\frac{dW}{d\omega dT}=-\omega\frac{4q^2}{\pi}\int_0^T dt\Bigg(\frac{1-v^2(T)+\omega_0^2(T-t)^2/2}{T-t}\\ \times\sin[\omega(1-v)(T-t)]+\omega_0^2(T-t)^3/24\Bigg)\\ \times[1-J_0(\omega\theta(T)d(T))].\end{aligned} \tag{2.39}$$

Here

$$\theta(T)=m/E\qquad T\ll m/F,$$

$$\theta(T)\sim v_t(T)\sim m/E+FT/E,\qquad E/F\gg T\gg m/F. \tag{2.40}$$

In order to obtain the full radiation reaction we must integrate the latter formulas over $\omega$. We thus obtained the classical radiation reaction of the dipole using simple wave mechanics. In parallel we understood the nature of the interference in the transverse plane, which we shall use in the quantum case.

## III. RADIATION REACTION FOR THE RELATIVISTIC DIPOLE: RECOIL EFFECTS

### A. Single particle

In the preceding section we studied, using the relativistic quantum mechanics method, the classical radiation from the classical dipole. Let us now move to quantum effects. There are two types of quantum effects [9]. First, there are effects due to the quantum character of the particle motion in the external field. This effect is characterized by the parameter $F/E^2$ [9]. Second, there are quantum effects specifically due to the motion of the quantum dipole [16,17,19]. We will not take these effects into account. Third, there are recoil effects, that arise if we take $E_i\neq E_f$ into account. The general theory of such effects was first derived in Ref. [7]. Recently a new approach was derived by Akhiezer and Shulga [9,10]. Let us briefly review the idea of Ref. [9]. We return to the derivation of the matrix element of the radiation of photons (2.3). We still use the representation (2.7) for the quasiclassical wave functions, but when we substitute them into the matrix element (2.3) we take into account that the corresponding integral over $\vec{p}$ is saturated not at $\vec{p}=\vec{p}_f$, as we assumed when we neglected recoil, but at $\vec{p}\sim\vec{p}_f+\vec{k}$, where as usual $\vec{k}$ is the wave vector of the emitted photon. Then it is possible to prove that the generalized action $S=S_f-(\omega t-\vec{k}\cdot\vec{r})$ satisfies the generalized Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t}=(\vec{\nabla}S-q\vec{A}-\vec{k})^2+m^2. \tag{3.1}$$

Solving this equation and substituting the solution into the matrix element (2.3), where we use the representation (2.7) for the wave function $\phi_i$, one obtains the quasiclassical matrix element of the photon radiation where the recoil is taken into account:

$$\begin{aligned}M_{\rm fi}=-iq\int_0^T dt\int d^3\vec{r}\sqrt{\frac{2\pi}{\omega}}\sqrt{\frac{E_i}{E_f}}\\ \times\left[\vec{e}\cdot\vec{v}(t)\exp\left(i\frac{E_i}{E_f}[\omega t-\vec{k}\cdot\vec{r}(t)]\right)\right].\end{aligned} \tag{3.2}$$

Here we can set $E_f = E_i - \omega$.

The corresponding radiation reaction will be the same as that for the single particle in the preceding section, except for the rescaling of the frequency

$$\omega \to \omega \frac{E_i}{E_f}$$

in the exponent and the general multiplier $E_i/E_f$:

$$dW_{\rm fi} = q^2 \frac{2}{\pi^2} \frac{E}{E_f} d^3k \int_0^T \int_0^T dt dt' \times [\vec{e}\cdot\vec{v}(t)][\vec{e}^*\cdot\vec{v}(t')] \exp\left(\frac{E}{E_f}\right) \times \{ i\omega(t-t') - i\vec{k}\cdot[\vec{r}(t) - \vec{r}(t')]\}. \tag{3.3}$$

After averaging over the photon polarizations and integrating by parts we obtain, as in the preceding section, the equation for the radiation reaction of the single particle including the recoil effects:

$$\frac{dW}{dT} = \omega q^2 \frac{4}{\pi} d\omega \left( \int_0^T dt [\vec{v}(T)\cdot\vec{v}(t) - 1] \cos\left(\frac{E}{E_f}\omega(T-t)\right) \frac{\sin[(E/E_f)\omega|\vec{r}(T)-\vec{r}(t)|]}{|\vec{r}(T)-\vec{r}(t)|} + \frac{2}{(E/E_f)\omega} \frac{\cos[(E/E_f)(\omega T - \omega r(T))] - \cos[(E/E_f)(\omega T + \omega r(T))]}{r(T)} + \frac{2}{(E/E_f)\omega} \int_0^T \frac{d}{dT} \frac{\cos[(E/E_f)(\omega s) - \omega|\vec{r}(T) - \vec{r}(T-s)|] - \cos(E/E_f)[\omega s + \omega|\vec{r}(T)-\vec{r}(T-s)|]}{|\vec{r}(T)-\vec{r}(T-s)|} - \frac{d}{dT} \frac{2}{[(E/E_f)\omega]^2} \left(1 - \frac{\sin(E/E_f)\omega[T + r(T)] - \sin[(E/E_f)(\omega T - \omega r(T))]}{r(T)}\right)\right). \tag{3.4}$$

All other formulas from the Sec. II A are transformed in the same way: $\omega$ is rescaled except in the measure, and the general multiplier is added, $E/E_f$. Note that the terms that arise from integration by parts (boundary effects) are suppressed now, even stronger, as $\omega TE/(E-\omega)$.

We keep the terms due to integration by parts so that we shall be able to check explicitly that they are small in our analysis of Eq. (3.4). For convienience, let us write the latter equation without the back reaction term, that is, the result that will be used in the calculations:

$$\frac{dW}{dT} = q^2 \frac{4}{\pi} \omega\, d\omega \left[ \int_0^T dt [\vec{v}(T)\cdot\vec{v}(t) - 1] \cos\left(\frac{E}{E_f}\omega(T-t)\right) \times \frac{\sin[(E/E_f)\omega|\vec{r}(T)-\vec{r}(t)|]}{|\vec{r}(T)-\vec{r}(t)|}\right]. \tag{3.5}$$

The recoil effects lead to the qualitative change of the spectrum of the single particle. The maximum of the radiation reaction will be shifted to $\omega_m \sim 0.4E$ for large $\chi$, and will be virtually $\chi$ independent. For the opposite limit of small $\chi$ the maximum will remain at the classical value of $\sim E\chi$.

### B. Recoil effects in the dipole radiation

It is clear from the previous sections that taking recoil into account will mean just rescaling $\omega$ in the preceding section. Consequently, we obtain

$$\frac{dW}{d\omega\, dT} = \frac{4q^2}{\pi} \int_0^T dt \cos\left(\frac{E}{E_f}\omega(T-t)\right) [\vec{v}_P(T)\cdot\vec{v}_P(t) - 1] \frac{\sin|\vec{r}_P(T) - \vec{r}_P(t)|}{|\vec{r}_P(T) - \vec{r}_P(t)|} \left[1 - J_0\left(\frac{E}{E_f}\omega\theta(T)d(T)\right)\right], \tag{3.6}$$

where the function $\theta(T)$ is given by Eq. (2.40). Since the main contibution still comes from $s \sim T$, we obtain

$$\frac{dW}{d\omega dT} = -\omega \frac{4q^2}{\pi} \int_0^T dt \frac{1 - v^2(T) + \omega_0^2 s^2/2}{s} \sin\left[\frac{E}{E_f}(\omega[(1-v) \times (s)] + \omega_0^2 (s)^3/24)\right]\left[1 - J_0\left(\frac{E}{E_f}\omega\theta(T)d(T)\right)\right]. \tag{3.7}$$

One can obtain the full radiation reaction by integrating the above equation over all frequencies.

## IV. RADIATION REACTION FOR THE VERY SMALL DIPOLE

Let us analyze the above equations for different regimes discussed in Sec. III. We consider in this section the case of the very small dipole: $1/\omega \ll T \ll m/F$. First, let us check what time scales contribute to Eq. (2.39) in this case. For the linear term in the argument of the cosine in Eq. (2.39) to be dominant we need

$$(1-v)\gg \omega_0^2 s^2/24$$

or

$$s \ll 2\sqrt{6}\frac{m}{\sqrt{2}E}(E/F)=2\sqrt{3}\ \frac{m}{F}\sim 3.5\frac{m}{F}.$$

Here $s=T-t$, Since the latter condition is satisfied for the very small dipole for the entire integration region in $s$, we can neglect the cubic terms in the arguments of the cosine as well as the nonleading terms in the preexponentials. The integrals over $s$ in Eq. (2.39) can be taken explicitly. As explained in Appendix A, in this case we can discard the terms in Eq. (2.39) proportional to $1-v^2$, as well as the terms originating from the integration by parts. The reason is that up to the terms suppressed as $m^2/E^2$ these terms correspond to the radiation reaction of the free charged particle moving with a constant velocity. The latter is of course a nonphysical phenomena (see discussion in the Appendix A) and must be subtracted. We start with an integral

$$\frac{dW}{d\omega}=-\frac{4q^2}{\pi}\int_0^T dt\int_0^t ds\frac{\omega_0^2}{2}s\sin\left[\frac{E}{E_f}(\omega[(1-v)(s)])\right]\left[1-J_0\left(\frac{E}{E_f}\omega\frac{m}{E}d(T)\right)\right]. \tag{4.1}$$

Note that the latter integral, as is well known from the classical theory [1], is proportional to $F^2$, i.e., to the square of the acceleration.

The latter integral can be taken eplicitly under the assumption that the interference multiplier weakly depends on time $T$ in the limit of integration, and thus can be taken outside of the integrand. We obtain

$$\frac{dW}{d\omega}=\omega_0^2\omega\frac{q^2}{\pi}\left(\frac{2\{1-\cos[\omega'(1+v)T]-\omega(1+v)T\sin[\omega'(1+v)T]\}}{\omega' 3(1+v)^3}-\frac{2\{1-\cos[\omega'(1-v)T]-\omega(1-v)T\sin[\omega'(1-v)T]\}}{\omega'^3(1-v)^3}\right)\left[1-J_0\left(\frac{E}{E_f}\omega\frac{m}{E}d(T)\right)\right]. \tag{4.2}$$

The corresponding spectral curve for the single particle (without taking into account radiation) is depicted in graph (a) of the Fig. 1. The spectral curve for the same energy and field, but for the dipole, whose transverse motion velocity is $v_t \ll 1$ and the interference is taken into account is depicted in the graph (b) of Fig. 1. We see a drastic decrease of the radiation for all frequencies. We also see that increasing the factor $\gamma$ (i.e., decreasing the mass for a given energy) leads the maximum to be shifted further to the end point of the spectrum [graph (c) of Fig. 1]. Indeed, the classical maximum of radiation is at

$$\omega_{\rm cl}\sim\frac{1}{T}\frac{E^2}{m^2} \tag{4.3}$$

and for sufficiently big energies is beyond the end point. This is the case when the recoil effects are most important. Note that at $T\sim m/F$ the classical radiation maximum $\omega_{\rm cl}$ of Eq. (4.3) reaches $\omega_H$—the classical radiation maximum for the small (but not very small) time regime.

We see that the effects of interference are the largest if the transverse motion is nonrelativistic. For $v_{0t}\sim 1$ the interference effects are small [see graph (c) of Fig. 1].

The latter equation can be differentiated in $T$ and then integrated over $\omega$ to obtain the total radiation reaction of the very small dipole:

$$\frac{dE}{dT}=\frac{q^2}{\pi}T^2E^2\omega_0^2\int_0^\infty dx\frac{1}{x*(1+x)^3}\sin(xb)-b*x\cos(b*x)\Big/ b^2-[\sin(xa)-a*x\cos(a*x)/a^2][1-J_0(xmd(T))] \tag{4.4}$$

Here $a=ET(1-v)$, $b=ET(1+v)$. The latter intergal, contrary to the single-particle case (see Appendix A), cannot be taken explicitly. In order to estimate this integral, it is worthwhile to get rid of oscillating terms using the representation

$$\frac{1}{(1+x)^3}=0.5\int_0^\infty p^2\exp(-p)$$

and Eqs. (B7) and (B8) from Appendix B. We obtain

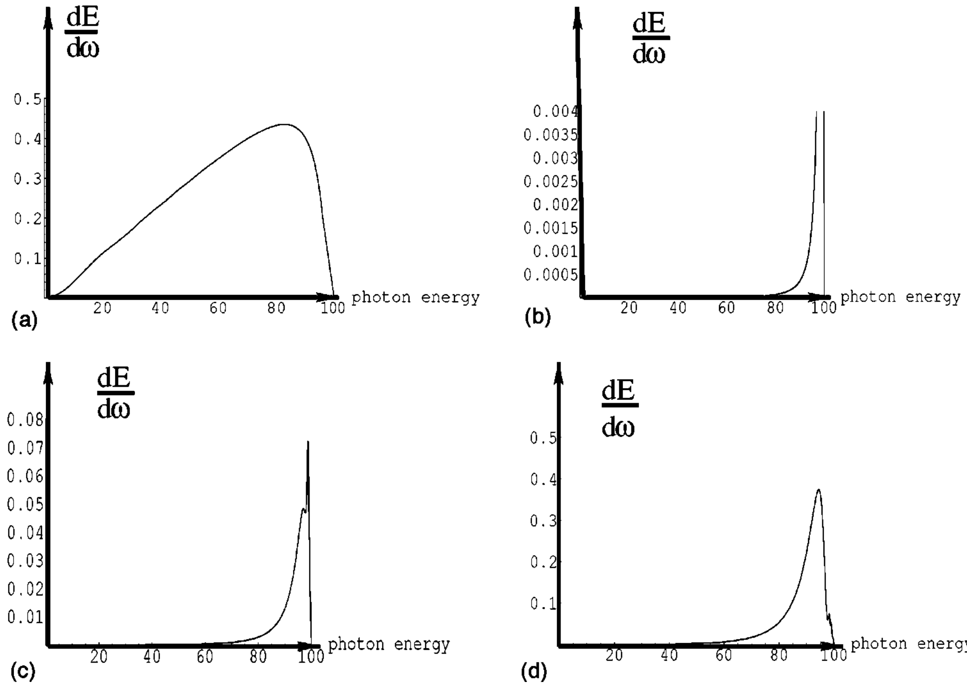


FIG. 1. The spectral distribution curves for the radiated energy $dW/d\omega$ (normalized by $q^2/\pi$) for the the very small dipole regime ($T \leqslant m/F$). For all graphs $E=100$ GeV, the field $F=100$ GeV$^2$, $T=0.1$ GeV$^{-1}$. (a) The spectral distribution of a single particle, $v=0.99$, $m=14$ GeV. (b) The dipole, $v=0.99$, the center of mass transverse velocity $v_{0t}=0.2$. (c) The dipole, $v=0.999$, $v_{0t}=0.2$. (d) The dipole, $v=0.99$, $v_{0t}=0.9$.

$$\frac{dE}{dT}=0.5\frac{q^2}{\pi}\int_0^\infty p^2 \exp(-p)[G_1(p,b,md(T)) - G_2(p,b,md(T))]. \tag{4.5}$$

The corresponding time dependence is given in graph (b) of the Fig. 1. We put this graph along the radiation reaction curve for the single particle [graph (a) of Fig. 2]. The radiation reaction decreases drastically due to interference. Finally, note that for $T\sim m/F$ the radiation reaction for $\chi\to\infty$ behaves like $1/\chi$, while like $\sqrt{\chi}$ for $\chi\to 0$.

Numerically, it is easy to see that the condition for the interference to decrease the total radiation reaction significantly is that the radiation maximum must occur for the frequencies where the interference is still strong, i.e., the frequency $\omega_m$ is such that

$$\omega_m E/(E-\omega_m)md(T)\leqslant 1. \tag{4.6}$$

It is easy to see that this condition is equivalent to

$$E/m \leqslant T/d(T)\sim 1/v_t. \tag{4.7}$$

In other words, the interference decreases the dipole radiation by an order of magnitude if it is nonrelativistic in its c.m. reference frame, while the interference influences the total radiation reaction only slightly if the dipole transverse motion is relativistic ($v_{0t}\sim 1$).

It is worthwhile to describe qualitatively the position of the radiation maximum and the structure of the spectral curve for different $\chi$. First, consider $\chi \ll 1$. In this case we see that at $T\sim E/m^2 \ll m/F$ the maximum of radiation will be near the end point of the spectrum. The radiation itself will be negligible. For larger times the total radiation slowly increases, while the radiation maximum moves to $\sim E\chi$, where it reaches at times $\sim m/F$. Then the curve smoothly transforms itself into the curve for the small dipole that is studied in the next section. The radiation maximum no longer moves. Afterwards the radiation reaction quickly increases, while the interference diminishes. The reason why the radia-

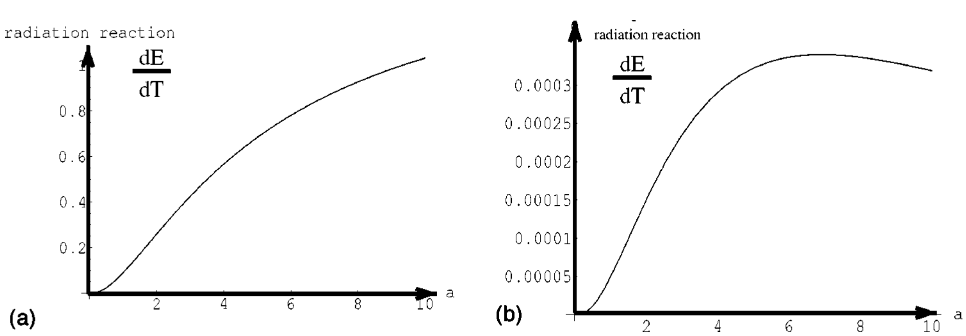


FIG. 2. Radiation reaction of the single particle (a) and for dipole (b) as a function of $a=m^2T/2E$ for the very small time regime $T\leqslant m/F$. For the picture $m^2/2E=1$ GeV, $E=100$ GeV, $F=100$ GeV$^2$. The radiation eaction $dE/dT$ is normalized by $q^2/\pi$. For the dipole $v_{0t}=0.2$.

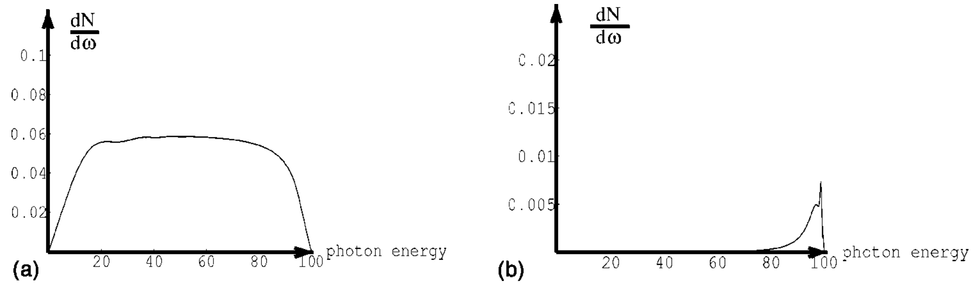


FIG. 3. The spectral curve for the spectral distribution of the number of the radiated photons $dN/d\omega$ (normalized by $q^2/\pi$) versus the photon energy $\omega$. (a) Photon number for a single particle. The velocity $v=0.99$, $E=100$ GeV, $m=14$ GeV, field $F=100$ GeV$^2$, $T=0.1$ GeV$^{-1}$. (b) The dipole with the same parameters and c.m. transverse velocity $v_{0t}=0.2$.

tion is still suppressed at times $\sim m/F$ for the nonrelativistic transverse motion is that the suppression factor in the maximum is $1-J_0(v_{0t}x_m)$.

For the opposite case $\chi \gg 1$ the situation quite different. There is a complete suppression of radiation and the radiation maximum remains near the end point well into the small dipole regime (see the next section).

The results for the radiation reaction are in correspondence with the situation with the total number of the photons. Without interference [see graph (a) in Fig. 3] the total number of radiated photons remains finite, and has a maximum. Including the interference cuts off the soft photons and decreases the total number of photons drastically [see graph (b) of Fig. 3]. The maximum in the distribution of the number of radiated photons, as seen from the figure, is parametrically located at the same frequencies as that of radiation reaction, i.e., most of radiated photons are hard photons.

In this discussion we did not take into account the important effects of Sudakov form factors and wave-function renormalization that generally tend to cancel out. For the number of photons we expect these factors to be more important than for the energy. Consequently, our discussion is just a conjecture that needs further calculation.

## V. SMALL DIPOLE

In the preceding section we discussed the case of very small dipoles, corresponding to $T \ll m/F$. The goal of this section will be to consider the opposite limiting case $T \gg m/F$. In the latter case we can substitute the integration limits by infinity and discard the terms due to integration by parts. We immediately obtain for spectral density

$$\frac{dE}{dTd\omega}=\frac{2q^2}{\sqrt{\pi}}\frac{m^2}{E^2}\omega\left(\frac{1}{2}\int_a^\infty \Phi(u)du+\frac{1}{a}\frac{\partial\Phi(a)}{\partial a}\right)\times[1-J_0(\omega'\theta(T)d(T))], \tag{5.1}$$

where

$$a=(\omega/\omega_H)^{2/3}, \quad \omega_H=\omega_0(E/m)^3, \tag{5.2}$$

$$\omega'=\omega E/(E-\omega). \tag{5.3}$$

Function $\Phi$ is the standard Airy function (see Appendix B). This result is the single-particle answer times the interference multiplier.

Let us first consider the case of $\chi \ll 1$. The corresponding graphs are depicted in Fig. 4. Figure 4(b) corresponds to time $T\sim m/F$, while the graph in Fig. 4(c) to time $T\sim E/F$. We see that for the first graph the interference is very strong, while for the second case it is only slight. For comparison we also present the analogous graph for a single charged particle [graph (a) in Fig. 4].

The opposite limiting case $\chi \gg 1$ is depicted in Fig. 5. We see that for $T\sim m/F$ the interference decreases the radiation reaction dramatically, for $T\sim E/F$ the decrease is only slight (if fact we see a very slight increase). Most interesting, we see that interference remains important numerically even at $T\sim(E/F^2)^{1/3}$, where we see that it decreases the maximum

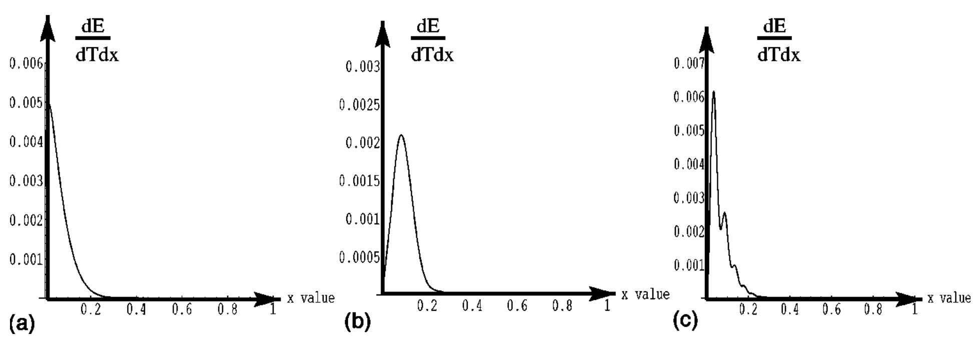


FIG. 4. Spectral curve as a function of $x=\omega/E$. We choose here $F=100$ GeV$^2$, $E=100$ GeV, $v=0.8$, $\chi=0.04$. [All graphs in this and the next figure depict $dE/(dTd\omega)$, normalized by $q^2/\sqrt{\pi}$.] (a) The spectral curve for a single charged particle. (b) Spectral curve for the dipole in a small time regime, $T=m/F=25$ GeV$^{-1}$. (c) Spectral curve for the dipole in a small time regime, $T=m/F=100$ GeV$^{-1}$.

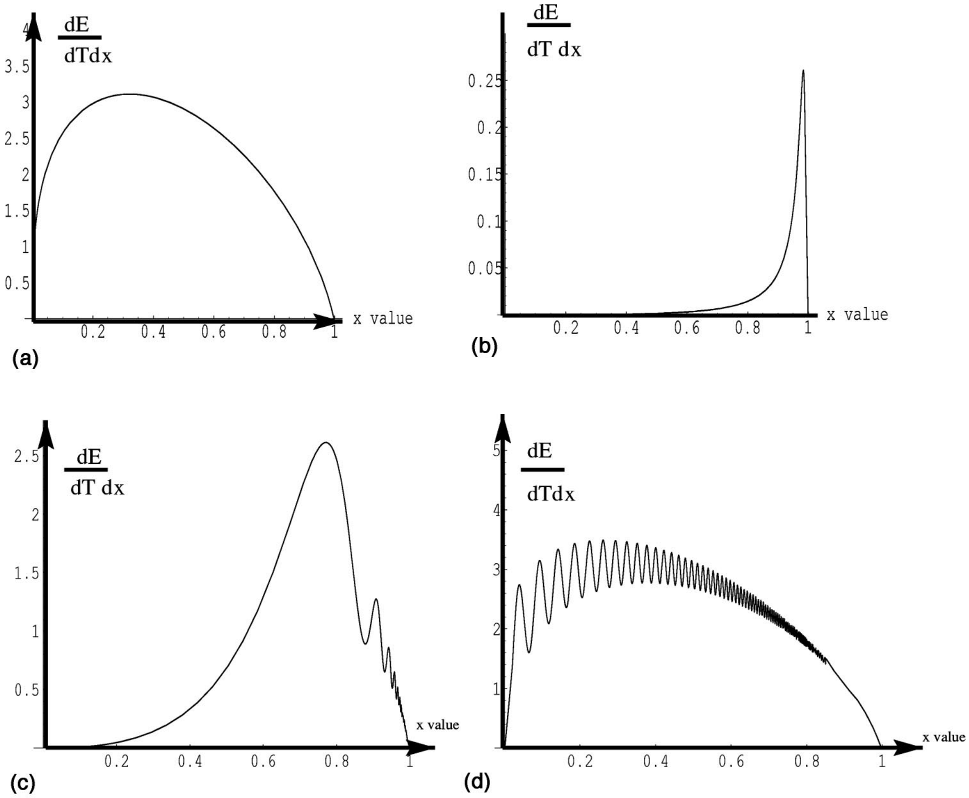


FIG. 5. Spectral curves as a function of $x=\omega/E$, as in the previous figure. We choose here $F=100\ \mathrm{GeV}^2$, $E=100$ GeV, $v=0.999$, $\chi=111.6$. (a) Single charged particle. (b) The dipole in the small time regime, $T=m/F=0.045\ \mathrm{GeV}^{-1}$. (c) The dipole in the small time regime, $T=(E/F^2)^{1/3}=0.21\ \mathrm{GeV}^{-1}$. (d) The dipole in the small time regime, $T=E/F=100\ \mathrm{GeV}^{-1}$.

by the order of 1.5 and clearly significantly decreases the total radiation reaction (the spectral curve is still ''dipole like''). Moreover, the interference leads to a further shift of the radiation maximum to the end point of the spectrum.

Let us now consider the total radiation reaction. Integrating Eq. (5.1) we obtain

$$\frac{dE}{dT}=\frac{2q^2}{\sqrt{\pi}}m^2\int_0^\infty dx\frac{x}{(1+x)^3}\left(0.5\int_u^\infty \Phi(u)+\Phi'(u)/u\right)\times[1-J_0(xEd(T)\theta(T))], \tag{5.4}$$

where the function $\theta(T)$ is given by Eq. (2.40) above. The expression in the case without interference differs slightly from that for the single particle, since we did not do the usual integration by parts:

$$u=\frac{x^{2/3}}{\chi^{2/3}}. \tag{5.5}$$

In order to understand qualitatively the influence of the dipole interference let us change the integration variable to $u$. Then

$$\frac{dE}{dT}=\frac{3q^2\chi^2}{\sqrt{\pi}}m^2\int_0^\infty du\frac{u^2}{(1+\chi u^{3/2})^3}\left(0.5\int_u^\infty \Phi(s)ds+\Phi'(u)'u\right)[1-J_0(u^{3/2}\chi Ed(T)\theta(T))]. \tag{5.6}$$

The radiation reaction is the function of two parameters: the relativistic invariant $\chi$ and relativistic invariant $b(\tau)$, which is equal to

$$b(\tau)=E\,\theta(T)d(T)=m\frac{dd^2(\tau)}{d\tau} \tag{5.7}$$

for the small dipole and

$$b(\tau)=md(\tau) \tag{5.8}$$

for the very small dipole. The radiation reaction can be written as

$$\frac{dE}{dT}=F(\chi,c(\tau)), \tag{5.9}$$

where

$$c(\tau)=b(\tau)\chi. \tag{5.10}$$

Here $\tau$ is the proper time in the c.m. reference frame of the dipole.

In order to understand qualitatively the influence of the dipole interference consider two limits: $\chi\ll 1$ and $\chi\gg 1$. For $\chi\ll 1$ we can use Eq. (5.6). For the time $T\gg m/F$ we can estimate,

$$dd^2(T)/dT\sim F^2T^3/E,$$

and the argument of the Bessel function in Eq. (5.6) is just $u^{3/2}(T/T_0)^3$, where $T_F=m/F$. We then put $\chi=0$ in the denominator in the latter equation and obtain

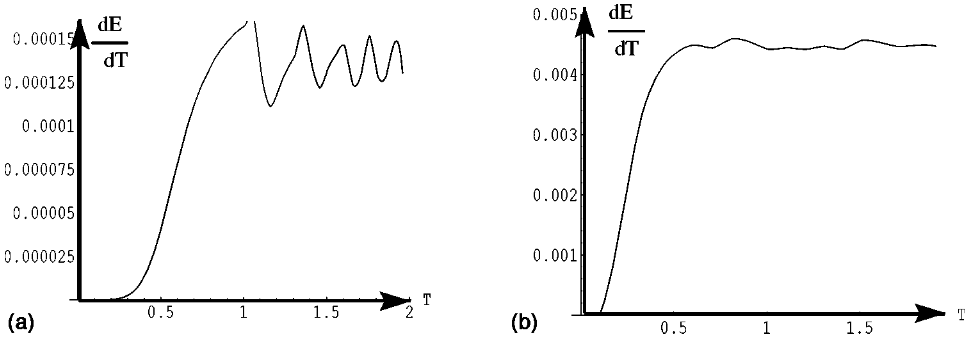


FIG. 6. Radiation reaction of the dipole as a function of $T(\mathrm{GeV})^{-1}$ for the small time regime $E/F \gg T \gg m/F$. The radiation reaction $dE/dT$ is normalized by $q^2/\sqrt{\pi}$. (a) $\chi=0.04$, $E=100$ GeV, $F=100\ \mathrm{GeV}^2$. (b) $\chi=111.4$, $E=100$ GeV, $F=100\ \mathrm{GeV}^2$, $T^*\sim 0.22\ \mathrm{GeV}^{-1}$.

$$dE/dT=-\frac{2}{3}\frac{F^2E^2}{m^4}G(T/T_F), \tag{5.11}$$

where the first term corresponds to the classical Pomeranchuk effect, and the function $G(T/T_F)$ is defined by

$$G(s)=\frac{9}{2\sqrt{pi}}\int_0^\infty u^2\left(0.5\int_u^\infty[\Phi(u)+\Phi'(u)/u]\right)[1-J_0(u^{3/2}(T/T_F)^3)]. \tag{5.12}$$

It is clear that for $T\gg T_0$, $G(T)\rightarrow 1$. The radiation reaction in this case is depicted in the first graph in Fig. 6, where we see the sharp increase of the radiation reaction at $T\sim T_F$. We see that it becomes weakly time dependent numerically at $T\sim E/F$.

For the opposite case $\chi\gg 1$ the main contribution in the integral of Eq. (5.6) comes from the $u\rightarrow 0$. In this case we can put the argument in the Airy functions to zero, and use $\Phi'(0)\sim -0.5$. We then obtain

$$dE/dT=-\frac{q^2m^2\chi^{2/3}\Phi'(0)}{\pi}F(T/T^*), \tag{5.13}$$

where

$$F(T/T^*)=\int_0^\infty x^{1/3}/(1+x)^3[1-J_0(x(T/T^*)^3)], \tag{5.14}$$

i.e., $T^*=(E/F^2)^{1/3}$.

Note that for $\chi\gg 1 E/F\gg T^*\gg T_F=m/F$. Thus the argument $x$ of the Bessel function is multiplied by a number less than 1. Consequently, as it is clear from the second graph in Fig. 6, the radiation reaction remains negligible up to $T\sim T^*$. Afterwards, it increases rapidly, and at $T\sim E/F$ becomes approximately time independent and a sum of radiation reactions of the components of the dipole. In both cases the qualitative dependence on the parameter $\chi$ for the dipole is the same as that for the single particle.

It is worthwhile, as in the preceding section, to follow the position of the maximum of radiation. For small $\chi$ we saw that it does not change, and the interference in this regime is small. The interesting case is the case of large $\chi$. We see that up to $T\sim T^*$ the spectrum is shifted to the end point. This is in contrast with the single particle, where, as it seen from the first graph in Fig. 5 (see also Refs. [9,10]) the spectrum is concentrated near $\omega_m\sim 0.4E$. The strong interference in the maximum is the reason for the suppression of the total backreaction. When $T\gg T^*$, the maximum, as it is clear from the figures above, starts to move to the position of the maximum of the single particle, i.e., $0.4E$, where it comes by $T\sim E/F$, and the radiation reaction quickly increases.

Consider now the number of radiated photons in the $q^2$ approximation of the perturbation theory. It is clear that this number is drastically decreased by interference. We see in Fig. 7(a) the number of emitted photons without the interference, while in Fig. 7(b) the number of emitted photons is shown with the interference taken into account. We see that the interference qualitatively changes the spectrum: instead of being infinite in the limit of soft photons ($\omega\rightarrow 0$), now the spectrum has no infrared singularity for soft frequencies. Instead a number of radiated photons $\rightarrow 0$ at $\omega\rightarrow 0$ and has a finite maximum at finite frequency. Moreover, it is clear that

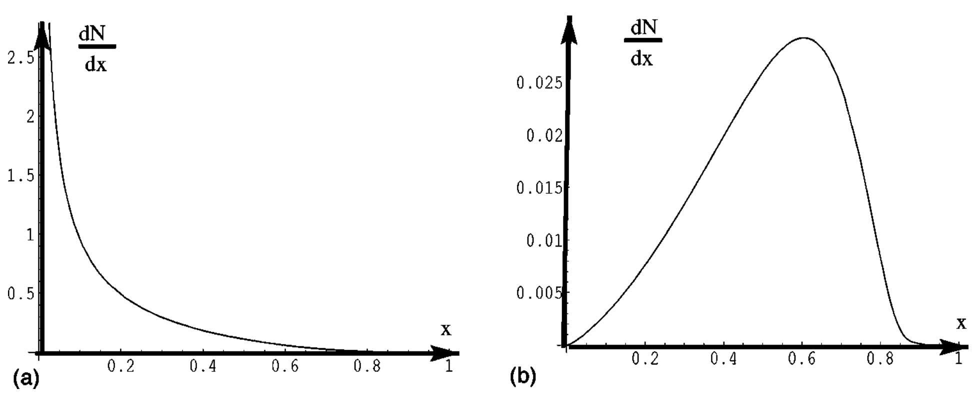


FIG. 7. Number of radiated photons in the small dipole regime as a function of frequency. (normalized by $q^2/\sqrt{\pi}$). (a) No interference. (b) Interference is taken into account.

up to a numerical coefficient of the order one, the position of this maximum will be the same as the frequency that corresponds to the maximum of the radiation reaction. In particular, for the ultrarelativistic dipole $\chi \gg 1$ the relevant frequency will shift to the end point of the spectrum. The photons will take almost the entire energy of the radiating electron in a single radiation event for the time interval $T^* \ll T \ll E/F$. For $\chi \sim 1$ one radiating event will take $\sim 1/2$ of the initial energy of the electron.

An even more interesting effect will take place for higher times $T \sim E/F$ (and $\chi \gg 1$). Since the soft photons will be cut by the dipole effects, the number of photons will have a maximum for finite frequencies. Numerical analysis shows that in this case

$$\omega_m \sim 2/T. \tag{5.15}$$

This means that there are two distinct groups of radiating events for large $T$: radiation of large numbers of soft photons with frequencies given by Eq. (5.15) and the radiating events where the dipole loses approximately half of its energy each time, this half being carried by a photon.

## VI. BACKREACTION AND EVOLUTION OF THE VERY SMALL DIPOLE

We can answer now how the backreaction influences the evolution of the very small dipole, and where the energy loss goes due to radiation: to the relative motion of the particles in the center of mass or to the loss of the total energy of the center of mass motion. Our results show, that for the single particle and for very small times the backreaction force behaves according to Eq. (A7) in Appendix A. For the dipole the backreaction force behaves in much less singular fashion. Note that for very small times one can write

$$dE/dT = \frac{q^2 a^3}{\pi(1-v)^2} \int_0^\infty ds[\sin(s) - s\cos(s)] * [1 - J_0(2v_{0t}s)]/[s(s+a)^3]. \tag{6.1}$$

Here as in the previous sections $a = ET(1-v)$. Equation (6.1) can be used to obtain the first several terms in the expansion of the backreaction force for small $T$:

$$dE/dT = \frac{q^2 T^3 m^2 F^2 v_{0t}^2}{2\pi E}\left(1 - \frac{3}{2}\frac{m^2 T \pi}{4E}\right) + O(T^5 \log(m^2 T/E)). \tag{6.2}$$

Recall that $v_{0t}$ is the transverse velocity in the c.m. frame.

The backreaction force for the dipole is smaller than that for the single particle, logarithmic terms are present only starting from $T^5$, and its leading term is proportional to $T^3$. The condition for the applicability of the expansion (6.2) is

$$T \ll \min(E/m^2, m/F). \tag{6.3}$$

Since the photons are radiated almost parallel to the direction of the dipole, the corresponding backreaction force is directed in the direction opposite to the direction of the dipole and leads to the decrease of its center of mass velocity.

There is an additional backreaction force that slows the expansion of the dipole in the orthogonal direction. For the very small time regime this force is evidently $\sim \sin^2\theta (dE/dT)/p_t$, where $\theta \sim m/E$ is the radiation cone angle, and $p_t \sim v_{0t} m$ is the transverse momentum. Consequently the orthogonal component of the backreaction force is

$$F_y(T) \sim \frac{q^2}{\pi}\frac{F^2 m^3 T^3 v_{0t}}{E^2} + O(T^4, T^5 \log(m^2 T/E)). \tag{6.4}$$

The influence of this force on the wave packet radius begins only from the terms of $\sim T^5$, i.e., for very small $T$ the expansion due to quantum diffusion and external field is dominant.

## VII. THE QUANTUM DIPOLE

In the latter analysis we did not take into account the quantum character of the dipole motion. In fact, there is an additional effect that influences the motion of dipole, and this is the non-Coulombic photon exchange between the components of dipole. This effect is significant if the distance between the components of the dipole is less then $1/m$ and leads to so-called quantum diffusion: the distance between the components of the dipole increases not linearly or quadratically as in the usual relativistic quantum mechanics, but in the diffusion way, i.e., as $\sim \sqrt{T}$ [16] (a simple qualitative explanation of this phenomena is contained in Ref. [27]). Moreover, the dipole motion along the coherence length may stop being quasiclassical, as assumed throughout this paper [16].

It is easy to see that the effect is important for the ultrarelativistic dipole with $\chi \gg 1$. Indeed, the diffusion is important until the distance between the components of the dipole is $\sim 1/m$, where $1/m$ is the scale of bound state in QED. In order to take into account the external field we need to write the wave functions in the external field, taking into account quantum non-Coulombic photon exchanges. This is beyond the scope of the current paper. Here we shall try to build a qualitative model to indicate the influence of the quantum effects. In order to estimate the field influence on the diffusion let us note that the diffusion law,

$$d^2(T) = \frac{2T}{E}, \tag{7.1}$$

can be obtained from the equation

$$\dot{y}/E = \eta(t), \tag{7.2}$$

where $\eta$ is the random external force such that

$$\langle \eta(t)\eta(t')\rangle = E\,\delta(t-t'). \tag{7.3}$$

In order to include the external field, we generalize this equation in an obvious way:

$$\ddot{y}(t)+\dot{y}/E=\eta(T)+F/E. \tag{7.4}$$

This equation can be easily solved with the result

$$d^2(T)=4\langle y^2(T)\rangle=2T/E+2F^2T^2/E^4+O(T^3) \tag{7.5}$$

and

$$\langle y(T)\rangle=FT/E^2,\quad \langle v_y(T)\rangle=F/E^2. \tag{7.6}$$

We see that quantum diffusion changes the velocity and the distance between charges. The average velocity is small, $F/E^2$, and constant. Diffusion is important until the distance between dipole components is $1/m$, i.e., $2T/E\sim 1/m^2$, or

$$T\sim(E/m)(2/m). \tag{7.7}$$

Note that for $\chi\gg 1$ this time is $\gg m/F$. Also note that for all reasonable times $\ll E/F$ the first term in Eq. (7.5) is dominant.

Consider now the interference in the case of diffusion. We need to average the product $\sin(\theta)\omega d$. There are two possibilities. If the angle between the direction of a component of the dipole and z axis is much bigger than $m/E$, we use $\sin\theta\sim v_y$. Then we need to average $\langle v_y d(T)\rangle = dd(T)^2/dT=2/E$, and we get as the argument of the Bessel function $2x=2\omega/E$ ($\omega'/E$ if we also take into account recoil effects). The interference multiplier will be

$$1-J_0(2x). \tag{7.8}$$

If the angle is $m/E$, we shall get the argument $\omega m\langle d(T)\rangle/E\sim 2\omega mFT/E^3$. We must choose the larger of two arguments. It is easy to see that the first argument will be larger up to times $T\sim E^2/(mF)=(E/F)(E/m)$, i.e., for all times where the dipole notion has meaning. Thus, in our simple model, in the diffusion regime, which lasts parametrically longer, as $\chi$ increases, the interference depends on time only weakly.

For small $\chi$ the diffusion law holds only for very small times $\ll m/F$. Thus the largest influence seems to occur for $\chi\gg 1$, when the interference multiplier may significantly change the radiation.

We have developed above a simple phenomenological model, indicating the effects connected with the quantum character of the dipole motion. Unfortunately, at the moment we can, on the basis of this model, only indicate that they may be very important for large $\chi$ and that they lead to the suppression of the dipole radiation, as in the quasiclassical dipole.

The reason for the difficulties we encounter is the inadequacy of the classical approximation. It is possible to estimate the area of reliability of the quasiclassical approximation: we must demand that the transverse velocity acquired in the classical approximation due to the action of the external field is larger than the velocity due to quantum diffusion. Quite remarkably this gives us the condition $T\gg T^*$, i.e., the quantum diffusion effects are important for large $\chi$ up to the scale when quasiclassically, radiation suppression stops and the radiated energy quickly increases as we saw in Sec. IV. This result is consistent with the conclusion above, that in the time interval when the quantum effects in the dipole motion are important, there is still a suppression of the radiation reaction (the charge transparency). The origin of the difficulty is clear. The parameter $\chi=l_c/l_F$, where $l_c\sim E/m^2$ is the coherence length, while $l_F\sim m/F$ is the field regeneration length, in other words the average distance the dipole must travel before colliding with the external field photon. It is clear that the external field does not break coherence. Thus we are in the situation when we have multiple coherence-conserving collisions along the coherence length. In this situation it is well known (see, e.g., Ref. [21]) that the classical approximation is, generally speaking, not applicable. The quasiclassical approximation corresponds to neglecting the coherence conservation and thus can lead to the wrong results. Further analysis along the lines of Ref. [21] is therefore needed.

## VIII. CONCLUSION

We have studied the backreaction and its influence on the evolution of the relativistic dipole in the arbitrary strong external field using the quasiclassical approximation. We have taken into account the quantum recoil effects in radiation, but not quantum effects in the motion of the dipole, i.e., quantum diffusion. We found that the dipole motion is governed by two invariant parameters; one of them describes the longitudinal motion and is equal to $\chi=EF/m^3$; another describes the motion in the transverse plane and is equal to

$$b(\tau)=md(\tau)\quad \text{if}\quad T\ll m/F,$$

$$b(\tau)=m\frac{dd^2(\tau)}{d\tau}\quad \text{if}\quad E/F\gg T\gg m/F.$$

It is quite possible that there exists a single formula for $b$, although we were not able to obtain it.

We have studied the pattern of charge transparency in the external field. We have found that the interference effects can be taken into account by the use of the general interference multiplier $1-J_0(xb(\tau))$, where $x=\omega/(E-\omega)$. For arbitrary times the radiation reaction is given by Eq. (2.39).

We have seen that there are three different time scales. First, is the very small dipole regime, $T\ll m/F$. This time scale exists if the dipole transverse velocity $v_{0t}\ll 1$. In this regime the radiation reaction is strongly suppressed by interference, leading to the strong decrease of the backreaction, i.e., the fast moving dipole does not lose its energy. In this case we were able to calculate analytically the backreaction force analytically for both the entire regime, and for very small times [Eq. (6.2)]. For larger time scales $E/F\gg T\gg m/F$ the influence of interference on backreaction depends on the value of the parameter $\chi=EF/m^3$. If $\chi\ll 1$, the radiation reaction quickly increases starting from $T\sim m/F$, and by the time $T\sim E/F$ it is a sum of radiation reaction of the components of the dipole [see graph (a) in Fig. 6]. However, for the opposite case $\chi\gg 1$, the radiation reaction starts to increase only from the times $T \sim T^* = (E/F^2)^{1/3}$, and then once again goes quickly to the sum of the radiation reactions of the components.

For the third regime $T \gg E/F$ the components of the dipole can be considered as independent particles. For each of the regimes we obtained the analytical expressions for both the spectral distribution of the radiation and the total back force. The results are qualitatively shown in Figs. 1–7.

The results for the radiation reaction are in the correspondence with the influence of the interference on the number of radiated photons and on the scattering cross sections. These physical quantities are qualitatively influenced by interference up to times $\sim E/F$. Without interference the number of photons is maximum (infinite) at $\omega \to 0$. As a result of the interference the number of photons goes to zero when $\omega \to 0$. The photon-number distribution has maximum at $\omega^* \sim \gamma/d(T)$ for the very small dipole regime, i.e., for the ultrarelativistic dipole the maximum of the number of radiated photons shifts to the end point of the spectrum.

For larger times (small dipole regime) the maximum in the number of the radiated photons is finite and parametrically lies at the same frequencies as the maximum of the radiation reaction $-E\chi$ for $\chi \ll 1$, $0.4E$ for $\chi \gg 1$. Moreover, we have seen that for $\chi \gg 1$ the radiated particles carry $\sim 1/2$ of the dipole energy for arbitrary times $T \gg E/F$, when the particles move as independent ones. There are two distinct maxima and two groups of photons. One group is responsible for the energy loss, and its spectral curve maximum is at $\omega \sim 0.4E$ for large $\chi$. Another group is the soft photons, responsible for the total number of photons emitted (and they may give the main contribution to the cross sections). These photons in the regime under discussion are the soft photons, with the maximum of the spectral curve located at $\omega \sim 2/T$ for large $\chi$.

It is also interesting to summarize the behavior of the radiation spectrum for different $\chi$. For $\chi \ll 1$, the relevant maximum lies near the end point if $T \leqslant E/m^2$, but the radiation is strongly suppressed. However, if it occurs, the dipole will be immediately destroyed, since the photon takes all of its energy. Then it moves to $E\chi$ by the time $m/F$, and only afterwards does the radiation begin to increase.

In the opposite case $\chi \gg 1$ the maximum is near the end point until $T \sim T^*$, and only then begins to move to saturation, $0.4E$, which corresponds to the single particle maximum. For $T \leqslant T^*$ the radiation is suppressed, but if occurs it destroys the dipole (the photon carries its entire energy). For times $T \gg E/F$ the radiation reaction is a sum of the component radiation events, and at each radiation event on average half of the electron energy is taken by the photon.

We have seen that our results, although they were obtained for the simple model of the constant transverse field, can be reformulated in a model-independent way. The parameter $\chi = l_c/l_F$ is the ratio of the coherence and the field generation length. The very small dipole regime corresponds to the situation when the dipole travels a distance less than $l_F$. There exists a charge transparency in this region independently of $\chi$. However, for large times the parameter $\chi$ starts to play an important role. If $\chi \ll 1$ (this is the situation considered in Refs. [16,17,19]) one can see the external field as a small perturbation. The region of the quantum diffusion is small (since $l_c \ll l_F$) and charge transparency (i.e., radiation suppression) continues up to $T \sim m/F$, well beyond the quantum diffusion range. However, once $\chi \geqslant 1$, we are in a completely different situation. In the quasiclassical approach we have here the situation is quite similar to the Landau-Pomeranchuk effect in the single particle dynamics of the fast particle moving through the amorphous media. The radiation reaction continues to be suppressed even after the field regeneration time, thus extending parametrically the charge transparency interval to times $T \sim T^*$. However, as it was noted in the preceeding section, the area of $T \leqslant T^*$ must be studied beyond the quasiclassical approximation, since we must take into account multiple coherent collisions. We expect that the radiation will still be strongly suppressed in this time interval, but further analysis is needed to make qualitative statements, and to compare the results with those from the quasiclassical approach.

The main possible drawback of our paper is the validity of the quasiclassical approximation. For $\chi \leqslant 1$ the quasiclassical approximation works for all times larger than $T_c \sim E/m^2 \leqslant T_F$. For $\chi \geqslant 1$ for the dipole one may expect significant corrections to the quasiclassical approximation for all times in light of the results of Ref. [16] (see also the preceding section). Nevertheless, quasiclassical analysis is still important as a first step for understanding the radiation patterns in this regime of parameters.

Our work certainly leaves a number of questions open. First, there is the question about the influence of the quantum effects in the dipole motion on the radiation reaction. This is important for the study of the quantum dipole. We have seen that such effects for the dipole may be much more important than those for a single particle, and may require analysis beyond the quasiclassical approximation due to the coherent multiple scattering.

Second, it will be interesting to study further the dependence of the number of the radiated photons on $\chi$, in particular taking the multiple photon radiation into account. Our results suggest that, since the electrons are created in pairs, i.e., as a dipole, the infrared photons are always cut off, and the evolution continues by a series of radiative events, such that in each of these events the electron loses approximately half of its energy. This is true at least if $\chi \gg 1$, i.e., the dipole is ultrarelativistic or the field is very strong. This is opposite of the scenario when the fast electron loses its energy by radiating soft photons, with a small energy loss in each of the radiating events. This result can be important for carrying out the next-to-leading order logarithmic calculations. The results of this paper imply, roughly, that such a dipole moves until times $T^*$ without radiation, then after the transition period (up to $T \sim E/F$), starts to radiate, losing at each event $\sim 1/2$ of its energy.

Moreover, the numerical analysis shows that for large times there are two parallel processes for the ultrarelativistic dipole. First, it emits soft photons. The maximum of the photon number distribution for large $\chi$, as the numerical analysis of Eq. (4.1) shows, lies at

$$\omega \sim 2/T, \quad T \gg E/F. \tag{8.1}$$

These photon numbers make significant contributions to cross sections. But the energy loss of the dipole occurs via a series of different events, when $\sim 0.4E$ is lost in each event, and the relevant photons are hard for the ultrarelativistic dipole.

It will be interesting to see if the different regimes of radiation discussed in this paper are connected with the theory of the production of the $e^+$-$e^-$ pairs by fast particles in the external field discussed in Ref. [28]. Finally, it will be interesting to study the implications of our results for QCD. In particular, our results are clearly relevant to the studies of the color transparency phenomena, first discussed in Refs. [16,17,19]. As it was noted in the Introduction for the case of the deep inelastic scattering on the longitudinal photons, the charge transparency is directly translated into color transparency [20]. Our results give qualitative bounds on the color transparency for the arbitrary external fields, and indicate the direction of the research one needs to extend the color transparency for the case of the arbitrary external field.

It will be especially interesting to extend our results to the gluon color dipole radiation, since then the shift of the spectrum to the end point will mean that the dipole loses all its energy by a single radiative event for a small time. It will also imply that the color dipole loses its energy by a series of events in each of which the gluon looses half of its energy. Note, however, that the extension to the color dipole is nontrivial since the mass of the gluon is zero, and we need the additional regularization. Moreover, the definition of the QCD dipole is slightly different then the one in this paper. In this paper the dipole is a system of two oppositely charged particles with interfernce, and the QCD dipole is only a quantum dipole, i.e., the times considered are always less than the time interval that corresponds to the coherence length. For such a case, as we saw above there may be significant corrections to the quasiclassical approximation that need further study. Nevertheless our results imply that the recoil effects may be very important also for the color dipole, i.e., for the small deep inelastic scattering.

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## APPENDIX A: RADIATION REACTION FOR A SINGLE QUANTUM PARTICLE FOR SMALL TIMES (SMALL DEFLECTION ANGLES)

Although the article is devoted to the radiation reaction of a dipole, in this section we shall discuss the radiation reaction of a single particle for small times, taking into account recoil effects. Although such a problem may look unphysical, since charged particles are created by pairs, exactly the same problem appears if the particle goes through the line of the constant external field of the length $L \leqslant m/F$. The purely classical case was discussed by Landau and Lifshits [1]. However, we were not able to find the quantum case in the literature.

We start from Eq. (2.10) for total energy radiated by a single particle during the time interval $T$. Using the approximation of Eqs. (2.38) and (2.39) we see that the radiation reaction is the sum of three terms: the term proportional to $1-v^2$, the term proportional $\omega_0^2$, and the term due to the integration by parts. Note that all cubic terms in the arguments are negligible and thus can be omitted. The terms that arise due to the integration by parts do not depend on the external field, up to the terms additionally suppressed as $m^2/E^2$, and are the same as that for the free particle. So only the term proportional to $\omega_0^2$ remains. It is straightforward to find

$$\frac{dW}{d\omega} = \omega_0^2 \frac{q^2}{2\pi} \int_0^T \int_0^T ds\, ds' \int_{-1}^1 d(\cos\theta)(s-s')^2 \times \cos[\omega(s-s')] \exp\{i[\omega v(s-s')\cos\theta]\}. \quad \text{(A1)}$$

The latter triple integral can be easily taken. We obtain

$$\frac{dW}{d\omega} = -\omega_0^2 \frac{q^2}{v\pi} \frac{2\{1-\cos[\omega(1+v)T] - \omega(1+v)T \sin[\omega(1+v)T]\}}{\omega^2(1+v)^3} + \omega_0^2 \frac{q^2}{\pi} \frac{2\{1-\cos[\omega(1-v)T] - \omega(1-v)T \sin[\omega(1-v)T]\}}{\omega^2(1-v)^3}. \quad \text{(A2)}$$

This is the classical formula. The recoil is taken by first rewriting

$$\frac{d\omega}{\omega^2} = \frac{\omega d\omega}{\omega^3}.$$

Then we need to rescale $\omega$ in the right-hand side (r.h.s.) of Eq. (A2), except in the product $\omega d\omega$, as discussed in the text:

$$\omega \to \omega' = \omega E/(E-\omega). \quad \text{(A3)}$$

We obtain:

$$\frac{dW}{d\omega}=-\omega_0^2\frac{q^2}{v\pi}\omega\frac{2\{1-\cos[\omega'(1+v)T]-\omega'(1+v)T\sin[\omega'(1+v)T]\}}{\omega'3(1+v)^3}$$
$$+\omega_0^2\frac{q^2}{\pi}\omega\frac{2\{1-\cos[\omega'(1-v)T]-\omega'(1-v)T\sin[\omega'(1-v)T]\}}{\omega'^3(1-v)^3}. \tag{A4}$$

The typical spectral curve is depicted in the first graph of Fig. 1. [In the figure we also added the contribution of the term proportional to $(1-v^2)$.]

It is straightforward to integrate the latter equation over $\omega$ from 0 to $E$ [for this we change the integration variable to $y=\omega/(E-\omega)$]. After trivial integration we obtain

$$\frac{dE}{dT}=\frac{q^2}{\pi(1-v)^2}\omega_0^2([(1-v)^2/(1+v)^2]\{-b+\mathrm{Ci}(b)[b\cos b-\sin(b)+b^2\sin(b)/2-b^3\cos(b)/2]$$
$$+\mathrm{Si}(b)[\cos(b)-b\sin b-b^2\cos(b)/2-b^3\sin(b)/2]\})+\pi b(-1/2+b^2/4)\sin b+\pi(-1/2+b^2/4)\cos b$$
$$-\{-a+\mathrm{Ci}(a)[a\cos a-\sin(a)+a^2\sin(a)/2-a^3\cos(a)/2]$$
$$+\mathrm{Si}(a)[\cos(a)-a\sin a-a^2\cos(a)/2-a^3\sin(a)/2]+\pi a(-1/2+a^2/4)\sin a+\pi(-1/2+a^2/4)\cos a\}. \tag{A5}$$

Here

$$a=(1-v)ET\sim m^2T/(2E),\quad b=(1+v)ET\sim 2ET. \tag{A6}$$

The corresponding typical radiation reaction curve is depicted in Fig. 2.

Expanding the latter equation in powers in $T$ we see that the backreaction force for small times is very small:

$$\frac{dE}{dT}=-\frac{q^2}{6\pi}\frac{m^2F^2T^3}{E}[\log(m^2T/2E)+\gamma+1]+O(T^4), \tag{A7}$$

where $\gamma\sim 0.55$ is the Euler constant. This force is directed against the direction of the particle.

The latter equation works for the whole range of $T\leqslant m/F$ if $\chi\gg 1$. For $T\sim m/F$, $\chi\ll 1$, the parameter $a\sim 1/\chi\gg 1$, and we have to use the whole equation (A5) in the limit $a\gg 1$. For $\chi\gg 1$ the expansion parameter $a\sim m^2T/E\sim 1/\chi$ and is still small for $T\sim m/F$. Then the backreaction force is for this time scale:

$$F_{\mathrm{b.r.}}\sim\frac{q^2m^5}{6\pi EF}\log(E^2/m^2)=\frac{q^2m^2}{6\pi\chi}\log 1/\chi.$$

We see that the backreaction is strongly suppressed for single particles in the ultrarelativistic case.

Finally, let us make a comment on the discarded terms proportional to $(1-v^2)$, and those originating from integration by parts. It may be strange from first sight that these terms really exist, since they are nonzero for a particle moving with constant velocity and finite mass, i.e., a particle that does not emit any radiation field. In fact this situation is usual in quantum mechanics and quantum field theory. Indeed, when we calculate the transition rate due to photon radiation in standard perturbation theory between stationary states we encounter the multiplier

$$\sin^2(E_f-E_i-\omega)T/T(E_f-E_i-\omega). \tag{A8}$$

For infinite $T$ this term gives a delta function $\delta(E_f-E_i-\omega)$, ensuring the law of the energy conservation. However, for finite $T$ and nonfinite $\Delta E=E_f-E_i-\omega$ this will be a function of $T$ decreasing as a function of $T$ for fixed $\Delta E$. This decrease, as it is well known, just expresses the energy uncertainty principle. If we consider the system for a finite time, the energy cannot be measured unambiguously: $\Delta ET\geqslant\hbar$. The discarded terms in the radiation reaction have exactly the same origin and the same character. They decrease for large $T$ as $1/T$ or faster and thus disappear at infinite $T$ altogether. They exhibit the ambiguity in the measurement of the electromagnetic field of the free particle due to the finite time of our process. In practice this leads to the finite width of spectral lines for finite times. It is interesting to study these terms in more detail in connection with the Landau-Pierels inequalities [29]. However, it is clear from the above that these terms must be discarded if we are interested in the radiation in external field. In other words, all quantum calculations must contain renormalization, meaning that a free particle does not radiate.

## APPENDIX B: SOME USEFUL INTEGRALS AND THEIR PROPERTIES

Here we shall collect some useful integrals and asymptotic expansions, as given in Refs. [22–25]. We shall also collect the definitions of several special functions that differ in the literature by normalization constants. We use the following integrals, directly expressible through Airy functions:

$$\frac{1}{\sqrt{\pi}}\int_0^\infty ds\, s\,\sin(as+s^3)=-\frac{d}{da}\mathrm{Ai}(a), \tag{B1}$$

$$\int_0^\infty ds\,\frac{1}{\sqrt{\pi}}\sin(as+s^3)/s=-\int_a^\infty dz\,\mathrm{Ai}(z). \tag{B2}$$

Here $\mathrm{Ai}(z)$ is an Airy function [22,23]:

$$\int_0^\infty ds\,\frac{1}{\sqrt{\pi}}\cos(as+s^3/3)=\mathrm{Ai}(a). \tag{B3}$$

Note that the Airy function decreases as $\sim\exp(-z^{3/2})/z^{1/4}$ for the positive $z\to\infty$.

We use integral of the Airy function:

$$\int_0^\infty z^{b-1}\mathrm{Ai}(z)dz=3^{(4b-1)/6-1}\Gamma(a/3)\Gamma((a+1)/3). \tag{B4}$$

We define the integral sinus and cosinus as

$$\mathrm{Si}(x)=\int_0^x\frac{\sin(x)}{x}, \tag{B5}$$

$$\mathrm{Ci}(x)=-\int_x^\infty\frac{\cos(x)}{x}, \tag{B6}$$

While studying the dipole radiation we used some formulas for the integrals of Bessel functions [26]. We use

$$G_2(p,a,b)=\int_0^\infty \exp(-px)\sin(bx)J_0(ax)/x=\arcsin(2b/r), \tag{B7}$$

$$G_1(p,a,b)=\int_0^\infty \exp(-px)\cos(bx)J_0(ax)$$
$$=\frac{1}{\sqrt{p^2+(b+a)^2}*\sqrt{p^2+(b-a)^2}}*\sqrt{(r^2/4-b^2)}. \tag{B8}$$

Here

$$r=\sqrt{(b+a)^2+p^2}+\sqrt{p^2+(b-a)^2}. \tag{B9}$$


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