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Thermal leptogenesis and gauge mediation

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We show that a minithermal inflation occurs naturally in a class of gauge mediation models of supersymmetry (SUSY) breaking, provided that the reheating temperature T_R of the primary inflation is much higher than the SUSY-breaking scale, say $T_R > 10^{10}$ GeV. The reheating process of the thermal inflation produces an amount of entropy, which dilutes the number density of relic gravitinos. This dilution renders the gravitino to be the dark matter in the present Universe. The abundance of the gravitinos is independent of the reheating temperature T_R, once the gravitinos are thermally produced after the reheating of the primary inflation. We find that the thermal leptogenesis takes place at $T_L \simeq 10^{12}$–10^{14} GeV for $m_{3/2} \simeq 100$ keV–10 MeV without any gravitino problem.

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I. INTRODUCTION

The baryon-number asymmetry in the Universe is one of the fundamental parameters in cosmology. A number of mechanisms have been proposed for producing the baryon asymmetry in the early Universe. Among them, leptogenesis [1] is the most attractive and fruitful mechanism, since it may have a connection to low-energy observations, that is, neutrino masses and mixings. In fact, a detailed analysis on thermal leptogenesis [2] gives an upper bound on all neutrino masses of 0.1 eV, which is consistent with data of neutrino oscillation experiments. Thermal leptogenesis requires a reheating temperature $T_R \gtrsim 10^{10}$ GeV, which, however, leads to the overproduction of unstable gravitinos. The decays of gravitinos produced after inflation destroy the success of nucleosynthesis [3,4]. This problem is not solved even if one raises the gravitino mass $m_{3/2}$ up to 30 TeV [5] and hence thermal leptogenesis seems to conflict with the gravity mediation model of supersymmetry (SUSY) breaking. In the gauge mediation model [6], on the other hand, the gravitino is the lightest SUSY particle (LSP) and one does not need to worry about gravitino decay. However, we have an even stronger constraint on the reheating temperature $T_R \lesssim 10^8$ GeV for $m_{3/2} \lesssim 1$ GeV to avoid the overproduction of the gravitinos [8] (otherwise, the stable gravitinos overclose the present Universe).

It has been recently pointed out [9] that the above problem is naturally solved in the gauge mediation model. The crucial observation is that the gravitinos are in the thermal equilibrium at high temperatures such as $T_R \gtrsim 10^{10}$ GeV for $m_{3/2} \lesssim 1$ GeV. Thus, the number density of gravitinos is independent of the reheating temperature once they are in thermal equilibrium, while the maximal lepton (baryon) asymmetry depends linearly on T_R. Therefore, if a suitable amount of entropy is provided at later time to dilute the number density of gravitinos, one may account for both the dark matter abundance and the baryon asymmetry in the present Universe.

The required dilution factor Δ_r to explain the observed dark matter abundance by gravitinos is given by [9]

$$\Delta_r \simeq 3.0\times 10^3 \left(\frac{m_{3/2}}{1\ \ \text{MeV}}\right)\left(\frac{350}{g_*(T_f)}\right)\left(\frac{0.11}{\Omega_{\rm DM}h^2}\right), \qquad (1)$$

where T_f denotes the gravitino freeze-out temperature. Once the above dilution factor is provided, the baryon asymmetry produced by the thermal leptogenesis [9] is estimated as

$$\Omega_{\rm B}h^2 = 0.03\times\left(\frac{1\ \ \text{MeV}}{m_{3/2}}\right)\left(\frac{g_*(T_f)}{350}\right)\left(\frac{350}{g_*(T_R)}\right) \times\left(\frac{M_{R1}}{10^{13}\ \ \text{GeV}}\right)\left(\frac{m_{\nu 3}}{0.05\ \ \text{eV}}\right)\kappa\,\delta_{\rm eff}. \qquad (2)$$

Here M_{R1} is a mass of the lightest heavy right-handed Majorana neutrino (and hence, T_R must be higher than M_{R1} for the thermal leptogenesis to work) and $m_{\nu 3}$ a mass of the heaviest left-handed Majorana neutrino. The coefficient $\delta_{\rm eff} = O(1)$ is an effective CP-violating phase and $\kappa \simeq 0.05$–0.3 denotes the fraction of the produced asymmetry that survives washout processes by lepton-number-violating interactions after the decay of the right-handed neutrino. As can be seen, the thermal leptogenesis at $T_R \simeq 10^{12}$–10^{14} GeV for $m_{3/2} \simeq 100$ keV–10 MeV naturally explains the observed baryon asymmetry $\Omega_B h^2 \simeq 0.02$ without any gravitino problem.

Reference [9] shows that the required entropy given in Eq. (1) is naturally supplied by decays of messenger particles in the gauge mediation model if the SUSY-breaking transmission to the SUSY standard-model (SSM) sector is direct. In this paper we show that the late-time entropy production takes place even if the mediation of SUSY breaking is not of the direct type. This is because the gauge mediation model

TABLE I. R charges of the fields in the DSB, the messenger and the SSM sectors.

Fields	Z^{ij}	Q_i	S	E^i	$d_M, \overline{\ell}_M$	$\overline{d}_M, \ell_M$	$\overline{d}$	ℓ
R charges	2	0	2/3	2/3	-1	7/3	1	1

we discuss in this paper has a flat potential and a minithermal inflation occurs naturally producing the required amount of entropy.

II. THE GAUGE MEDIATION MODEL

We consider an extension of the gauge mediation model proposed in [10]. The reason why we take this model is that it has the unique true vacuum of SUSY breaking. Otherwise, it seems very difficult to choose a SUSY-breaking false vacuum in the evolution of the Universe, since we assume the reheating temperature T_R much higher than the SUSY-breaking scale.

The dynamical SUSY-breaking (DSB) sector is based on a SUSY $\mathrm{SU}(2)_H$ hypercolor gauge theory with four doublet chiral superfields Q^i called hyperquarks and six singlet ones $Z^{ij}=-Z^{ji}$ [11,12]. Here, the indices $\alpha=1,2$ denote the $\mathrm{SU}(2)_H$ ones and the indices $i,j=1,\ldots,4$ are flavor ones. We impose, for simplicity, a flavor symmetry SP(4) and write the superpotential as

$$W_{\mathrm{tree}}=\lambda Z(QQ)+\lambda' Z^a(QQ)_a, \tag{3}$$

where Z and (QQ) are singlets of the flavor SP(4) and Z^a and $(QQ)_a$ are **5** representations of the SP(4).[1] It should be noted here that we have a global $\mathrm{U}(1)\times\mathrm{U}(1)_R$ in addition to the flavor SP(4) at the classical level, where the $\mathrm{U}(1)_R$ represents a R symmetry. We choose R-charges of relevant superfields so that the $\mathrm{U}(1)_R$ has no $\mathrm{SU}(2)_H$ gauge anomaly. The R-charges for the Q^i_α and $Z_{(a)}$ are given in Table I. The flavor U(1) breaks down to a discrete Z_4 symmetry at the quantum level, under which Q^i_α transforms as $Q^i_\alpha\rightarrow iQ^i_\alpha$ and $Z_{(a)}$ as $Z_{(a)}\rightarrow -Z_{(a)}$.

We show that for $\lambda'>\lambda$ the low-energy effective superpotential is approximately given by

$$W_{\mathrm{effective}}\simeq\frac{\lambda}{(4\pi)^2}\Lambda_H^2 Z, \tag{4}$$

where Λ_H is a dynamical scale of the $\mathrm{SU}(2)_H$ gauge interaction and

$$\langle(QQ)\rangle\equiv\left\langle\frac{1}{2}(Q_1Q_2+Q_3Q_4)\right\rangle\simeq\left(\frac{\Lambda_H}{4\pi}\right)^2. \tag{5}$$

The superfield Z has a nonvanishing F term $\langle F_Z\rangle\simeq\lambda\Lambda_H^2/(4\pi)^2$ and hence SUSY is spontaneously broken

[1] Z and Z^a are linear combinations of the original Z^{ij}.

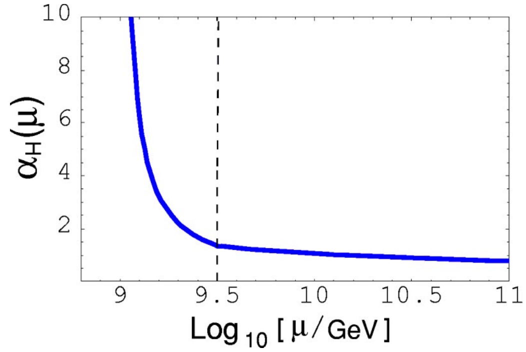

FIG. 1. The running of gauge coupling constant α_H. μ denotes the renormalization point. Here, we assume $n_q=3$, $M_{Q'}=10^{9.5}$ GeV and $\alpha_H(M_G)=0.25$. The vertical dashed line denotes the mass scale $M_{Q'}$.

[11,12].[2] The condensation of QQ does not break the R symmetry, but causes the breaking of the discrete Z_4 symmetry down to a discrete Z_2. This breaking generates unwanted domain walls and hence we should introduce explicit breaking terms of the Z_4. Here, we introduce a nonrenormalizable interaction in the Kahler potential, $K=(k/M_G^2)QQZZ^\dagger$, to eliminate the domain walls before they dominate the early universe,[3] where M_G is the gravitational scale $M_G\simeq 2.4\times10^{18}$ GeV. We see that this nonrenormalizable interaction does not affect the dynamics of SUSY breaking.

We now introduce $2n_q$ massive hypercolor quarks $Q'^{a(j)}_\alpha$ $(j=1,\ldots,n_q)$ and assume that each pair of the $Q'^{a(j)}_\alpha$ form doublets $(a=1,2)$ of a new gauge group $\mathrm{SU}(2)_m$. Namely, the massive hyperquarks are $(\mathbf{2},\mathbf{2})$ representations of $\mathrm{SU}(2)_H\times\mathrm{SU}(2)_m$. In the original model in [10] a $\mathrm{U}(1)_m$ subgroup of the SP(4) is gauged. The reason why we introduce the $\mathrm{SU}(2)_m$ gauge interaction becomes clear in the next section. Notice that the introduction of the above massive hyperquarks does not affect the dynamics of the SUSY breaking [14]. The $\mathrm{SU}(2)_m$ gauge interaction and the massive hyperquarks $Q'^{a(j)}_\alpha$ play a role of transmitting the SUSY breaking effects to the messenger sector. In the present analysis we take the masses M'_Q of the hyperquarks $Q'^{a(j)}_\alpha$ at the dynamical scale of the hypercolor $\mathrm{SU}(2)_H$ gauge interaction, that is $M_{Q'}\simeq\Lambda_H$. As we see from Fig. 1, this assumption is natural since the running of the gauge coupling constant α_H becomes very fast below the mass scale $M_{Q'}$.

The messenger sector consists of $2n$ chiral superfields E^i_a with $i=1,\ldots,2n$ which are doublets of the $\mathrm{SU}(2)_m$, a singlet superfield S, and vector-like messenger quark and lepton superfields, d_M, $\overline{d}_M$, ℓ_M, and $\overline{\ell}_M$. The messenger fields,

[2] The integration of the hypercolor sector induces a nonminimal Kahler potential of the Z superfield, which determines the vacuum-expectation value of the Z field. However, one cannot calculate the Kahler potential due to the strong hypercolor gauge interaction. Thus, we postulate $\langle Z\rangle=0$, for simplicity.

[3] We find that this breaking term with $k=O(1)$ is strong enough to eliminate the unwanted domain walls (see [13]).

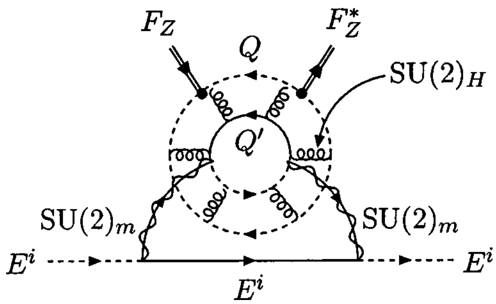

FIG. 2. A typical example of the Feynman diagrams which give the soft SUSY-breaking masses for E^i.

$(\overline{d}_M,\ell_M)$ and $(d_M,\overline{\ell}_M)$ transform as $\mathbf{5}^*$ and $\mathbf{5}$ of the grand unification group $\mathrm{SU}(5)_{\mathrm{GUT}}$, respectively. For $n\leqslant 2$, the $\mathrm{SU}(2)_m$ symmetry is broken after the DSB sector is integrated out. On the other hand, for $3<n<6$, the $\mathrm{SU}(2)_m$ gauge theory is in the conformal window, and we cannot discuss behaviors of the fields of the messenger sector around origin. Thus, we take $n=3$ in the present analysis.[4] Other cases will be discussed elsewhere.[5]

The most general superpotential for the messenger sector without any dimensional parameters is

$$W_{\mathrm{mess}}=\sum_{i\neq j} k_{ij}SE^iE^j+\frac{f}{3}S^3+k_dSd_M\overline{d}_M+k_\ell S\ell_M\overline{\ell}_M, \tag{6}$$

where $k_{ij}=-k_{ji}$ $(i,j=1,\ldots,6)$, and we have omitted indices of the messenger gauge $\mathrm{SU}(2)_m$. This W_{mess} is natural, since we have a R symmetry that forbids other possible terms in the superpotential. R charges for relevant superfields are given in Table I. It is clear that the superpotential W_{mess} possesses a discrete Z_3 symmetry where S, E, d_M, and ℓ_M have the Z_3 charge $+1$.

The $\mathrm{SU}(2)_m$ gauge interaction [together with the hypercolor $\mathrm{SU}(2)_H$ interaction] transmits the SUSY-breaking effects of the DSB sector to the messenger sector and generates soft SUSY-breaking masses for the E^i_α superfields, $(m_E^{\mathrm{soft}})^2$ (for an example of Feynman diagrams see Fig. 2). A straightforward calculation [10] shows (see also [14])

$$m_E^{\mathrm{soft}}\simeq\sqrt{\frac{3n_q}{2}}\left(\frac{\alpha_m}{4\pi}\right)\frac{\lambda F_Z}{\Lambda_H}\simeq\sqrt{\frac{3n_q}{2}}\left(\frac{\alpha_m}{4\pi}\right)\frac{\lambda^2}{16\pi^2}\Lambda_H, \tag{7}$$

where $\alpha_m=g_m^2/4\pi$ with g_m being the $\mathrm{SU}(2)_m$ gauge coupling constant. As shown in [10], effects of E^i loops give rise to a negative soft SUSY-breaking mass squared, $-m_S^2$, for the singlet superfield S. The m_S^2 is estimated as

$$m_S^2\simeq\frac{16}{16\pi^2}\sum_{i\neq j}k_{ij}^2(m_E^{\mathrm{soft}})^2\ln\frac{\Lambda_H}{M_E}, \tag{8}$$

where M_E is a SUSY-invariant mass for the superfields E^i which is given by the condensation of the superfield S (see the following discussion).

Now we have a potential for the scalar fields in the messenger sector,

$$\begin{aligned}V_{\mathrm{mesenger}}\simeq&\left|\sum_{i\neq j}k_{ij}E^iE^j+fS^2+k_dd_M\overline{d}_M+k_\ell\ell_M\overline{\ell}_M\right|^2\\&+\sum_{i=1}^{6}\left|\sum_{j\neq i}k_{i,j}SE^j\right|^2+|k_dSd_M|^2+|k_dS\overline{d}_M|^2\\&+|k_lS\ell_M|^2+|k_lS\overline{\ell}_M|^2\\&+(m_E^{\mathrm{soft}})^2\sum_{i=1}^{6}|E^i|^2-m_S^2|S|^2,\end{aligned} \tag{9}$$

where all fields represent corresponding scalar boson fields. We see that this potential has a global minimum at

$$\langle S^*S\rangle=\frac{m_S^2}{2f^2},\quad \langle E^i\rangle=\langle d_M\rangle=\langle\overline{d}_M\rangle=\langle\ell_M\rangle=\langle\overline{\ell}_M\rangle=0,$$

$$\langle|F_S|\rangle=\frac{m_S^2}{2f}, \tag{10}$$

for $k_d,k_\ell\gg f$. We show in Sec. IV that this condition for the Yukawa coupling constants is naturally realized. The superfields E^i, d_M $(\overline{d}_M)$, and l_M $(\overline{l}_M)$ have SUSY-invariant masses as $M_E=2k_{ij}\langle S\rangle$, $M_d=k_d\langle S\rangle$, and $M_l=k_l\langle S\rangle$, respectively. The SUSY-breaking effects are transmitted to the messenger quark and lepton superfields through $\langle F_S\rangle$ and Yukawa coupling in Eq. (6).

The condensation of the S field, $\langle S\rangle\neq 0$, breaks the R symmetry which generates a R axion (the phase component of the complex S boson). The axion mass is usually induced by a constant term in the superpotential, since the constant term breaks the R symmetry explicitly. However, in the present model the induced axion mass vanishes at the tree level, and hence we need another explicit breaking term of the R symmetry to give a sufficiently large mass to the R axion. We introduce a nonrenormalizable interaction in the superpotential, $W_{\mathrm{mess}}=(1/M_G)QQS^2$.[6] This new term induces the R axion mass as $m_{\mathrm{axion}}\simeq 10\ \mathrm{GeV}\sqrt{(m_{3/2}/\mathrm{MeV})(m_S/10^5\ \mathrm{GeV})}$. Notice that this nonrenormalizable interaction breaks also the discrete Z_3 symmetry explicitly and hence we have no domain-wall problem.

[4]For $n_q\geqslant 4$ the $\mathrm{SU}(2)_m$ gauge interaction becomes nonasymptotic free. We prefer to use rather strong coupling constant at around the gauge-mediation scale in order to make the DSB scale low enough, which leads to rather light gravitino. Thus, we regard the case with $n_q<4$ is preferable.

[5]It is an interesting possibility to take $n=5$, since one may assign E^i_α $(i=1\sim 10)$ to be $\mathbf{5}+\mathbf{5}^*$ of the $\mathrm{SU}(5)_{\mathrm{GUT}}$. In this case one does not need to introduce the messenger quarks and leptons.

[6]We assume, throughout this paper, that the R symmetry is explicitly broken by nonrenormalizable interactions suppressed by the gravitational scale.

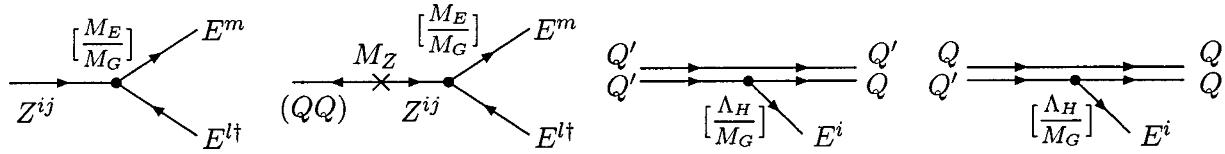

FIG. 3. The super Feymnan diagrams for decay processes of Z^{ij}, (QQ), $(Q'Q')$ and (QQ').

The SSM gauginos acquire soft SUSY-breaking masses through messenger loop diagrams, and at the one-loop level they can be written as [6]

$$m_{\tilde{g}_i} = c_i \frac{\alpha_i}{4\pi} M_{\rm mess}, \tag{11}$$

where $c_1 = 5/3$, $c_2 = c_3 = 1$, and $m_{\tilde{g}_i}$ $(i=1,2,3)$ denote the B-ino, W-ino, and gluino masses, respectively. Similarly, the soft SUSY-breaking masses for the squarks, sleptons, and Higgs bosons, $\tilde{f}$, in the SSM sector are generated at the 2-loop level as [6]

$$m_{\tilde{f}}^2 = 2M_{\rm mess}^2 \left[C_3 \left(\frac{\alpha_3}{4\pi} \right)^2 + C_2 \left(\frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} Y^2 \left(\frac{\alpha_1}{4\pi} \right)^2 \right], \tag{12}$$

where $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets, and Y is the SM hypercharge, $Y = Q_{\rm em} - T_3$. Here, $M_{\rm mess}$ is an effective messenger scale defined as

$$M_{\rm mess} \equiv \frac{\langle |F_S| \rangle}{\langle S \rangle} = \frac{m_S}{\sqrt{2}}, \tag{13}$$

and in terms of SUSY-breaking scale $\sqrt{F_Z}$, it can be written as

$$M_{\rm mess} \simeq \frac{2\sqrt{3 n_q}}{(4\pi)^3} \alpha_m \sqrt{\sum_{i \neq j} k_{ij}^2 \lambda^3 \ln \frac{\Lambda_H}{M_E}} \sqrt{F_Z}. \tag{14}$$

To have the SSM gaugino and sfermion masses at the electroweak scale, the effective messenger scale $M_{\rm mess}$ must be $\sim 10^4$–10^5 GeV. Then, the SUSY-breaking scale $\sqrt{F_Z}$ becomes $\simeq 10^7$–10^8 GeV for $\alpha_m(\Lambda_H) = 0.5$, $\lambda = \sqrt{\Sigma_{i\neq j} k_{ij}^2} = O(1)$, and $\sqrt{\ln \Lambda_H / M_E} = O(1)$.[7] This corresponds to the gravitino mass,

$$m_{3/2} \simeq \frac{F_Z}{\sqrt{3} M_G} \simeq 100 \ \text{keV–10 MeV}. \tag{15}$$

Thus, we consider that the dynamical scale of hypercolor gauge interaction, $\Lambda_H \simeq 10^8$–10^9 GeV, and the SUSY-breaking masses for E_α^i and S, $m_E^{\rm soft} \simeq 10^4$–$10^5$ GeV and $m_S \simeq 10^4$–10^5 GeV. We should note here that the SUSY-invariant masses for messenger quarks, leptons, and E^i are about 10^6–10^7 GeV (see the discussion in Sec. IV).

III. DECAY PROCESSES OF THE QUASISTABLE STATES

We are now at the point to discuss decays of all quasistable particles in the DSB and the messenger sectors and show that their lifetimes are short enough not to produce extra entropy at the decay times (except for the R axion). We first consider the quasistable particles in the DSB sector, that are the fields Z^{ij}, the lightest bound states $Q^i Q^j$, $Q'^i Q'^j$, and $Q^i Q'^j$.

A. The DSB sector

The $SU(2)_H$ singlets Z^{ij} may decay into pairs of the $SU(2)_m$ doublets $E^m + E^{l\dagger}$ through nonrenormalizable interactions in the Kahler potential, $K = (h/M_G) Z^{ij} (E^l E_\alpha^{m\dagger}) + \text{H.c.}$, with $i,j = 1,\ldots,4$ and $l,m = 1,\ldots,6$ (Fig. 3), where h is of order of unity. The decay rates are estimated as $\Gamma_Z \simeq 6^2 (h^2/4\pi)(h M_E / M_G)^2 M_Z$ and $M_Z \simeq (\lambda^{(\prime)}/4\pi) \Lambda_H$ is a mass[8] of the Z and QQ. Notice that the bound states QQ form massive multiplets together with the Z [see Eq. (3)]. The decay temperature is $T_d^Z \simeq O(100)$ GeV for $M_E \simeq 10^7$ GeV and $\Lambda_H \simeq 10^9$ GeV.[9] The QQ bound states which are mass partners of the Z^{ij} fields decay similarly. On the other hand, they decouple from the thermal bath when the rate of the annihilation $\langle \sigma v \rangle n_Z$ drops below the Hubble expansion rate H, where $\langle \sigma v \rangle$ is a thermally averaged annihilation cross section and n_Z a number density of Z^{ij} and QQ. Thus, the relic abundance of Z^{ij} and QQ is $n_Z \simeq H / \langle \sigma v \rangle$ after the decoupling from the thermal bath, and the total energy density of the universe is given by

$$\rho = \frac{\pi^2}{30} g_*(T) T^4 + M_Z n_Z(T), \tag{16}$$

where $g_*(T)$ is the degree of freedoms of effective massless particles at a temperature T. Then, if they were stable, they

[7]The $SU(2)_m$ gauge coupling constant at $\mu \simeq M_E$ is estimated as $\alpha_m(M_E)/(4\pi) \simeq 0.2$ for $\alpha_m(\Lambda_H) \simeq 0.5$ and hence the perturbative calculation for the $SU(2)_m$ gauge interaction at $\mu \simeq M_E$ is valid. The dynamical scale of the $SU(2)_m$ gauge interaction is estimated as $\Lambda_m \simeq 10^6$–10^7 GeV$\lesssim M_E$.

[8]We use a naive dimensional analysis for the hypercolor dynamics [7]. Therefore, we may have a $O(1)$ ambiguity in the estimations on masses and couplings of composite bound states.

[9]The scalar component of the flat direction Z receives a SUSY-breaking mass of the order of $\Lambda_H/4\pi$ through one-loop corrections in the Kahler potential and decays also into a pair of $E + E^\dagger$. On the contrary, its fermion partner is nothing but the Goldstino component of the gravitino.

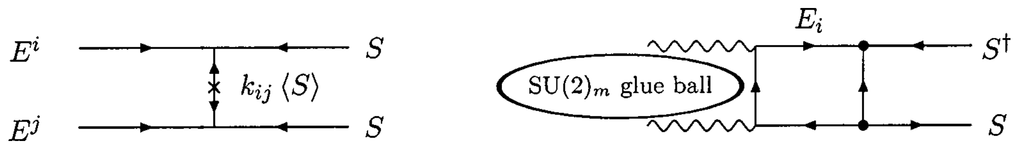

FIG. 4. The super Feymnan diagrams for decay processes of the E^iE^j bound states and the SU(2)m glue balls.

could dominate the energy density of the Universe $((\pi^2/30)g_*(T)T^4 \lesssim M_Z n_Z(T))$ at the temperature

$$T_c^Z \simeq \frac{4}{3}M_Z\left(\frac{n_Z}{s(T_f)}\right)\frac{1}{\Delta_S} \simeq \frac{M_Z}{\langle\sigma v\rangle M_G T_f \Delta_S} \simeq \frac{30M_Z^2}{4\pi\alpha_m^2 M_G \Delta_S}, \tag{17}$$

where $s(T)=(2\pi^2/45)g_*(T)T^3$ is an entropy density, $\Delta_S \simeq 10^2-10^4$ the dilution factor which is discussed in the next section [Eq. (20)] and T_f their freeze-out temperature. We have used $\langle\sigma v\rangle \simeq 4\pi\alpha_m^2/M_Z^2$ and $T_f \simeq M_Z/30$.[10] We can see that T_c^Z is much lower than T_d^Z, and hence Z^{ij} and QQ decay before they dominate the Universe producing no significant entropy.

The SU(2)$_m$ singlet bound states $Q'Q'$ decay into $QQ'+E^i$, and then the doublet bound states QQ' decay into $QQ+E^i$ through the Kahler potential $K=(h/M_G)(Q'Q^\dagger)E^{i\dagger}$ (Fig. 3). The decay rates are given by $\Gamma_{\rm hyper} \simeq (h^2/8\pi)(\Lambda_H/M_G)^2\Lambda_H$ for both decays and the corresponding decay temperature is $T_d \simeq O(10)$ TeV for $\Lambda_H \simeq 10^9$ GeV. When they were stable, the bound states $Q'Q'$ and QQ' could dominate the energy density at the temperature T_c^Z in Eq. (17), since their annihilation cross sections and masses are about the same as those of QQ bound states. We can see that T_c^Z is much lower than the decay temperatures of $Q'Q'$ and QQ' and hence they also produce no extra entropy.

B. The messenger sector

We turn to the messenger sector. First of all, the lightest E^iE^j bound states can decay into a pair of S fields through the diagrams in Fig. 4, and decay rates are given by $\Gamma \simeq (k_{ij}^4/4\pi)M_E$. The SU(2)$_m$ glue balls decay into a pair of S and $S^\dagger$ through 1-loop diagrams of intermediate E_α^i particles (Fig. 4), and their decay rates are estimated as $\Gamma \simeq (6^2/4\pi)(k_E^2\alpha_m/4\pi)^2(\Lambda_m^3/M_E^2)$. We easily see that the decay rates of the E^iE^j bound states and SU(2)$_m$ glue balls are large enough not to produce extra entropy at their decay times.

In the original model in [10], the E^i particles overclose the Universe, since they are stable particles. In the present model, however, they have no cosmological problem, since the E^i particles form necessarily bound states owing to the non-Abelian gauge dynamics and the bound states can decay sufficiently fast as we see above. This is the main reason why we adopt the non-Abelian SU(2)$_m$ instead of the U(1)$_m$.

The S fermion and the radial component of the complex S boson (called R saxion) can decay into a pair of the SSM gauge multiplets through the messenger quark (lepton) loops, and the decay rate is $\Gamma \simeq (1/8\pi)(\alpha_3/4\pi)^2(cm_S/\langle S\rangle)^2 cm_S$ ($c=1/\sqrt{2}$ for S fermion and $c=\sqrt{2}$ for R saxion) (see Fig. 5). In addition to the above decay modes, the R saxions can also decay into R axion pairs with the decay rate $\Gamma \simeq (1/64\pi)(\sqrt{2}m_S/\langle S\rangle)^2\sqrt{2}m_S$, which is the dominant decay mode of the R saxion [16]. Thus, the decay rates of those particles are sufficiently large and hence the S fermion causes no entropy production, while, as discussed in the next section, the R saxion plays a role of "flaton" in the thermal inflation, which produces a large entropy at the reheating epoch despite the large decay rate. The R axion decays into a QCD gluon pair through the diagram in Fig. 5, and the decay rate is estimated as $\Gamma \simeq (1/4\pi)(\alpha_3/4\pi)^2(m_{\rm axion}/\langle S\rangle)^2 m_{\rm axion}$. Thus, their decay temperature is given by

$$T_d^{\rm axion} \simeq 1\ \ {\rm GeV}\left(\frac{m_{\rm axion}}{10\ \ {\rm GeV}}\right)^{3/2}\left(\frac{10^5\ \ {\rm GeV}}{m_S}\right)\left(\frac{f}{10^{-3}}\right). \tag{18}$$

As discussed in the next section, the R axion produces a small but non-negligible amount of entropy at its decay time and the dilution factor from the R axion decay is about 10 [see Eq. (21)].

The remaining stable particles are now messenger quarks and leptons. They can mix with the SSM quarks $\overline{d}$ and leptons ℓ through nonrenormalizable interactions, $W=(1/M_G^2)\langle W\rangle d_M\overline{d}+(1/M_G^2)\langle W\rangle\overline{\ell}_M\ell$, where $\langle W\rangle \simeq m_{3/2}M_G^2$ is a constant term in the superpotential which is needed to tune the vacuum energy vanishing. The R charges of messenger fields can be chosen so that these interactions are allowed. The messenger quarks and leptons decay into the SSM particles through the mixings [9]. The decay rate is estimated as

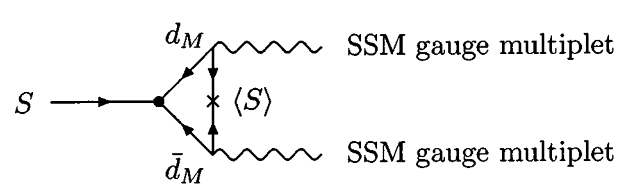

FIG. 5. The super Feymnan diagrams for decay processes of S.

[10] The QQ bound states can annihilate into a pair of the SU(2)$_m$ gauge multiplets through $Q'Q'$ loop diagrams. The Z^{ij} which are mass partners of QQ also annihilate into a pair of the SU(2)$_m$ gauge multiplets through their mass terms. Thus, annihilation cross section is given by $\langle\sigma v\rangle \simeq (\eta^2/4\pi)(\alpha_m^2/M_Z^2)$, where η is a factor which comes from the strong dynamics and naturally expected to be $O(1)$. Even if η is much smaller than $O(1)$ for some reasons, the following discussion does not change for $\eta \gtrsim 0.01$.

$\Gamma \simeq (\alpha_{2,3}^2/4\pi)(m_{3/2}/M_d)^2 M_d$ and the resultant decay temperature is $T_d^{\rm mess} \simeq 10$ GeV $\sqrt{(f/10^{-3})(k_d/10^{-1})(m_{3/2}/{\rm MeV})}$. As discussed in the next section, the relic abundance of the messenger quarks and leptons are given by Eq. (29) after the reheating of the thermal inflation. Then the temperature at which the messenger quarks and leptons could begin to dominate the energy density if they were stable is given by

$$T_c^{\rm mess} \simeq M_{d,l}\frac{4}{3}\left(\frac{n_{d,l}^{\rm after}}{s(T_c^{\rm mess})}\right)$$
$$\simeq 10^{-4}\ {\rm GeV}\left(\frac{M_{d,l}}{10^7\ {\rm GeV}}\right)^3\left(\frac{10^5\ {\rm GeV}}{m_S}\right)\left(\frac{f}{10^{-3}}\right), \tag{19}$$

where $n_{d,l}^{\rm after}$ are the number density of the messenger quarks and leptons [see Eq. (29)]. We see that $T_c^{\rm mess}$ is much smaller than $T_d^{\rm mess}$, and hence the messenger quarks and leptons produce no significant entropy in the present scenario.

We conclude that none of the quasistable particles (except for the R axion) in the DSB and the messenger sectors produces extra entropy after the freeze-out time of the gravitino.[11]

IV. A MINITHERMAL INFLATION AND THE ENTROPY PRODUCTION

In this section we discuss the thermal history of the present system. We consider the reheating temperature T_R is much higher than the DSB scale, $\Lambda_H \simeq 10^8$–10^9 GeV, and all particles in the DSB and the messenger sectors as well as the SSM sector including the gravitino are in the thermal bath. Then, the expectation value of the field S is set at the origin by the thermal effects. When the temperature T cools down to the messenger scale, the radial component of the scalar field S starts rolling down to the true minimum from the origin. We call it the "flaton" S. From Eq. (9) it is clear that if the coupling constant f is small, the potential of S is very flat and a thermal inflation [15] takes place. We assume, for the time being, that it is the case and calculate how much the entropy is produced after the thermal inflation. And we show, later on, that the coupling f is naturally small as $f \simeq 10^{-2}$–10^{-4}, while other Yukawa coupling constants, k_E, $k_d, k_l = O(1)$.

When the temperature T reaches $T \simeq m_S/(2\sqrt{f})$, the energy density of the field S, $\rho_{\rm start} \simeq m_S^4/(4f^2)$, begins to dominate the total energy density of the Universe and the thermal inflation starts. It ends when the temperature falls down to $T \simeq m_S$, and the "flaton" S starts to oscillate around the minimum of the potential. The "flaton" S decays into R axions dominantly as explained in the preceding section. Thus, the decay of "flaton" S only reheats up the temperature of the R axion, while the temperature of the SSM sector unreheated [16]. Although the decay of the "flaton" S occurs sufficiently fast in the vacuum, the reheating temperature $T_{\rm th}$ of R axion cannot exceed the mass of the "flaton" S, and hence the reheating temperature $T_{\rm th}$ is fixed by the mass of the "flaton" S field, that is $T_{\rm th} \simeq \sqrt{2} m_S$ [17]. The resultant yield of the gravitino is given by

$$Y_{3/2}^{\rm after} \equiv \frac{n_{3/2}^{\rm after}}{s^{\rm after}} = \frac{1}{s^{\rm after}}\left(\frac{\rho^{\rm after}}{\rho^{\rm before}}\right) n_{3/2}^{\rm before} = \frac{3}{4}T_{\rm th}\left(\frac{s^{\rm before}}{\rho^{\rm before}}\right)\frac{n_{3/2}^{\rm before}}{s^{\rm before}}$$
$$\simeq \left(\frac{\pi^2 g_*^{\rm before}}{30}\right)(4\sqrt{2}f^2)Y_{3/2}^{\rm before}, \tag{20}$$

where $Y_{3/2}^{\rm after,before}$ are the yields of the gravitino after/before the decay of the "flaton" S, $n_{3/2}^{\rm after,before}$ the number densities of the gravitino, $\rho^{\rm after,before}$ the energy densities and $s^{\rm after,before}$ the entropy densities.[12] Here we have used the fact that the universe is matter dominated during the decay of the "flaton" S, $\rho^{\rm after}/s^{\rm after} = (3/4)T_{\rm th}$, $s^{\rm before} \simeq (2\pi^2 g_*^{\rm before}/45)m_S^3$ and $\rho^{\rm before} \simeq m_S^4/4f^2$. After the decay of the "flaton" S, the R axions dominate the energy density of the Universe and when the temperature of the R axion cools down to its decay temperature $T_d^{\rm axion}$, they decay into pairs of SM gluons. Then, the SM particles are reheated up and the yield of the gravitino is further diluted as

$$Y_{3/2}^{\rm af.decay} = \frac{1}{s^{\rm af.decay}}\left(\frac{\rho^{\rm af.decay}}{\rho^{\rm bef.decay}}\right) n_{3/2}^{\rm bef.decay}$$
$$= \frac{3}{4}T_d^{\rm axion}\left(\frac{s^{\rm bef.decay}}{\rho^{\rm bef.decay}}\right) Y_{3/2}^{\rm bef.decay}$$
$$\simeq \frac{3}{4}\frac{T_d^{\rm axion}}{m_{\rm axion}Y_{\rm axion}} Y_{3/2}^{\rm bef.decay}, \tag{21}$$

where superscript af.decay/bef.decay means the after/before the R axion decay and $Y_{\rm axion} \simeq 0.3$ is a yield of the R axion. Here, we have used that the Universe is R axion-matter dominated when the R axion decays and also used $\rho^{\rm bef.decay} = s^{\rm bef.decay}Y_{\rm axion}m_{\rm axion}$. From Eqs. (20) and (21), we obtain the resultant dilution factor of the gravitino as

$$\Delta \equiv \left(\frac{Y_{3/2}^{\rm before}}{Y_{3/2}^{\rm af.decay}}\right) = \left(\frac{Y_{3/2}^{\rm before}}{Y_{3/2}^{\rm after}}\right)\left(\frac{Y_{3/2}^{\rm bef.decay}}{Y_{3/2}^{\rm af.decay}}\right)$$
$$\simeq \frac{4}{3}\frac{m_{\rm axion}Y_{\rm axion}}{T_d^{\rm axion}}\left(\frac{30}{\pi^2 g_*^{\rm before}}\right)\frac{1}{4\sqrt{2}f^2}, \tag{22}$$

[11]The next-to-lightest superparticle (NLSP) is in the SSM sector. Its decay process ends well before the big bang nucleosynthesis (BBN), and hence, the NLSP is cosmologically harmless.

[12]The $g_*^{\rm before} \simeq 230$ is the degree of freedoms of effective massless particles just after the end of the thermal inflation, which corresponds to the number of the SSM particles. On the other hand, we use $g_*^{\rm after} \simeq 1$ for the degree of freedoms of massless particles in the R axion (radiation) dominated era.

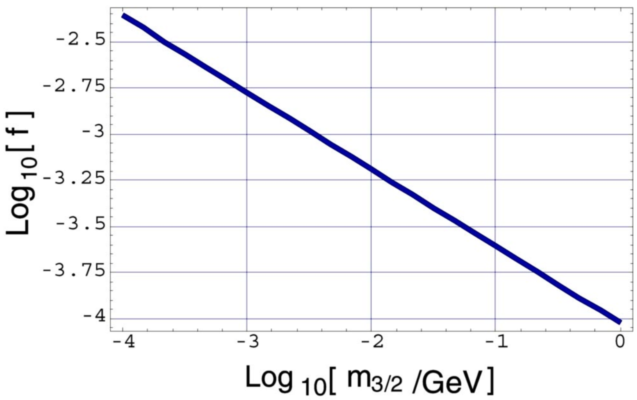

FIG. 6. The required coupling constant f which leads to the sufficient thermal inflation making the gravitino density $\Omega_{3/2}h^2 \simeq 0.11$.

where we have used $Y_{3/2}^{\text{bef.decay}} = Y_{3/2}^{\text{after}}$, since no extra entropy for the SSM particles is produced when the ''flaton'' S decays and the yield of the gravitino does not change in the R axion dominated era.

From Eqs. (1), (18), and (22), we find that f is required to be

$$f \simeq 10^{-2.8} \left(\frac{m_{3/2}}{\text{MeV}} \right)^{-5/12} \left(\frac{m_S}{10^5\ \text{GeV}} \right)^{1/4} \left(\frac{\Omega_{\text{DM}}h^2}{0.11} \right)^{1/3}, \quad (23)$$

to explain the mass density of the dark matter by the stable gravitinos.[13] We show the required f to realize the gravitino dark matter as a function of the gravitino mass $m_{3/2}$ for $m_S \simeq 10^5$ GeV in Fig. 6. We see that for $m_{3/2} = 100$ keV–1 GeV we need $f \simeq 10^{-2}$–10^{-4}, which, as we will see, is naturally obtained in the present gauge mediation model.

In the following, we show that the coupling constant f is naturally small at the messenger scale m_S. We assume that all Yukawa coupling constants in the messenger sector is of order of unity at the gravitational scale M_G and $n_q = 3$. All Yukawa coupling constants including f at the messenger scale m_S are determined by solving the following renormalization group equations (RGEs):

$$\frac{\partial}{\partial \ln \mu}\left(\frac{1}{\alpha_f} \right) = -\left[\frac{6}{2\pi} + \frac{6}{2\pi}\left(\frac{\alpha_l}{\alpha_f} \right) + \frac{9}{2\pi}\left(\frac{\alpha_d}{\alpha_f} \right) + \frac{24}{2\pi} \sum_{i>j} \left(\frac{\alpha_E^{ij}}{\alpha_f} \right) \right], \quad (24)$$

$$\frac{\partial}{\partial \ln \mu}\left(\frac{1}{\alpha_d} \right) = -\left[\frac{5}{2\pi} + \frac{2}{2\pi}\left(\frac{\alpha_l}{\alpha_d} \right) + \frac{2}{2\pi}\left(\frac{\alpha_f}{\alpha_d} \right) + \frac{8}{2\pi} \sum_{i>j} \left(\frac{\alpha_E^{ij}}{\alpha_l} \right) - \frac{2}{15\pi}\left(\frac{\alpha_1}{\alpha_d} \right) - \frac{8}{3\pi}\left(\frac{\alpha_3}{\alpha_d} \right) \right], \quad (25)$$

$$\frac{\partial}{\partial \ln \mu}\left(\frac{1}{\alpha_l} \right) = -\left[\frac{4}{2\pi} + \frac{3}{2\pi}\left(\frac{\alpha_d}{\alpha_l} \right) + \frac{2}{2\pi}\left(\frac{\alpha_f}{\alpha_l} \right) + \frac{8}{2\pi} \sum_{i>j} \left(\frac{\alpha_E^{ij}}{\alpha_l} \right) - \frac{3}{10\pi}\left(\frac{\alpha_1}{\alpha_l} \right) - \frac{3}{2\pi}\left(\frac{\alpha_3}{\alpha_l} \right) \right], \quad (26)$$

$$\begin{aligned} \frac{\partial}{\partial \ln \mu}\left(\frac{1}{\alpha_E^{ij}} \right) = -\left(\frac{1}{\alpha_E^{ij}} \right) \Bigg[& \frac{2}{2\pi}\alpha_f + \frac{3}{2\pi}\alpha_d + \frac{2}{2\pi}\alpha_l \\ & + \frac{8}{2\pi}\sum_{i>j} \alpha_E^{ij} + \frac{4}{2\pi}\sum_{\ell \neq i}^{6} \alpha_E^{i\ell} \\ & + \frac{4}{2\pi}\sum_{\ell \neq j}^{6} \alpha_E^{j\ell} - \frac{3}{2\pi}\alpha_m \Bigg] \\ & + \frac{4}{2\pi}\sum_{\ell \neq i,j}^{6} \left(\frac{1}{\alpha_E^{ij}} \right)^{3/2} \Bigg[\sum_{m \neq \ell, i}^{6} \sqrt{\alpha_E^{j\ell}\alpha_E^{\ell m}\alpha_E^{mi}} \\ & + \sum_{m \neq \ell, j}^{6} \sqrt{\alpha_E^{i\ell}\alpha_E^{\ell m}\alpha_E^{mj}} \Bigg], \end{aligned} \quad (27)$$

where $\alpha_f = f^2/4\pi$, $\alpha_d = k_d^2/4\pi$, $\alpha_l = k_l^2/4\pi$, and $\alpha_E^{ij} = k_{ij}^2/4\pi$. We can see that the RGE of the coupling constant f has no effect from the gauge coupling constants which slacken the speed of the Yukawa-coupling running. Thus, we can expect that the coupling constant f becomes much smaller than the other Yukawa coupling constants at the low energy scale.

The result on the coupling f is shown in Fig. 7. Here we have assumed

$$k_{ij},\ \ k_d,\ \ k_\ell,\ \ \ f = 0.3 - 3, \quad (28)$$

at the gravitational scale M_G. We find that the desired coupling $f \simeq 10^{-3}$ is obtained at the messenger scale $\mu = m_S$. We also show the obtained coupling constants for $\sqrt{\Sigma_{i\neq j} k_{ij}}$, k_d, k_ℓ in Fig. 7. We see that the assumptions on the Yukawa coupling constants made in the preceding section are realized naturally [for instance, $\sqrt{\Sigma_{i\neq j} k_{ij}} = O(1)$ and $k_d, k_l \gg f$].

Before closing this section, we should comment on the reproductions of gravitinos from the thermal background after the thermal inflation. We find that the gravitino reproduction rate from the thermal R axion bath is small enough not to spoil the successful dilution of the gravitino. The

[13] In the present scenario, the temperature where the gravitinos are thermalized is higher than the reheating temperature T_{th} of the thermal inflation. At temperature which is higher than the reheating temperature T_{th}, the expectation value of the field S is set at the origin, and the fields of the messenger sector are also massless. Therefore, the degree of freedoms of the effective massless particles, $g_*(T_f)$, is enhanced as $g_*(T_f) \simeq 350$ for $T_f > T_{\text{th}}$.

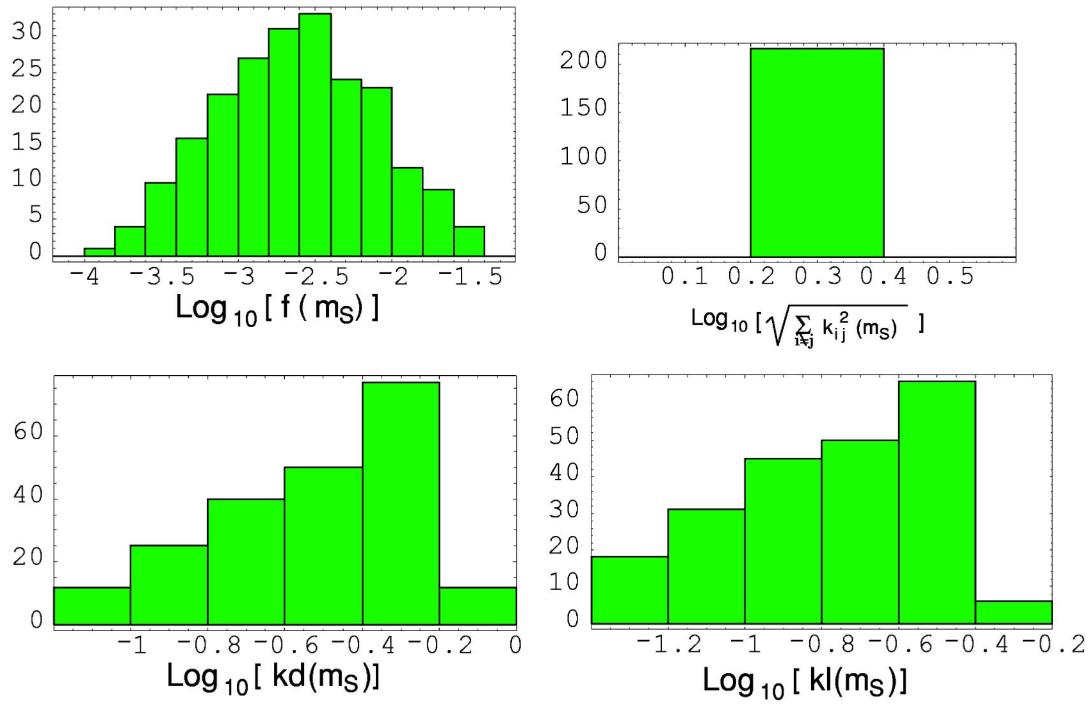

FIG. 7. The histograms for Yukawa coupling constants at the DSB scale with $n_q=3$. We have varied the coupling constants k_{ij}, k_d, k_l, and f from 0.3 to 3 with a logarithmic measure at the gravitational scale M_G. Here $\alpha_m(\Lambda_H)=0.5$, and we have set all the k_{ij} equal for simplicity.

reheating temperature of the SSM sector is $T_d^{\rm axion} \simeq 1\ {\rm GeV}(m_{\rm axion}/10\ {\rm GeV})^{3/2}$ and hence the gravitino reproduction from the SSM background is also negligible.

Finally, we estimate the relic abundances of the messenger quarks and leptons. When the "flaton" S stay at the origin, the messenger quarks and leptons are massless, and their annihilation processes take place during the reheating epoch of the thermal inflation. When the annihilation rates $\langle\sigma v\rangle n_{d,l}$ become smaller than the Hubble expansion rate H, the messenger quarks and leptons are frozen out from the thermal bath with the number density $n_{d,l}^f \simeq H_f/\langle\sigma v\rangle$, where $\langle\sigma v\rangle$ is an annihilation cross section of the messenger quarks and leptons and the sub(super)script f denotes the "freeze-out" time. Since the annihilation processes are instantaneous, H_f is estimated as $H_f \simeq \sqrt{\rho^{\rm before}}/M_G$. Thus, the resultant relic abundances of the messenger quarks and leptons after the reheating process are given by

$$\frac{n_{d,l}^{\rm after}}{s^{\rm after}} \simeq \frac{1}{s^{\rm after}}\left(\frac{\rho^{\rm after}}{\rho^{\rm before}}\right) n_{d,l}^f \simeq \frac{2f}{\langle\sigma v\rangle M_G m_S} \simeq \frac{2fM_{d,l}^2}{4\pi\alpha_{2,3}^2 M_G m_S}, \tag{29}$$

where we have used the fact that the Universe is matter dominated during the decay of the "flaton" S, $\rho^{\rm after}/s^{\rm after} = (3/4)T_{\rm th}$, $\rho^{\rm before} \simeq m_S^4/4f^2$, and $\langle\sigma v\rangle \simeq 4\pi\alpha_{2,3}^2/M_{d,l}^2$. As discussed in the preceding section, the messenger quarks and leptons cannot dominate the energy density of the Universe before their decay times, and hence they produce no extra entropy.[14]

V. CONCLUSIONS

We assume, in this paper, that the reheating temperature T_R of the primary inflation is $T_R > 10^{10}$ GeV so that the thermal leptogenesis takes place. With this reheating temperature the gravitinos of mass $m_{3/2} \lesssim 1$ GeV are thermally produced and they overclose the Universe if there is no entropy production after their freeze-out time. We find, however, that a minithermal inflation occurs naturally in a class of gauge meditation models we discuss in this paper. The reheating process of the thermal inflation produces an amount of entropy, which dilutes the number density of the relic gravitinos avoiding the overclosure. This dilution makes the gravitino to be the dark matter in the present Universe. The dilution factor depends on a Yukawa coupling constant f. From Fig. 6, we see that for $m_{3/2} = 100$ keV–1 GeV we need $f \simeq 10^{-2}$–10^{-4}, which is naturally obtained in the present gauge mediation model.

The abundance of the gravitino dark matter is independent of the reheating temperature T_R of the primary inflation,

[14]The bound states of the hyperquarks are heavy even before the thermal inflation. Thus, their annihilation processes finish before the thermal inflation. Therefore, their relic abundances are diluted by the thermal inflation as well as the relic abundance of the gravitino [Eq. (17)].

once the gravitinos are in the thermal equilibrium. Therefore, there is no upper bound on the reheating temperature T_R from the overproduction of gravitinos and hence the thermal leptogenesis takes place without any gravitino problem [9]. The temperature T_L of the leptogenesis is found as $T_L \simeq 10^{12}$–10^{14} GeV for $m_{3/2} \simeq 100$ keV–10 MeV (see [9]).

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