$\gamma\gamma^*$ production of two ρ^0 mesons

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We present a theoretical estimation for the cross-section of exclusive two ρ-meson production in two photon collision when one of the initial photons is highly virtual. The compatibility of our analysis with recent experimental data obtained by the L3 Collaboration at (CERN) LEP is discussed. We show that these data prove the scaling behavior of the exclusive production amplitude. They are thus consistent with a partonic description of the exclusive process $\gamma\gamma^*\to\rho^0\rho^0$ when $Q^2\sim 2-20$ GeV^2.

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I. INTRODUCTION

Two-photon collisions provide a tool to study a variety of fundamental aspects of QCD and have long been a subject of great interest (cf. e.g. [1–3] and references therein). A peculiar facet of this interesting domain is exclusive two hadron production in the region where one initial photon is highly virtual (its virtuality being denoted as Q^2) but the overall energy (or invariant mass of the two hadrons) is small [4]. This process factorizes [5,6] into a perturbatively calculable, short-distance dominated scattering $\gamma^*\gamma\to q\bar{q}$ or $\gamma^*\gamma\to gg$, and nonperturbative matrix elements measuring the transitions $q\bar{q}\to AB$ and $gg\to AB$. These matrix elements have been called generalized distribution amplitudes (GDAs) to emphasize their close connection to the distribution amplitudes introduced many years ago in the QCD description of exclusive hard processes [7].

In this paper, we focus on the process $\gamma^*\gamma\to\rho^0\rho^0$ which has not been discussed much theoretically (see however Ref. [8]) but has recently been observed at CERN e^+e^- collider LEP in the right kinematical domain [9].

Since $\gamma^*\gamma\to\rho\rho$ is the crossed channel of virtual Compton scattering on a spin-1 meson, the physics considered here is closely related to deeply virtual Compton scattering (DVCS) on a spin-1 target [10], which has recently attracted some attention in the context of generalized (skewed) parton distributions of the deuteron [11].

II. KINEMATICS

The reaction which we study here is (see Fig. 1)

$$e(k)+e(l)\to e(k')+e(l')+\rho^0(p_1)+\rho^0(p_2) \qquad (1)$$

where the initial electron $e(k)$ radiates a hard virtual photon with momentum $q=k-k'$; in other words, the square of virtual photon momentum $q^2=-Q^2$ is very large. This means that the scattered electron $e(k')$ is tagged. To describe reaction (1), it is useful to consider, at the same time, the subprocess

$$e(k)+\gamma(q')\to e(k')+\rho^0(p_1)+\rho^0(p_2). \qquad (2)$$

Regarding the other photon momentum $q'=l-l'$, we assume that, first, its momentum is collinear to the electron momentum l and, secondly, that q'^2 is approximately equal to zero, which is a usual approximation when the second lepton is untagged.

Let us now pass to a short discussion of kinematics in the $\gamma\gamma^*$ center of mass system. We adopt the z axis directed along the three-dimensional vector $\mathbf{q}$, and the ρ-meson momenta lie in the (x,z) plane. It means that we ignore the azimuthal dependence of the cross section, which happens to be absent in the approximation we use. So, we write for the momenta in the c.m. system:

$$q=(q_0,0,0,\mathbf{q}), \quad p_1=(p_1^0,\mathbf{p}_1\sin\theta,0,\mathbf{p}_1\cos\theta). \qquad (3)$$

Also, we need to write down the Mandelstam S-variables for the electron-positron (1) and electron-photon (2) collisions:

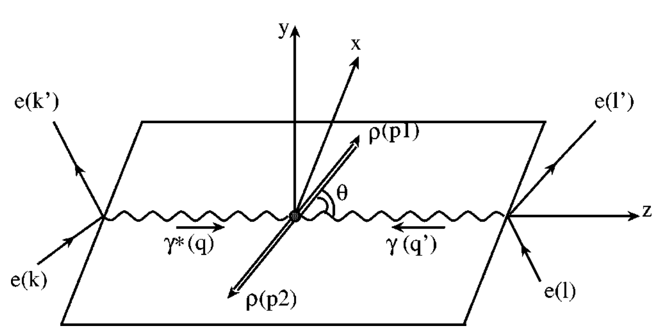

FIG. 1. Kinematics of the process $e(k)+e(l)\to e(k')+e(l')+\rho^0(p_1)+\rho^0(p_2)$ in the c.m. system of the two mesons.

$$S_{ee}=(k+l)^2,\quad S_{e\gamma}=(k+q')^2. \tag{4}$$

Neglecting the lepton masses, these variables can be rewritten as

$$S_{ee}\approx 2(k\cdot l),\quad S_{e\gamma}\approx 2(k\cdot q')=x_2 S_{ee}, \tag{5}$$

where the fraction x_2 defined as $q_0'=x_2 l_0$ is introduced (see [12]).

III. PARAMETRIZATION OF ρ-MATRIX ELEMENTS AND THEIR PROPERTIES

Let us first introduce the basis light-cone vectors. We adopt a basis consisting of two lightlike vectors p and n of mass dimension 1 and -1, respectively. In other words, they obey the following conditions:

$$p^2=n^2=0,\quad (p\cdot n)=1. \tag{6}$$

With the help of this basis, the ρ-mesons momenta p_1 and p_2 can be written as

$$p_1^\mu=\zeta p^\mu+(1-\zeta)\frac{W^2}{2}n^\mu-\frac{\Delta_T^\mu}{2},$$

$$p_2^\mu=(1-\zeta)p^\mu+\zeta\frac{W^2}{2}n^\mu+\frac{\Delta_T^\mu}{2}. \tag{7}$$

As usually for the two meson generalized distribution amplitude case, we introduce the sum and difference of hadronic momenta which take the form in the light-cone decomposition:

$$\Delta^\mu=p_2^\mu-p_1^\mu,\quad P^\mu=p_2^\mu+p_1^\mu. \tag{8}$$

The skewedness parameter ζ is defined by

$$\zeta=\frac{p_1^+}{P^+}=\frac{1+\beta\cos\theta}{2},\quad \beta=\sqrt{1-\frac{4m_\rho^2}{W^2}}. \tag{9}$$

Note also that within the c.m. frame the transverse component of the transfer momentum $\Delta_T=(0,\mathbf{\Delta}_T,0)$ is given by

$$\Delta_T^2=-\mathbf{\Delta}_T^2=(4m_\rho^2-W^2)\sin^2\theta. \tag{10}$$

Let us now write down the decomposition for the longitudinal and transverse ρ-meson polarization vectors:

$$e_{1\,\mu}^{(0)}=\frac{1}{m_\rho}\left(p_{1\mu}-\frac{m_\rho^2}{\zeta}n_\mu\right),$$

$$e_{1\,\mu}^{(1)}=\frac{2}{\sqrt{\mathbf{\Delta}_T^2}}\left(\zeta p_{2\,\mu}-(1-\zeta)p_{1\mu}-\frac{\zeta(2\zeta-1)W^2+\mathbf{\Delta}_T^2}{2\zeta}n_\mu\right),$$

$$e_{1\mu}^{(2)}=-\frac{2}{\sqrt{\mathbf{\Delta}_T^2}}\varepsilon_{\mu\alpha\beta\gamma}p_{2\alpha}p_{1\,\beta}n_\gamma, \tag{11}$$

for the ρ meson with momentum p_1, and

$$e_{2\mu}^{(0)}=\frac{1}{m_\rho}\left(p_{2\mu}-\frac{m_\rho^2}{1-\zeta}n_\mu\right),$$

$$e_{2\mu}^{(1)}=-\frac{2}{\sqrt{\mathbf{\Delta}_T^2}}\left((1-\zeta)p_{1\mu}-\zeta p_{2\mu}+\frac{(1-\zeta)(2\zeta-1)W^2-\mathbf{\Delta}_T^2}{2(1-\zeta)}n_\mu\right),$$

$$e_{2\mu}^{(2)}=\frac{2}{\sqrt{\mathbf{\Delta}_T^2}}\varepsilon_{\mu\alpha\beta\gamma}p_{2\,\alpha}p_{1\,\beta}n_\gamma, \tag{12}$$

for the other ρ meson with momentum p_2. As usual,

$$e_i\cdot p_i=0,\quad e_i^{(\lambda)}\cdot e_i^{(\lambda')}=-\delta^{\lambda\lambda'}. \tag{13}$$

Besides, for each meson, the following polarization vectors can be introduced to define the light-cone helicity:

$$e(0)=e^{(0)},\quad e(\pm)=\frac{\mp e^{(1)}-ie^{(2)}}{\sqrt{2}}. \tag{14}$$

We now come to the parametrization of the relevant matrix elements. Keeping the terms of leading twist 2, the vector and axial correlators can be written as

$$\langle p_1,\lambda_1;p_2,\lambda_2|\bar\psi(0)\gamma_\mu\psi(\lambda n)|0\rangle \overset{\mathcal{F}}{=} p_\mu\sum_i e_1^\alpha e_2^\beta V_{\alpha\beta}^{(i)}(p_1,p_2,n)H_i^{\rho\rho,\,V}(y,\zeta,W^2), \tag{15}$$

$$\langle p_1,\lambda_1;p_2,\lambda_2|\bar\psi(0)\gamma_\mu\gamma_5\psi(\lambda n)|0\rangle \overset{\mathcal{F}}{=} p_\mu\sum_i e_1^\alpha e_2^\beta A_{\alpha\beta}^{(i)}(p_1,p_2,n)H_i^{\rho\rho,\,A}(y,\zeta,W^2). \tag{16}$$

Here λ_1 and λ_2 are the helicities of ρ mesons and $\overset{\mathcal{F}}{=}$ denotes the Fourier transformation with measure $(z_1=\lambda n, z_2=0)$ [16]:

$$d\mu(y)=dy\,e^{-iypz_1+i(1-y)pz_2}. \tag{17}$$

With the help of parity invariance we can show that the vector tensors $V_{\alpha\beta}^{(i)}$ may be written in terms of five tensor structures while the axial tensors $A_{\alpha\beta}^{(i)}$ are linear combinations of four independent structures. In complete analogy with the analysis of the deuteron generalized parton distributions [11] we write

$$\sum_i e_1^\alpha e_2^\beta V_{\alpha\beta}^{(i)}(p_1,p_2,n)H_i^{\rho\rho,V}(y) = -(e_1\cdot e_2)H_1^{\rho\rho,V}(y) + ((e_1\cdot n)(e_2\cdot p_1)+(e_2\cdot n)(e_1\cdot p_2))H_2^{\rho\rho,V}(y) - \frac{(e_1\cdot p_2)(e_2\cdot p_1)}{2m_\rho^2}H_3^{\rho\rho,V}(y) + ((e_1\cdot n)(e_2\cdot p_1)-(e_2\cdot n)(e_1\cdot p_2))H_4^{\rho\rho,V}(y) + \left(4m_\rho^2\,(e_1\cdot n)(e_2\cdot n)+\frac{1}{3}(e_1\cdot e_2)\right)H_5^{\rho\rho,V}(y) \tag{18}$$

for the vector tensor structures and

$$\sum_i e_1^\alpha e_2^\beta A_{\alpha\beta}^{(i)}(p_1,p_2,n)H_i^{\rho\rho,A}(y) = -i\epsilon_{\alpha\beta}^T e_{1\alpha}^T e_{2\beta}^T\, H_1^{\rho\rho,A}(y) + i\epsilon_{\alpha\beta}^T\Delta_\alpha\frac{e_{1\beta}(e_2\cdot p_1)+e_{2\beta}(e_1\cdot p_2)}{m_\rho^2}H_2^{\rho\rho,A}(y) + i\epsilon_{\alpha\beta}^T\Delta_\alpha\frac{e_{1\beta}(e_2\cdot p_1)-e_{2\beta}(e_1\cdot p_2)}{m_\rho^2}H_3^{\rho\rho,A}(y) + i\epsilon_{\alpha\beta}^T\Delta_\alpha(e_{1\beta}(e_2\cdot n) + e_{2\beta}(e_1\cdot n))H_4^{\rho\rho,A}(y) \tag{19}$$

for the axial tensor structures. In Eqs. (18) and (19), the standard notation $\epsilon_{\alpha\beta}^T=\epsilon_{\alpha\beta\gamma\delta}\,p_\gamma n_\delta$ was used and the dependence of parametrizing functions (GDA) on ζ and W^2 is implied.

Let us now turn to the consideration of symmetry properties. The $\gamma\gamma^*$ subprocess is selecting the parts with the following symmetry:

$$H_i^{\rho\rho,V}(y) = -H_i^{\rho\rho,V}(1-y),\quad H_i^{\rho\rho,A}(y)=H_i^{\rho\rho,A}(1-y). \tag{20}$$

Note that the scale dependence of generalized distribution amplitudes acquired in the process of factorization of the scattering amplitude has been studied in [12] and there are no essential differences between the $\pi\pi$ channel discussed there and vector contributions to our $\rho\rho$ case. At the same time, the evolution of axial contributions are similar to the case of the distribution amplitudes of singlet axial mesons. The W^2 behavior of 2π GDA's has been recently explored in terms of an impact representation of the hadronization process $q\bar{q}\to MM$ [13].

IV. QUARK-HADRON HELICITY AMPLITUDES

Although the main objects of our investigation are the generalized distribution amplitudes (GDA) let us begin from the discussion of the helicity amplitudes related to the generalized parton distribution (GPD), which are related to GDAs by means of $s\leftrightarrow t$ crossing symmetry [14,15]. One considers the quark helicity conserving distributions parametrizing the combination of the vector and axial matrix elements:

$$\mathcal{A}_{(\lambda_2\pm;\lambda_1\pm)} = \frac{1}{2}\langle p_2,\lambda_2|\bar{\psi}(0)\,\gamma^+(1\pm\gamma_5)\,\psi(z)|p_1,\lambda_1\rangle, \tag{21}$$

where λ and $\mu=\pm$ denote the helicities of initial (final) hadrons and quarks, respectively. The matrix elements in Eq. (21) are taken between two ρ mesons. In the forward limit when $p_1=p_2$ helicity conservation takes place and requires $\lambda_1+\mu_1=\lambda_2+\mu_2$. The parity transformation, $\lambda_i\to-\lambda_i$ etc., invariance leads to nine independent helicity amplitudes (21). At the same time the time reversal transformation, $(\mu_1,\lambda_1)\leftrightarrow(\mu_2,\lambda_2)$, invariance does not lead to any reduction of the number of independent structures owing to ζ dependence (see, for instance, [11]).

As noted before, the crossing transformation relates the helicity amplitudes referring to the GDAs to the corresponding GPDs helicity amplitudes. Indeed, the crossing transformations imply that the initial hadron is replaced by a final hadron with the opposite momentum: $p_1\to-p_1$ and, therefore, opposite helicities $\lambda_1\to-\lambda_1$. Similarly, we implement the crossing replacement for the quark fields. Namely, the final quark with helicity μ_2 is replaced by the initial antiquark with helicity $-\mu_2$. Thus, the quark helicity nonflip amplitudes in t-channel come to the amplitudes in s-channel where the initial quark and antiquark helicities are opposite, and vice versa:

$$\mathcal{A}^{(t)}_{(\lambda_2\pm;\lambda_1\pm)}\to A^{(s)}_{(\lambda_2-\lambda_1;\mp\pm)}. \tag{22}$$

Note that the first two indices labeling the helicity amplitudes in Eq. (22) or Eq. (21) correspond to the helicities in the final state and the last two indices — to the initial state.

Let us now focus on the amplitudes in s-channel. As the helicity and chirality of antiquarks are distinguished by sign (in this paper, we consider the case of massless quarks), the chirality conserving quark-antiquark operator will give the combination of quark and antiquark fields with opposite helicities: $\bar{v}_+\gamma^+u_+\leftrightarrow\bar{v}_{(-)}\gamma^+u_{(+)}$, where the bracketed subscripts denote the helicity while the unbracketed ones denote the chirality. Therefore, using Eqs. (15) and (16), we write, forgetting from now on the (s) subscript,

$$A_{(\lambda_2\lambda_1;\ \pm\mp)}=\frac{1}{2}\langle p_2,\lambda_2;\ p_1,\lambda_1|\overline{\psi}(0)\gamma^+(1\pm\gamma_5)\psi(z)|0\rangle. \tag{23}$$

Further, a straightforward calculation derives the expressions for the helicity amplitudes (cf. [11]):

$$2A_{(++,+-)}=H_1^{\rho\rho,V}+BH_3^{\rho\rho,V}-\frac{1}{3}H_5^{\rho\rho,V}+H_1^{\rho\rho,A}+2B[H_2^{\rho\rho,A}+(2\zeta-1)H_3^{\rho\rho,A}],$$

$$\begin{aligned}2A_{(0+,+-)}=\sqrt{\frac{2B\zeta}{1-\zeta}}\Bigg(&H_1^{\rho\rho,V}-\zeta[H_2^{\rho\rho,V}+H_4^{\rho\rho,V}]\\&+\left[\frac{(2\zeta-1)W^2}{4m_\rho^2}+2(1-\zeta)\right]H_3^{\rho\rho,V}\\&-\frac{1}{3}H_5^{\rho\rho,V}+(1-\zeta)H_1^{\rho\rho,A}+(1-\zeta)\\&\times\left[\frac{(2\zeta-1)W^2}{m_\rho^2}+4(1-\zeta)B\right][H_2^{\rho\rho,A}\\&-H_3^{\rho\rho,A}]+\zeta(1-\zeta)H_4^{\rho\rho,A}\Bigg),\end{aligned}$$

$$2A_{(-+,+-)}=-2B\left(\frac{1}{2}H_2^{\rho\rho,V}-(2\zeta-1)H_2^{\rho\rho,A}-H_3^{\rho\rho,A}\right),$$

$$\begin{aligned}2A_{(00,+-)}=\Bigg[&\left(\frac{1}{2}-\zeta(1-\zeta)\right)\frac{W^2}{m_\rho^2}-4(1-\zeta(1-\zeta))B\Bigg]\\&\times\left[H_1^{\rho\rho,V}-\frac{1}{3}H_5^{\rho\rho,V}\right]-\frac{(2\zeta-1)W^2}{2m_\rho^2}\\&\times[H_4^{\rho\rho,V}+(2\zeta-1)H_2^{\rho\rho,V}]+2B[H_2^{\rho\rho,V}\\&+(2\zeta-1)H_4^{\rho\rho,V}]+\Bigg[\frac{(2\zeta-1)^2W^4}{8m_\rho^4}\\&-2B\left(\frac{(2\zeta-1)^2W^2}{2m_\rho^2}+4\zeta(1-\zeta)B\right)\Bigg]H_3^{\rho\rho,V}\\&+4\zeta(1-\zeta)H_5^{\rho\rho,V}.\end{aligned} \tag{24}$$

In Eq. (24) the following notation has been used:

$$B=\frac{\mathbf{\Delta}_T^2}{16m_\rho^2\zeta(1-\zeta)}. \tag{25}$$

V. AMPLITUDE OF $\gamma\gamma^*\to\rho^0\rho^0$ SUBPROCESS

In this section, we consider the $\gamma(q')\gamma^*(q)\to\rho^0(p_1)\rho^0(p_2)$ subprocess. Following [16], the amplitude of this subprocess including the leading twist-2 terms can be written as

$$\begin{aligned}T_{\mu\nu}^{\gamma\gamma^*\to\rho^0\rho^0}=\frac{1}{2}\sum_{q=1}^{n_f}e_q^2\int_0^1 dy[&g_{\mu\nu}^T E_-(y)\mathbf{V}_q(y,\cos\theta,W^2)\\&-i\epsilon_{\mu\nu}^T E_+(y)\mathbf{A}_q(y,\cos\theta,W^2)],\end{aligned} \tag{26}$$

where

$$E_\pm=\frac{1}{1-y}\pm\frac{1}{y}. \tag{27}$$

In Eq. (26), the scalar and pseudoscalar functions $(\mathbf{V},\mathbf{A})$ denote the following contractions:

$$\mathbf{V}_q(y,\cos\theta,W^2)=\sum_i e_1^\alpha e_2^\beta V_{\alpha\beta}^{(i)}H_{i,q}^{\rho\rho,V}(y,\zeta(\cos\theta),W^2),$$

$$\mathbf{A}_q(y,\cos\theta,W^2)=\sum_i e_1^\alpha e_2^\beta A_{\alpha\beta}^{(i)}H_{i,q}^{\rho\rho,A}(y,\zeta(\cos\theta),W^2).$$

The helicity amplitudes are obtained from the usual amplitudes after multiplying by the photon polarization vectors

$$A_{(i,j)}=\varepsilon_\mu'^{(i)}\varepsilon_\nu^{(j)}T^{\mu\nu}_{\gamma\gamma^*\to\rho^0\rho^0}. \tag{28}$$

Here, in the $\gamma\gamma^*$ c.m. frame, the photon polarization vectors read

$$\varepsilon_\mu'^{(\pm)}=\left(0,\frac{\mp1}{\sqrt{2}},\frac{+i}{\sqrt{2}},0\right),$$

$$\varepsilon_\mu^{(\pm)}=\left(0,\frac{\mp1}{\sqrt{2}},\frac{-i}{\sqrt{2}},0\right),\quad \varepsilon_\mu^{(0)}=\left(\frac{|q|}{\sqrt{Q^2}},0,0,\frac{q_0}{\sqrt{Q^2}}\right), \tag{29}$$

for the real and virtual photons, respectively.

VI. DIFFERENTIAL CROSS SECTIONS

We will now concentrate on the calculation of the differential cross section of Eq. (1). The amplitude of this process can be written as

$$\begin{aligned}\mathcal{A}_{ee\to ee\rho^0\rho^0}=\sum_{i,j}[\overline{u(l')}\gamma^\mu u(l)\overset{*}{\varepsilon}{}_\mu'^{(i)}]\frac{1}{q'^2}A_{(i,j)}^{\gamma\gamma^*\to\rho^0\rho^0}\frac{1}{q^2}\\\times[\overset{*}{\varepsilon}{}_\nu^{(j)}\overline{u(k')}\gamma^\nu u(k)].\end{aligned} \tag{30}$$

This amplitude depends on the polarization states of the produced ρ mesons. Due to parity invariance there are only three independent sets of helicity (photon) amplitudes which we put to be $A_{(+,+)}$, $A_{(+,-)}$ and $A_{(+,0)}$. Let us focus on the leading twist-2 helicity amplitude in the unpolarized electrons case, i.e., the $A_{(+,+)}$ amplitude. In this case, the square of the modulus of the amplitude (30) can be presented in the "factorized" form:

$$|\mathcal{A}_{ee\to ee\rho^0\rho^0}|^2 = |\mathcal{A}_{e\gamma\to e\rho^0\rho^0}|^2 \frac{1}{q'^4} |\mathcal{A}_{e\to e\gamma}|^2 \tag{31}$$

and the scattering cross section is

$$d\sigma_{ee\to ee\rho^0\rho^0} = \frac{1}{2S_{ee}} \frac{d^3l'}{(2\pi)^3 2l'_0} \frac{d^3p_1}{(2\pi)^3 2p_1^0} \times \frac{d^3p_2}{(2\pi)^3 2p_2^0} \frac{d^3k'}{(2\pi)^3 2k'_0} |\mathcal{A}_{e\gamma\to e\rho^0\rho^0}|^2 \frac{1}{q'^{\,4}} \times |\mathcal{A}_{e\to e\gamma}|^2. \tag{32}$$

From now on, we will sum over the polarization states of the ρ mesons. Separating the differential cross section for $e\gamma \to e\rho^0\rho^0$ subprocess, we are able to rewrite Eq. (32) in the form

$$d\sigma_{ee\to ee\rho^0\rho^0} = \frac{d^3l'}{(2\pi)^3 2l'_0} \frac{x_2}{q'^4} |\mathcal{A}_{e\to e\gamma}|^2 d\sigma_{e\gamma\to e\rho^0\rho^0}, \tag{33}$$

where

$$d\sigma_{e\gamma\to e\rho^0\rho^0} = \frac{1}{2S_{e\gamma}} \frac{d^3p_1}{(2\pi)^3 2p_1^0} \frac{d^3p_2}{(2\pi)^3 2p_2^0} \frac{d^3k'}{(2\pi)^3 2k'_0} \times |\mathcal{A}_{e\gamma\to e\rho^0\rho^0}|^2. \tag{34}$$

Using the equivalent photon approximation we find the expression for the corresponding cross section:

$$\frac{d\sigma_{ee\to ee\rho^0\rho^0}}{dQ^2} = \int \cdots \int dW^2\, d\cos\theta\, d\phi\, dx_2 \frac{\alpha}{\pi} F_{WW}(x_2) \times \frac{d\sigma_{e\gamma\to e\rho^0\rho^0}}{dQ^2 dW^2 d\cos\theta\, d\phi}, \tag{35}$$

where the Weizsacker-Williams function F_{WW} is defined as usual as:

$$F_{WW}(x_2) = \frac{1+(1-x_2)^2}{2x_2} \ln \frac{Q'^2(x_2)}{m_e^2} - \frac{1-x_2}{x_2}, \tag{36}$$

and the value $Q'^{\,2}$ is defined as

$$Q'^2 = -q'^2_{max} = \frac{(1-x_2)}{4} S_{ee} \sin^2\alpha_{max}. \tag{37}$$

The angle α_{max} in Eq. (37) is determined by the acceptance of a lepton in the detector (see, for instance, [12]) and the value of the c.m. energy of the ee collision $\sqrt{S_{ee}}$ is 91 GeV at LEP1 and 195 GeV at LEP2.

In Eq. (35), the cross section for the subprocess can be calculated directly; we have

$$\frac{d\sigma_{e\gamma\to e\rho^0\rho^0}}{dQ^2 dW^2 d\cos\theta\, d\phi} = \frac{\alpha^3}{16\pi} \frac{\beta}{S_{e\gamma}^2} \frac{1}{Q^2} \left(1 - \frac{2S_{e\gamma}(Q^2+W^2-S_{e\gamma})}{(Q^2+W^2)^2} \right) \times |A_{(+,+)}(\cos\theta, W^2)|^2 \tag{38}$$

where

$$|A_{(+,+)}(\cos\theta, W^2)|^2 = (|\mathbf{V}(\cos\theta, W^2)|^2 + |\mathbf{A}(\cos\theta, W^2)|^2). \tag{39}$$

In Eq. (39), the squared and ρ meson polarizations summed functions $|\mathbf{V}|^2$ and $|\mathbf{A}|^2$ read

$$|\mathbf{V}(\cos\theta, W^2)|^2 = \frac{1}{4} P^{\alpha_1\alpha_2}(p_1) P^{\beta_1\beta_2}(p_2) \sum_{i,\,q} e_q^2 V^{(i)}_{\alpha_1\beta_1} \times \int dy_1 E_-(y_1) H^{\rho\rho,\,V}_{i,q}(y_1, \cos\theta, W^2) \times \sum_{j,\,q} e_q^2 V^{(j)}_{\alpha_2\beta_2} \int dy_2 E_-(y_2) \times H^{\rho\rho,V}_{j,q}(y_2, \cos\theta, W^2) \tag{40}$$

$$|\mathbf{A}(\cos\theta, W^2)|^2 = \frac{1}{4} P^{\alpha_1\alpha_2}(p_1) P^{\beta_1\beta_2}(p_2) \sum_{i,q} e_q^2 A^{(i)}_{\alpha_1\beta_1} \times \int dy_1 E_+(y_1) H^{\rho\rho,A}_{i,q}(y_1, \cos\theta, W^2) \times \sum_{j,q} e_q^2 A^{(j)}_{\alpha_2\beta_2} \int dy_2 E_+(y_2) \times H^{\rho\rho,A}_{j,q}(y_2, \cos\theta, W^2), \tag{41}$$

where

$$P_{\alpha\beta}(p) = \sum_\lambda e_\alpha^{(\lambda)} e_\beta^{*\,(\lambda)} = -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_\rho^2}. \tag{42}$$

As mentioned before, the expressions (40) and (41) do not depend on the azimuth ϕ.

The helicity squared amplitude when the integration over $\cos\theta$ is implemented may be written as

$$F_{(+,+)}(W^2) = \int_0^1 d\cos\theta |A_{(+,+)}(\cos\theta, W^2)|^2, \tag{43}$$

and the cross section takes the form

$$\frac{d\sigma_{ee\to ee\rho^0\rho^0}}{dQ^2}=\frac{25\alpha^4}{36\pi}\int_0^1 dx_2 F_{WW}(x_2)\times\left(\frac{1}{x_2^2S_{ee}^2Q^2}\int_{W^2_{min}}^{W^2_{max}}dW^2\beta\, F_{(+,+)}(W^2)-\frac{2}{x_2S_{ee}Q^2}\int_{W^2_{min}}^{W^2_{max}}dW^2\frac{\beta\, F_{(+,+)}(W^2)}{Q^2+W^2}+\frac{2}{Q^2}\int_{W^2_{min}}^{W^2_{max}}dW^2\frac{\beta\, F_{(+,+)}(W^2)}{(Q^2+W^2)^2}\right). \tag{44}$$

In Eq. (44), the magnitudes of W^2_{min} and W^2_{max} are defined by the interval covering most of detected two ρ^0 events, which in the L3 case at LEP [9] is:

$$1.21\ \mathrm{GeV}^2<W^2<9.0\ \mathrm{GeV}^2. \tag{45}$$

Since the ρ meson width is large, the lower limit is a matter of convention, but it can be less than $4m_\rho^2$. Notice, also, that the integrated function $F_{(+,+)}$ is independent of Q^2 up to logarithms. Besides, the exact W^2 dependence of this quantity remains unknown unless some modeling is used. However, the mean value theorem gives the possibility to reduce the three different integrals over W^2 in Eq. (44) to one integration. Indeed, the mean value theorem reads

$$\int_{W^2_{min}}^{W^2_{max}}dW^2\frac{\beta\, F_{(+,+)}(W^2)}{Q^2+W^2}=\frac{1}{Q^2+\langle W_1\rangle^2}\int_{W^2_{min}}^{W^2_{max}}dW^2\beta\, F_{(+,+)}(W^2), \tag{46}$$

$$\int_{W^2_{min}}^{W^2_{max}}dW^2\frac{\beta\, F_{(+,+)}(W^2)}{(Q^2+W^2)^2}=\frac{1}{(Q^2+\langle W_2\rangle^2)^2}\int_{W^2_{min}}^{W^2_{max}}dW^2\beta\, F_{(+,+)}(W^2) \tag{47}$$

with two phenomenological parameters $\langle W_1\rangle$ and $\langle W_2\rangle$. Let us now discuss the status of these parameters. In principle, each of these parameters is a function of Q^2, but owing to the unknown W^2 dependence of hadronic function $F_{(+,+)}$ the Q^2 dependences of given parameters stay out of the exact computations. Therefore we will deal with $\langle W_1\rangle$ and $\langle W_2\rangle$ which will be considered in the sense of average value on the whole interval of Q^2. Notice also that the values of our parameters have actually the same order of magnitude as Q^2, therefore we need to keep $\langle W\rangle$ parameters in the prefactors of Eqs. (46) and (47).

From Eqs. (46) and (47), one can see that it is useful to introduce a third phenomenogical parameter:

$$C_1=\int_{W^2_{min}}^{W^2_{max}}dW^2\beta F_{(+,+)}(W^2). \tag{48}$$

The normalization of our 2ρGDA's and consequently the value of our C_1 parameter [see Eq. (61)] is difficult to guess. In the $\pi\pi$ case (see, for instance [12]), the relation of the second moment of the operator defining the GDA to the energy momentum tensor, allowed to relate the value of the second moment of the GDA to the total energy carried by quarks in the π meson, which is known from the study of parton distributions in the meson. This analysis required a modest extrapolation of the meson-pair energy to zero, which was legitimate in the two π meson case with small W, but is very dangerous in the present two ρ meson case, due to the much larger threshold W_{min}. Still, one may expect, that some order of magnitude estimate may be provided by such an extrapolation, as will be confirmed by the numerical results below. Moreover, such an analysis may give indirect access to the parton distributions inside a ρ meson, and, in particular, to the spin and orbital angular momenta carried by quarks.

Finally, the cross section (44) is expressed through these three phenomenological parameters in the following simple way:

$$\frac{d\sigma_{ee\to ee\rho^0\rho^0}}{dQ^2}=\frac{25\alpha^4}{36\pi}C_1\int_0^1 dx_2F_{WW}(x_2)\times\left(\frac{1}{x_2^2S_{ee}^2Q^2}+\frac{2}{Q^2(Q^2+\langle W_2\rangle^2)^2}-\frac{2}{x_2S_{ee}Q^2(Q^2+\langle W_1\rangle^2)}\right). \tag{49}$$

In the L3 Collaboration analysis, the value W belongs to the interval (45). Hence we are able to conclude that the phenomenological parameters $\langle W_1\rangle$ and $\langle W_2\rangle$ may take any values inside this interval.

VII. COMPARISON OF ρ MESONS AND LEPTON PAIR PRODUCTIONS

In this section we compare the $\rho^0\rho^0$ production with the production of a lepton pair $\mu^+\mu^-$ in the same kinematics. The cross section of the process $e\gamma\to e\mu^+\mu^-$ coming through the $\gamma\gamma^*$ subprocess is well known and has the form:

$$\frac{d\sigma^{\gamma\gamma^*}_{e\gamma\to e\mu\mu}}{dQ^2}=\frac{\alpha^3}{4S^2_{e\gamma}\,Q^2}\int_{4m_\mu^2}^{W^2_{max}}dW^2\times\left(1-\frac{2S_{e\gamma}(Q^2+W^2-S_{e\gamma})}{(Q^2+W^2)^2}\right)f^{\mu^+\mu^-}(W^2), \tag{50}$$

where

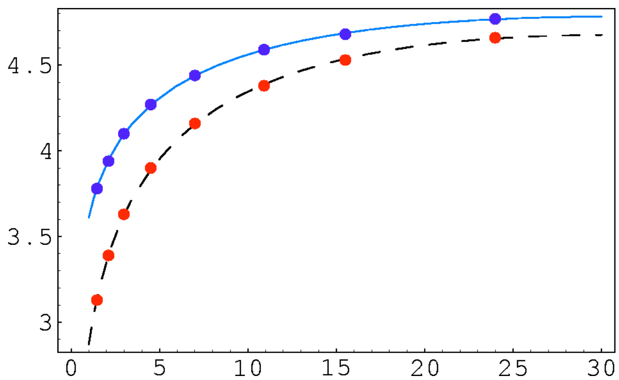

FIG. 2. The muonic parameters $\langle W_{1,(\mu)}\rangle^2$ (solid) and $\langle W_{2,(\mu)}\rangle^2$ (dashed) as functions of Q^2.

$$f^{\mu^+\mu^-}(W^2)=8\left(\ln\frac{1+\beta^{(\mu)}}{1-\beta^{(\mu)}}-\beta^{(\mu)}\right),$$

$$\beta^{(\mu)}=\sqrt{1-\frac{4m_\mu^2}{W^2}}. \tag{51}$$

In Eq. (50), we can also apply the mean value theorem. But now, by virtue of the known form of the muon function (51), the analogues of $\langle W_i\rangle$ can be explicitly calculated. So, the mean value theorem for the muon case reads

$$\int_{4m_\mu^2}^{W_{max}^2}dW^2\,\frac{f^{\mu^+\mu^-}(W^2)}{Q^2+W^2}=\frac{1}{Q^2+\langle W_{1,(\mu)}\rangle^2}\times\int_{4m_\mu^2}^{W_{max}^2}dW^2 f^{\mu^+\mu^-}(W^2), \tag{52}$$

$$\int_{4m_\mu^2}^{W_{max}^2}dW^2\,\frac{f^{\mu^+\mu^-}(W^2)}{(Q^2+W^2)^2}=\frac{1}{(Q^2+\langle W_{2,(\mu)}\rangle^2)^2}\times\int_{4m_\mu^2}^{W_{max}^2}dW^2 f^{\mu^+\mu^-}(W^2). \tag{53}$$

Whence, we obtain

$$\langle W_{1,(\mu)}\rangle^2(Q^2)=R_1(Q^2)-Q^2,$$

$$(Q^2+\langle W_{2,(\mu)}\rangle^2)^2=R_2(Q^2), \tag{54}$$

where the following notations are introduced:

$$R_n(Q^2)=\frac{\mathcal{K}}{\mathcal{L}_n(Q^2)},\quad \mathcal{K}=\int_{4m_\mu^2}^{W_{max}^2}dW^2 f^{\mu^+\mu^-}(W^2),$$

$$\mathcal{L}_n(Q^2)=\int_{4m_\mu^2}^{W_{max}^2}dW^2\,\frac{f^{\mu^+\mu^-}(W^2)}{(Q^2+W^2)^n}. \tag{55}$$

The Q^2 dependences of $\langle W_{i,(\mu)}\rangle^2$ are shown in Fig. 2. The solid line corresponds to the function $\langle W_{1,(\mu)}\rangle^2(Q^2)$ and the dashed one to the function $\langle W_{2,(\mu)}\rangle^2(Q^2)$. The weak Q^2 dependence of $\langle W_{i,(\mu)}\rangle^2$ justifies the possibility to use the averaged $\langle W_{i,(\mu)}\rangle^2$ when fitting the data. Besides, a fitting procedure gives us the following representation for these functions:

$$\langle W_{1,(\mu)}\rangle^2(Q^2)=3.63-0.015Q^2+0.47\ln Q^2,$$

$$\langle W_{2,(\mu)}\rangle^2(Q^2)=2.90-0.024Q^2+0.73\ln Q^2.$$

Further, the cross section of two ρ meson production can be written as

$$\frac{d\sigma_{ee\to ee\rho^0\rho^0}}{dQ^2}=\int_0^1 dx_2F_{WW}(x_2)\frac{\alpha^4}{4\pi}\frac{C_1}{Q^2S_{e\gamma}^2}\mathcal{N}(x_2)\times\int_{4m_\mu^2}^{W_{max}^2}dW^2\left(1-\frac{2S_{e\gamma}(Q^2+W^2-S_{e\gamma})}{(Q^2+W^2)^2}\right)\times f^{\mu^+\mu^-}(W^2), \tag{56}$$

where the function $\mathcal{N}(x_2)$ is defined by the ratio [one reminds here that $S_{e\gamma}$ is proportional to the x_2 fraction, see Eq. (5)]:

$$\mathcal{N}(x_2)=\frac{\mathcal{I}_1(x_2)}{\mathcal{I}_2(x_2)},$$

$$\mathcal{I}_1(x_2)=\frac{25}{9}\left(1-\frac{2S_{e\gamma}}{Q^2+\langle W_1\rangle^2}+\frac{2S_{e\gamma}^2}{(Q^2+\langle W_2\rangle^2)^2}\right),$$

$$\mathcal{I}_2(x_2)=\left(1-\frac{2S_{e\gamma}}{Q^2+\langle W_{1,(\mu)}\rangle^2}+\frac{2S_{e\gamma}^2}{(Q^2+\langle W_{2,(\mu)}\rangle^2)^2}\right)\mathcal{K}. \tag{57}$$

If one considers the case where Q^2 is large with respect to the invariant mass squared W^2, we can omit the terms of $O(1/Q^2)$ and $O(\ln Q^2/Q^4)$ in Eq. (57) and obtain that $\mathcal{N}$ becomes independent of x_2:

$$\mathcal{N}\approx\frac{25}{9\ \mathcal{K}}=0.01\ \text{GeV}^{-2}. \tag{58}$$

We stress that this value has been obtained providing the value of upper limit W_{max}^2 is fixed [see Eq. (45)]. Actually, the value $\mathcal{N}$ is a function of W_{max}^2, owing to the W_{max}^2-dependence of the integral $\mathcal{K}$ [see Eq. (55)], and this dependence is pretty strong. For example, enhancing W_{max}^2 up to 16.0 GeV^2 the value $\mathcal{N}$ will be halved.

Thus the cross section of our process is related to the cross section of lepton pair production as

$$\frac{d\sigma_{ee\to ee\rho^0\rho^0}}{dQ^2}\sim 0.01C_1\frac{d\sigma_{ee\to e\mu\mu}^{\gamma\gamma^*}}{dQ^2}. \tag{59}$$

In the next section we will show that C_1 is close to 1.0 GeV^2. Hence, one can see that the cross section of two ρ

meson production is suppressed compared to the cross section of two muon production by a factor which is approximately equal to 100. Note that a factor of suppression of the same order was present in the case of two π mesons production [12].

VIII. COMPARISION WITH EXPERIMENTAL DATA

In the previous section we obtained a simple expression for the two mesons cross section as a function of the three parameters $\langle W_1 \rangle$, $\langle W_2 \rangle$ and C_1. Let us now make a fit of these phenomenological parameters in order to get the best description of experimental data. The best values of the parameters can be found by the method of least squares, χ^2 method, which flows from the maximum likelihood theorem (see, for instance, [17]). As usual, the χ^2 sum as a function of parameters is written in the form

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{\sigma_i^{exp} - \sigma_i^{th}(\mathbf{P})}{\delta\sigma_i} \right)^2, \tag{60}$$

where $\mathbf{P} = \{\langle W_1 \rangle, \langle W_2 \rangle, C_1\}$ denotes the set of fitted parameters; σ_i^{exp} and σ_i^{th} are the experimental measurements of the cross section and its theoretical estimations; $\delta\sigma_i$ are the statistical errors. The experimental data for the cross section of the exclusive double ρ^0 production were taken from the measurement of the L3 Collaboration at LEP [9]. Minimizing χ^2 sum in Eq. (60) with respect to the parameters $\mathbf{P}$ we find that the set of solutions $\mathbf{P}_{min}$ with their confidence intervals are the following:

$$C_1 = 1.20 \pm 0.23 \text{ GeV}^2, \quad \langle W_1 \rangle = 3.0 \pm 1.9 \text{ GeV},$$
$$\langle W_2 \rangle = 1.50 \pm 0.09 \text{ GeV}. \tag{61}$$

With this the magnitude of χ^2 is equal to 1.40 and, therefore, we have

$$\frac{\chi^2}{\text{degree of freedom}} = 0.28 < 1. \tag{62}$$

The confidence intervals were defined for the case of one-standard deviation. Figure 3 shows the experimental data with the theoretical fit.

We can see from Eq. (61) that the confidence interval for the parameter $\langle W_1 \rangle$ covers the whole available interval for W. Moreover the obtained values of $\langle W_2 \rangle$ is quite small. This analysis shows the compatibility of the data with the leading twist analysis which we developed. At the same time, one cannot exclude the existence of sizeable higher twist contributions to the production amplitude. More theoretical and experimental studies are required to make a definite conclusion on higher twist contributions.

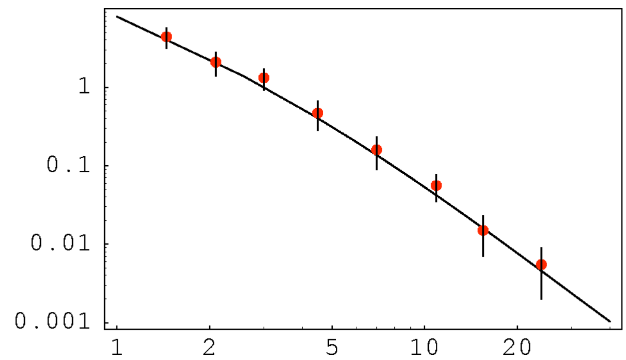

FIG. 3. Cross-section $d\sigma_{ee \to ee\rho^0\rho^0}/dQ^2$[pb/GeV^2] as a function of Q^2. The theoretical cross section is plotted for the best fitted parameters which are $C_1 = 1.2$ GeV^2, $\langle W_1 \rangle = 3.0$ GeV and $\langle W_2 \rangle = 1.5$ GeV.

IX. CONCLUSION

Data thus are in full agreement with the theoretical expectation on the Q^2 behavior of the cross section. Since the effective structure function $F_{(+,+)}$ is expected to be independent of Q^2 up to logarithms, our analysis of the data is a strong indication of the relevance of the partonic description of the process $\gamma\gamma^* \to \rho^0\rho^0$ in the kinematics of the L3 experiment.

Much more can be done if detailed experimental results are collected. For instance the angular dependence of the final state is a good test of the validity of the asymptotic form of the generalized distribution amplitudes. The spin structure of the final state, if elucidated, would allow to disentangle the roles of the nine generalized distribution amplitudes. The W^2 behavior of the cross section may have some interesting features. It depends much on the possible resonances which are able to couple to two ρ mesons. The $\rho^+\rho^-$ channel may be calculated along the same lines. In that case a bremsstrahlung subprocess where the mesons are radiated from the lepton line must be added. The charge asymmetry then comes from the interference of the two processes. These items will be discussed in a forthcoming publication. More data may be collected at other energies, in particular, in e^+e^- experiments around 10 GeV such as BABAR in SLAC and BELLE in KEK.

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