Neutrino production of resonances

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We take a fresh look at the analysis of resonance production by neutrinos. We consider three resonances P_{33}, P_{11} and S_{11} with a detailed discussion of their form factors. The paper presents results for free proton and neutron targets and discusses the corrections which appear on nuclear targets. The Pauli suppression factor is derived in the Fermi gas model and shown to apply to resonance production. The importance of the various resonances is demonstrated with numerical calculations. The Δ resonance is described by two form factors and its differential cross sections are compared with experimental data. The article is self-contained and could be helpful to readers who wish to reproduce and use these cross sections.

I. INTRODUCTION

The excitation of the resonances by electrons and neutrinos has been studied for a long time. The earlier articles [1–5] tried to determine the $p\Delta$ transition form factors in terms of basic principles, such as conserved vector current (CVC), partially conserved axial-vector currents (PCAC), dispersion relations, etc. These and subsequent papers introduced dipole form factors and in various cases other functional forms with additional kinematic factors in order to reproduce the data. The result was the presentation of cross sections (differential and integrated) in terms of several parameters [6–9]. The relatively large number of parameters and the limited statistics of the experiments provided qualitative comparisons but an accurate determination of the terms is still missing. A new generation of experiments is now under construction aiming to measure properties of the neutrino oscillations and they present the opportunity for precise tests of the standard model.

For these reasons we decided to improve the calculation of the excitation of resonances with isospin $I=3/2$ and $I=1/2$ looking into the various terms that enter the calculations and trying to determine them, as accurately as possible. For reasons of economy we shall study four resonances. We shall give explicit formulas and form factors for $P_{33}(1232)$, $P_{11}(1440)$, and $S_{11}(1535)$ and use a functional form for $D_{13}(1520)$ obtained in electroproduction [11]. We calculate the cross section for the production of each resonance alone (Fig. 3) and found that $P_{33}(1232)$ dominates. Next we shall study the vector current in electroproduction data and then use it for neutrino reactions. For the axial-vector current we shall discuss the constraints introduced by PCAC for several form factors. Then we shall calculate the differential cross section in Q^2 and also in W (invariant mass of the final pion-nucleon system). We shall restrict the numerical analysis to the $p\pi^+$ channel and to $W \lesssim 1.6$ GeV where there are accurate data and in order to avoid the influence of higher resonances. For example the $P_{33}(1640)$ resonance is expected to provide a small contribution, at low energies, because it is further away and has a small elasticity [8]. For our kinematic region the higher resonances with $I=1/2$ provide a smooth background to the $p\pi^0$ and $n\pi^+$ channels. Their effects will show up in the W distributions and the overall scale of the integrated cross sections. This approach was demonstrated [10] to agree with the available data. In this paper we extend the calculations giving more details which will be helpful for future comparisons. Cross sections will be presented primarily for free protons and neutrons. In the future we will extend this study by including higher resonances contributing to other channels. It will be interesting to include the inelasticity of higher resonance in order to estimate their contribution to the multi-pion events.

Neutrino experiments, however, use medium-heavy and heavy nuclei which brings additional corrections. Several studies so far identified nuclear effects from (i) the Pauli exclusion principle, (ii) the Fermi motion, and (iii) the absorption and charge exchange of the produced pions in nuclei. In this paper we discuss in greater detail the Pauli effect and show that it brings a modification to the data which should be identified and checked in the experiments. It may be important in producing the Q^2 distribution.

We shall adopt a notation similar to the one in deep inelastic scattering

$$\nu(k)+p(p)\rightarrow\mu^-(k')+\Delta^{++}(p')\hookrightarrow p(p'')+\pi(p_\pi). \tag{1.1}$$

The momentum transfer will be denoted by $q=k-k'$, which is space-like with

$$Q^2=-q^2=-m_\mu^2+4E(E'-[E'^2-m_\mu^2]^{1/2}\cos\theta). \tag{1.2}$$

The energy of the current in the laboratory and in the rest frame of the resonance, to be denoted by CM, is given by

$$q_L^0=\nu=\frac{q\cdot p}{M}=\frac{W^2+Q^2-M^2}{2M}$$

$$q_{CM}^0=\frac{W^2-Q^2-M^2}{2W}$$

$$q_{CM} = [(q_{CM}^0)^2 + Q^2]^{1/2}. \quad (1.3)$$

Finally, we give the energy of the pion in the rest frame of the resonance

$$p_\pi^0(W) = \frac{W^2 + m_\pi^2 - M^2}{2W}. \quad (1.4)$$

We shall try to give explicit formulas for the cross sections so that the interested reader will be able to reproduce the results.

The paper is planned as follows. In Sec. II we present the general formalism for the production of the Δ resonance emphasizing the minimal input, which is necessary. In Sec. III we give explicit formulas for the other two resonances P_{11} and S_{11} where we find the contribution to be small. Since nuclear corrections were calculated a long time ago we decided to present an explicit calculation of the Pauli suppression factor in order to examine its validity in various processes. We emphasize in Sec. IV how the Pauli factor should be applied to quasi-elastic scattering, as well as in the production of resonances. In Sec. V we apply the formalism to the production of Δ^{++} and rely on the connection with electroproduction data. Finally, in Sec. VI is presented a summary of the results.

II. CROSS SECTIONS AND THEIR FORM FACTORS

Photoproduction and electroproduction data in the Δ region are well reproduced by a single form factor plus a smooth background representing the tails of higher resonances [11]. We shall use these results by adopting a one vector form factor, i.e. the magnetic dipole term.

For neutrino-induced reactions, on the other hand, there are many models, some of which introduce many resonances. This way one accounts for the data at the cost of introducing many parameters. Other models restrict the analysis in the Δ region, $W \lesssim 1.4$ GeV, where the Δ resonance dominates and the main issue is the selection of the axial-vector form factors and their parametrization. We shall use form factors which satisfy PCAC and have a modified dipole Q^2 dependence. This approach provides a relatively simple formalism for single pion production and was shown recently to reproduce the existing data [10]. We shall adopt this formalism and use it to calculate differential and total cross sections comparing it with data in order to determine overall validity or need for modifications and extensions. In this paper the formulas for the excitation of the Δ resonance are simpler than in Ref. [5] because we take into account special properties of the form factors, which have been accumulated in the meanwhile. We give explicit formulas for the $I = 1/2$ resonances and show that in the energy region of the experiments their contribution is small.

For the matrix element of the vector current we use the general form

$$\langle \Delta^{++} | V_\mu | p \rangle = [\bar\psi_\mu A_\lambda q^\lambda - \bar\psi_\lambda q^\lambda A_\mu$$
$$+ C_6^V(q^2)\bar\psi_\mu]\gamma_5 u(p)f(W) \quad (2.1)$$

with $A_\lambda = (C_3^V/M)\gamma_\lambda + (C_4^V/M^2)p'_\lambda + (C_5^V/M^2)p_\lambda$, M the proton mass and C_i^V, $i = 3, \ldots, 6$ the vector form factor, ψ_μ is the Rarita-Schwinger wave function of the Δ resonance and $f(W)$ is the s-wave Breit-Wigner resonance, given explicitly in Eq. (2.5). The conservation of the vector current (CVC) gives $C_6^V(q^2) = 0$ and the other form factors are determined from electroproduction experiments, where the magnetic form factor dominates. This dominance leads to the conditions

$$C_4^V(q^2) = -\frac{M}{W}C_3^V(q^2) \quad \text{and} \quad C_5^V(q^2) = 0. \quad (2.2)$$

With these conditions the electroproduction data depend only on one vector form factor $C_3^V(q^2)$. Precise electroproduction data determined the form factor, which can be parametrized in various forms.

Early articles describe the static theory [12] and the quark model [13] predicting the form factor for the $\gamma N \Delta$ vertex to be proportional to the isovector part of the nucleon form factors. Subsequent data [11] showed that the form factor for Δ electroproduction falls faster with increasing Q^2 than the nucleon form factor which motivated some authors to introduce other parametrizations, including exponentials [14] and modified dipoles [15]. The functional form

$$C_3^V(Q^2) = \frac{C_3^V(0)}{\left[1 + \dfrac{Q^2}{M_V^2}\right]^2} \left(\frac{1}{1 + \dfrac{Q^2}{4M_V^2}}\right) \quad (2.3)$$

gives an accurate representation. In this paper we adopt this vector form factor and use CVC to determine its contribution to the neutrino-induced reactions. Details of the vector and axial-vector contributions are presented in Sec. V, where we shall estimate the contribution of C_3^V from the electroproduction data.

The matrix element of the axial-vector current has a similar parametrization

$$\langle \Delta^{++} | A_\mu | p \rangle = [\bar\psi_\mu B_\lambda q^\lambda - \bar\psi_\lambda q^\lambda B_\mu + \bar\psi_\mu C_5^A$$
$$+ \bar\psi_\lambda q^\lambda q_\mu C_6^A]u(p)f(W) \quad (2.4)$$

with $B_\lambda = (C_3^A/M)\gamma_\lambda + (C_4^A/M^2)p'_\lambda$ and $\Gamma(W) = \Gamma_0[q_\pi(W)/q_\pi(W_R)]$ and

$$f(W) = \frac{\sqrt{\Gamma(W)/2\pi}}{(W_R - W) - \frac{1}{2}i\Gamma(W)} \quad (2.5)$$

and $\Gamma_0 = 120$ MeV.

The PCAC condition gives the relation $C_5^A(q^2) = -(C_6^A/M^2)q^2$ which for small $q^2 = 0$ leads to the numerical value $C_5^A(0) = 1.2$ [5]. The contribution of the form factor C_6^A to the cross section is proportional to the lepton mass and will be ignored.

The Q^2 dependence of the form factors varies among the publications giving different cross sections and different Q^2

distributions even when the same M_A is used. For this dependence we shall use a modified dipole form

$$C_5^A(Q^2) = \frac{1.2}{\left[1 + \dfrac{Q^2}{M_A^2}\right]^2} \left(\frac{1}{1 + \dfrac{Q^2}{3M_A^2}}\right). \qquad (2.6)$$

The proton has a charge distribution reflected in the form factor. To build the resonance we must add a pion to the proton which creates a bound state with a larger physical extent. If the overlap of the wave functions has a larger mean-square radius then the form factor will have a steeper Q^2 dependence as is indicated by the electromagnetic form factor for the excitation of Δ [11]. Since the effect is geometrical we expect a similar behavior for the vector and axial-vector form factors. For this reason we replace another factor used in previous articles [5] by the modified dipole in Eq. (2.6) with $3M_A^2 \sim 4M_V^2$. For the other two form factors $C_3^A(Q^2)$ and $C_4^A(Q^2)$ we shall use $C_3^A = 0$ and $C_4^A(Q^2) = -\frac{1}{4}C_5^A$ [5]. It is evident that there is still arbitrariness in the form factors with C_3^A and C_4^A being small. We show in Fig. 4 the relative contributions of the various terms, where it is clear that the contributions of C_4^A and C_4^V are indeed very small.

The differential cross section is finally given by

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{16\pi M^2} \left(\sum_{i=1}^{3} K_i W_i\right) g(Q^2, W) \qquad (2.7)$$

with the kinematic factors K_i and the structure functions $W_i(Q^2, W)$ defined in Ref. [5]. For our simplified case we collected the relevant formulas and kinematics in the Appendix, so that the paper is self-contained. Most of the neutrino data give the integrated cross section as function of the neutrino energy. There are recent compilations of data [10,16] which have been compared with a theoretical calculation [10]. Differential cross sections $d\sigma/dQ^2$ or $d\sigma/dQ^2 dW$ were also reported by several experiments. In a high statistics experiment at Brookhaven Laboratory, neutrino-deuterium interactions [17] were measured in the bubble chamber. Their data are precise and lead to the differential cross section shown in Fig. 6. The cross section falls off with increasing Q^2 and there is a dip at $Q^2 \lesssim 0.2$ GeV. We shall investigate the distributions which are available in the simplified model described above.

III. HIGHER RESONANCES

In addition to the Δ contribution there are higher mass resonances $P_{11}(1440)$ and $S_{11}(1535)$ which contribute to the background under Δ and, of course, at higher invariant masses. It is interesting to estimate their contribution as a function of neutrino energy, momentum-transfer-squared and invariant mass of the final state. These are new resonances whose form factors are not known precisely. We know accurately masses and widths of the resonances which are important for the calculations.

For the P_{11} resonance we introduce the amplitude

$$\mathcal{M}_P = \frac{G_F}{\sqrt{2}} \cos\theta_c l^\mu \bar{u}(p') \gamma_5 (\not{p} + \not{q} + M_R)$$

$$\times \gamma_\mu (g_V - g_A \gamma_5) u(p) g_P f_P(W), \qquad (3.1)$$

where g_P denotes the coupling at the pion-nucleon resonance vertex and the Breit-Wigner factor

$$f_R(w) = \frac{1}{(W^2 - M_R^2) + i M_R \Gamma(W)} \qquad (3.2)$$

with $\Gamma_R(W) = \Gamma_R^0 (q_\pi(W)/q_\pi(M_R))^3$ and $q_\pi(W) = (1/2W)[(W^2 - M^2 - m_\pi^2)^2 - 4M^2 m_\pi^2]^{1/2}$ in the pion momentum in the rest frame of the resonance and l^μ is the leptonic tensor. The other form factors g_V and g_A are defined below.

Similarly, the amplitude for the production of S_{11} is

$$\mathcal{M}_S = \frac{G_F}{\sqrt{2}} \cos\theta_c l^\mu \bar{u}(p') (\not{p} + \not{q} + M_R)$$

$$\times \gamma_\mu (g_V' - g_A' \gamma_5) u(p) g_S f_S(W), \qquad (3.3)$$

the resonance factor $f_S(W)$ and the form factors g_V' and g_A' being now appropriate for the S_{11} resonance. The functional form of the width is the same as for the Δ resonance: $\Gamma_S(W) = \Gamma_S^0 (q_\pi(W)/q_\pi(M_S))$.

The calculation of the new resonances requires knowledge of the form factors which are not known accurately. Usually, the form factors are obtained from model calculations.

Whenever we use $I = 1/2$ resonances, we use the form factors [6]

$$g_P^V(Q^2) = 0, \qquad (3.4)$$

$$g_S^V(Q^2) = -\frac{Q^2 g^{2V}(Q^2)}{M_N(M_N - M_R)} \quad \text{with} \qquad (3.5)$$

$$g^{2V}(Q^2) = \frac{g^V(0)}{\left(1 + \dfrac{Q^2}{4.3 \text{ GeV}^2}\right)^2} \frac{1}{1 + \dfrac{Q^2}{(M_R - M_N)^2}}, \qquad (3.6)$$

$$g_R^A(Q^2) = \frac{g_R^A(0)}{\left(1 + \dfrac{Q^2}{M_A}\right)^2}. \qquad (3.7)$$

The differential cross section has the general form

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{16\pi^3} \frac{q_\pi}{(k \cdot p)^2} V_R \frac{1}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2} \cdot g_R^2 \qquad (3.8)$$

where the function V_R is given by

$$V_R = \cos^2\theta_c \left(-(g_V^2 - g_A^2) M_N [2M_R p' \cdot \tilde{k} \mp M_N (W^2 + M_R^2)] \frac{Q^2}{2} \right.$$

$$+ (g_V^2 + g_A^2) \left\{ (p' \cdot \tilde{k} \mp M_N M_R) M_N [Q^2(E_f - E_i) + 4M_N E_i E_f] + (M_R^2 - W^2) \left[M_N \left(\frac{Q^2}{2} (E_f - E_i) + 2E_i E_f (M_N - E_\pi) \right) \right] \right\}$$

$$+ g_V g_A \left\{ 2(p' \cdot \tilde{k} \mp M_N M_R) \left(\frac{1}{4} Q^2 (Q^2 - 2M_N E_f) \right) + (M_R^2 - W^2) \left(\frac{1}{4} Q^2 (Q^2 - 2M_N E_f + 2E_i E_\pi) \right) \right\} \right) \tag{3.9}$$

with $p' \cdot \tilde{k} = W^2 - WE_\pi$ and the upper and lower sign corresponding to the P and S resonances, respectively. The subscript $R = P, S$ signifies the P_{11} and S_{11} resonances. We shall use these cross sections for calculating effects of higher resonances. We obtain an overview of their importance by looking at the relevant contributions to the W and Q^2 distributions. The results are shown for $E_\nu = 1.5$ GeV in Figs. 3(a) and 3(b). The contribution of the S_{11} resonance is very small everywhere. The P_{11} resonance gives a small contribution for $W > 1.3$ GeV and may influence the Q^2 contribution. The presence of a background at $W > 1.3$ GeV shows up in the electroproduction data as well as indicated also in Fig. 5. In fact in a previous analysis of the electroproduction of the Δ resonance [11] a polynomial dependence in W was introduced to represent a background contribution. The form factors and the other quantities, which we use, are summarized in Table I.

IV. THE PAULI EFFECT

Among the nuclear effects we shall describe in detail the Pauli suppression factor. We shall assume the Fermi gas model with the nucleons enclosed in a sphere with maximal momentum p_F. The Pauli exclusion principle requires the final wave functions to be anti-symmetric in the exchange of two identical particles. We assume the final wave functions to be plane waves of the form

$$\psi(r_1, r_2) = e^{i(\vec{k_1}\vec{r_1} + \vec{k_2}\vec{r_2})} - e^{i(\vec{k_1}\vec{r_2} + \vec{k_2}\vec{r_1})}. \tag{4.1}$$

The incoming current also brings a momentum $\vec{q}$ so that the relevant matrix element is

$$\mathcal{M} = \int \psi^*(r_1, r_2) e^{-iq \cdot r_1} \varphi(r_1) \varphi(r_2) d^3 r_1, d^3 r_2 \tag{4.2}$$

TABLE I. Parameters.

	M_R (GeV)	g_R	Γ_R^0 (GeV)	$g_R^V(0)$	$g_R^A(0)$
P_{11}	1.440	4.45	0.35	0.0	0.35
S_{11}	1.535	0.48	0.15	−0.28 (p)	0.16
				0.14 (n)	

where $\varphi(r_1)$ and $\varphi(r_2)$ are the wave functions of two bound nucleons. We can regroup the terms and write the matrix element as

$$\mathcal{M} = F(k_1 + q) F(k_2) - F(k_2 + q) F(k1) \tag{4.3}$$

with $F(k_1 + q) = \int e^{-i(\vec{k_1} + \vec{q}) \cdot \vec{r}} \varphi(r) dr$ and $F(k_1) = \int e^{-i\vec{k_1} \cdot \vec{r}} \varphi(r) dr$. It is evident that the matrix element vanishes for $\vec{q} = 0$, which we shall use as a condition later on. The problem now is to carry out the integrals and express the suppression factor in terms of $b = |\vec{q}|/p_F$, the ratio of momentum transfer to the Fermi momentum.

To obtain the cross section we square the amplitude and integrate over final momenta

$$\Sigma = \int |\mathcal{M}|^2 d^3 k_1 d^3 k_2$$

$$= \int |F(k_1 + q)|^2 d^3 k_1 \int |F(k_2)|^2 d^3 k_2$$

$$+ \int |F(k_1)|^2 d^3 k_1 \int |F(k_2 + q)|^2 d^3 k_2$$

$$- 2 \operatorname{Re} \int F^*(k_1 + q) F(k_1) d^3 k_1$$

$$\times \int F^*(k_2 + q) F(k_2) d^3 k_2. \tag{4.4}$$

The terms $\int |F(k_2)|^2 d^3 k_2 = 1$ involve the spectator nucleon and will be normalized to one. Similarly the integral $\int^\infty |F_1(k + q)|^2 d^3 k = 1$ provided that the integral falls very fast with $|\vec{k} + \vec{q}|^2 \to \infty$. This holds for the atomic form factors which are Gaussian or fall off exponentially for large momentum transfers.

The third integral in Eq. (4.4) is called the exchange term and it is convenient to change the order of integration. Performing the momentum integral first

$$I_0(r_1, r_2) = \int_{k_1=0}^{p_f} e^{i\vec{k_1} \cdot \vec{r_{21}}} d^3 k_1$$

$$= 4\pi \frac{p_f r_{21} \cos p_F r_{21} - \sin p_F r_{21}}{r_{21}^3} \tag{4.5}$$

with $r_{21} = |\vec{r_2} - \vec{r_1}|$ and p_f the momentum of the Fermi sea. The remaining integrations over r_1 and r_2 are organized so that one of them is over $(\vec{r_1} + \vec{r_2})$ giving a volume term and the other is over $r_{21} = (\vec{r_2} - \vec{r_1})$. The final result is

$$\Sigma = |\mathcal{M}|^2 d^3 k_1 d^3 k_2$$

$$= 2 - 2V(2\pi)^4 \int \cos(\vec{q} \cdot \vec{r_{21}})$$

$$\times \frac{(p_F r_{21} \cos p_F r_{21} - \sin p_F r_{21})^2}{r_{21}^6} d^3 r_{21} \quad (4.6)$$

with the last integral still over the 3-dimensional $\vec{r_{21}}$ space. The volume V comes from the integration over $(\vec{r_1} + \vec{r_2})$ and is at this stage unspecified. Defining $b = q/p_F$ and $z = p_F r_{21}$ the last integral attains the form

$$\int_0^\infty \frac{\sin bz (\sin z - z \cos z)^2}{z^5} dz = \frac{\pi}{24} \left[\frac{1}{4} b^4 - 3b^2 + 4b \right] \quad (4.7)$$

for $0 < b < 2$ and is equal to zero for $b = 2$. The evaluation of this integral is not immediately evident and can be calculated with computer programs (MATHEMATICA) or with the help of the residue theorem as described by Gatto [18].

As we collect the various terms together, we must still deal with the volume V appearing in Eq. (4.6). We combine the volume together with other multiplicative constants in a new constant K, so that the cross section attains the form

$$\Sigma = 2 - 2K p_F \frac{1}{b}(b^4 - 3b^2 + 4b) \quad (4.8)$$

$$= (2 - 8K p_F) + 2K p_F(3b - b^3). \quad (4.9)$$

Now we impose the condition that Σ vanishes for $b = 0$. This gives

$$2K p_F = \frac{1}{2} \quad \text{and} \quad \Sigma = \frac{3}{2}b - \frac{1}{2}b^3. \quad (4.10)$$

Σ represents the fraction of the nucleons which can contribute for a given momentum transfer q. It has a geometrical interpretation frequently used in articles: when a momentum q is transferred to a nucleon, the center of the Fermi sea is displaced by this momentum. The fraction of the nucleons contributing to a cross section is the fraction of the displaced sphere which lies outside the original Fermi surface. The allowed region is the shaded volume shown in Fig. 1.

The above derivation of the Pauli factor depends on the approximation of treating the nucleus as a collection of independent protons and neutrons. The suppression factor depends on the ratio of the momentum transfer to the Fermi momentum and has a simple geometric interpretation. It does not depend on the specific process and should hold for elastic and resonance production, provided we calculate the ratio $b = q/p_F$ appropriate for each process.
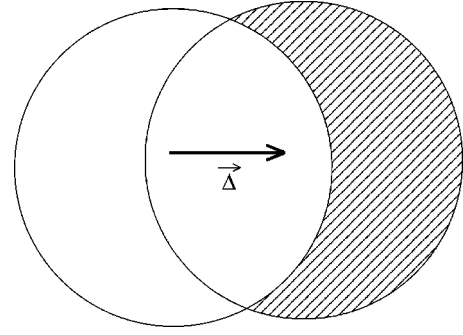

FIG. 1. Geometrical description of the Pauli suppression factor.

For quasielastic scattering the Pauli factor has been used in several articles and it was found to be necessary. One assumes that there is one Fermi surface for protons and neutrons for which the factor Σ from (4.10) applies. Some authors assumed that there are distinct Fermi surfaces for neutrons and protons and obtained two expressions [19,20]. A detailed discussion for elastic scattering is given by Berman [19] and is also in the review article of Llewellyn-Smith [4].

For the production of the Δ resonance a similar correction applies. After the scattering of the current on a neutron the Δ resonance is produced which travels and decays within the nucleus. For most of the kinematic region the Δ is non-relativistic and it takes 5 to 10 lifetimes to travel through the nucleus. When it decays it transfers momentum $|\vec{q}|$ to the proton which is still bound. For the Δ resonance special attention is required since it propagates in a medium. However, once it decays we have a proton which seeks to find an empty state. In the independent particle of the Fermi model the unoccupied levels are above the Fermi surface. The kinematics were considered analytically and the blocking was given explicitly in Ref. [21]. For completeness we transcribe the formulas here introducing the notation familiar from deep inelastic scattering. We define in the rest frame of the Δ resonance

$$p_\pi = \frac{W^2 + m_\pi^2 - M^2}{2W}, \quad q_0 = \frac{W^2 - M^2 - Q^2}{2W} \quad \text{and} \quad R = p_F.$$

(i) For $2 p_F \geq |\vec{q}| + p_\pi > |\vec{q}| - p_\pi$

$$g(W, |\vec{q}|) = \frac{1}{2|\vec{q}|} \left(\frac{3|\vec{q}|^2 + p_\pi^2}{2R} - \frac{5|\vec{q}|^4 + p_\pi^4 + 10|\vec{q}|^2 q_\pi}{40R^3} \right). \quad (4.11)$$

(ii) For $|\vec{q}| + p_\pi > 2 p_F$

$$g(W, |\vec{q}|) = \frac{1}{4 p_\pi |\vec{q}|} \left((|\vec{q}| - p_\pi)^2 - \frac{4}{5}R^2 \right.$$

$$\left. - \frac{(|\vec{q}| - p_\pi)^3}{2R} + \frac{(|\vec{q}| - p_\pi)^5}{40R^3} \right). \quad (4.12)$$
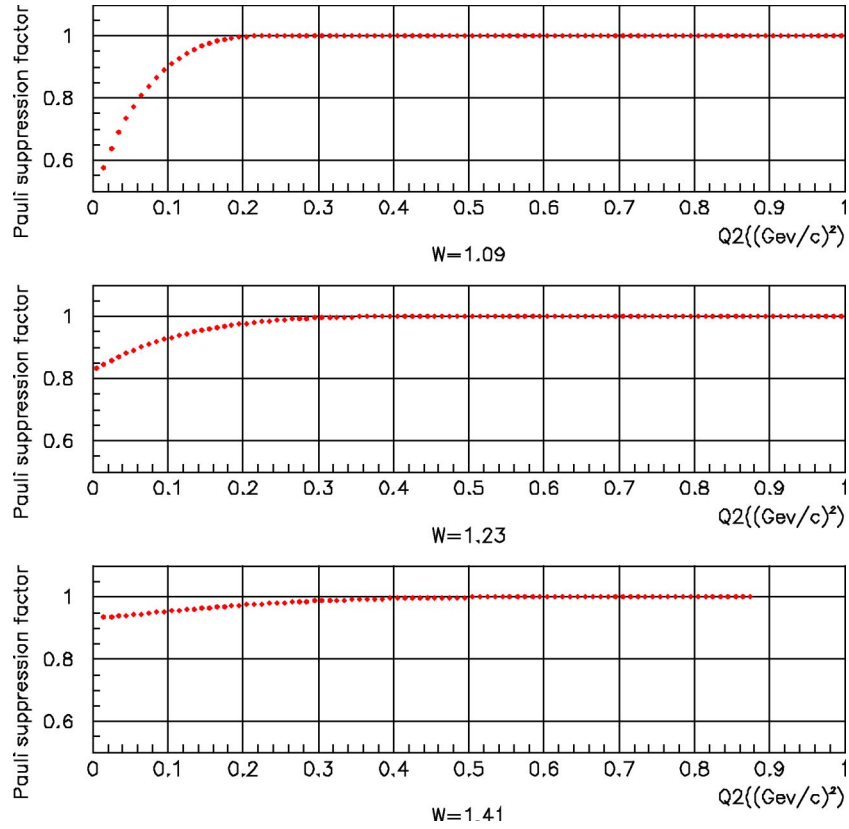
(iii) For $|\vec{q}| - p_\pi \geq 2 p_F$

FIG. 2. The Pauli suppression factor as a function of Q^2 and for various values of W, for the case of $P_F = 0.226$ GeV/c.

$$g(W, |\vec{q}|) = 1. \tag{4.13}$$

The complicated formulas result from integrations over the angles. For most of the phase space the first term in the bracket is dominant. We plot in Fig. 2 the Pauli suppression factor as a function of Q^2 and for the various values of W. The suppression appears for $Q^2 \lesssim 0.2$ GeV^2.

In the Monte Carlo method, the Pauli exclusion effect is taken into account by requiring the recoiling nucleon momentum to be greater than p_F. They obtain similar results [22].

The effects of Fermi motion are easily included. Since the cross sections in Eqs. (2.7) and (3.8) are written in a Lorentz invariant way, they are valid in any frame. In the laboratory frame we give the proton a small momentum within its Fermi sea

$$p_\mu = (\sqrt{p^2 + M_N^2}, p_x, p_y, p_z) \tag{4.14}$$

and write the inner products $k \cdot p$, $k' \cdot p$ accordingly. Then one integrates numerically for all momenta inside the Fermi surface $|\vec{p}| < p_F$.

V. NUMERICAL RESULTS

With the formalism described in Secs. II, III and the Appendix we can study the contributions of the various terms to the cross sections. We decided to include the resonances $\Delta(1232)$, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$. The last two resonances are narrow and further way so that we expect

a smaller contribution at lower energies. For typical values of the parameters given in this article and using for $D_{13}(1520)$ couplings close to those of the Δ resonance, we computed the invariant mass and Q^2 contributions for two energies $E_\nu = 1.5$ and 2.0 GeV and found the results shown in Fig. 3. The curves show the cross section for P_{33} going to $I = 3/2$, and S_{11}, P_{11} separately going to the state with $I = 1/2$. The same situation prevails for $E_\nu = 2.0$ GeV. We can interpret this result as indicating the fact that the P_{11} is closest to the Δ resonance and has a larger width. For the relative low energies of the neutrino beams $E_\nu < 2.0$ GeV and $W < 1.4$ GeV the dominant contribution comes from the Δ resonance with an $I = 1/2$ background from the other resonances and perhaps part of a continuum. The $I = 1/2$ terms contribute only to the $p\pi^0$ and $n\pi^+$ final states.

For the next issue we consider the contribution of the various form factors. We show in Fig. 4 the relative importance of the various form factors, where C_3^V and C_5^A dominate the cross section. The cross section from the axial-vector form factors has a peak at $Q^2 = 0$, while the cross section from C_3^V turns to zero. The zero from the vector form factor is understood, because in the configuration where the muon is parallel to the neutrino, the leptonic current is proportional to q_μ and takes the divergence of the vector current, which vanishes by the CVC. The contributions from C_4^V and C_4^A are very small as shown in Fig. 4. Thus the excitation of the Δ resonance, to the accuracy of present experiments, is described by two form factors.
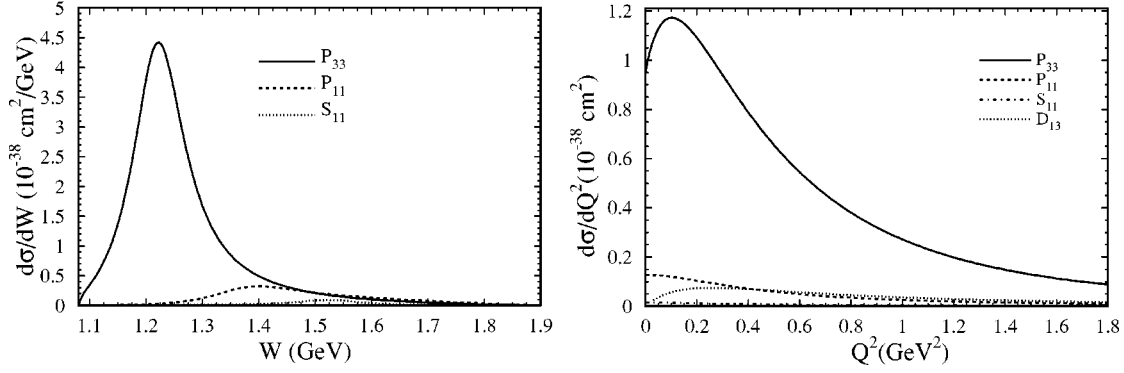
An estimate of the vector contribution is also possible

FIG. 3. Spectra of (a) invariant mass $d\sigma/dW$ and (b) $d\sigma/dQ^2$ for P_{33}, P_{11} and S_{11} resonance with neutrino energy $E_\nu = 1.5$ GeV. The curves correspond to each resonance alone without interferences.

using electroproduction data. There are precise data for the electroproduction of the Δ and other resonances [11], including their decays to various pion-nucleon modes. In the data of Galster *et al.* cross sections for the channels $(p+\pi^0)$ and $(n+\pi^+)$ are tabulated from which we conclude that both $I=3/2$ and $I=1/2$ amplitudes are present. For instance, for $W=1.232$ GeV the $I=1/2$ background is 10% of the cross section.

For our comparison we shall take the electroproduction data after subtraction of the background, as shown in Fig. 5, and then use the CVC to obtain the contribution of V_μ^+ to neutrino-induced reactions. We use the formula

$$\frac{dV^\nu}{dQ^2 dW} = \frac{G^2}{\pi} \frac{3}{8} \frac{Q^4}{\pi \alpha^2} \frac{d\sigma^{em,I=1}}{dQ^2 dW} \qquad (5.1)$$

to convert the observed [11] cross sections for the sum of the reactions $e+p \rightarrow e + \left\{ {n\,\pi^+ \atop p\,\pi^0} \right.$ to the vector contribution in the reaction $\nu + p \rightarrow \mu^- + p + \pi^+$ denoted in Eq. (5.1) by V^ν. The factor 3/8 originates from the Clebsch-Gordan coefficients relating the matrix elements of the two channels in the electromagnetic case to the matrix element of the weak charged current. We use the data of Galster *et al.* [11] at $Q^2 = 0.35$ GeV^2 and subtract the background as suggested by them. Then we converted the points to the vector contribu-
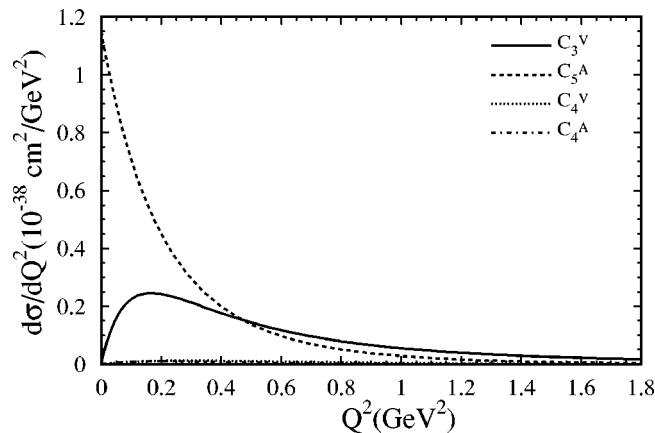
tion for the neutrino reaction according to Eq. (5.1). In the same figure we show the neutrino cross section with $C_3^V(0) = 2.05$ (solid), $C_3^V(0) = 2.0$ (dotted), $C_3^V(0) = 2.1$ (dot-dashed) and the contribution of the background (dashed) and all other form factors equal to zero. Before leaving this topic we mention that the analysis of the electroproduction data [11] included a contribution from the $D_{13}(1520)$ resonance which was found to be small.

For the axial-vector form factor we use the form given in Eq. (2.6) and we must keep an open mind to notice whether a modification will become necessary. With the method described in this paper we have all parameters for the Δ resonance. We may still change the couplings by a few percent and vary M_V and M_A. For the other resonances we can use the results of Sec. III which for the low energies introduce an $I=1/2$ background. For higher energies and for other channels the additional resonances play a significant role because they influence the overall scale of the integrated cross section; they also contribute to the multi-pion events since the inelasticities are large.

There is still the Q^2 distribution [16] to be accounted for. As mentioned already, the data are from the Brookhaven experiment [17] where the experimental group presented a histogram averaged over the neutrino flux and with an unspecified normalization. We use the formalism of this article
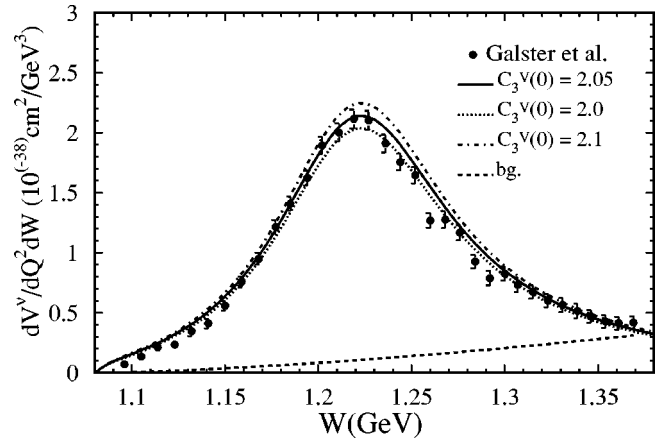

FIG. 4. Form factors.

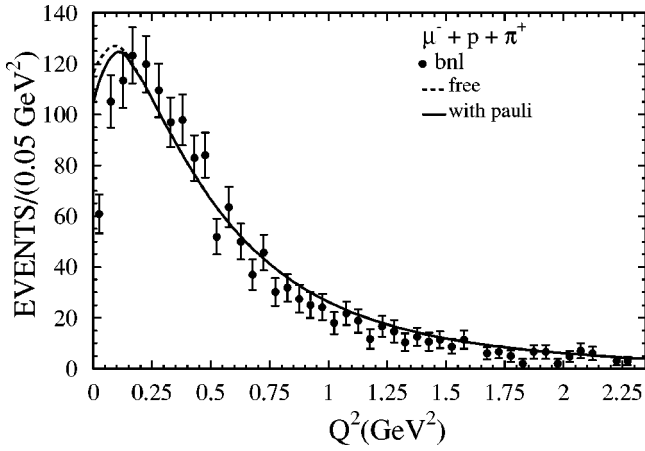FIG. 5. Cross section $dV^\nu/dQ^2 dW$ for electroproduction in the Δ resonance.

FIG. 6. Q^2-spectrum of the process $\nu p \rightarrow \mu^- p \pi^+$.

with the Δ resonances plus the correction from the Pauli factor. For the relative normalization, we normalized the area under the theoretical curve for $Q^2 \gtrsim 0.2$ GeV^2 to the corresponding area under the data. For the other parameters we found the values

$$C_3^V(0) = 1.95, \quad C_5^A(0) = 1.2,$$

$$M_V = 0.84 \text{ GeV}, \quad M_A = 1.05 \text{ GeV} \quad (5.2)$$

which produce the curves shown in Fig. 6. The solid curve includes the Pauli factor with $P_F = 0.160$ GeV and the broken curve is without the Pauli factor. In the theoretical curves we averaged over the neutrino flux for the BNL experiment [23]. It is evident that the Pauli exclusion introduces a small difference at $Q^2 < 0.20$ GeV^2. There is a discrepancy between theoretical curves and the two experimental points with $Q^2 < 0.20$ GeV^2. The fit of all of the data with $Q^2 > 0.20$ GeV^2 gives a $\chi^2 = 1.04$ per degree of freedom. Including all points we obtain $\chi^2 = 1.76$ per degree of freedom. The same theoretical parameters give an integrate cross section consistent with the data.

Taking the experimental points for granted we are presented with the problem of explaining the two points at the small Q^2 region. Among the possibilities is the modification of the normalization or the functional form of the form factors. In this paper we determined $C_3^V(0)$ from the electroproduction data. Changing the value $C_5^A(0)$ will not help because it has a different shape as shown in Fig. 4 and we must also change the functional form of $C_5^A(Q^2)$ for which we have no clear motivation. The Pauli suppression factor used in the article was based on the Fermi gas model and is appropriate for medium and heavy nuclei. It is still interesting to study whether a calculation with a deuteron wave function will produce different results [24]. These topics should be investigated in detail as we wait for results from the new experiments.

Finally we calculated the integrated cross sections as functions of the neutrino energy for the various channels. All cross sections reach constant values at higher energies. The asymptotic value for the $p \pi^+$ channel depends on the excitation of the Δ and the input parameters. For the other chan-

nels, however, the shape of the integrated cross sections and the constant asymptotic value also depends on the $I = 1/2$ contribution. In fact, comparisons of the available compilations with theoretical estimates show different values [10,13]. This topic will be investigated theoretically and in the new experiments, where additional contributions from inelastic channels and other nuclear effects must be considered.

VI. SUMMARY

We adopted in this paper a simplified formulation for the production of the Δ resonance which depends on two independent form factors. One form factor for the vector current was determined from electroproduction data, and the other axial-vector form factor was determined by PCAC and neutrino data. We plotted the vector and axial-vector contributions separately in order to understand and locate their characteristic properties. Our numerical results agree qualitatively with previous analysis [8].

We analyzed the differential cross section $d\sigma/dQ^2$ in terms of the two form factors. The vector contribution has a modified dipole form, as determined from electroproduction data and the axial form factor is also modified. Since we had to search diligently for the values in Eq. (5.2), we find that the values at low and high Q^2 are correlated, and the curves are shown in Fig. 6.

Fitting the large Q^2 region gives a differential cross section which is too high at $Q^2 \lesssim 0.15$ GeV^2. This may be due to the scanning efficiency [25] or may be a genuine property and must be followed up in the future. The present analysis points to the direction that a modified axial-vector form factor may be preferable. Similar tendencies were reported for Q^2 distributions of other articles [25–27].

We also included the Pauli factor which brings a small correction at $Q^2 \lesssim 0.20$ GeV^2. In order to justify its application to light nuclei and for the decay of a resonance within a nucleus we rederived the Pauli factor in the Fermi gas model and showed that it agrees with the standard geometrical interpretation.

Now, that the Δ^{++} resonance can be accounted for there is interest to predict the other channels $p \pi^0$ and $n \pi^+$, where $I = 1/2$ resonances also contribute. In Sec. III we give formulas for the differential cross section for P_{11} and S_{11} resonances. They will influence the cross sections for the other channel especially at energies $E_\nu > 2$ GeV.

In this paper we have demonstrated that the excitation of the Δ resonance can be accounted for by two form factors. This result forms the basis for the analysis of new data which will confirm the adequacy of this minimal set or require additional form factors or alternative parametrizations. The work will be extended to the other final states $p \pi^0$ and $n \pi^+$ where $I = 1/2$ will be included. The extension to higher energies $E_\nu > 2$ GeV will reveal the significance of additional resonances, especially those with large inelasticities, because they may reveal characteristics for the transition to the inelastic region where multi-pion production is important.

APPENDIX: FORMULAS FOR THE AMPLITUDES AND THE CROSS SECTIONS

In this appendix we give a summary of the formulas used for the calculation of the cross section. We set the lepton mass equal to zero and write the differential cross section as

$$\frac{d\sigma}{dQ^2 dW} = \frac{G^2}{16\pi M^2} \sum_{i=1}^{3} K_i W_i.$$

The kinematic factors are

$$K_1 = \frac{2Q^2}{E_\nu^2}$$

$$K_2 = 4\left(1 - \frac{Q^2}{4E_\nu^2} - \frac{q_L^0}{E_\nu} \right)$$

$$K_3 = \frac{MQ^2}{E_\nu W q_{CM}} \left(2 - \frac{q_L^0}{E_\nu} \right)$$

with the kinematic variables being those used in deep inelastic scattering.

The structure functions W_i are expressed in terms of helicity amplitudes.

$$W_1 = \frac{W}{q_{CM}} (|T_{3/2}|^2 + |T_{1/2}|^2 + |U_{3/2}|^2 + |U_{1/2}|^2)$$

$$W_2 = \frac{M^2 Q^2}{W q_{CM}^3} (|T_{3/2}|^2 + |T_{1/2}|^2 + |U_{3/2}|^2 + |U_{1/2}|^2)$$
$$+ \frac{2M^2 Q^4}{W q_{CM}^5} (|T_C|^2 + |U_C|^2)$$

$$W_3 = \frac{4W}{q_{CM}} (\operatorname{Re} T_{3/2}^* U_{3/2} - \operatorname{Re} T_{1/2}^* U_{1/2}).$$

Several remarks are now in order. The structure functions W_1 and W_2 contain the square of the vector current plus the square of the axial-vector currents. The structure W_3 is the vector $\otimes$ axial interference term. The last term in W_2 is regular as $Q^2 \to 0$. Finally, T_i and U_i are real so that the symbol for a real part is superfluous.

The helicity amplitudes are given in terms of form factors

$$T_{3/2} = y \left[\left(\frac{W+M}{M} \right) C_3^V + \frac{W q_{CM}^0}{M^2} C_4^V \right]$$

$$T_{1/2} = \frac{y}{\sqrt{3}} \left[\frac{q_{CM}^0 - p_{CM}^0 - M}{M} C_3^V + \frac{W q_{CM}^0}{M_N^2} C_4^V \right]$$

$$T_C = -\sqrt{\frac{2}{3}} y \frac{q_{CM}}{M} \left(C_3^V + \frac{W}{M} C_4^V \right).$$

The amplitude T_C vanishes at $W = M_\Delta$. Similarly the axial-vector current contributions are

$$U_{3/2} = z \left(\frac{W q_{CM}^0}{M^2} C_4^A + C_5^A \right)$$

$$U_{1/2} = \frac{1}{\sqrt{3}} U_{3/2}$$

$$U_C = -\sqrt{3/2} z \frac{q_{CM}}{M} \left(\frac{W}{M} C_4^A - \frac{M q_{CM}^0}{Q^2} C_5^A \right).$$

The kinematic factors are

$$y = f(W) N_{RS} q_{CM}, \quad z = f(W) N_{RS} (p_{CM}^0 + M),$$

$$N_{RS} = -i \left[\frac{q_{CM}}{4W(p_{CM}^0 + M)} \right]^{1/2}, \quad p_{CM}^0 = [q_{CM}^2 + M^2]^{1/2}$$

and the rest are defined in Sec. I.

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