Ground-state scalar $\bar{q}q$ nonet: SU(3) mass splittings and strong, electromagnetic, and weak decay rates

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By comparing SU(3)-breaking scales of linear mass formulas, it is shown that the lowest vector and scalar mesons all have a $\bar{q}q$ configuration, while the ground-state octet and decuplet baryons are qqq. Also, the quark-level linear σ model is employed to predict similar $\bar{q}q$ and qqq states. Furthermore, the approximate mass degeneracy of the scalar $a_0(985)$ and $f_0(980)$ mesons is demonstrated to be accidental. Finally, it is shown that various strong, electromagnetic, and weak mesonic decay rates are successfully explained within the framework of the quark-level linear σ model.

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I. INTRODUCTION

In the quark model, one usually assumes that pseudoscalar ($\mathcal{P}$) and vector ($\mathcal{V}$) mesons are $\bar{q}q$, whereas octet ($\mathcal{O}$) and decuplet ($\mathcal{D}$) baryons are qqq states. However, it is now argued [1] that the light scalar ($\mathcal{S}$) mesons are non-$\bar{q}q$ candidates, in view of their low masses and broad widths. In this paper, we shall show that the ground-state meson nonets $\mathcal{P}$, $\mathcal{S}$, and $\mathcal{V}$ are *all* $\bar{q}q$, hence including the light scalars, while the lowest $\mathcal{O}$ and $\mathcal{D}$ baryons are qqq states.

In Sec. II, SU(3) mass splittings for loosely bound $\mathcal{V}$ and $\mathcal{S}$ states are shown to have symmetry-breaking scales of 13% and 18%, respectively, using linear mass formulas. We apply the latter formulas to qqq $\mathcal{O}$ and $\mathcal{D}$ states in Sec. III, leading to SU(3)-breaking scales of 13% and 12%, respectively. Then in Sec. IV, we employ the quark-level linear σ model (LσM) to predict similar $\bar{q}q$ and qqq states as in Secs. II and III. Next in Sec. V, we study the $\mathcal{S}\,\bar{q}q$ states and argue why the $\mathcal{V}$ states have slightly higher masses, on the basis of the nonrelativistic quark model. Moreover, the approximate mass degeneracy of the $\mathcal{S}\,a_0(985)$ and $f_0(980)$ mesons is shown to be just accidental. Finally, in Secs. VI, VII, and VIII we successfully determine, in an LσM framework, mesonic decay rates for strong, electromagnetic, and weak processes, respectively. In Sec. IX we summarize our results and draw some conclusions.

II. MASS SPLITTINGS FOR U(3)×U(3) $\mathcal{V}$ AND $\mathcal{S}\,\bar{q}q$ MESONS

Although meson masses are expected to appear *quadratically* in model Lagrangians, while they must appear so for $\mathcal{P}$ states [2], for $\mathcal{V}$ and $\mathcal{S}$ states a Taylor-series linear form for SU(3) mass splittings is also possible. Thus, consider a Hamiltonian density $H=H(\lambda_0)+H_{ss}(\lambda_8)$ using Gell-Mann matrices. Then the vector-meson-nonet masses $m_{\mathcal{V}}=\sqrt{2/3}m^0_{\mathcal{V}}-d_{\bar{i}8i}\delta m_{\mathcal{V}}$ are[1]

$$m_{\rho,\omega}=\sqrt{\frac{2}{3}}m^0_{\mathcal{V}}-\frac{1}{\sqrt{3}}\delta m_{\mathcal{V}}\approx 776\text{ MeV},$$

$$m_{K*}=\sqrt{\frac{2}{3}}m^0_{\mathcal{V}}+\frac{1}{2\sqrt{3}}\delta m_{\mathcal{V}}\approx 894\text{ MeV},\qquad (1)$$

$$m_{\phi}=\sqrt{\frac{2}{3}}m^0_{\mathcal{V}}+\frac{2}{\sqrt{3}}\delta m_{\mathcal{V}}\approx 1020\text{ MeV},$$

with $\phi\approx\bar{s}s$. Measured vector masses [1] suggest average mass splittings

$$m^0_{\mathcal{V}}\approx 1048\text{ MeV},\quad \delta m_{\mathcal{V}}\approx 141\text{ MeV},\qquad (2)$$

giving an SU(3)-breaking scale of $\delta m_{\mathcal{V}}/m^0_{\mathcal{V}}\approx 13\%$.

Such considerations can be repeated for axial-vector mesons as well, even though it is now hard to draw any decisive conclusions, also in view of the experimental situation. This is why regarding these mesons we limit ourselves to the following observations. In the case of axial-vector a_1 states, we

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[1]Recall that $d_{0ij}=\sqrt{2/3}\delta_{ij}$, $d_{\bar{n}8n}=1/\sqrt{3}$, $d_{\bar{s}8n}=d_{K8K}=-1/(2\sqrt{3})$, and $d_{\bar{s}8s}=-2/\sqrt{3}$.

assume the $f_1(1420)$ is mostly $\bar{s}s$, because the Particle Data Group (PDG) [1] reports $f_1(1420)\to KK\pi$, K^*K as dominant, while $f_1(1285)\to KK\pi$, K^*K are almost absent. Thus, $f_1(1285)$ is mostly $\bar{n}n$, like the nonstrange $a_1(1260)$ [with $a_1\to\sigma\pi$ seen, but $a_1\to f_0(980)\pi$ not seen, because $f_0(980)$ is mostly $\bar{s}s$].

Also the scalar masses (not incompatible with Ref. [1]) predicted from the LσM discussed in Sec. IV obey the mass-splitting pattern [for the chiral limit (CL) in SU(2) and SU(3), see Refs. [3] and [4], respectively]

$$m_{\sigma_n}=\sqrt{\frac{2}{3}}m_{\mathcal{S}}^0-\frac{1}{\sqrt{3}}\delta m_{\mathcal{S}}\overset{\text{CL}}{\to}2\hat{m}_{\text{CL}}=650\ \text{MeV},$$

$$m_\kappa=\sqrt{\frac{2}{3}}m_{\mathcal{S}}^0+\frac{1}{2\sqrt{3}}\delta m_{\mathcal{S}}\overset{\text{CL}}{\to}2\sqrt{\hat{m}_{\text{CL}}m_{s,\text{CL}}}=780\ \text{MeV},\tag{3}$$

$$m_{\sigma_s}=\sqrt{\frac{2}{3}}m_{\mathcal{S}}^0+\frac{2}{\sqrt{3}}\delta m_{\mathcal{S}}\overset{\text{CL}}{\to}2m_{s,\text{CL}}=940\ \text{MeV}.$$

Here, $m_{\sigma_n(650)}$ is near the PDG average [1] $m_{f_0(600)}$, $m_{\kappa(780)}$ is near the E791 value [5] 797 ± 19 MeV, and $m_{\sigma_s(940)}$ is near the PDG value $m_{f_0(980)}$, which is thus mostly $\bar{s}s$. The masses from Eqs. (3) then give the CL average mass splittings

$$m_{\mathcal{S}}^0\overset{\text{CL}}{\to}922\ \text{MeV},$$

$$\delta m_{\mathcal{S}}\overset{\text{CL}}{\to}167\ \text{MeV},\tag{4}$$

$$\frac{\delta m_{\mathcal{S}}}{m_{\mathcal{S}}^0}\overset{\text{CL}}{\to}18\%.$$

The fact that the $\bar{q}q$ scalars have an SU(3)-breaking CL scale of 18%, larger than the 13% scale of $\mathcal{V}$ ground states, further suggests that, whereas the $\mathcal{V}$ are $\bar{q}q$ loosely bound states, the $\bar{q}q$ $\mathcal{S}$ states [with quarks touching in the Nambu–Jona-Lasinio (NJL) scheme [6]] are ‘‘barely’’ elementary-particle partners of the tightly bound $\mathcal{P}$ states (discussed in Sec. IV).

III. LOOSELY BOUND qqq BARYONS

In this same Taylor-series spirit, the octet ($\mathcal{O}$) baryon SU(3) mass splitting $m_{\mathcal{O}}=m_{\mathcal{O}}^0-\delta m_{\mathcal{O}}(d_{ss}d^{\bar{i}8i}+f_{ss}if^{\bar{i}8i})$, with $d_{ss}+f_{ss}=1$, predicts (the index *ss* means semistrong)

$$m_N=m_{\mathcal{O}}^0-\frac{\delta m_{\mathcal{O}}}{2\sqrt{3}}(-d_{ss}+3f_{ss})\approx939\ \text{MeV},$$

$$m_\Lambda=m_{\mathcal{O}}^0+\frac{\delta m_{\mathcal{O}}}{\sqrt{3}}d_{ss}\approx1116\ \text{MeV},$$

$$m_\Sigma=m_{\mathcal{O}}^0-\frac{\delta m_{\mathcal{O}}}{\sqrt{3}}d_{ss}\approx1193\ \text{MeV},\tag{5}$$

$$m_\Xi=m_{\mathcal{O}}^0+\frac{\delta m_{\mathcal{O}}}{2\sqrt{3}}(d_{ss}+3f_{ss})\approx1318\ \text{MeV}.$$

The $(d/f)_{ss}$ ratio can be found from Eqs. (5) as

$$\left(\frac{d}{f}\right)_{ss}=-\frac{3}{2}\frac{m_\Sigma-m_\Lambda}{m_\Xi-m_N}\approx-0.305,\quad d_{ss}\approx-0.44,$$

$$f_{ss}\approx1.44.\tag{6}$$

Thus, Eqs. (5) predict the average mass splittings

$$m_{\mathcal{O}}^0\approx1151\ \text{MeV},\quad \delta m_{\mathcal{O}}\approx150\ \text{MeV},\quad \frac{\delta m_{\mathcal{O}}}{m_{\mathcal{O}}^0}\approx13\%.\tag{7}$$

The SU(3) $\mathcal{D}$ baryon masses $m_{\mathcal{D}}=m_{\mathcal{D}}^0+\delta m_{\mathcal{D}}$ have $m_{\mathcal{D}}^0$ weighted by wave functions

$$\overline{\Psi}^{(abc)}\Psi_{(abc)}=\bar{\Delta}\Delta+\bar{\Sigma}^*\Sigma^*+\bar{\Xi}^*\Xi^*+\bar{\Omega}\Omega,\tag{8}$$

and $\delta m_{\mathcal{D}}$ is weighted by

$$3\overline{\Psi}^{(ab3)}\Psi_{(ab3)}=\bar{\Sigma}^*\Sigma^*+2\bar{\Xi}^*\Xi^*+3\bar{\Omega}\Omega.\tag{9}$$

Then the SU(3) $\mathcal{D}$ masses are predicted (in MeV) to be

$$m_\Delta=m_{\mathcal{D}}^0\approx1232,$$

$$m_{\Sigma^*}=m_{\mathcal{D}}^0+\delta m_{\mathcal{D}}\approx1385,\ \text{with}\ \delta m_{\mathcal{D}}\approx153,$$

$$m_{\Xi^*}=m_{\mathcal{D}}^0+2\,\delta m_{\mathcal{D}}\approx1533,\ \text{with}\ \delta m_{\mathcal{D}}\approx151,\tag{10}$$

$$m_\Omega=m_{\mathcal{D}}^0+3\,\delta m_{\mathcal{D}}\approx1672,\ \text{with}\ \delta m_{\mathcal{D}}\approx147.$$

This corresponds to average mass splittings

$$m_{\mathcal{D}}^0\approx1232\ \text{MeV},\quad \delta m_{\mathcal{D}}\approx150\ \text{MeV},\quad \frac{\delta m_{\mathcal{D}}}{m_{\mathcal{D}}^0}\approx12\%.\tag{11}$$

It is interesting that both loosely bound qqq $\mathcal{O}$ and $\mathcal{D}$ symmetry-breaking scales of about 150 MeV are near the $\bar{q}q$ $\mathcal{V}$, $\mathcal{S}$ mean mass-splitting scale of $\delta m=141$ MeV, 167 MeV. However, the CL SU(3)-breaking scale of 18% for scalars is about 50% greater than the 12–13% scales of $\mathcal{V}$, $\mathcal{O}$, $\mathcal{D}$ states. This suggests that $\mathcal{V}$, $\mathcal{O}$, $\mathcal{D}$ $\bar{q}q$ or qqq states are all loosely bound, in contrast with the elementary-particle $\bar{q}q$ $\mathcal{S}$ and, of course, the $\mathcal{P}$ states (see above). In fact, the latter

Nambu-Goldstone $\mathcal{P}$ states are massless in the CL $p^2=m_\pi^2=0$, $p^2=m_K^2=0$, as the tightly bound measured [1] π^+ and K^+ charge radii indicate [7].

IV. CONSTITUENT QUARKS AND THE QUARK-LEVEL LσM

Formulating the $\mathcal{P}$ and $\mathcal{S}\bar{q}q$ states as elementary chiral partners [8], the Lagrangian density of the SU(2) quark-level linear σ model (LσM) has (see Appendices A and B for a more detailed summary of the quark-level LσM), after the spontaneous-symmetry-breaking shift, the interacting part [9] [for $f_\pi=(92.42\pm0.27)$ MeV$\approx$93 MeV]

$$\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}}^{\mathrm{int}}=g\bar{\psi}(\sigma+i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi+g'\sigma(\sigma^2+\pi^2)-\frac{\lambda}{4}(\sigma^2+\pi^2)^2 - f_\pi g\bar{\psi}\psi, \tag{12}$$

with tree-order CL couplings related as ($f_\pi^{\mathrm{CL}}\to 90$ MeV)

$$g=\frac{m_q}{f_\pi},\quad g'=\frac{m_\sigma^2}{2f_\pi}=\lambda f_\pi. \tag{13}$$

The SU(2) and SU(3) chiral Goldberger-Treiman relations (GTRs) are

$$f_\pi g=\hat{m}=\frac{1}{2}(m_u+m_d),\quad f_K g=\frac{1}{2}(m_s+\hat{m}). \tag{14}$$

Since $f_K/f_\pi\approx1.22$ [1], the constituent-quark-mass ratio from Eq. (14) becomes

$$1.22\approx\frac{f_K}{f_\pi}=\frac{1}{2}\left(1+\frac{m_s}{\hat{m}}\right)\Rightarrow\frac{m_s}{\hat{m}}\approx1.44, \tag{15}$$

which is independent of the value of g. In loop order, Eqs. (13) are recovered, along with [3,7]

$$m_\sigma=2m_q,\quad g=\frac{2\pi}{\sqrt{N_c}},\quad \text{for } N_c=3. \tag{16}$$

Here, the first equation is the NJL relation [6], now true for the LσM as well. The second equation in Eqs. (16) was first found via the $Z=0$ compositeness relation [10], separating the elementary π and σ particles from the bound states ρ, ω, and a_1.

We first estimate the nonstrange and strange constituent quark masses from the GTRs (14), together with the LσM loop-order result (16):[2]

$$\hat{m}\approx gf_\pi\approx\frac{2\pi}{\sqrt{3}}(93\text{ MeV})\approx337\text{ MeV}\xrightarrow{\mathrm{CL}}325\text{ MeV},$$

$$m_s=\left(\frac{m_s}{\hat{m}}\right)\hat{m}\approx1.44\hat{m}\approx485\text{ MeV}\xrightarrow{\mathrm{CL}}470\text{ MeV}. \tag{17}$$

These quark-mass scales in turn confirm the mass-splitting scales found in Secs. II and III,

$$\delta m_{\mathcal{V}}\approx\delta m_{\mathcal{S}}\approx\delta m_{\mathcal{O}}\approx\delta m_{\mathcal{D}}\approx(485-337)\text{ MeV}=148\text{ MeV} \xrightarrow{\mathrm{CL}}(470-325)\text{ MeV}=145\text{ MeV}, \tag{18}$$

near 141, 167, 150, 150 MeV, respectively. Also the SU(3) nonvanishing masses are predicted as

$$m_{\mathcal{V}}^0\approx\sqrt{\frac{3}{2}}(m_s+\hat{m})\approx1007\text{ MeV},$$

$$m_{\mathcal{O}}^0=m_{\mathcal{D}}^0\approx m_s+2\hat{m}\approx1160\text{ MeV}, \tag{19}$$

near the 1048, 1151, and 1232 MeV m^0 masses in Secs. II and III.

To verify that the pion and kaon are tightly bound $\bar{q}q$ mesons, we compute the π^+ and K^+ charge radii as [7] $r_\pi=1/\hat{m}_{\mathrm{CL}}=0.61$ fm and $r_K=2/(m_s+\hat{m})_{\mathrm{CL}}=0.50$ fm, near data [1] 0.672 ± 0.008 fm and 0.560 ± 0.031 fm, respectively. Likewise, to verify that the proton is a qqq touching pyramid, we estimate the proton charge radius as $R_p=(1+\sin30°)r_\pi\approx0.9$ fm, near data [1] 0.870 ± 0.008 fm.

V. $\mathcal{S}$ SCALARS AND ACCIDENTAL DEGENERACIES

We begin with the non-CL NJL-LσM scalar masses $m_{\sigma_n}=2\hat{m}=674$ MeV, $m_\kappa=2\sqrt{\hat{m}m_s}=809$ MeV, and $m_{\sigma_s}=2m_s=970$ MeV.

An almost degenerate case in the nonrelativistic quark model (NRQM) is [12], in the context of QCD,[3]

$$m_{\mathcal{S}}\approx m_{\mathcal{V}}+\frac{2\alpha_s}{m_{\mathrm{dyn}}^2}\left(\frac{\vec{L}\cdot\vec{S}}{r^3}\right)=780\text{ MeV}-140\text{ MeV}=640\text{ MeV}, \tag{20}$$

where the ground-state vector mesons have $L=0$ and so no spin-orbit contribution to the mass. This corresponds to $m_{\sigma(650)}\approx m_{\omega(782)}-140$ MeV$=642$ MeV. Equivalently, invoking the $I=1/2$ CGC of 1/2, one predicts via the NRQM $m_{\kappa(800)}\approx m_{K^*(892)}-70$ MeV$=822$ MeV. Or invoking instead the $\bar{s}s$ CGC of 1/3, one gets $m_{\sigma_s(970)}\approx m_{\phi(1020)}-47$ MeV$=973$ MeV. In a similar way we obtain also $m_{a_0(985)}=m_{\rho(770)}+(3/2)140$ MeV$=980$ MeV.

[2]The resulting quark masses are well in agreement with the values obtained on the basis of the magnetic moments of the respective baryons (see, e.g., Ref. [11]). The proton magnetic moment $\mu_p\simeq2.7928$, e.g., yields $\hat{m}=m_p/\mu_p=336$ MeV.

[3]Note that we follow Ref. [12], and use $\alpha_s(m_\sigma^2)\simeq\pi/4$ (see also Ref. [13]), $\vec{L}\cdot\vec{S}=-2$, $m_{\mathrm{dyn}}=315$ MeV, while $\langle r^{-3}\rangle=4\beta^3/(3\sqrt{\pi})$ is obtained employing harmonic-oscillator wave functions with $\beta\simeq180$ MeV.

As an alternative way to examine the latter, in the case of the elementary-particle $\mathcal{P}$ and $\mathcal{S}$ states, one should invoke the infinite-momentum-frame (IMF, see Appendix C) scalar-pseudoscalar SU(3) equal-splitting laws (ESLs), reading [14]

$$m_\sigma^2 - m_\pi^2 \approx m_\kappa^2 - m_K^2 \approx m_{a_0}^2 - m_{\eta_{\rm avg}}^2 \approx 0.40\ \text{GeV}^2, \quad (21)$$

where $m_{\eta_{\rm avg}}$ is the average η, η' mass 753 MeV. These ESLs hold for the non-CL NJL-LσM scalar mass values. Using the ESLs (21) to predict the a_0 mass, one finds

$$m_{a_0} = \sqrt{0.40\ \text{GeV}^2 + m_{\eta_{\rm avg}}^2} \approx 983.4\ \text{MeV}, \quad (22)$$

very close to the PDG value 984.7±1.2 MeV. Thus, the nearness of the $a_0(985)$ and $f_0(980)$ masses, the latter scalar being mostly $\bar{s}s$ and so near the vector $\bar{s}s$ $\phi(1020)$ (see above), is indeed an accidental degeneracy. Note that a similar (approximate) degeneracy is found in the dynamical unitarized quark-meson model of Ref. [15], where the same $\bar{q}q$ assignments are employed as here.

This ground-state scalar 0^+ nonet [$\sigma(650)$, $\kappa(800)$, $f_0(980)$, $a_0(985)$] is about 500–700 MeV below the 0^+ nonet [1,16] [$f_0(1370)$, $K_0^*(1430)$, $f_0(1500)$, $a_0(1450)$], just as the ground-state 1^- vector nonet [$\rho(770)$, $\omega(782)$, $K^*(892)$, $\phi(1020)$] is about 600–800 MeV below the 1^- nonet [1] [$\rho(1450)$, $\omega(1420)$, $K^*(1680)$, $\phi(1680)$].

VI. STRONG-INTERACTION SCALAR-MESON DECAY RATES

Given the above scalar-meson nonet $\sigma(650)$, $\kappa(800)$, $f_0(980)$, $a_0(985)$, compatible with present data and also with the SU(3) mass splittings in Secs. II, III, V and the quark-level LσM in Sec. IV, we now predict LσM decay rates based on the SU(3) Lagrangian density $\mathcal{L}_{\rm L\sigma M}^{\rm int} = g_{\sigma\pi\pi} d_{ijk} S_i P_j P_k$, with L$\sigma$M coupling $g_{\sigma\pi\pi} = (m_\sigma^2 - m_\pi^2)/(2 f_\pi) \approx 2.18$ GeV, where $f_\pi = (92.42 \pm 0.27)$ MeV and $m_\sigma \approx 650$ MeV (the latter stems from the CL $m_q \approx 325$ MeV [3]). Thus, the $\sigma \to 2\pi$ decay rate, for $p_{cm} = 294$ MeV and $\phi_s = \pm 18°$,[4] becomes

$$\Gamma_{\sigma\pi\pi} = \frac{p_{cm}}{8\pi m_\sigma^2}\left(\frac{3}{2}\right)[2 g_{\sigma\pi\pi} \cos\phi_s]^2 \approx 714\ \text{MeV}. \quad (23)$$

Here the factor of 2 is due to Bose statistics (see, e.g., Ref. [17]), and this broad width $\Gamma_\sigma \simeq m_\sigma$ is expected from data [18] and from phenomenology [19].

Next, the $a_0(985) \to \eta\pi$ width for $p_{cm} = 321$ MeV is

$$\Gamma_{a_0\eta\pi} = \frac{p_{cm}}{8\pi m_{a_0}^2}[2 g_{\sigma\pi\pi} \cos\phi_{ps}]^2 \approx 138\ \text{MeV}, \quad (24)$$

where $\phi_{ps} \approx 42°$ is in the quark nonstrange($\bar{n}n$)–strange($\bar{s}s$) basis [20]. This predicted LσM width is not incompatible with the high-statistics decay rate [21] $\Gamma_{a_0\eta\pi} = (95 \pm 14)$ MeV.

Furthermore, the $\kappa \to K\pi$ decay rate, for $p_{cm} = 218$ MeV and $m_\kappa = 800$ MeV, is

$$\Gamma_{\kappa K\pi} = \frac{p_{cm}}{8\pi m_\kappa^2}\left(\frac{3}{4}\right)[2 g_{\sigma\pi\pi}]^2 \approx 193\ \text{MeV}, \quad (25)$$

which is of the same order as the E791 data [5]

$$\Gamma_{\kappa K\pi}^{\rm E791} = (410 \pm 43 \pm 87)\ \text{MeV},$$
$$m_\kappa = (797 \pm 19 \pm 43)\ \text{MeV}, \quad (26)$$

and especially the very recent data of the BES Collaboration [22]

$$\Gamma_{\kappa K\pi}^{\rm BES} = (220\ {}^{+225}_{-169} \pm 97)\ \text{MeV},$$
$$m_\kappa = (771\ {}^{+164}_{-221} \pm 55)\ \text{MeV}. \quad (27)$$

Last, we estimate (see, e.g., Ref. [16]) the $f_0(980) \to \pi\pi$ rate, assuming again that the $f_0(980)$ is mostly $\bar{s}s$, with mixing angle ±18° in the quark basis [23], for $p_{cm} = 470$ MeV,

$$\Gamma_{f_0 2\pi} = \frac{p_{cm}}{8\pi m_{f_0}^2}\left(\frac{3}{2}\right)[2 g_{\sigma\pi\pi} \sin\phi_s]^2 \approx 53\ \text{MeV}, \quad (28)$$

not too distant from the recent E791 measurement [5]

$$\Gamma_{f_0 2\pi}^{\rm E791} = (44 \pm 2 \pm 2)\ \text{MeV}, \quad m_{f_0} = (977 \pm 3 \pm 2)\ \text{MeV}. \quad (29)$$

VII. ELECTROMAGNETIC MESON DECAY RATES INVOLVING $\bar{q}q$ SCALARS

Next we study the five electromagnetic meson decays $\sigma \to 2\gamma$, $a_0 \to 2\gamma$, $f_0 \to 2\gamma$, $\phi \to f_0\gamma$, and $\phi \to a_0\gamma$. Again assuming $m_\sigma \approx 650$ MeV (because $\hat{m} \approx 325$ MeV$\simeq m_N/3$ in the CL, so that the NJL-LσM scalar mass is $m_\sigma = 2\hat{m} \approx 650$ MeV), the quark-loop amplitude magnitude is, for $f_\pi = (92.42 \pm 0.27)$ MeV [20] [see, e.g., Eq. (11a) in Ref. [24], and the considerations in Ref. [16]]

$$|M(\sigma \to 2\gamma)| \approx \frac{5}{3}\frac{\alpha}{\pi f_\pi} + \frac{1}{3}\frac{\alpha}{\pi f_\pi} \approx 5.0 \times 10^{-2}\ \text{GeV}^{-1}. \quad (30)$$

Here, the first term is due to the nonstrange quark triangle, while the second term stems from the charged-kaon and -pion triangle graphs. This result (30) is compatible with the data estimate [25]

$$\Gamma_{\sigma 2\gamma} = \frac{m_\sigma^3}{64\pi}|M(\sigma \to 2\gamma)|^2 = (3.8 \pm 1.5)\ \text{keV}, \quad (31)$$

[4] For convenience, we use here the same value of the mixing angle ϕ_s as in Ref. [16], i.e., $\phi_s = \pm 18°$.

or (for $m_\sigma \simeq 650$ MeV)

$$|M(\sigma \to 2\gamma)| \simeq (5.3 \pm 1.0) \times 10^{-2} \ \text{GeV}^{-1}. \quad (32)$$

Now we examine $a_0(985) \to 2\gamma$. A nonstrange-quark triangle loop predicts the gauge-invariant induced amplitude magnitude [26] [for $m_{a_0} \simeq (984.7 \pm 1.2)$ MeV]

$$\begin{aligned} |M(a_0 \to 2\gamma)|_{\text{quark-loop}} &= \left| 2\xi[2 + (1-4\xi)I(\xi)] \frac{\alpha}{\pi f_\pi} \right| \\ &= |2.03 \pm 0.07 + i(1.89 \pm 0.03)| \\ &\quad \times 10^{-2} \ \text{GeV}^{-1} \\ &= (2.78 \pm 0.06) \times 10^{-2} \ \text{GeV}^{-1}, \end{aligned} \quad (33)$$

for $\xi = \hat{m}^2/m_{a_0}^2 \approx 0.109 \pm 0.004 < 1/4$ in the CL, with (see, e.g., Ref. [16])

$$\begin{aligned} I(\xi) &= \int_0^1 dy \int_0^1 dx \frac{y}{\xi - xy(1-y)} \\ &\overset{\xi < 1/4}{=} \frac{\pi^2}{2} - 2\ln^2\left[\frac{1}{\sqrt{4\xi}} + \sqrt{\frac{1}{4\xi} - 1}\right] \\ &\quad + 2\pi i \ln\left[\frac{1}{\sqrt{4\xi}} + \sqrt{\frac{1}{4\xi} - 1}\right] \\ &= 3.03 \pm 0.08 + i(6.13 \pm 0.13). \end{aligned} \quad (34)$$

However, adding to Eq. (33) the charged-kaon-loop amplitude [26] 0.97×10^{-2} GeV^{-1} (as required by the LσM), which has the opposite sign as compared to the fermionic quark-loop amplitude, in turn predicts [27]

$$\begin{aligned} |M(a_0 \to 2\gamma)| &\approx |M(a_0 \to 2\gamma)_{\text{quark-loop}} + M(a_0 \to 2\gamma)_{\text{kaon-loop}}| \\ &= |1.07 \pm 0.44 + i(1.89 \pm 0.03)| \times 10^{-2} \ \text{GeV}^{-1} \\ &= (2.17 \pm 0.22) \times 10^{-2} \ \text{GeV}^{-1}. \end{aligned} \quad (35)$$

The latter result is too large as compared to data, assuming $a_0 \to \eta\pi$ is dominant [1],

$$\Gamma_{a_0 2\gamma} = \frac{m_{a_0}^3}{64\pi} |M(a_0 \to 2\gamma)|^2 = (0.24 \pm 0.08) \ \text{keV} \quad (36)$$

or

$$|M(a_0 \to 2\gamma)| = (0.7 \pm 0.2) \times 10^{-2} \ \text{GeV}^{-1}. \quad (37)$$

However, upon disregarding the imaginary part of the quark-loop amplitude, which is reasonable in view of quark confinement, we come much closer to the data, as

$$\begin{aligned} &\text{Re}[M(a_0 \to 2\gamma)_{\text{quark-loop}} + M(a_0 \to 2\gamma)_{\text{kaon-loop}}] \\ &\quad = (1.07 \pm 0.44) \times 10^{-2} \ \text{GeV}^{-1}. \end{aligned} \quad (38)$$

Next we study $f_0 \to 2\gamma$. Assuming for the moment that $f_0(980)$ is purely $\bar{s}s$, the strange-quark loop gives, for $N_c = 3$ [28] (see also Ref. [16]),

$$|M(f_0 \to 2\gamma)|_{\text{quark-loop}} = \frac{\alpha N_c g_{f_0 \bar{s}s}}{9\pi m_s} \simeq 8.19 \times 10^{-3} \ \text{GeV}^{-1}, \quad (39)$$

taking the LσM value $m_s = 485$ MeV from Eq. (17), with the LσM coupling $g_{f_0 \bar{s}s} = 2\pi\sqrt{2/3} \approx 5.13$. In fact, Eq. (39) is surprisingly near the observed amplitude [1]

$$\Gamma_{f_0 2\gamma} = \frac{m_{f_0}^3}{64\pi} |M(f_0 \to 2\gamma)|^2 = (0.39 \pm 0.12) \ \text{keV}, \quad (40)$$

or [with $m_{f_0} \simeq (980 \pm 10)$ MeV]

$$|M(f_0 \to 2\gamma)| = (9.1 \pm 1.5) \times 10^{-3} \ \text{GeV}^{-1}. \quad (41)$$

Nevertheless, a more detailed analysis based on kaon and pion loops, and allowing a small $\bar{n}n$ admixture in the $f_0(980)$, essentially confirms this nice result [16].

Let us now analyze the decay $\phi(1020) \to f_0(980)\gamma$. Since the $\phi(1020)$ is known to be dominantly $\bar{s}s$, just as we assume the $f_0(980)$ to be, the s-quark loop gives (with $g_\phi = 13.43$ from $\Gamma_{\phi ee}$ and $e = \sqrt{4\pi\alpha} = 0.302\,82 \cdots$)

$$|M(\phi \to f_0\gamma)|_{\text{quark-loop}} = \frac{2 g_\phi e g_{f_0 \bar{s}s}}{4\pi^2 m_s} \cos\phi_s \simeq 2.07 \ \text{GeV}^{-1}. \quad (42)$$

However, the charged-kaon loop is known to give the rate [29]

$$\Gamma_{\phi f_0 \gamma}|_{\text{kaon-loop}} = 8.59 \times 10^{-4} \ \text{MeV} \quad (43)$$

or

$$|M(\phi \to f_0\gamma)|_{\text{kaon-loop}} = 0.75 \ \text{GeV}^{-1}. \quad (44)$$

Subtracting this kaon-loop amplitude (44) from the quark-loop amplitude (42) predicts in turn

$$|M(\phi \to f_0\gamma)| \approx 2.07 \ \text{GeV}^{-1} - 0.75 \ \text{GeV}^{-1} = 1.32 \ \text{GeV}^{-1}, \quad (45)$$

near the recent KLOE data [30], for $p_{cm} \simeq (38.69 \pm 9.62)$ MeV,

$$\Gamma_{\phi f_0 \gamma}|_{\text{KLOE}} = \frac{p_{cm}^3}{12\pi} |M(\phi \to f_0\gamma)|^2 \approx (19 \pm 1) \times 10^{-4} \ \text{MeV} \quad (46)$$

or

$$|M(\phi \to f_0\gamma)| \approx (1.11 \pm 0.42) \ \text{GeV}^{-1}, \quad (47)$$

as the branching rate for $\phi \to f_0\gamma$ is $(4.47 \pm 0.21) \times 10^{-4}$.

Last we note that the KLOE observed branching ratio (BR) is [31]

$$\mathrm{BR}(\phi\to f_0\gamma/a_0\gamma)=6.1\pm0.6. \tag{48}$$

Because we know that ϕ is dominantly $\bar{s}s$, this BR Eq. (48) being much greater than unity strongly suggests that $a_0(985)$ is mostly $\bar{n}n$ and $f_0(980)$ is mostly $\bar{s}s$. The latter assumption we have continually made throughout this paper, while it had been a conclusion of Ref. [16] (see also Ref. [32]).

VIII. *W*-EMISSION WEAK DECAY RATES

In this section we study the five weak decays $K^+\to\pi^0\pi^+$, $D^+\to\bar{K}^0\pi^+$, $D^+\to\sigma\pi^+$, $D^+\to\pi^0\pi^+$, and $D_s\to f_0(980)\pi^+$, via tree-level *W*-emission graphs. Recalling from Refs. [16] and [32], the amplitudes due to *W* emission are,[5] for $f_\pi=(92.42\pm0.27)$ MeV,

$$|M(K^+\to\pi^0\pi^+)|=\frac{G_F|V_{ud}||V_{us}|}{2\sqrt{2}}f_\pi(m^2_{K^+}-m^2_{\pi^0})$$
$$=(1.837\pm0.020)\times10^{-8}\ \mathrm{GeV}, \tag{49}$$

near data [1] $(1.832\pm0.007)\times10^{-8}$ GeV,

$$|M(D^+\to\bar{K}^0\pi^+)|=\frac{G_F|V_{ud}||V_{cs}|}{2}f_\pi(m^2_{D^+}-m^2_{\bar{K}^0})$$
$$=(177\pm27)\times10^{-8}\ \mathrm{GeV}, \tag{50}$$

near data [1] $(136\pm6)\times10^{-8}$ GeV, and

$$|M(D_s^+\to f_0\pi^+)|=\frac{G_F|V_{ud}||V_{cs}|}{2}f_\pi(m^2_{D_s^+}-m^2_{f_0})$$
$$=(159\pm25)\times10^{-8}\ \mathrm{GeV}, \tag{51}$$

near data [1] $(178\pm40)\times10^{-8}$ GeV. In the latter case we have assumed that $f_0(980)$ is all $\bar{s}s$.

Now we also consider $D\to\pi^0\pi^+$ and $D\to\sigma\pi^+$ (with $m_\sigma=650$ MeV), again in this *W*-emission scheme, predicting

$$|M(D^+\to\pi^0\pi^+)|=\frac{G_F|V_{ud}||V_{cd}|}{2\sqrt{2}}f_\pi(m^2_{D^+}-m^2_{\pi^0})$$
$$=(28.9\pm2.1)\times10^{-8}\ \mathrm{GeV}, \tag{52}$$

near data [1] $(38.6\pm5.4)\times10^{-8}$ GeV (also see Ref. [33], with $p_{cm}=925$ MeV), and

$$|M(D^+\to\sigma\pi^+)|=\frac{G_F|V_{ud}||V_{cd}|}{2\sqrt{2}}f_\pi(m^2_{D^+}-m^2_\sigma)$$
$$\simeq25.5\times10^{-8}\ \mathrm{GeV}, \tag{53}$$

near[6] recent data [1] $(37.6\pm4.5)\times10^{-8}$ GeV. This latter amplitude follows from the decay rate (with $p_{cm}=815$ MeV, $\tau=1051\times10^{-15}$ s)

$$\Gamma_{D^+\sigma\pi^+}=\frac{p_{cm}}{8\pi m^2_{D^+}}|M(D^+\to\sigma\pi^+)|^2$$
$$=\frac{h}{2\pi\tau}(2.1\pm0.5)\times10^{-3}$$
$$=(1.32\pm0.31)\times10^{-15}\ \mathrm{GeV}. \tag{54}$$

Not only are the above $D^+\to\pi^0\pi^+$ and $D^+\to\sigma\pi^+$ *W*-emission amplitudes near data, they are even of about the same magnitude. This is another example of the σ and π being chiral partners [8].

IX. SUMMARY AND CONCLUSIONS

Throughout this paper we have dealt with all ground-state mesons as $\bar{q}q$ nonets in the context of the LσM. In Sec. II we studied SU(3) mass splittings for $\mathcal{V}$ and $\mathcal{S}\bar{q}q$ mesons, with $\mathcal{V}$ loosely bound states, and $\mathcal{P}$, $\mathcal{S}$ tighter $\bar{q}q$ elementary particles. In Sec. III we reviewed *qqq* octet and decuplet baryons. In Sec. IV we briefly summarized the quark-level LσM theory, while in Sec. V we explained the accidental degeneracy of the $a_0(985)$ and $f_0(980)$ scalars. In Sec. VI we computed a few strong scalar-meson decay widths, while in Sec. VII we performed a similar analysis for some electromagnetic decays involving scalar mesons. Finally, in Sec. VIII we employed *W*-emission graphs to describe several hadronic weak-decay processes.

The usual field-theory picture is that meson masses should appear quadratically and baryon masses linearly in Lagrangian models based on the Klein-Gordon and Dirac equations. However, in Secs. II and III we studied both mesons and baryons in a *linear*-mass SU(3)-symmetry Taylor-series sense. Instead, in Sec. V we studied symmetry breaking in the IMF, with $E=[p^2+m^2]^{1/2}\approx p[1+m^2/2p^2+\cdots]$. Here, between brackets, the 1 indicates the symmetry limit, and the *quadratic* mass term means that both meson and baryon masses are *squared* in the mass-breaking IMF for $\Delta S=1$ ESLs. While the former mass-splitting approach (with linear masses) fits all $\mathcal{V}$, $\mathcal{S}$, $\mathcal{O}$, and $\mathcal{D}$ ground-state SU(3)-flavor multiplets, so does the latter (with quadratic masses) for the IMF-ESLs. Nevertheless, Nambu-Goldstone pseudoscalars $\mathcal{P}$ *always* involve *quadratic* masses. Both approaches suggest that all ground-state mesons ($\mathcal{P}$, $\mathcal{S}$, $\mathcal{V}$) are $\bar{q}q$ states, while baryons ($\mathcal{O}$, $\mathcal{D}$) are *qqq* states. This picture is manifest

[5]We use here $G_F=1.166\,39(1)\times10^{-5}\ \mathrm{GeV}^{-2}$, $|V_{ud}|=0.9735\pm0.0008$, $|V_{us}|=0.2196\pm0.0023$, $|V_{cd}|=0.224\pm0.016$, $|V_{cs}|=1.04\pm0.16$, $m_{D^+}=(1869.4\pm0.5)$ MeV, and $m_{D_s^+}=(1969.0\pm1.4)$ MeV.

[6]At this point we should keep in mind that the uncertainty in m_σ is of the order of m_σ!

in the quark-level LσM of Sec. IV. The accidental scalar degeneracy between the $\bar{s}s$ $f_0(980)$ and the $\bar{n}n$ $a_0(985)$ was explained in Sec. V, via the IMF quadratic-mass ESLs—also compatible with mesons being $\bar{q}q$ and baryons qqq states.

Concerning the mass splittings in general, we observed the remarkable feature that the real parts of masses of resonances in mesonic and baryonic ground-state multiplets nicely follow an SU(3) splitting pattern, despite the enormous disparities in decay widths and thus in the imaginary parts. This may be understood in the unitarized picture of Ref. [15], in which both real and virtual decay channels contribute to the physical masses of, e.g., the scalar mesons as dressed $\bar{q}q$ states. We also verified in Secs. VI, VII, and VIII that mesonic decay rates can be simply explained on the basis of the flavor and chiral symmetry underlying the quark-level LσM. This is another indication that the lowest lying mesons are all $\bar{q}q$, while the considered baryons are qqq.

So far we have taken the mass and coupling parameters of the quark-level LσM—in particular m_σ—to be real numbers (“narrow-width approximation”). A recently developed formalism [34] may allow us to go beyond this approximation in the near future.

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APPENDIX A: BOOTSTRAPPING $G_{\sigma\pi\pi}\to G'$ AND $\lambda_{\rm box}\to\lambda_{\rm tree}$

The $\sigma\pi\pi$ or $\sigma\sigma\sigma$ u,d quark triangle graphs [3,35] induced by $\mathcal{L}^{\rm int}_{\rm L\sigma M}$ in Eq. (12) implies in the CL

$$g_{\sigma\pi\pi}=-8ig^3N_c m_q\int đ^4p[p^2-\hat{m}^2]^{-2}=2gm_q, \tag{A1}$$

due to the logarithmically divergent gap equation (LDGE) [36,37]

$$1=-4ig^2N_c\int đ^4p[p^2-\hat{m}^2]^{-2}. \tag{A2}$$

Then the GTR Eq. (14), together with $m_\sigma=2m_q$, reduces Eq. (A1) to

$$g_{\sigma\pi\pi}=2gm_q=\frac{m_\sigma^2}{2f_\pi}=g', \tag{A3}$$

the tree-level cubic meson LσM coupling in Eq. (13). Also the $\pi\pi\pi\pi$ (or $\sigma\sigma\sigma\sigma$, $\pi\pi\sigma\sigma$) quark box graph [3,35] generates in the CL

$$\lambda_{\rm box}=-8ig^4N_c\int đ^4p[p^2-\hat{m}^2]^{-2}=2g^2=\frac{g'}{f_\pi}=\lambda_{\rm tree}, \tag{A4}$$

again due to the LDGE (A2). Note that the cubic and quartic LσM tree couplings in Eq. (13) are dynamically loop-generated in Eqs. (A1) and (A4), respectively. Both are analytic, nonperturbative bootstrap procedures [3].

APPENDIX B: DIM-REG LEMMA GENERATING QUARK AND σ MASS

The Nambu $\delta m_q=m_q$ (constituent-) quark mass-gap tadpole graph [3,35] generates quark mass. However, this quadratically divergent term, subtracted from the LDGE (A2), in fact scales to quark mass *independently* of quadratically divergent terms, by virtue of the dimensional-regularization (dim-reg) lemma [3]

$$I=\int đ^4p\left[\frac{m^2}{(p^2-m^2)^2}-\frac{1}{p^2-m^2}\right]$$
$$=\lim_{\ell\to 2}\frac{im^{2\ell-2}}{(4\pi)^\ell}[\Gamma(2-\ell)+\Gamma(1-\ell)]=-im^2(4\pi)^{-2}, \tag{B1}$$

due to the gamma-function *identity* $\Gamma(2-\ell)+\Gamma(1-\ell)=\Gamma(3-\ell)/(1-\ell)\to-1$ as $\ell\to 2$. To reconfirm this dim-reg-lemma “trick” (B1), we invoke the partial-fraction *identity*

$$\frac{m^2}{(p^2-m^2)^2}-\frac{1}{p^2-m^2}=\frac{1}{p^2}\left[\frac{m^4}{(p^2-m^2)^2}-1\right], \tag{B2}$$

integrated via $\int đ^4p$ as in the I integral on the left-hand side of Eq. (B1). Then dropping the massless-tadpole integral $\int đ^4p/p^2=0$ (as done in dimensional, analytic, zeta-function, and Pauli-Villars regularizations [3,38]), and Wick rotating $d^4p=i\pi^2p_E^2dp_E^2$, the Euclidean integral becomes

$$I=-\frac{im^4}{(4\pi)^2}\int_0^\infty\frac{dp_E^2}{(p_E^2+m^2)^2}=-\frac{im^2}{(4\pi)^2}, \tag{B3}$$

identical to the right-hand side of Eq. (B1).

In order to further justify the neglect of $\int đ^4p/p^2$, we invoke the Karlson trick [39] (long advocated by Schwinger)

$$\frac{d}{dm^2}\int\frac{d^4p}{p^2-m^2}=\int\frac{d^4p}{(p^2-m^2)^2}, \tag{B4}$$

and compute [40]

$$(2\pi)^4\frac{dI}{dm^2}=\int\frac{d^4p}{(p^2-m^2)^2}+2m^2\int\frac{d^4p}{(p^2-m^2)^3}$$
$$-\frac{d}{dm^2}\int\frac{d^4p}{p^2-m^2}, \tag{B5}$$

with the first and third terms cancelling due to Eq. (B4). Then the remaining, finite second term in Eq. (B5) gives

$$(2\pi)^4 \frac{dI}{dm^2} = 2m^2\left(-\frac{i\pi^2}{2m^2}\right) = -i\pi^2, \tag{B6}$$

which is the *same* result as differentiating the dim-reg lemma (B1):

$$(2\pi)^4 \frac{dI}{dm^2} = (-i\pi^2)\frac{dm^2}{dm^2} = -i\pi^2. \tag{B7}$$

So far we have only assumed $\int đ^4p/p^2$ is independent of m^2, so that $(d/dm^2)\int đ^4p/p^2 = 0$.

But to demonstrate that the $\int dm^2$ integration constant *vanishes*, i.e., $\int d^4p/p^2 = \Lambda^2 = 0$, we invoke the implied dimensional-analysis relations

$$\int \frac{d^4p}{p^2} = 0, \quad \int \frac{d^4p}{p^2 - m^2} \propto m^2, \quad \int \frac{d^4p}{p^2 - m_\sigma^2} \propto m_\sigma^2 \tag{B8}$$

to solve Lee's null-tadpole sum [41], which characterizes the true vacuum for $N_f = 2$ as [3]

$$(2m_q)^4 N_c = 3m_\sigma^4 \tag{B9}$$

(with the factor of 3 due to σ-σ-σ combinatorics) in the CL $m_\pi = 0$, meaning $N_c = 3$ when $m_\sigma = 2m_q$. Thus, $\int d^4p/p^2$ indeed vanishes as suggested [3,38].

APPENDIX C: KINEMATIC INFINITE-MOMENTUM FRAME

The infinite-momentum frame (IMF) has two virtues: (i) $E = [p^2 + m^2]^{1/2} \approx p + m^2/2p + \cdots$, for $p \to \infty$, requires *squared* masses when the lead term p is eliminated, using SU(3) formulas with coefficients $1+3=2+2$, as, e.g., the Gell-Mann–Okubo linear mass formula $\Sigma + 3\Lambda = 2N + 2\Xi$, valid to 3%; (ii) when $p\to\infty$, dynamical tadpole graphs are suppressed [42]. In fact, $\Sigma^2 + 3\Lambda^2 = 2N^2 + 2\Xi^2$ is also valid empirically to 3%. This squared qqq baryon mass formula can be interpreted as a $\Delta S = 1$ ESL, which holds for both $\mathcal{O}$ and $\mathcal{D}$ baryons [14],

$$\Sigma\Lambda - N^2 \approx \Xi^2 - \Sigma\Lambda \approx \frac{1}{2}(\Xi^2 - N^2) \approx 0.43 \ \text{GeV}^2,$$

$$\Sigma^{*2} - \Delta^2 \approx \Xi^{*2} - \Sigma^{*2} \approx \Omega^2 - \Xi^{*2} \approx \frac{1}{2}(\Omega^2 - \Sigma^{*2}) \approx 0.43 \ \text{GeV}^2. \tag{C1}$$

However, the $\bar{q}q$ pseudoscalar and vector $\Delta S = 1$ ESLs have about one-half this scale (also empirically valid to 3%), viz.

$$m_K^2 - m_\pi^2 \approx m_{K*}^2 - m_\rho^2 \approx m_\phi^2 - m_{K*}^2 \approx \frac{1}{2}(m_\phi^2 - m_\rho^2) \approx 0.22 \ \text{GeV}^2, \tag{C2}$$

as roughly do the $\bar{q}q$ scalars found in Sec. II, i.e.,

$$m_{\kappa(800)}^2 - m_{\sigma_n(650)}^2 \approx m_{\sigma_s(940)}^2 - m_{\kappa(800)}^2 \approx 0.22\cdots 0.24 \ \text{GeV}^2. \tag{C3}$$

This approximate factor of 2 between Eqs. (C1) and Eqs. (C2) and (C3) is because there are two $\Delta S = 1$ qqq transitions, whereas there is only one $\Delta S = 1$ transition for $\bar{q}q$ configurations.

So if we take Eq. (B3) as physically meaningful, we may write

$$2m_\kappa^2 \approx m_{\sigma(600)}^2 + m_{f_0(980)}^2 \approx m_{\sigma_n(650)}^2 + m_{\sigma_s(940)}^2 \approx 1.31\cdots 1.32 \ \text{GeV}^2, \tag{C4}$$

yielding $m_\kappa \approx 811$ MeV close to experiment, which again suggests these scalars are $\bar{q}q$ states.

These IMF quadratic mass schemes, along with the non-CL NJL-LσM κ mass $m_{\kappa(809)} = 2\sqrt{\hat{m}m_s} = 809$ MeV or the averaged[7] mass value of 800 MeV, again suggest [as do the empirical scales of Eqs. (C2) and (C3) vs Eqs. (C1)] that *all* ground-state meson nonets are $\bar{q}q$, whereas the baryon octet and decuplet are qqq states.

[7] We average here approximately between the non-CL NJL-LσM mass value and the respective value in the CL.

[1] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002); see p. 754 for non-$\bar{q}q$ states.

[2] For quadratic-mass pseudoscalars ($\mathcal{P}$), the Gell-Mann–Okubo formula $m_\pi^2 + 3m_{\eta_8}^2 = 4m_K^2$ implies $m_{\eta_8} = 567$ MeV, 3% greater than the observed [1] $m_\eta = 547$ MeV. Then the η-η'-mixing sum rule $m_\eta^2 + m_{\eta'}^2 = m_{\eta_1}^2 + m_{\eta_8}^2$ requires $m_{\eta_1} = 947$ MeV, 3% greater than the U(1)-anomaly mass 917 MeV. For the U(1) problem, see e.g., A.N. Patrascioiu and M.D. Scadron, Phys. Rev. D **22**, 2054 (1980); R. Delbourgo and M.D. Scadron, *ibid.* **28**, 2345 (1983); S.R. Choudhury and M.D. Scadron, Mod. Phys. Lett. A **1**, 535 (1986); D. Kekez, D. Klabucar, and M.D. Scadron, J. Phys. G **26**, 1335 (2000).

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$$M(a_0\to 2\gamma)_{\text{kaon-loop}}=\frac{2\alpha}{\pi m_{a_0}^2}\,\frac{m_{a_0}^2-m_{K^\pm}^2}{2f_K}\left[-\frac{1}{2}+\xi_K I(\xi_K)\right].$$

With $\xi_K=m_{K^\pm}/m_{a_0}^2\approx 0.251\pm 0.008>1/4$ and $f_K=(113\pm 1.04)$ MeV, we get

$$I(\xi_K)\overset{\xi_K>1/4}{=}2\arcsin^2(\sqrt{4\xi_K})^{-1}=4.48\pm 1.25,\text{ and}$$
$$M(a_0\to 2\gamma)_{\text{kaon-loop}}=(0.97\pm 0.43)\times 10^{-2}\text{ GeV}^{-1}.$$

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