Neutrino electroweak radius

Kazuo Fujikawa
Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

Robert Shrock
C. N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794, USA

(Received 29 September 2003; published 30 January 2004)

We study a combination of amplitudes for neutrino scattering that can isolate a (gauge-invariant) difference of chirality-preserving neutrino electroweak radii for ν_μ and ν_τ. This involves both photon and Z_μ exchange contributions. It is shown that the construction singles out the contributions of the hypercharge gauge field B_μ in the standard model. We comment on how gauge-dependent terms from the charge radii cancel with other terms in the relative electroweak radii.

DOI: 10.1103/PhysRevD.69.013007 PACS number(s): 13.15.+g, 11.15.−q, 12.15.Lk

I. INTRODUCTION

Electromagnetic properties of neutrinos are of fundamental importance and serve as a probe of whether neutrinos are Dirac or Majorana particles and of new physics beyond the standard model (SM) [1–30]. In general, the matrix element of the electromagnetic current between initial and final neutrinos ψ_j and ψ_k with 4-momenta p and p' is

$$\begin{aligned}\langle\psi_k(p')|J_\lambda|\psi_j(p)\rangle &= e\bar{\psi}_k(p')\Big\{\gamma_\lambda[F_1^V(q^2)+F_1^A(q^2)\gamma_5]\\ &+i\frac{\sigma_{\lambda\rho}q^\rho}{m_{\nu_k}+m_{\nu_j}}[F_2^V(q^2)+F_2^A(q^2)\gamma_5]\\ &+q_\lambda[F_3^V(q^2)+F_3^A(q^2)\gamma_5]\Big\}\psi_j(p), \end{aligned}\tag{1.1}$$

where e is the electromagnetic coupling and $q=p-p'$. The form factors $F_n^{V,A}$ are matrices in the space of neutrino mass eigenstates, and their (kj) elements appear in the above amplitude [31]. In the diagonal case $j=k$, $eF_2^V(0)_{jj}/(2m_{\nu_j})$ is the magnetic dipole moment of the mass eigenstate ν_j and $-ieF_2^A(0)_{jj}/(2m_{\nu_j})$ gives the electric dipole moment. A Dirac neutrino in the standard model generalized to include such masses has a magnetic moment $\mu_{\nu_j}=3eG_Fm_{\nu_j}/(8\pi^2\sqrt{2})$ [9], while in models with right-handed charged currents, this quantity also involves terms depending on charged lepton masses [5,8]. A Dirac neutrino may also have a *CP*-violating electric dipole moment (e.g. [14]). For a Majorana neutrino, with $\psi_j=\psi_j^c$, these operators vanish identically, so that $\mu_{\nu_j}=d_{\nu_j}=0$. (Below, for pedagogical purposes we shall consider a special case where neutrino masses vanish.)

It is also of interest to consider the chirality-preserving terms $\bar{\psi}_k\gamma_\lambda\psi_j$ and $\bar{\psi}_k\gamma_\lambda\gamma_5\psi_j$ in Eq. (1.1) and the associated form factors $F_1^V(q^2)$ and $F_1^A(q^2)$. Although the electric neutrality of the neutrino means that $F_1^V(0)_{jj}=0$, one may consider the Taylor series expansions of $F_1^{V,A}(q^2)_{jj}$ as functions of q^2, in particular,

$$\langle r^2\rangle_{\nu_j}=6\frac{dF_1^V(q^2)_{jj}}{dq^2}\bigg|_{q^2=0}\tag{1.2}$$

which is the neutrino charge radius squared. This is often called simply the neutrino charge radius, and we shall follow this convention. By itself, the neutrino charge radius is gauge-dependent and hence is not a physical observable [1,7] (contrary to the recent claim in Refs. [28,29]). Explicit calculations in unified renormalizable electroweak gauge theories [7] using the R_ξ gauge [37,2] clearly displayed the gauge dependence of various quantities including the charge radius.

It is useful to consider the construction of a gauge-independent set of amplitudes involving the chirality-preserving terms in Eq. (1.1) that can serve to characterize neutrino properties. In this paper we shall discuss the construction of a set obtained from differences of neutrino scattering amplitudes, giving details of our note [30] and how this relates to, and differs from, the recent approach of Refs. [27–29]. Since our focus here is on constraints from gauge invariance and since for the chirality-preserving terms under consideration, neutrino masses do not play as important a role as they do in the chirality-flipping terms in Eq. (1.1), we shall make the simplification of working within the standard model with massless neutrinos. (These simplifications were also made in Refs. [27–29] and [30].) For this massless neutrino case, Dirac and Majorana neutrinos are equivalent, and there is no lepton mixing, so that the neutrino mass and group eigenstates coincide. From Eq. (1.1) the relevant matrix element is then

$$\langle\psi_j(p')|J_\lambda|\psi_j(p)\rangle=e\bar{\psi}_j(p')\gamma_\lambda(1-\gamma_5)F_1(q^2)_j\psi_j(p),\tag{1.3}$$

where $F_1^V(q^2)_{jj}=F_1^A(q^2)_{jj}\equiv F_1(q^2)_j$. (We will often drop the subscript j where it is obvious from the context.)

II. NEUTRINO SCATTERING REACTIONS

To begin, consider the tree and one-loop scattering amplitudes for the reaction

$$\nu_\mu(p)+e(k)\to\nu_\mu(p')+e(k') \tag{2.1}$$

(where all the particles appearing in the above process are physical, on-shell particles). The four-momentum transfer squared is denoted $t=q^2$ and the center-of-mass energy squared is denoted s. Consider next the high-energy limit $s/m_e^2\gg1$ and specialize to the amplitude for the reaction

$$\nu_\mu(p)+e_R(k)\to\nu_\mu(p')+e_R(k'), \tag{2.2}$$

where $e_R=P_Re$ denotes a right-handed electron (to be precise the helicity plus electron which does not coincide with the right-handed electron specified by P_R in the presence of the nonvanishing electron mass), where $P_{R,L}=(1/2)(1\pm\gamma_5)$ are chirality projection operators. The tree-level diagram for this reaction involves the exchange of a Z boson in the t-channel. An important simplifying approximation is that the electron mass is neglected for this high-energy limit even for $q^2=0$, except for an infinitesimal electron mass on internal fermion lines to control infrared divergences. With this approximation of neglecting the electron mass, so that a positive-helicity electron is equivalent to a right-handed electron, the contribution of the box diagram with $2W$ exchange, shown in Fig. 1, vanishes. As a result, one can extract the contributions from the one-photon and one-Z exchange diagrams with one-loop self-energy and vertex corrections together with $2Z$ exchange box diagrams, shown in the Appendix. This amplitude with right-handed massless electrons is gauge-independent and constitutes a part of the physical S-matrix.

We remark on a related but different approach to the analysis of the matrix element. In Refs. [27–29], the authors write the Feynman diagrams defined in the R_ξ gauge and then apply the "pinch technique," previously discussed in [32,33], to the complete set of Feynman diagrams contributing to this S-matrix element. This technique involves a rearrangement of various Feynman diagrams for an element of the physical S-matrix before the calculation of loop integrals. For a tree-level amplitude contributing to a physical S-matrix element, it is trivial to redefine the Feynman rules given by the R_ξ gauge to those of the background Feynman gauge. If one uses dimensional regularization, for example, all the Feynman diagrams are finite at the one-loop level also, and thus the rearrangement of Feynman diagrams is justified. The starting Feynman rules are identical in the conventional formulation and with this pinch technique, and thus the result is also identical, provided that the method is well-defined.

After this rearrangement, one obtains the amplitudes written in terms of the Feynman gauge in the background field method [34–36,19], which exhibits the $U(1)_{em}$ symmetry of the electromagnetic interaction explicitly. This rearrangement of various Feynman diagrams and the cancellation of the gauge parameter ξ for the physical process is consistent with the general formulation of gauge theory. It therefore follows that physical results obtained via the conventional

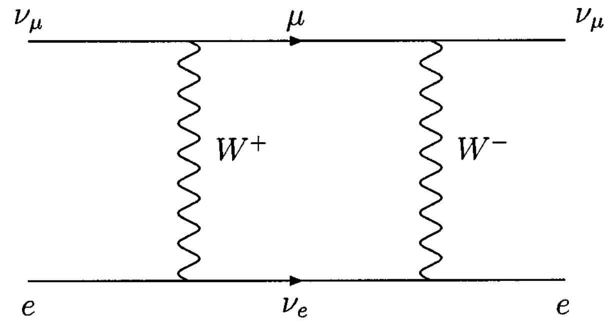

FIG. 1. $2W$-exchange graph, whose contribution vanishes for $e=e_R$ in the SM.

formulation of gauge theories and via the use of this pinch technique must be the same, since the Feynman gauge in the background field method is one of the allowed gauge conditions. However the Feynman gauge in the background field method has no privileged position among various possible gauge conditions. For example, the $U(1)_{em}$ symmetry of the electromagnetic interaction remains intact in a nonlinear R_ξ gauge [37] also.

III. METHOD USING DIFFERENCES OF NEUTRINO REACTIONS

One way to separate the contributions of the $2Z$ exchange box diagrams from those of the one-photon and one-Z exchanges involves the "neutrino-antineutrino method" utilized in Refs. [28,29] (see Ref. [15] for earlier related work), in which one considers the sum $d\sigma(\nu_\mu e_R\to\nu_\mu e_R)/dq^2+d\sigma(\overline{\nu}_\mu e_R\to\overline{\nu}_\mu e_R)/dq^2$, specifically the $q^2\to0$ limit.[1]

The idea underlying the neutrino-antineutrino method is simplified and the basic physical idea is precisely stated in a gauge-independent way if one compares the process (2.2) to the process with the charge-conjugated electron (i.e., positron) (see also Ref. [15])

$$\nu_\mu(p)+e_L^+(k)\to\nu_\mu(p')+e_L^+(k'). \tag{3.3}$$

In the massless electron case, the only interaction of the right-handed electron is given by

$$\mathcal{L}_{int}=-g'\overline{e_R}\gamma^\mu B_\mu e_R \tag{3.4}$$

[1]The crucial Eq. (2) in Ref. [28], which is the basis of the analysis of Refs. [28,29]

$$\overline{v}(p_1)\gamma_\mu P_L v(p_2)=-\overline{u}(p_2)\gamma_\mu P_R u(p_1), \tag{3.1}$$

is however not justified, as can be confirmed by considering the time component of the current for $p_1=p_2$; in this case, this relation leads to

$$\|P_L v(p_1)\|^2=-\|P_R u(p_1)\|^2 \tag{3.2}$$

and thus $P_Lv(p_1)=P_Ru(p_1)=0$.

where B_μ stands for the hypercharge gauge field associated with the $U(1)_Y$ factor of the standard model. Recall that B_μ is a linear combination of the photon A_μ and Z_μ fields satisfying

$$g'B_\mu=\frac{g'}{\sqrt{g^2+(g')^2}}(gA_\mu-g'Z_\mu)=eA_\mu-G\sin^2\theta_W Z_\mu \tag{3.5}$$

with g and g', respectively, standing for the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants and $G=\sqrt{g^2+(g')^2}$. If one rewrites the interaction (3.4) with the charge-conjugated variables defined by

$$e(x)=-C^{-1}(\overline{e^c})^T(x),\quad \overline{e}(x)=(e^c)^T(x)C, \tag{3.6}$$

it can be expressed equivalently (considering a suitable limit of the point-splitting definition of the current and the anticommuting property of the electron field) as

$$\mathcal{L}_{int}=g'\overline{e^c}_L\gamma^\mu B_\mu e^c_L. \tag{3.7}$$

If one compares the above two processes (2.2) and (3.3) with the identical kinematical configurations, one can distinguish the amplitudes with odd powers in $g'B_\mu$ (in the electron sector) from the amplitudes with even powers in $g'B_\mu$ (i.e., the $2Z_\mu$ exchange box diagrams), except for the wave function $P_Lu(k)$ for e^c_L replacing $P_Ru(k)$ for e_R, by noting that

$$\langle e^c_L(x)\overline{e^c}_L(y)\rangle=P_L\langle e^c(x)\overline{e^c}(y)\rangle P_R=P_L\langle e(x)\overline{e}(y)\rangle P_R \tag{3.8}$$

in the Dyson expansion of the S-matrix. The appearance of the positive-energy solutions in both cases[2] and the difference between $P_Lu(k)$ and $P_Ru(k)$ do not matter in the evaluation of the forward cross section [28,29]. The interference term of box diagrams with the Z_μ-exchange tree diagram, which is the relevant quantity in the lowest order process beyond the tree process, thus changes sign between the above two processes. One can thus eliminate the Z_μ box diagram contributions by a physical operation by considering the sum of the two cross sections (in the limit $q^2\to 0$)

$$\frac{d\sigma(\nu_\mu+e_R\to\nu_\mu+e_R)}{dq^2}+\frac{d\sigma(\nu_\mu+e^c_L\to\nu_\mu+e^c_L)}{dq^2}. \tag{3.9}$$

That is, since $d\sigma/dq^2=|A_{tree}|^2+2\,\mathrm{Re}(A_{tree}A^*_{1-loop})$ plus higher-order terms, and since for the second term, A_{tree} reverses sign for the reaction with e_R replaced by e^c_L while the $2Z$ exchange graphs do not, this sum removes terms from the $2Z$ exchange box diagrams. These terms from the $2Z$ box diagrams are gauge-independent by themselves, as can be explicitly confirmed; the gauge parameter for Z_μ cancels among the box and crossed diagrams (see also [20,23]). The physical separation of the $2Z$ exchange contributions is thus perfectly consistent with the basic principle of gauge theory.

This physical separation of these box diagrams is an important operation, but it is clear that what one measures after this separation is the form factor of the neutrino detected by the hypercharge gauge field B_μ, which is a linear combination of the physical fields A_μ and Z_μ. The neutrino-antineutrino method with a massless electron gives a physical basis for writing the gauge-independent one-loop amplitude (by excluding the $2Z$ box diagrams and neglecting the wave function renormalization factors for simplicity)

$$e^2\overline{u}_R\left[\frac{F_1(q^2)}{q^2}+\frac{F_Z^{\nu\nu}(q^2)+F_Z^{ee}(q^2)}{q^2-M_Z^2}-\frac{\Pi_{ZZ}(q^2)}{(q^2-M_Z^2)^2}-\frac{\Pi_{\gamma Z}(q^2)}{q^2(q^2-M_Z^2)}\right]\gamma^\alpha u_R\overline{\nu}_\mu\gamma_\alpha(1-\gamma_5)\nu_\mu. \tag{3.10}$$

Here $F_1(q^2)$ and $F_Z^{\nu\nu}(q^2)$ denote the ν_μ vertex functions measured by B_μ, and $F_Z^{ee}(q^2)$ is the electron vertex function measured by Z_μ. The self-energy corrections $\Pi_{ZZ}(q^2)$ and $\Pi_{\gamma Z}(q^2)$ stand for the two-point functions for B_μ-Z_μ coupling. The gauge independence of Eq. (3.10) is established by a simple argument of the gauge independence of the physical S-matrix without referring to the technical details such as the pinch technique or the background field method.

IV. RELATIVE ELECTROWEAK RADII

One can further consider the difference of Eq. (3.10) and the corresponding amplitude with ν_μ replaced by ν_τ by measuring the difference [28,29]

$$\left[\frac{d\sigma(\nu_\mu+e_R\to\nu_\mu+e_R)}{dq^2}+\frac{d\sigma(\nu_\mu+e^c_L\to\nu_\mu+e^c_L)}{dq^2}\right]-\left[\frac{d\sigma(\nu_\tau+e_R\to\nu_\tau+e_R)}{dq^2}+\frac{d\sigma(\nu_\tau+e^c_L\to\nu_\tau+e^c_L)}{dq^2}\right]. \tag{4.1}$$

---

[2] We expand the generic Dirac field as

$$\psi(x)=\sum_s\int\frac{d^3p}{\sqrt{(2\pi)^3 2p_0}}[u(p,s)b(p,s)e^{-ipx}+v(p,s)d^\dagger(p,s)e^{ipx}]$$

with the charge conjugation relations

$$C\gamma^\mu C^{-1}=-(\gamma^\mu)^T,\quad C\gamma_5C^{-1}=\gamma_5^T,$$
$$v^T(p,s)C=\overline{u}(p,s),\quad -C^{-1}\overline{v}^T=u(p,s),$$
$$C^\dagger C=1, C^T=-C.$$

We then have

$$\psi^c(x)=-C^{-1}\overline{\psi}^T=\sum_s\int\frac{d^3p}{\sqrt{(2\pi)^3 2p_0}}[v(p,s)b^\dagger(p,s)e^{ipx}+u(p,s)d(p,s)e^{-ipx}].$$

In this way one can eliminate the common terms $F_Z^{ee}(q^2)$, $\Pi_{ZZ}(q^2)$, and $\Pi_{\gamma Z}(q^2)$ in the two amplitudes, since one can measure the amplitude (3.10) and its ν_τ analogue themselves as interference terms with the leading tree-level Z exchange diagram. One thus arrives at the physical quantity measured by the hypercharge gauge field B_μ

$$e^2 \lim_{q^2\to 0}\left[\frac{F_1(q^2)}{q^2}+\frac{F_Z^{\nu\nu}(q^2)}{q^2-M_Z^2}\right]\Bigg|_{\nu_\mu} - e^2 \lim_{q^2\to 0}\left[\frac{F_1(q^2)}{q^2}+\frac{F_Z^{\nu\nu}(q^2)}{q^2-M_Z^2}\right]\Bigg|_{\nu_\tau} \tag{4.2}$$

which is gauge-independent. The gauge independence of Eq. (4.2) is equivalent to the gauge independence of Eq. (3.10) and its ν_τ analogue. However, the physical separation of the photon exchange contributions from the Z_μ exchange contributions in Eq. (4.2), namely the separation of $F_1(q^2)$ from $F_Z^{\nu\nu}(q^2)$, is not obvious in the standard model. Indeed, previous detailed calculations [7,18] of $F_1(q^2)/q^2|_{q^2=0}$ show that the subleading term of the order m_l^2/M_W^2 with $l=\mu$ or τ in the relative charge radius (squared)

$$\Delta\langle r^2\rangle = 6 \lim_{q^2\to 0}\left[\frac{F_1(q^2)}{q^2}\Bigg|_{\nu_\mu} - \frac{F_1(q^2)}{q^2}\Bigg|_{\nu_\tau}\right] \tag{4.3}$$

depends on the gauge parameter in R_ξ gauge and diverges as $\xi\to 0$ like

$$\frac{3g^2}{128\pi^2 M_W^2}\left[\frac{m_\mu^2-m_\tau^2}{M_W^2}\right]\ln\left(\frac{1}{\xi}\right) \tag{4.4}$$

[see Eqs. (2.30) and (2.54) in [7] and also Eq. (4a) in [18], which confirmed the calculation in [7]]. This shows that a gauge-independent separation of $F_1(q^2)$ from $F_Z^{\nu\nu}(q^2)$ in Eq. (4.2) is not possible, since if it were possible, the above gauge parameter would not appear. The leading contribution to the relative charge radius (squared) in Eq. (4.3) (see, for example, [7,18])

$$\Delta\langle r^2\rangle_{leading} = \frac{g^2}{16\pi^2 M_W^2}\left[\ln\frac{M_W^2}{m_\mu^2} - \ln\frac{M_W^2}{m_\tau^2}\right] \tag{4.5}$$

is formally gauge-independent, but we emphasize that the separation of the leading term from the subleading term is not well-defined for the gauge-dependent relative charge radius, since the subleading term can be made arbitrarily large by gauge choice, as is evident in Eq. (4.4).

The relative electroweak radius defined in Eq. (4.2) as a combination of $F_1(q^2)$ and $F_Z^{\nu\nu}(q^2)$ is gauge-independent, and only in this combination can one separate the leading term from the subleading term. Our analysis of Eq. (4.5) is consistent with the result in Refs. [28,29], which obtains $\Delta F_Z^{\nu\nu}(0)=0$ up to terms of the order [38] m_l^2/M_W^2 with $l=\mu$ or τ; that is, the leading term of $\Delta F_Z^{\nu\nu}(0)=0$ vanishes and is thus gauge independent. The leading term of the gauge independent electroweak radius in Eq. (4.2) thus agrees with the value (4.5), which is, in fact, gauge-independent. The subleading term in $\Delta F_Z^{\nu\nu}(0)$ is nonvanishing and is required to cancel the gauge dependence of the subleading term in $\Delta F_1(q^2)/q^2$ at $q^2=0$ in Eq. (4.2), namely, Eq. (4.4).

We emphasize that the nonvanishing subleading term in $\Delta F_Z^{\nu\nu}(0)$ is a manifestation of the gauge dependence of the relative neutrino charge radius in the pinch technique; that is, one cannot eliminate the Z_μ contamination in a gauge-invariant manner, since only the combination (4.2) is gauge-independent. The nonexistence of a strictly gauge-independent relative neutrino charge radius thus persists both in the conventional formulation and with the pinch technique. Our analysis in the framework of conventional gauge theory thus clearly explains what is going on in the complicated pinch technique analysis in [28,29]. Moreover, our analysis shows that the result for the gauge-independent relative neutrino charge radius in [28,29] arises from the extra approximation of neglecting the subleading terms and not from the use of the pinch technique. If one neglects the subleading terms, one can readily establish the gauge independence of the relative neutrino charge radius in the conventional formulation also. The pinch technique does not produce any result different from that of the conventional formulation.

Our analysis clearly shows that the relative neutrino charge radius is not gauge-independent, much less the charge radius for an individual species of neutrino. As for the detailed analysis on the basis of the pinch technique [39], this may be useful to confirm the gauge independence of the physical S-matrix element. But one cannot infer that each part of the total amplitude separately has a gauge-independent physical meaning simply because the gauge parameter formally disappeared in the operation of the pinch technique; the Feynman rules in the Feynman gauge would always be free of gauge parameters in such a sense. Only the total amplitude for the S-matrix is gauge-independent. Without the photon pole in the above neutrino scattering process, no general principle can be used to argue for the gauge independence of the one-photon exchange amplitude. To establish the gauge independence of the relative neutrino charge radius, one would need to restore the gauge parameter in the photon exchange diagrams without changing the neutrino charge radius, as is demonstrated for the case of the muon magnetic moment in Ref. [2]. This is, however, equivalent to the gauge parameter independence of the vertex corrections to the photon exchange diagrams by themselves before one applies the pinch technique.

Our analysis suggests that a useful quantity is the "relative electroweak radius" in Eq. (4.2) measured by the hypercharge gauge field B_μ in the standard model. This relative electroweak radius is gauge-independent, and its leading term agrees with the leading term of the relative neutrino charge radius in Refs. [27–29] defined by the Feynman gauge in the background field method. We believe that our definition of the relative electroweak radius is conceptually simple and clear.

V. DISCUSSION AND CONCLUSION

The notion of the physical neutrino charge radius (squared) would be important in the analysis of observables in gauge theory if this quantity were gauge-invariant. However, as was established long ago [1,7], it is not gauge-invariant and hence is not a physical observable (contrary to the recent claim in Refs. [28,29]). In this paper, in the limit $s/m_e^2 \gg 1$, we have constructed for the standard model a combination of terms that provides a gauge-invariant quantity that may be regarded as a relative electroweak chirality-preserving quantity, a sort of gauge-invariant generalization of a neutrino charge radius. We have shown that this involves the hypercharge gauge field B_μ in a natural way. But, as in [30], we do not find that the relative neutrino charge radius (measured by the photon) is gauge-invariant; the result in Refs. [27–29] is primarily a result of the neglect in these references of subleading terms.

Since one motivation for studying the neutrino charge radius would be to probe for new physics, we note that, in general, using the reaction $\nu_\mu + e_R \to \nu_\mu + e_R$ does not simplify the analysis. Consider models beyond the standard model with strong-electroweak gauge groups $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $G_{422} = SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, where PS stands for Pati-Salam [40]. Here the first step in the extraction process fails; the $A_L^\pm$ and $A_R^\pm$ mix to form the mass eigenstates $W_{1,2}^\pm$, and hence one is not able to remove the $2W$ exchange diagrams by considering $\nu_\mu + e_R \to \nu_\mu + e_R$. The standard model is thus special in allowing the separation of single-particle, i.e., B_μ, exchange diagrams from the box diagrams.

Neutrino masses affect neutrino electromagnetic properties, and our analysis also applies to the gauge-dependence of both the vector neutrino charge radius $dF_1^V(q^2)/dq^2|_{q^2=0}$ (which vanishes anyway for Majorana neutrinos) and the axial-vector analogue $dF_1^A(q^2)/dq^2|_{q^2=0}$.

ACKNOWLEDGMENTS

We thank P. A. Grassi for useful discussions. K.F. thanks the members of C.N. Yang Institute for Theoretical Physics at Stony Brook for hospitality during visits. The research of R.S. was partially supported by the grant NSF-PHY-00-98527. R.S. thanks the theory group of the University of Tokyo and the organizers of the Fuji-Yoshida Summer Institute 2003 for hospitality during visits.

APPENDIX

In this appendix we display one-loop diagrams in Figs. 2–6 for the reaction $\nu_\mu + e_R \to \nu_\mu + e_R$ in the SM, in addition to the $2W$-exchange diagram, whose contribution vanishes for $m_e \to 0$. We show these graphs for unitary gauge, but have analyzed the process in the full R_ξ gauge.

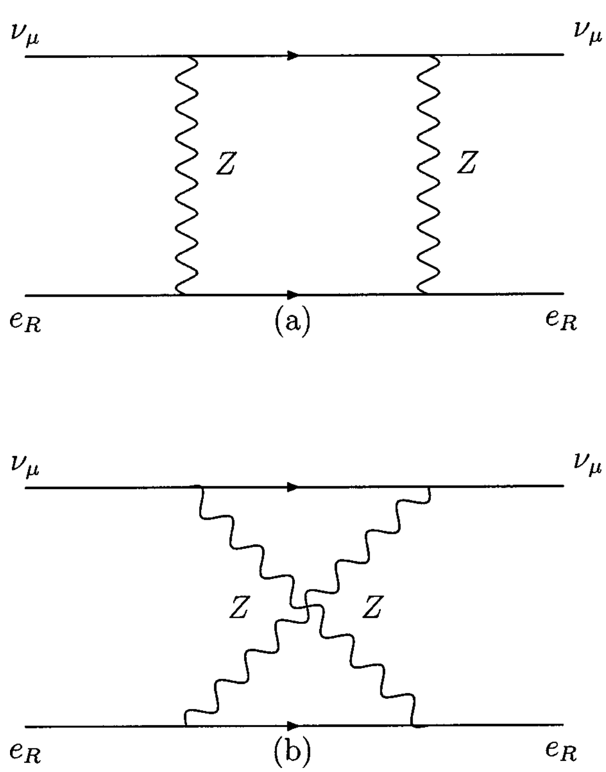

FIG. 2. $2Z$-exchange graphs.

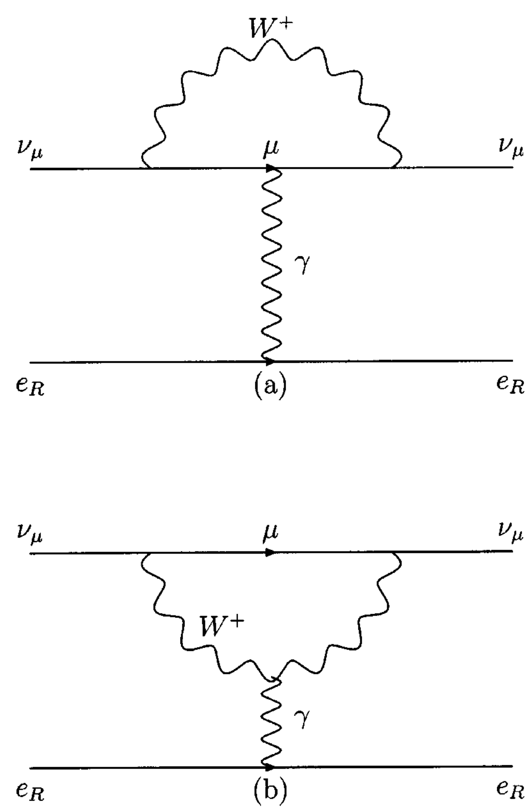

FIG. 3. Graphs contributing to $F_\gamma(t)$.

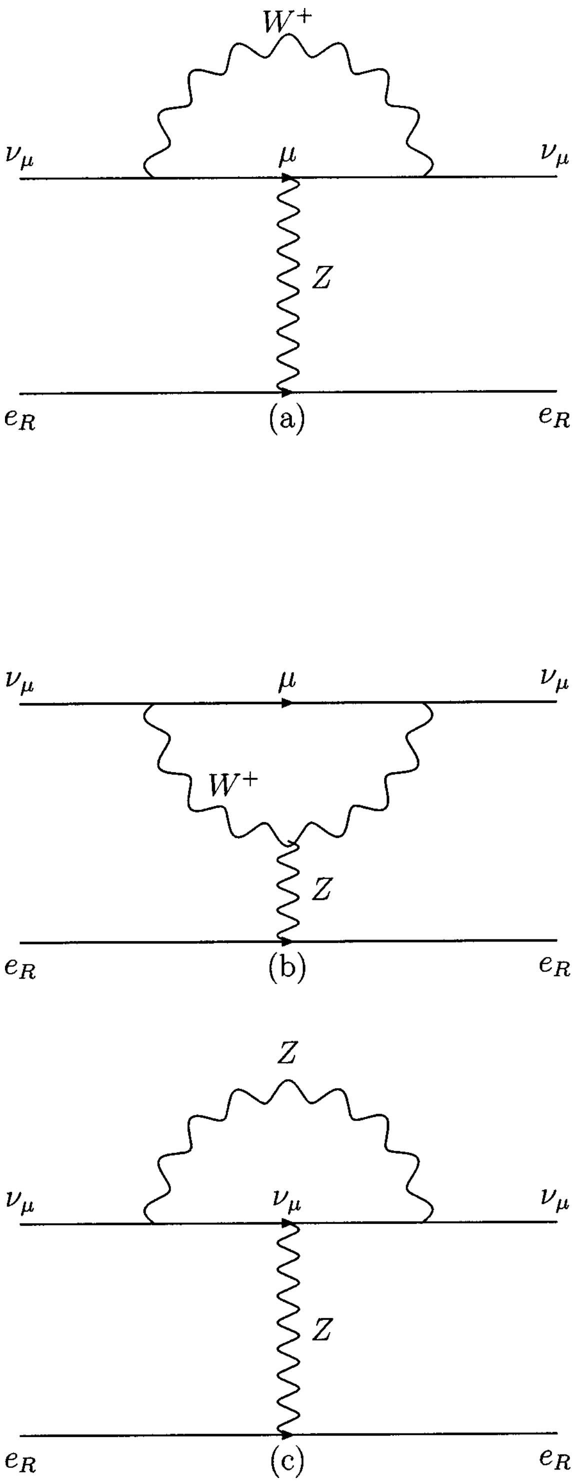

FIG. 4. Graphs contributing to $F_Z^{\nu\nu}(t)$.

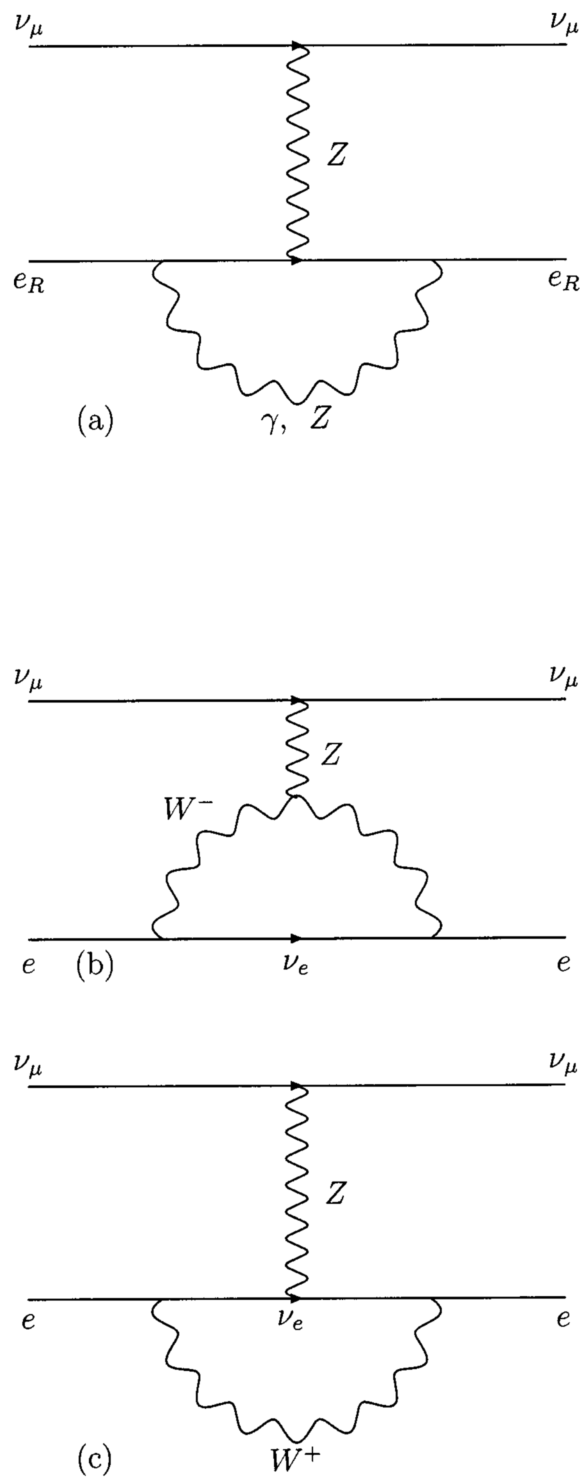

FIG. 5. Graph (a) contributing to $F_Z^{ee}(t)$. For $e = e_R$, the contributions from graphs (b) and (c) vanish in the SM.

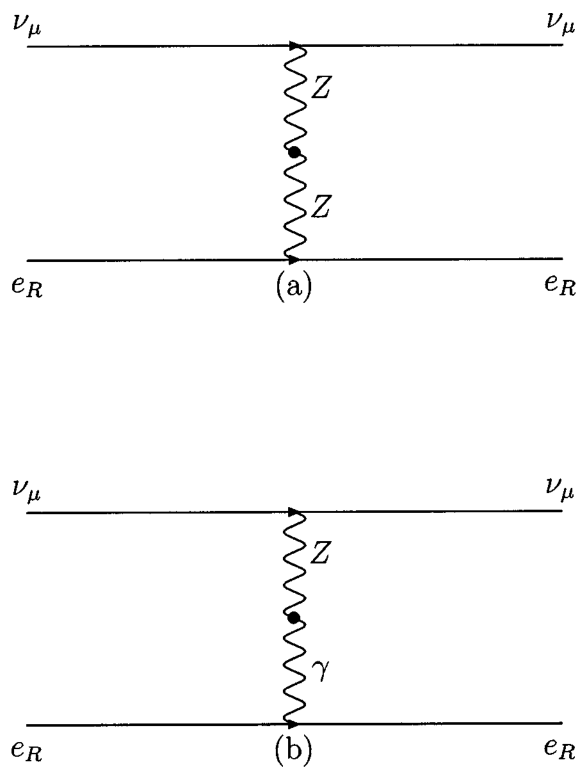

FIG. 6. Graphs contributing to (a) $\Pi_{ZZ}(t)$ and (b) $\Pi_{Z\gamma}(t)$. The filled dots denote (a) diagonal and (b) nondiagonal vector boson propagator corrections.

[1] W. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. **B46**, 319 (1972).
[2] K. Fujikawa, B.W. Lee, and A.I. Sanda, Phys. Rev. D **6**, 2923 (1972).
[3] R. Shrock, Phys. Rev. D **9**, 743 (1974).
[4] J. Smith and S.Y. Pi, Phys. Rev. D **9**, 1498 (1974).
[5] J.E. Kim, Phys. Rev. D **14**, 3000 (1976).
[6] W.J. Marciano and A.I. Sanda, Phys. Lett. **67B**, 303 (1977).
[7] B.W. Lee and R. Shrock, Phys. Rev. D **16**, 1444 (1977).
[8] M.A.B. Bég, W.J. Marciano, and M. Ruderman, Phys. Rev. D **17**, 1395 (1978).
[9] K. Fujikawa and R. Shrock, Phys. Rev. Lett. **45**, 963 (1980).
[10] W.J. Marciano and A. Sirlin, Phys. Rev. D **22**, 2695 (1980).
[11] S. Sakakibara, Phys. Rev. D **24**, 1149 (1981).
[12] B. Kayser, Phys. Rev. D **26**, 1662 (1982).
[13] J. Nieves, Phys. Rev. D **26**, 3152 (1982).
[14] R. Shrock, Nucl. Phys. **B206**, 359 (1982).
[15] S. Sarantakos, A. Sirlin, and W.J. Marciano, Nucl. Phys. **B217**, 84 (1983).
[16] J.L. Lucio, A. Rosado, and A. Zepeda, Phys. Rev. D **29**, 1539 (1984).
[17] N.M. Monyonko, J.H. Reid, and A. Sen, Phys. Lett. **136B**, 265 (1984).
[18] J.L. Lucio, A. Rosado, and A. Zepeda, Phys. Rev. D **31**, 1091 (1985).
[19] N.M. Monyonko and J.H. Reid, Prog. Theor. Phys. **73**, 734 (1985).
[20] G. Degrassi, A. Sirlin, and W.J. Marciano, Phys. Rev. D **39**, 287 (1989).
[21] P. Vogel and J. Engel, Phys. Rev. D **39**, 3378 (1989).
[22] J.A. Grifols and E. Masao, Phys. Rev. D **40**, 3819 (1989).
[23] J. Papavassiliou, Phys. Rev. D **41**, 3179 (1990).
[24] M.J. Musolf and B.R. Holstein, Phys. Rev. D **43**, 2956 (1991).
[25] J. Beacom and P. Vogel, Phys. Rev. Lett. **83**, 5222 (1999).
[26] G. Raffelt, Phys. Rep. **320**, 319 (1999).
[27] J. Bernabeu, L.G. Cabral-Rosetti, J. Papavassiliou, and J. Vidal, Phys. Rev. D **62**, 113012 (2000).
[28] J. Bernabeu, J. Papavassiliou, and J. Vidal, Phys. Rev. Lett. **89**, 101802 (2002); **89**, 229902(E) (2002).
[29] J. Bernabeu, J. Papavassiliou, and J. Vidal, hep-ph/0210055.
[30] K. Fujikawa and R. Shrock, hep-ph/0303188.
[31] Recall that electromagnetic current conservation implies the relations $(m_{\nu_k}-m_{\nu_j})F_1^V(q^2)_{kj}+q^2F_3^V(q^2)_{kj}=0$ and $-(m_{\nu_k}+m_{\nu_j})F_1^A(q^2)_{kj}+q^2F_3^A(q^2)_{kj}=0$.
[32] J.M. Cornwall, Phys. Rev. D **26**, 1453 (1982); J.M. Cornwall and J. Papavassiliou, *ibid.* **40**, 3474 (1989).
[33] G. Degrassi and A. Sirlin, Phys. Rev. D **46**, 3104 (1992).
[34] B.S. DeWitt, Phys. Rev. **162**, 1195 (1967).
[35] L.F. Abbott, Nucl. Phys. **B185**, 189 (1981); L.F. Abbott, M.T. Grisaru, and R.K. Schaeffer, *ibid.* **B229**, 372 (1983).
[36] P.A. Grassi, Nucl. Phys. **B462**, 524 (1996); **B537**, 527 (1999); **B560**, 301 (1999).
[37] K. Fujikawa, Phys. Rev. D **7**, 393 (1973).
[38] In general, $F_Z^{\nu\nu}(q^2)$ requires renormalization and thus

$[F_Z^{\nu\nu}(0)_{\nu_\mu} - F_Z^{\nu\nu}(0)_{\nu_\tau}]/M_Z^2$ gives the subleading order after renormalization. The leading term is absorbed into the renormalization convention of $F_Z^{\nu\nu}(q^2)$ for the on-shell neutrino ν_e and $q^2=0$, for example, and the difference above becomes subleading.

[39] We are, of course, not criticizing the pinch technique itself. This may be a useful technique to evaluate the gauge invariant S-matrix element. However, one cannot assign a physical (i.e., gauge-independent) significance to each element of a Feynman amplitude, such as the charge radius, *separately* after applying the pinch technique simply because the gauge parameter formally disappeared.

[40] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, *ibid.* **11**, 566 (1975).