Detailed analysis of one-loop neutrino masses from the generic supersymmetric standard model

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In the generic supersymmetric standard model which has no global symmetry enforced by hand, lepton number violation is a natural consequence. Supersymmetry, hence, can be considered the source of experimentally demanded properties for the neutrinos beyond the standard model. With an efficient formulation of the model, we perform a comprehensive detailed analysis of all (fermion-scalar) one-loop contributions to neutrino masses.

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I. INTRODUCTION

Low-energy supersymmetry (SUSY) is the most popular candidate theory for physics beyond the standard model (SM). The most extensively studied version, called the minimal supersymmetric standard model (MSSM), has an extra *ad hoc* discrete symmetry, called *R* parity, imposed on the Lagrangian. It is defined in terms of baryon number, lepton number, and spin as, explicitly, $R=(-1)^{3B+L+2S}$. The consequence is that the accidental symmetries of baryon number and lepton number in the SM are preserved, at the expense of making particles and superparticles having a categorically different quantum number, *R* parity. The latter is actually not the most effective discrete symmetry to control superparticle mediated proton decay [1], but is most restrictive in terms of what is admitted in the Lagrangian, or the superpotential alone.

R parity also forbids neutrino masses in the supersymmetric SM. However, the recent data from the solar and atmospheric neutrino experiments can be interpreted in terms of massive neutrino oscillations. Thus, the strong experimental hints for the existence of (Majorana) neutrino masses [2] is an indication of lepton number violation, and is, hence, suggestive of *R*-parity violation (RPV). Giving up *R* parity, a tree-level neutrino mass can be generated through diagonalization of the neutrino-neutralino mass matrix. At the 1-loop level, all three neutrinos will become massive. There is then no need to introduce extra superfields beyond what is required by the SM itself to describe neutrino phenomenology.

There is certainly no lack of studies on various RPV models in the literature. However, such models typically involve strong assumptions on the form of *R*-parity violation. In most cases, no clear statement on what motivates the assumptions taken is explicitly given. In fact, there are quite some confusing, or even plainly wrong, statements on the issues concerned. It is important to distinguish among the different RPV "theories," and, especially, between such a theory and the unique general supersymmetric standard model (GSSM) [3,4]. The latter is the *complete* theory of SUSY without *R*-parity, one which admits all the RPV terms without *a priori* bias. In the GSSM, RPV terms come in many different forms. In order not to miss any plausible RPV phenomenological features, it is important that all of the RPV parameters be taken into consideration. A clear listing and discussion of all these is recently presented in Ref. [5], under the framework of the single-VEV parametrization (SVP) [6,7]. The latter, summarized below, is an optimal choice of flavor bases that helps to guarantee a consistent and unambiguous treatment of all kind of admissible RPV terms with complete RPV effects on tree-level mass matrices for all states including scalars and fermions maintaining the simplest structure. Following the formulation, we present here a complete list of all the neutrino masses contributions up to 1-loop level.

A (Majorana) neutrino mass term violates SM lepton number by two units. The experimental evidence for neutrino masses comes in through indications of flavor oscillations, which requires mass mixings of the flavor states, ν_e, ν_μ, and ν_τ. Hence, we want neutrino mass terms that have lepton flavor violation (LFV). The latter is a generic consequence of *R*-parity violation. To put it in another way, the GSSM in fact contains many couplings that has one unit of LFV. Any combination of two of such couplings may be able to give rise to a neutrino mass term. Since the expected sub-eV neutrino masses are essentially the strongest source of upper bounds on such couplings [8] up to the present moment, we have no way to tell which particular combinations of couplings do saturate the bounds and give a dominant contribution to a neutrino mass term. In fact, each term also depends on a set of (*R*-parity conserving) SUSY or MSSM parameters. We do not have much knowledge on the SUSY parameters beyond some lower bounds on a set of related experimental parameters (mainly) from collider machines. In relation to the neutrino mass contributions discussed here, the set of SUSY parameters are typically taken as fixed by one generic SUSY mass scale. Changing the latter of course changes the actually neutrino masses resulted.

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More importantly, it is not totally clear whether some phenomenological hierarchy among values of the different SUSY parameters, may be together with some hierarchy among the values of the parameters with LFV, would not give a picture on the relative importance of the various neutrino mass terms different from what one may expect from such the kind of highly simplified analyses. Thus, it would be useful to have a complete list of such neutrino mass terms without much *a priori* assumption involved.

Guided by theoretical prejudices or otherwise, many different pieces of such neutrino mass terms have been studied [9,10] (see also Ref. [11] for a more updated list of references). More recently, there are attempts to give the more complete story. In particular, Ref. [12], gives the general formulas for neutrino mass contributions up to the full 1-loop level. However, the latter analysis is not formulated under the SVP and any detailed discussion is limited to a scenario where the ''third generation couplings dominate.'' Among the trilinear RPV couplings, this amounts to admitting only nonzero λ'_{i33}'s and λ_{i33}'s, though all nonzero bilinear RPV are indeed included. The maximal mixing result from Super-Kamiokande may bring that wisdom of ''third generation domination'' under question. References [10] and [13], for example, illustrate how no (family) hierarchy, or even an anti-hierarchy, among the RPV couplings may be preferred. More important to our perspective here, the study has assumption on the B_i parameters and is interested in the numerical study of a specific high-energy scenario. Here, we aim at a more detailed analytical study on the different pieces of contribution instead. With the help of a more simple theoretical framework, the SVP, we follow the basic approach of Ref. [12] and give a more transparent list of formulas, as well as pushing on to give much more detailed analytical results of each individual neutrino mass term.

The basic approach of Ref. [12] is to give each 1-loop neutrino mass diagram in terms of effective couplings of the mass eigenstates of various scalars and fermions running inside the loop, using a formula from the so-called ''effective mixing matrix'' method [14]. Details of all the admissible RPV contributions to all the scalar, as well as fermion, mass terms under the SVP framework are very manageable [16,4]. The complete expressions, together with useful perturbative diagonalization formulas for the interesting elements of the mixing matrices are listed in Ref. [4]. We use below exactly the same notation as presented in details in the latter reference, which is taken as the background of the present presentation. Our goal is to present the exact analytical expression for each neutrino mass term, and the approximate dominating result from each term under very mild assumption. The major part of the approximation is the perturbative diagonalization formulas of the mass matrices, which are well founded on the smallness of the neutrino masses. The approximation also helps to extract the major RPV parameter dependence of each mass term and, hence, is an important target of the present study.

There is actually a detailed analysis of all the neutrino mass terms pretty much in the same spirit of present study published [11]. The latter reference also essentially adopted the SVP framework. However, mass insertion approximation is used to obtain the results based on the use of MSSM states. Our approach here may be a more direct and transparent alternative. Having results from both approaches also serving as a counter checking and helps to illustrate more clearly some of the subtle points involved. Reference [11] also has some very different emphasis in their discussion. Hence, we consider the present study necessary to complete the story of neutrino masses in GSSM (or from *R*-parity violation). Moreover, our exact formulas, in terms of mass eigenstates running inside the loop allow direct numerical calculations of the neutrino mass results free from any approximation. We are also working on a detailed study of radiative neutrino decay within the model [19], to which the present paper also gives the necessary background.

It should be emphasized here that it is not our intention to discuss scenarios within the general model that could fit the experimental date. There being such a large number of lepton number violating parameters within the GSSM, phenomenologically viable scenarios will not be difficult to find. The beauty of the GSSM in explaining the neutrino data is that the parameters responsible will also give a rich collections of other experimental signals. More studies of various aspects of the model, and constraints from various SUSY and LFV searches in the future may give much better guideline for picking the real interesting scenarios. The goal of the present study is to provide a useful better reference for such efforts.

It should also be noted that we do not include here results of the gauge boson loop contributions. Such contributions have been studied (see, for example, the paper by Hempfling in Ref. [9]). They represent a small correction to the tree-level results which could be absoluted into a renormalization of the tree-level lepton flavor violating parameters, as also pointed out in Ref. [11]. Hence, we focus only on the finite 1-loop contributions that will give new structure to the neutrino mass matrix (same strategy was adopted in Ref. [11]). Namely, we focus on the fermion-scalar loop. Our formulation in terms of mass eigenstates for the fermions and scalars inside the loop allow us to identify explicitly the Goldstone mode. Any Goldstone mode contribution is taken out from the summation over mass eigenstates running inside the loop. In fact, the Goldstone modes are of course unphysical and calculation of their contribution gauge dependent. The diagrams with the Goldstone modes only form gauge invariant sets with the corresponding gauge boson and ghost diagrams added together. If one wishes, one could take it that we are doing the fermion-scalar loop calculation in the unitary gauge. The gauge choice does not matter, as we are focusing on the sector of physical scalar bosons and fermions only. Including the Goldstone modes in our fermion-scalar loop calculations would rather be inconsistent.

In Sec. II below, we give a brief summary of the basic formulation of GSSM used. Readers are referred to Ref. [4] for details. Section III then starts on the neutrino mass discussion. While the tree-level neutrino–neutralino mass matrix is quite well known, we present some of the details here for completeness. The presentation also sets the stage for the discussion of the 1-loop contribution calculation. All the basics of the 1-loop analysis is presented in the latter parts of the section. The next section discusses some details of the results in the way outlined above. Some of the detailed listing of individual terms are, however, left to the Appendixes. In Sec. V, we present a brief discussion on the application of the results to numerical studies, while any detailed numerical studies will be left for future publications. Finally, we conclude the paper with some remarks in Sec. VI.

II. BACKGROUND OF THE GSSM

Let us start with summarizing our formulation and notations here; readers are referred to Ref. [4] for more details. The most general renormalizable superpotential with the spectrum of minimal superfields containing all the SM states can be written as

$$W=\varepsilon_{ab}\Big[\mu_\alpha \hat{H}_u^a \hat{L}_\alpha^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^C + \lambda'_{\alpha jk}\hat{L}_\alpha^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2}\lambda_{\alpha\beta k}\hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^C\Big] + \frac{1}{2}\lambda''_{ijk}\hat{U}_i^C \hat{D}_j^C \hat{D}_k^C, \tag{1}$$

where (a,b) are SU(2) indices, (i,j,k) are the usual family (flavor) indices (going from 1 to 3). The four $\hat{L}_\alpha$'s, with the (α,β) indices as extended flavor indices going from 0 to 3, include the usual leptonic doublets and the H_d doublet. Four doublet superfields with the same quantum number are needed for gauge anomaly cancelation. The four are not *a priori* distinguishable. The rest of the superfield notations are obvious. Note that λ is antisymmetric in the first two indices, as required by the SU(2) product rules, shown explicitly here with $\varepsilon_{12}=-\varepsilon_{21}=1$. Similarly, λ'' is antisymmetric in the last two indices from $\mathrm{SU}(3)_C$, though color contents are not shown here.

Doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters. As far as the SM itself is concerned, the extra 26 real parameters are simply redundant. There is simply no way to learn about the 36 real parameters of Yukawa couplings for the quarks in some generic flavor bases, so far as the SM is concerned. For instance, one can choose to write the SM quark Yukawa couplings such that the down-quark Yukawa couplings are diagonal, while the up-quark Yukawa coupling matrix is a product of (the conjugate of) the CKM and the diagonal quark masses, and the leptonic Yukawa couplings diagonal. Doing that has imposing no constraint or assumption onto the model. On the contrary, not fixing the flavor bases makes the connection between the parameters of the model and the phenomenological observables ambiguous.

In the case of the GSSM, the choice of flavor basis among the 4 $\hat{L}_\alpha$'s is a particularly subtle issue, because of the fact that they are superfields the scalar parts of which could bear VEVs. A parametrization called the single-VEV parametrization (SVP) has been advocated since Ref. [6]. The central idea is to pick a flavor basis such that only one among the $\hat{L}_\alpha$'s, designated as $\hat{L}_0$, bears a nonzero VEV. There is to say, the direction of the VEV, or the Higgs field H_d, is singled out in the four dimensional vector space spanned by the $\hat{L}_\alpha$'s. Explicitly, under the SVP, flavor bases are chosen such that: 1/$\langle\hat{L}_i\rangle\equiv 0$, which implies $\hat{L}_0\equiv\hat{H}_d$; 2/$y_{jk}^e(\equiv\lambda_{0jk}=-\lambda_{j0k}) = (\sqrt{2}/v_0)\mathrm{diag}\{m_1,m_2,m_3\}$; 3/$y_{jk}^d(\equiv\lambda'_{0jk}) = (\sqrt{2}/v_0)\times\mathrm{diag}\{m_d,m_s,m_b\}$; 4/$y_{ik}^u=(\sqrt{2}/v_u)V_{\mathrm{CKM}}^T\,\mathrm{diag}\{m_u,m_c,m_t\}$, where $v_0\equiv\sqrt{2}\langle\hat{L}_0\rangle$ and $v_u\equiv\sqrt{2}\langle\hat{H}_u\rangle$. A point to note is that the m_i's above are, conceptually, not the charged lepton masses. The parametrization is optimal, apart from some minor redundancy in complex phases among the couplings. We simply assume all the admissible nonzero couplings within the SVP are generally complex. The big advantage of the SVP is that it gives the complete tree-level mass matrices of all the states (scalars and fermions) the simplest structure [4].

Following our notation above, the soft SUSY breaking terms of the Lagrangian, can be written as follows:

$$\begin{aligned}V_{\mathrm{soft}}=&\,\epsilon_{ab}B_\alpha H_u^a\tilde{L}_\alpha^b\\&+\epsilon_{ab}[A_{ij}^U\tilde{Q}_i^a H_u^b\tilde{U}_j^C + A_{ij}^D H_d^a\tilde{Q}_i^b\tilde{D}_j^C + A_{ij}^E H_d^a\tilde{L}_i^b\tilde{E}_j^C]\\&+\mathrm{H.c.}+\epsilon_{ab}\Big[A_{ijk}^{\lambda'}\tilde{L}_i^a\tilde{Q}_j^b\tilde{D}_k^C+\frac{1}{2}A_{ijk}^{\lambda}\tilde{L}_i^a\tilde{L}_j^b\tilde{E}_k^C\Big]\\&+\frac{1}{2}A_{ijk}^{\lambda''}\tilde{U}_i^C\tilde{D}_j^C\tilde{D}_k^C+\mathrm{H.c.}+\tilde{Q}^\dagger\tilde{m}_Q^2\tilde{Q}+\tilde{U}^\dagger\tilde{m}_U^2\tilde{U}\\&+\tilde{D}^\dagger\tilde{m}_D^2\tilde{D}+\tilde{L}^\dagger\tilde{m}_{\tilde{L}}^2\tilde{L}+\tilde{E}^\dagger\tilde{m}_E^2\tilde{E}+\tilde{m}_{H_u}^2|H_u|^2\\&+\frac{M_1}{2}\tilde{B}\tilde{B}+\frac{M_2}{2}\tilde{W}\tilde{W}+\frac{M_3}{2}\tilde{g}\tilde{g}+\mathrm{H.c.},\end{aligned}\tag{2}$$

where we have used H_d in the place of the equivalent $\tilde{L}_0$ among the trilinear A-terms. Note that $\tilde{L}^\dagger\tilde{m}_{\tilde{L}}^2\tilde{L}$, unlike the other soft mass terms, is given by a 4×4 matrix. Comparing with the MSSM case, $\tilde{m}_{L_{00}}^2$ corresponds to $\tilde{m}_{H_d}^2$ while $\tilde{m}_{L_{0k}}^2$'s give new mass mixings. The other notations are obvious. The writing of the soft terms in the above form makes identification of the scalar mass terms straightforward. Recall that only the doublets H_u and H_d bear VEVs. The A-terms in the second line of Eq. (2) hence do not contribute to scalar masses.

The SVP formulation also gives the complex equations

$$B_i\tan\beta=\tilde{m}_{L_{0i}}^2+\mu_0^*\mu_i, \tag{3}$$

reflecting the removed redundancy of parameters in a generic $\hat{L}_\alpha$ flavor basis. They are nothing but the vanishing tadpole equations. They give consistence conditions among the involved parameters that should not be overlooked. The equations suggest that the B_i's are expected to be suppressed, with respect to the B_0, as the μ_i's are, with respect to μ_0. The $\tilde{m}_{L_{0i}}^2$ parameters in particular are missing in some of the relevant discussions in the literature. From a different perspective, one may tend to think that the parameters are similar to the $\tilde{m}_{L_{ij}}^2$ parameters linked to soft flavor mixings. However, fixing $\tilde{m}^2_{L_{0i}}$ in Eq. (3) leads to definite relations between a B_i and a μ_i term, which may not be satisfied. The parameters B_i, μ_i, and $\tilde{m}^2_{L_{0i}}$ are not independent free parameters, because of the fact that freely chosen values of the set of parameters in a top-down approach, in general, do not land the model automatically into the single-VEV basis. The tadpole equations are incorporated completely into the scalar mass matrices involved in our calculations [4].

III. NEUTRINO MASSES

The GSSM has seven neutral fermions corresponding to the three neutrinos and four, heavy, neutralinos. The heavy states are supposed to be mainly gauginos and Higgsinos, but there is now admitted (RPV) mixings among all seven neutral electroweak states. In the case of small μ_i's of interest, it is convenient to use an approximate seesaw block diagonalization to extract the effective neutrino mass matrix. Note that the effective neutrino mass here is actually written in a basis which is approximately the mass eigenstate basis of the charged leptons, i.e., the basis is roughly (ν_e,ν_μ,ν_τ). The tree-level result is very well-known [9,10].

A. Getting the neutrinos among the neutral fermions

We use the basis $(-i\tilde{B},-i\tilde{W},\tilde{h}_u^{0^C},\tilde{h}_d^0,l_1^0,l_2^0,l_3^0)$ to write the 7×7 neutral fermion mass matrix $\mathcal{M}_N$. Note that $\tilde{h}_d^0 \equiv l_0^0$, while $\tilde{h}_u^{0^C}$ is the charge conjugate of the Higgsino $\tilde{h}_u^0$. For small μ_i's, we have $(l_1^0,l_2^0,l_3^0)\approx(\nu_e,\nu_\mu,\nu_\tau)$ [4]. The symmetric, but generally complex, matrix can be diagonalized by using unitary matrix $\boldsymbol{X}$ such that

$$\boldsymbol{X}^T\mathcal{M}_N\boldsymbol{X}=\text{diag}\{M_{\chi_n^0}\}. \tag{4}$$

Again, the first part of the mass eigenvalues, $M_{\chi_n^0}$ for $n=1$–4 here, gives the heavy states, i.e. neutralinos. The last part, $M_{\chi_n^0}$ for $n=5$–7, hence gives the physical neutrino masses.

The mass matrix $\mathcal{M}_N$ can be written in the form of block submatrices

$$\mathcal{M}_N=\begin{pmatrix}\mathcal{M}_n & \xi^T\\ \xi & m_\nu^o\end{pmatrix}, \tag{5}$$

where $\mathcal{M}_n$ is the upper-left 4×4 neutralino mass matrix, ξ is the 3×4 block, and m_ν^o is the lower-right 3×3 neutrino block in the 7×7 matrix. In the interest of small neutrino masses, a perturbative (seesaw) block diagonalization can be applied. Explicitly, the diagonalizing matrix can be written approximately as

$$\boldsymbol{Z}\simeq\begin{pmatrix} I_{4\times 4} & (\mathcal{M}_n^{-1}\xi^T)\\ -(\mathcal{M}_n^{-1}\xi^T)^\dagger & I_{3\times 3}\end{pmatrix}. \tag{6}$$

The tree-level effective neutrino mass matrix may then be obtained as

$$(m_\nu)\simeq-(\mathcal{M}_n^{-1}\xi^T)^T\mathcal{M}_n(\mathcal{M}_n^{-1}\xi^T)=-\xi\mathcal{M}_n^{-1}\xi^T$$
$$\simeq\frac{M_Z^2\cos^2\beta(M_1\cos^2\theta_W+M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)}(\mu_i\mu_j), \tag{7}$$

where

$$\det(\mathcal{M}_n)=\mu_0[-\mu_0M_1M_2+M_Z^2\sin 2\beta \times(M_1\cos^2\theta_W+M_2\sin^2\theta_W)] \tag{8}$$

is equivalent in expression to the determinant of the MSSM neutralino mass matrix.

It is obvious that the 3×3 matrix $(\mu_i\mu_j)$ has only one nonzero eigenvalue given by

$$\mu_5^2=|\mu_1|^2+|\mu_2|^2+|\mu_3|^2. \tag{9}$$

We can define

$$R_5=\begin{pmatrix}\frac{\mu_1^*}{\mu_5} & 0 & \frac{\sqrt{|\mu_2|^2+|\mu_3|^2}}{\mu_5}\\ \frac{\mu_2^*}{\mu_5} & \frac{\mu_3}{\sqrt{|\mu_2|^2+|\mu_3|^2}} & -\frac{\mu_1\mu_2^*}{\mu_5\sqrt{|\mu_2|^2+|\mu_3|^2}}\\ \frac{\mu_3^*}{\mu_5} & -\frac{\mu_2}{\sqrt{|\mu_2|^2+|\mu_3|^2}} & -\frac{\mu_1\mu_3^*}{\mu_5\sqrt{|\mu_2|^2+|\mu_3|^2}}\end{pmatrix}. \tag{10}$$

Then, we have $R_5^T(\mu_i\mu_j)R_5=\text{diag}\{\mu_5^2,0,0\}$. Here, μ_5 and $\sqrt{|\mu_2|^2+|\mu_3|^2}$ are taken as real and positive. With this result, we can write the overall diagonalizing matrix $\boldsymbol{X}$ in the form

$$\boldsymbol{X}\simeq\begin{pmatrix} I_{4\times 4} & (\mathcal{M}_n^{-1}\xi^T)\\ -(\mathcal{M}_n^{-1}\xi^T)^\dagger & I_{3\times 3}\end{pmatrix}\begin{pmatrix} R_n & 0_{4\times 3}\\ 0_{3\times 4} & e^{i\zeta}R_5\end{pmatrix}$$
$$=\begin{pmatrix} R_n & e^{i\zeta}(\mathcal{M}_n^{-1}\xi^T)R_5\\ -(\mathcal{M}_n^{-1}\xi^T)^\dagger R_n & e^{i\zeta}R_5\end{pmatrix}, \tag{11}$$

where R_n is a 4×4 matrix with elements all expected to be of order 1, basically the diagonalizing matrix for the $\mathcal{M}_n$ block and $e^{i\zeta}$ is a constant phase factor put in to absorb the overall phase in the constant factor in the expression of Eq. (7) so that the resulted neutrino mass eigenvalue would be real and positive. The matrix $\boldsymbol{X}$ contains the important information of the gaugino and Higgsino contents of the physical neutrinos. This is given by the mixing elements in the off-diagonal blocks. The $\boldsymbol{Z}$ matrix in itself gives similar information for the effective SM neutrinos (flavor states). The latter matrix may be more useful in the analysis of neutrino phenomenology.

B. Approach to 1-loop neutrino masses calculations

Following Ref. [12], we use the 1-loop (renormalized) mass formula from the ‘‘effective mixing matrix’’ approach, giving a fermion mass matrix as

$$\mathcal{M}_N{}^{(1)}(p^2)=\mathcal{M}(Q)+\Pi(p^2) -\frac{1}{2}[\mathcal{M}(Q)\Sigma(p^2)+\Sigma(p^2)\mathcal{M}(Q)]. \quad (12)$$

Note that $\mathcal{M}(Q)$ is the $\overline{DR}$ renormalized tree-level mass (matrix), while Π and Σ the contributions from 1-loop self-energy diagrams with and without chirality flip. We have

$$\mathcal{M}_N{}^{(1)}(p^2)=\begin{pmatrix} \mathcal{M}_n & \xi^T \\ \xi & 0 \end{pmatrix}+\begin{pmatrix} \delta\mathcal{M}_n & \delta\xi^T \\ \delta\xi & \delta(m_\nu^o) \end{pmatrix}(p^2), \quad (13)$$

where

$$\delta\mathcal{M}_n(p^2)=\Pi_n(p^2)-\frac{1}{2}[\mathcal{M}_n\Sigma_n(p^2)+\Sigma_n(p^2)\mathcal{M}_n],$$

$$\delta\xi(p^2)=\Pi_\xi(p^2)-\frac{1}{2}[\Sigma_\nu(p^2)\xi+\xi\Sigma_n(p^2)+\Sigma_\xi(p^2)\mathcal{M}_n],$$

$$\delta m_\nu^o(p^2)=\Pi_\nu(p^2)-\frac{1}{2}[\xi\Sigma_\xi^T(p^2)+\Sigma_\xi(p^2)\xi^T], \quad (14)$$

with the explicit renormalization scale (Q-)dependence of the tree-level parameters dropped. Seesaw diagonalization of $\mathcal{M}_N{}^{(1)}$ yields the 1-loop result,

$$\begin{aligned}(m_\nu)^{(1)}\simeq & -\xi\mathcal{M}_n^{-1}\xi^T+\delta(m_\nu^o)-\delta\xi\mathcal{M}_n^{-1}\xi^T-\xi\mathcal{M}_n^{-1}\delta\xi^T \\ & +\xi\mathcal{M}_n^{-1}\delta\mathcal{M}_n\mathcal{M}_n^{-1}\xi^T \\ = & -\xi\mathcal{M}_n^{-1}\xi^T+\Pi_\nu+\Pi_\xi\mathcal{M}_n^{-1}\xi^T+\xi\mathcal{M}_n^{-1}\Pi_\xi^T \\ & +\frac{1}{2}\Sigma_\nu\xi\mathcal{M}_n^{-1}\xi^T+\frac{1}{2}\xi\mathcal{M}_n^{-1}\xi^T\Sigma_\nu^T \\ & +\xi\mathcal{M}_n^{-1}\Pi_n\mathcal{M}_n^{-1}\xi^T, \end{aligned} \quad (15)$$

where we have dropped the p^2 dependence. As discussed below, the p^2 should be taken as at the scale of the mass $\mathcal{M}_N$ itself. Hence, in the application here to calculate the neutrino masses, the p^2 in $(m_\nu)^{(1)}(p^2)$ may be taken as practically zero. An important point to note here is that the Σ_ξ and Σ_n terms all cancel out and disappear from our final result for $(m_\nu)^{(1)}$. We refer the reader to Refs. [12,14] for further discussion on the merits of the approach and references to related works.

At this point, some remarks on the renormalization issue are in order. The issue has been well-addressed in the papers by Hempfling and Hirsch *et al.* [9] on calculations starting from tree-level mass eigenstates. The analog formula to Eq. (12) above is, under the $\overline{DR}$ scheme,

$$\mathcal{M}^{\text{pole}}=\mathcal{M}^{\overline{DR}}(Q)+\Delta\mathcal{M}(p,Q), \quad (16)$$

where the 1-loop correction part is given as

$$\Delta\mathcal{M}(p,Q)=[\Delta\mathcal{M}(p)]_{\Delta=0}, \quad (17)$$

i.e., the two-point functions involved are calculated by subtracting the term proportional to the regulator $\Delta\equiv 2/(4-d)-\gamma_E+\ln 4\pi$ of dimensional reduction. There is some ambiguity in the choice of Q in the evaluation of the off-diagonal two-point functions. As pointed out in Hempfling's paper, the effect of the ambiguity is of higher order. In the "effective mixing matrix" approach [14], the equation is casted in the electroweak state basis instead to arrive at Eq. (12), which upon seesaw block-diagonalization yields Eq. (15). Now, p^2 is practically zero, as we are calculating only diagrams with neutrinos on the external legs of the two-point functions. The rest are only tree-level mass matrix entries (to $\mathcal{M}_N$) coming into the formula as mixing matrix elements between the neutralino and neutrino blocks. The result for $(m_\nu)^{(1)}$ is however Q-dependent. Apart from the Q-dependence in the former set of parameters, there is also the Q-dependence coming up from the calculation of the two-point functions under the $\overline{DR}$ scheme as in Eq. (17). Furthermore, there are the full set of couplings involved in such calculations, which should be taken as running couplings at the scale Q. In the straight formal sense, the pole mass formula gives result $\mathcal{M}^{\text{pole}}$ that is Q-independent. However, in the application to obtain $\mathcal{M}_N{}^{(1)}(p^2)$ and its subsequent use in any explicit calculations, some residual Q-dependence is difficult to avoid.

Since we are interested in radiative neutrino mass generation from superparticles, we may take Q as M_{SUSY}, or roughly the electroweak scale. Below the scale, the superparticles decouple and the neutrino mass terms can only be expressed in terms of five-dimensional operators of the SM. Strictly speaking, one get the correct pole mass for the neutrinos only by running the operators to the neutrino mass scale through the corresponding renormalization group equations. However, such effects are minimal [15]. Apart from yielding the neutrino mass matrix in the more interesting flavor basis, Eq. (15) also avoids the superficial singularity reflecting the arbitrariness in the diagonalization of the mass matrix with degenerate massless neutrinos at tree-level. Furthering, the MNS (neutrino mixing) matrix obtained from the diagonalization of $(m_\nu)^{(1)}$ maintains a full unitary matrix.

The full 7×7 neutral fermion mass matrix has four heavy and three very light mass eigenvalues, corresponding to the neutralinos and neutrinos; and we are essentially only interested in the neutrino states. For a general mass calculation at 1-loop, we must choose a renormalization prescription for each of the tree-level parameters appearing in the full mass matrix $\mathcal{M}_N$, and will have to worry about the renormalization scale dependence issues of such parameters. The "effective mixing matrix" formula [cf. Eq. (15)] avoid the complication as the neutrino mass results depend only on p^2 explicitly, which is practical zero, as pointed out in Ref. [12]. The parameters involved in the neutrino mass generation are then taken as running parameter at the scale of interest. Renormalization scale dependence comes in only through the Σ_ν part (refer to Ref. [14] for more details) and the effect is small. The Σ_ν part itself is mostly not very important, as can be easily seen in Eq. (15).

Our neutrino mass formula [Eq. (15)] calls for a seesaw type block diagonalization of the mass matrix $\mathcal{M}_N$ up to 1-loop order. The diagonalizing transformation corresponds to the matrix $\boldsymbol{Z}$ of expression (6). The tree-level contribution, given by the first term in the formula, is obviously seesaw suppressed (by the neutralino mass scale). The second term Π_ν gives the direct 1-loop contributions. However, there are parts of Π_ν that involved other suppression beyond the loop factor. A typical example is the pure gaugino loop, or GH-loop [20], diagram contribution which can be interpreted as requiring seesaw induced Majorana-like ''sneutrino'' mass to give a nonvanishing result [5]. They may be called pseudo-direct 1-loop contributions. For the rest of the terms in Eq. (15), are indirect 1-loop contributions, which has part of the basic seesaw suppression going along. These include results from 1-loop diagrams contributing to the off-diagonal blocks of the $\mathcal{M}_N$ matrix, from Σ_ν diagrams, as well as from diagrams contributing to the diagonal block $\mathcal{M}_n$. The last one, given by the last term in the formula, gives no interesting features. It can be absorbed, for instance, into the tree-level result (first term) by replacing $\mathcal{M}_n$ there with the 1-loop corrected result. And, from the related calculations within MSSM, we know that the correction is about 6% [14]. In fact, the flavor conserving part of the contributions involving Σ_ν is similarly uninteresting. However, the part of the latter with LFV may be of interest.

To calculate explicitly the various neutrino mass contributions using the above formula, we need to have the effective couplings of the electroweak state neutral fermions to possible scalar and fermion mass eigenstates running in the quantum loop. The neutral fermion themselves, together with the nine neutral scalars of the model, give a class of neutral loop contributions. Obviously, the loop with the neutralino states dominates here. The effective couplings, to be given below, involve diagonalizing matrix elements of the states contributing to the states running inside the loop. For the fermion part, it is the $\boldsymbol{X}$ matrix discussed above. Similar perturbative diagonalization expressions for all the other matrices, those for the charged fermion, charged scalar, down-squarks, as well as the neutral scalar sector are discussed in details in Ref. [4]. We refrain from repeating the long list of such formulas in this paper. Most parts of the notation used, as will appear below, are quite easy to appreciate. Readers interested in checking any details on the derivations of the results, however, would need to use Ref. [4] extensively.

C. Neutral loop contributions

For the neutral loop contributions, we start with the effective interaction for the external l_i^0's with internal mass eigenstates,

$$\mathcal{L}=g_2\overline{\Psi}(l_i^0)\left[\mathcal{N}^L_{inm}\frac{1-\gamma_5}{2}+\mathcal{N}^R_{inm}\frac{1+\gamma_5}{2}\right]\Psi(\chi_n^0)\phi_m^0+\text{H.c.},\tag{18}$$

where $\frac{1}{2}(1\mp\gamma_5)$ are the left (L) and right (R) handed projections. We have

$$\mathcal{N}^R_{inm}=\frac{1}{2}[\tan\theta_W\boldsymbol{X}^*_{1n}-\boldsymbol{X}^*_{2n}][\mathcal{D}^s_{(i+2)m}+i\mathcal{D}^s_{(i+7)m}],\tag{19}$$

and $\mathcal{N}^L_{inm}=\mathcal{N}^{R*}_{inm}$. The direct 1-loop contributions is given by[1]

$$\Pi^N_{\nu_{ij}}=-\frac{\alpha_{\rm em}}{8\pi\sin^2\theta_W}\mathcal{N}^{R*}_{inm}\mathcal{N}^{R*}_{jnm}M_{\chi_n^0}\mathcal{B}_0(p^2,M^2_{\chi_n^0},M^2_{S_m}),\tag{20}$$

where the loop function $\mathcal{B}_0$ is defined in the limit of $p^2\to 0$ by

$$\mathcal{B}_0(p^2,m_1^2,m_2^2)=-\frac{m_2^2}{m_1^2-m_2^2}\ln\frac{m_1^2}{m_2^2}-\ln\frac{m_1^2}{Q^2}+1.\tag{21}$$

As will be shown explicitly below, this result is the gauge loop contribution first discussed in Ref. [20]. Note that $\boldsymbol{X}$ is the matrix that diagonalizes the seven neutral fermions, as discussed explicitly above. Among the seven fermion (tree-level) mass eigenstates denoted by the sum over n here, contributions from the $n=5$–7 states are certainly negligible. The sum of m runs through the nine physical neutral scalar states. The states, together with the unphysical Goldstone mode, are obtained from the 10×10 neutral scalar mass-squared matrix to be diagonalized by $\mathcal{D}^s$. We refer readers to Ref. [4] for details on the scalar sector. The set of coupling vertices may also be combined to give contributions to the self energy function Σ_ν. We have

$$\Sigma^N_{\nu_{ij}}=-\frac{\alpha_{\rm em}}{8\pi\sin^2\theta_W}\mathcal{N}^{R*}_{inm}\mathcal{N}^{R}_{jnm}\mathcal{B}_1(p^2,M^2_{\chi_n^0},M^2_{S_m}),\tag{22}$$

where the loop function $\mathcal{B}_1$ is defined by in the limit of $p^2\to 0$ by

$$\mathcal{B}_1(p^2,m_1^2,m_2^2)=\frac{1}{2}\left[1-\ln\frac{m_2^2}{Q^2}-\left(\frac{m_2^1}{m_1^2-m_2^2}\right)^2\ln\frac{m_1^2}{m_2^2}+\frac{1}{2}\left(\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\right)\right].\tag{23}$$

For the indirect 1-loop contributions, we need

[1]Here we have all fermions involved being Majorana fermions. We compose the 4-spinor Ψ by $\Psi=\begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix}$ where we have $\psi_R=-i\sigma_2\psi_L^*$. A mass term has $\overline{\Psi}'\Psi=\psi_L'^\dagger\psi_R+\psi_R'^\dagger\psi_L$. For instance, the $\psi_R'^\dagger\psi_L$ part can be written as $\psi_L'^T i\sigma_2\psi_L$. The $\mathcal{N}^{R*}_{inm}$ vertex brings in a ψ_{L_i} while a matching $\mathcal{N}^L_{jnm}$ vertex brings in a $\psi^\dagger_{R_j}=\psi^T_{L_j}i\sigma_2$. With the proper handling of the fermion wavefunction, $\Pi^N_{\nu_{ij}}$ has contributions proportional to $\mathcal{N}^{R*}_{inm}\mathcal{N}^L_{jnm}$ which is equivalent to the $\mathcal{N}^{R*}_{inm}\mathcal{N}^{R*}_{jnm}$ used here. Just for the l_i^0 states, the use of $\mathcal{N}^R_{inm}$ can only be seen as the intrinsic left-handed nature of the states. However, the explicit use of their charge conjugate to compose the 4-spinors and derive the effective couplings is necessary to complete the formulation, say in the case of the $\mathcal{C}^L_{inm}$ for the charged loop discussed below.

$$\mathcal{N}^R_{0nm} = \frac{1}{2}[\tan\theta_W \boldsymbol{X}^*_{1n} - \boldsymbol{X}^*_{2n}][\mathcal{D}^s_{2m} + i\mathcal{D}^s_{7m}], \tag{24}$$

$$\mathcal{N}^R_{\tilde{h}nm} = -\frac{1}{2}[\tan\theta_W \boldsymbol{X}^*_{1n} - \boldsymbol{X}^*_{2n}][\mathcal{D}^s_{1m} + i\mathcal{D}^s_{6m}], \tag{25}$$

$$\mathcal{N}^R_{\tilde{W}nm} = \frac{1}{2}\boldsymbol{X}^*_{3n}[\mathcal{D}^s_{1m} + i\mathcal{D}^s_{6m}] - \frac{1}{2}\boldsymbol{X}^*_{(4+\alpha)n}[\mathcal{D}^s_{(2+\alpha)m} - i\mathcal{D}^s_{(7+\alpha)m}], \tag{26}$$

$$\mathcal{N}^R_{\tilde{B}nm} = -\frac{1}{2}\tan\theta_W \boldsymbol{X}^*_{3n}[\mathcal{D}^s_{1m} + i\mathcal{D}^s_{6m}] + \frac{1}{2}\tan\theta_W \boldsymbol{X}^*_{(4+\alpha)n}[\mathcal{D}^s_{(2+\alpha)m} - i\mathcal{D}^s_{(7+\alpha)m}]. \tag{27}$$

The list of extra $\mathcal{N}^{R*}$ terms each combines with the $\mathcal{N}^{R*}_{inm}$ to give a neutral loop contribution to Π_{ξ}.

D. Charged loop contributions

The effective interaction for the external l^0_i with (colorless) charged fermions and scalars inside the loop is given by

$$\mathcal{L} = g_2 \overline{\Psi}(l^0_i)\left[\mathcal{C}^L_{inm}\frac{1-\gamma_5}{2} + \mathcal{C}^R_{inm}\frac{1+\gamma_5}{2}\right]\Psi(\chi^-_n)\phi^+_m + \text{H.c.}, \tag{28}$$

where

$$\mathcal{C}^R_{inm} = \frac{y_{e_i}}{g_2}\boldsymbol{V}_{(i+2)n}D^{l*}_{2m} - \frac{\lambda^*_{ikh}}{g_2}\boldsymbol{V}_{(h+2)n}D^{l*}_{(k+2)m},$$

$$\mathcal{C}^L_{inm} = -\boldsymbol{U}_{1n}\mathcal{D}^{l*}_{(i+2)m} + \frac{y_{e_i}}{g_2}\boldsymbol{U}_{2n}\mathcal{D}^{l*}_{(i+5)m} - \frac{\lambda_{ihk}}{g_2}\boldsymbol{U}_{(h+2)n}\mathcal{D}^{l*}_{(k+5)m}. \tag{29}$$

Here, $\boldsymbol{V}^\dagger\mathcal{M}_C\boldsymbol{U} = \text{diag}\{M_{\chi^-_n}\}$ where $\mathcal{M}_C$ is the 5×5 charged fermion mass matrix. Matrix $\mathcal{D}^l$ diagonalizes the mass-squared matrix of eight scalars of unit negative charge (see Ref. [4] for details). The latter includes again the unphysical Goldstone mode to be dropped from the sum over m.

Charged fermion loop contribution to direct 1-loop neutrino mass could then be easily obtained as

$$\Pi^C_{\nu_{ij}} = -\frac{\alpha_{\text{em}}}{8\pi\sin^2\theta_W}\mathcal{C}^{R*}_{inm}\mathcal{C}^L_{jnm}M_{\chi^-_n}\mathcal{B}_0(p^2, M^2_{\chi^-_n}, M^2_{\tilde{\ell}_m}) \quad (i\leftrightarrow j). \tag{30}$$

Unlike the case for the neutral loop result, the $\Pi^C_{\nu_{ij}}$ matrix written through the $\mathcal{C}^{R*}_{inm}\mathcal{C}^L_{jnm}$ coupled-vertices is not symmetric with respect to i and j. Hence, an explicit symmetrization has to be performed, as indicated above. The symmetrization also takes care of the asymmetry with respect to L and R, automatically. Similarly, for the Σ_ν part, we have

$$\Sigma^C_{\nu_{ij}} = \frac{\alpha_{\text{em}}}{8\pi\sin^2\theta_W}\{\mathcal{C}^L_{inm}\mathcal{C}^{L*}_{jnm} + \mathcal{C}^{R*}_{inm}\mathcal{C}^R_{jnm}\}\mathcal{B}_1(p^2, M^2_{\chi^-_n}, M^2_{\tilde{\ell}_m}) \quad (i\leftrightarrow j). \tag{31}$$

To go on to discussions of the indirect 1-loop contributions, we need the corresponding expressions of the $\mathcal{C}^{L,R}_{inm}$ for the other four neutral fermions. These are given as follows, with obvious notations,

$$\mathcal{C}^R_{0nm} = -\frac{y_{e_k}}{g_2}\boldsymbol{V}_{(k+2)n}D^{l*}_{(k+2)m},$$

$$\mathcal{C}^L_{0nm} = -\boldsymbol{U}_{1n}\mathcal{D}^{l*}_{2m} - \frac{y_{e_k}}{g_2}\boldsymbol{U}_{(k+2)n}\mathcal{D}^{l*}_{(k+5)m}, \tag{32}$$

$$\mathcal{C}^R_{\tilde{h}nm} = -\boldsymbol{V}_{1n}\mathcal{D}^{l*}_{1m},$$

$$\mathcal{C}^L_{\tilde{h}nm} = 0, \tag{33}$$

$$\mathcal{C}^R_{\tilde{W}nm} = \frac{-1}{\sqrt{2}}\boldsymbol{V}_{2n}\mathcal{D}^{l*}_{1m},$$

$$\mathcal{C}^L_{\tilde{W}nm} = \frac{1}{\sqrt{2}}[\boldsymbol{U}_{2n}\mathcal{D}^{l*}_{2m} + \boldsymbol{U}_{(k+2)n}\mathcal{D}^{l*}_{(k+2)m}], \tag{34}$$

$$\mathcal{C}^R_{\tilde{B}nm} = \frac{-\tan\theta_W}{\sqrt{2}}[\boldsymbol{V}_{2n}\mathcal{D}^{l*}_{1m} + 2\boldsymbol{V}_{(k+2)n}\mathcal{D}^{l*}_{(k+5)m}],$$

$$\mathcal{C}^L_{\tilde{B}nm} = \frac{\tan\theta_W}{\sqrt{2}}[\boldsymbol{U}_{2n}\mathcal{D}^{l*}_{2m} + \boldsymbol{U}_{(k+2)n}\mathcal{D}^{l*}_{(k+2)m}]. \tag{35}$$

Combining a $\mathcal{C}^{R*}$ with a $\mathcal{C}^L$ gives half of the charged fermion loop contribution, to the corresponding mass term; the other half is given by flipping L and R. For instance, the l^0_i-$\tilde{B}$ mass term, or $\Pi_{\xi_{i1}}$, is given by substituting $\mathcal{C}^L_{\tilde{B}nm}$ for $\mathcal{C}^L_{jnm}$ in Eq. (30), i.e., by a $\mathcal{C}^{R*}_{inm}\mathcal{C}^L_{\tilde{B}nm}$ combination, as well as the combination $\mathcal{C}^{R*}_{\tilde{B}nm}\mathcal{C}^L_{inm}$.

There is also another type of contributions, namely the quark-squark loops. The direct 1-loop part of such contributions is among the most well discussed. We summarize them here, under our notation, for completeness. We have

$$\Pi^{D}_{\nu_{ij}} = -\frac{\alpha_{\rm em} N_c}{8\pi \sin^2\theta_W} \mathcal{C}'^{R*}_{inm} \mathcal{C}'^{L}_{jnm} m_{d_n} \mathcal{B}_0(p^2, m^2_{d_n}, M^2_{\tilde{d}_m}) \quad (i \leftrightarrow j), \tag{36}$$

where

$$\mathcal{C}'^{R}_{inm} = -\frac{\lambda'^{*}_{ikn}}{g_2} \mathcal{D}^{d*}_{km},$$

$$\mathcal{C}'^{L}_{inm} = -\frac{\lambda'_{ink}}{g_2} \mathcal{D}^{d*}_{(k+3)m}, \tag{37}$$

and $\mathcal{D}^d$ diagonalizes the 6×6 squark mass-squared matrix $\mathcal{M}^2_D$. The structure is to be compared directly with those from the λ-couplings above. For Σ_ν, we have

$$\Sigma^{D}_{\nu_{ij}} = \frac{\alpha_{\rm em}}{8\pi \sin^2\theta_W} \{ \mathcal{C}'^{L}_{inm} \mathcal{C}'^{L*}_{jnm} + \mathcal{C}'^{R*}_{inm} \mathcal{C}'^{R}_{jnm} \} \mathcal{B}_1(p^2, M^2_{d_n}, M^2_{\tilde{d}_m}) \quad (i \leftrightarrow j). \tag{38}$$

For the indirect 1-loop part, we need

$$\mathcal{C}'^{R}_{0nm} = -\frac{y_{d_n}}{g_2} \mathcal{D}^{d*}_{km},$$

$$\mathcal{C}'^{L}_{0nm} = -\frac{y_{d_n}}{g_2} \mathcal{D}^{d*}_{(k+3)m}, \tag{39}$$

$$\mathcal{C}'^{R}_{\tilde{W}nm} = 0,$$

$$\mathcal{C}'^{L}_{\tilde{W}nm} = \frac{1}{\sqrt{2}} \mathcal{D}^{d*}_{km}, \tag{40}$$

$$\mathcal{C}'^{R}_{\tilde{B}nm} = -\frac{\sqrt{2}}{3} \tan\theta_W \mathcal{D}^{d*}_{(k+3)m},$$

$$\mathcal{C}'^{L}_{\tilde{B}nm} = -\frac{\sqrt{2}}{6} \tan\theta_W \mathcal{D}^{d*}_{km}. \tag{41}$$

We get the indirect 1-loop contributions by combining $\mathcal{C}'^{R(L)*}$ with $\mathcal{C}'^{L(R)}_{inm}$, in the same way as we do in the above case of (colorless) charged fermion loop.

IV. MORE DETAILED ANALYTICAL RESULTS

In this section, we give more explicit details of the neutrino mass terms obtained by applying the formulas in the preceeding section. We list the result from different combinations of interaction vertices and go on to illustrate the content of these exact mass eigenstate results by extracting the dominating piece(s) in the mass eigenstate double sum. There, we give the ‘‘approximate’’ analytical results through the use of perturbative diagonalization expressions [4] for the elements of the various mixing matrices. Such perturbative diagonalizations have been illustrated to be very good approximations, which also serve to illustrate well the role of the various lepton flavor violating (LFV) couplings involved (see Refs. [17,18] for other illustrations).

Note that we focus our discussions below only on the parts of the results that are particularly interesting to our analytical study. For instance, in the

$$\Pi^{N}_{\nu_{ij}} = -\frac{\alpha_{\rm em}}{8\pi \sin^2\theta_W} \mathcal{N}^{R*}_{inm} \mathcal{N}^{R*}_{jnm} M_{\chi^0_n} \mathcal{B}_0(p^2, M^2_{\chi^0_n}, M^2_{S_m})$$

term, we focus on the $\mathcal{N}^{R*}_{inm}\mathcal{N}^{R*}_{jnm}M_{\chi^0_n}$ part. That is, we will drop the common prefactor $\alpha_{\rm em}/8\pi\sin^2\theta_W$ and the loop integral $\mathcal{B}_0$ from all the neutrino mass term results given below. The following discussions do not include the Σ_ν part. The results of the latter are left all to an appendix at the end. They are included here mainly for completeness. It does not look like there is any important off-diagonal contribution, while diagonal contributions, as discussed above, only represent a universal correction to the tree-level result.

A. Results for $\Pi^N_{\nu_{ij}}$ and $(\Pi_\xi \mathcal{M}_n^{-1} \xi^T)^N_{ij}$

The result here may be written in the form of a single term as

$$\frac{1}{4}[\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}]^2 M_{\chi^0_n} [\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}] \times [\mathcal{D}^s_{(j+2)m} - i\mathcal{D}^s_{(j+7)m}] \simeq \frac{B_i B_j \tan^2\beta}{M_s^3} \left[\frac{1}{4} (\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n})^2 \frac{M_{\chi^0_n}}{M_s} \right] \quad (n = 1\text{–}4 \ \text{ dominate}). \tag{42}$$

The scalar sum is dominated by $m = 1,2,$ and 7 contributions. We illustrate here only the dependence on the B_i parameters and $\tan\beta$, with M_s denoting a generic mass parameter at the slepton scale.[2] Note that we write the final result in the form such that the square bracket [ ] contains a factor of order 1 (a pattern we stick to below), so that the reader can have an

[2]Note that we have $[\mathcal{D}^s_{(i+2)1} - i\mathcal{D}^s_{(i+7)1}] \simeq [-{\rm Re}(B_i)/M_s^2 - i[{\rm Im}(B_i)/M_s^2][\tan\beta \sin\alpha - \cos\alpha] = -(B_i/M_s^2)[\tan\beta\sin\alpha - \cos\alpha]$ from our perturbative formulas on the $\mathcal{D}^s$ elements. One may also check the other pieces. Take the $m=2$ piece, for example, we have then $[\mathcal{D}^s_{(i+2)1} - i\mathcal{D}^s_{(i+7)1}] \simeq -(B_i/M_s^2)[\tan\beta\cos\alpha + \sin\alpha]$; for $m = 7$, $[\mathcal{D}^s_{(i+2)7} - i\mathcal{D}^s_{(i+7)7}] \simeq [-{\rm Im}(B_i) + i\,{\rm Re}(B_i)]/\cos\beta M_s^2 = i(B_i/M_s^2)(1/\cos\beta)$. The extra factor of i guarantees a cancelation with the $m=1$ and 2 terms if the $(m=7)$ ‘‘pseudoscalar’’ is mass degenerate with the latter ‘‘scalars,’’ as $[\tan\beta\sin\alpha - \cos\alpha]^2 + [\tan\beta\cos\alpha + \sin\alpha]^2 = 1/\cos^2\beta$. Hence, to illustrate the generic result, we write the dominating $[\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}]$ result as $-B_i\tan\beta/M_s^2$. This is used throughout the section.

idea on the major parameters (those before the square bracket) affecting the scale of the neutrino mass. The resultant proportionality of the mass term here to the product B_iB_j has been addressed and interpreted as the necessity for a Majorana-like scalar mass insertion to complete the diagram, in terms of complex scalars. When one follows such an interpretation to consider the scalar inside the loop as complex field with mass insertions put in on the line explicitly (as shown in Fig. 6 of Ref. [5] for example), a proportionality on B_iB_j would likewise be resulted. The different m pieces in the scalar sum, however, cannot be put together at this level. Each piece involves actually a different value for M_s and a different loop integral from a physical scalar of different mass running in the loop. In fact, if one naively takes a sum over m without considering the loop integrals, a zero result would be obtained for any $\mathcal{D}^s_{am}\mathcal{D}^s_{bm}$ with $a\neq b$. The lack of degeneracy among the scalar mass eigenstates is what makes a nonzero result possible. This is a common feature for the type of diagrams (see also Ref. [18]). Interestingly enough, for the present case under discussion, a pairwise degeneracy among the "scalar" and "pseudoscalar" parts of a complex scalar is enough to guarantee a null result. This is equivalent to the statement that the neutrino mass contribution is proportional to a Majorana-like mass term. It is illustrated here in our expressions as a consequence of the cancelation between $\mathcal{D}^s_{(i+2)m}\mathcal{D}^s_{(j+2)m}$ and $i^2\mathcal{D}^s_{(i+7)m}\mathcal{D}^s_{(j+7)m}$ as well as between $\mathcal{D}^s_{(i+2)m}\mathcal{D}^s_{(j+7)m}$ and $\mathcal{D}^s_{(i+7)m}\mathcal{D}^s_{(j+2)m}$ for each single m value, from our perturbative expressions for the mixing matrix elements.

Next, we come to the $(\Pi_\xi\mathcal{M}_n^{-1}\xi^T)^N_{ij}$ part. The dominating results from all the individual terms of the form have a common proportionality to the combination of LFV parameters

$$B_i\mu_j(\tan\beta).$$

Again, the contribution mainly involves diagrams with a (physical) neutralino, together with a neutral scalar, running in the loop. As noted above, the lack of mass degeneracy among the scalars is essential for a nontrivial result. Note that upon the necessary symmetrization not explicitly shown, we will have also the $B_j\mu_i(\tan\beta)$ parameter combination coming in.

All the different terms in this class have very similar structure. We discuss here only an illustrative term, and leave the rest to Appendix A below. Let us take a look at the term $\Pi^N_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$. It is given as

$$-\mu_j\frac{\mu_0M_1M_2-M_Z^2\sin\beta\cos\beta(M_1\cos^2\theta_W+M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)}\frac{1}{4}[\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n}]^2M_{\chi^0_n}\cdot[\mathcal{D}^s_{(i+2)m}-i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{2m}-i\mathcal{D}^s_{7m}]$$
$$\simeq\frac{B_i\mu_j\tan\beta}{M_s^2}\left[\frac{\mu_0M_1M_2-M_Z^2\sin\beta\cos\beta(M_1\cos^2\theta_W+M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)}\frac{1}{4}(\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n})^2M_{\chi^0_n}\right]. \quad (43)$$

Note that from the general flavor structure of the model, one expect $\Pi^N_{\nu_i0}$ to have an expression similar to $\Pi^N_{\nu_{ij}}$ above with index j replaced by a 0, i.e., $\Pi^N_{\nu_i0}\simeq(B_iB_0/M_s^4)\tan^2\beta\frac{1}{4}\times(\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n})^2M_{\chi^0_n}$. Observing that $B_0/\cos\beta$ is a parameter of the same order as the generic mass scale parameter $M_s^2\tan\beta$ we do see an agreement here.

B. Results for the charged and color loops

Similar to the neutral loop case above, each term in the charged loop contributions to the Π's has a scalar part involving $D^l_{am}\mathcal{D}^{l*}_{bm}$ which would give a null result for $a\neq b$ if summed over m naively. The different loop integrals from the lack of scalar degeneracy is what guarantees nontrivial results. The fermionic part is more interesting. For illustrative purpose, we take an expression of the form $\boldsymbol{V}^*_{(i+2)n}M_{\chi^-_n}\boldsymbol{U}_{1n}$. Here $n=1$ and 2 give the chargino state contributions, with a large $M_{\chi^-_n}$ mass but a more suppressed $\boldsymbol{V}^*_{(i+2)n}$ mixing. The results are given by $(R^*_{R_{21}}/M_{c1})m_i\mu_i\boldsymbol{U}_{1n}$ and $(R^*_{R_{22}}/M_{c2})m_i\mu_i\boldsymbol{U}_{2n}$, respectively. On the other hand, the $n=i+2$ term involves a small fermion mass $M_{\chi^-_{i+2}}=m_i$ but a less suppressed mixing of $\boldsymbol{U}_{1(i+2)}\simeq(\sqrt{2}M_W\cos\beta/M_0^2)\mu_i$. Dropping all the factors of order 1, we have all three terms giving contribution of roughly the same order of magnitude, all proportional to $m_i\mu_i/M_s$, where M_s again denotes a SUSY scale mass parameter here corresponds, more exactly, to a chargino mass. This kind of feature is quite common in the charged loop results below. We illustrate results by dropping all the order 1 parameters and using the generic mass parameter M_s representing chargino as well as slepton mass scale.

There are six terms to the $\Pi^C_{\nu_{ij}}$ result. We mostly just list them, while drawing attention to particularly interesting features. Note that the necessary symmetrization is not shown explicitly,

$$-\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi^-_n}\boldsymbol{U}_{1n}D^l_{2m}\mathcal{D}^{l*}_{(j+2)m}\simeq-\frac{y_{e_i}}{g_2}\frac{m_i\mu_iB_j\tan\beta}{M_s^3}. \quad (44)$$

The scalar part result here is mainly from $D^l_{2(2+j)}\simeq B_j\tan\beta/M_s^2$.

$$\frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{2m}\mathcal{D}^{l*}_{(j+5)m} \simeq \frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\frac{m_i m_j \mu_i \mu_j \tan\beta}{M_s^3}. \tag{45}$$

The scalar part result here is mainly from $D^l_{2(5+j)} \simeq m_j\mu_j \tan\beta/M_s^2$.

$$-\frac{y_{e_i}}{g_2}\frac{\lambda_{jhk}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{(h+2)n}D^l_{2m}\mathcal{D}^{l*}_{(k+5)m} \simeq -\frac{y_{e_i}}{g_2}\frac{\lambda_{jik}}{g_2}\frac{m_i m_k \mu_k \tan\beta}{M_s^2}. \tag{46}$$

Here, the result is from $n=i+2$ which is interesting only at $h=i$; hence, only that is shown in the sum over h. It is the SUSY analog of the Zee diagram, discussed in Refs. [13,5]. For $h\neq i$ parts, the result is much further suppressed (by another $\mu_i\mu_h^*/M_s^2$ factor). The scalar part result is the same as the previous case,

$$\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{(k+2)m}\mathcal{D}^{l*}_{(j+2)m} \simeq \frac{\lambda_{ijh}}{g_2}\frac{m_h\mu_h}{M_s} \quad \text{(symmetrization)}. \tag{47}$$

We note here that the result is actually very sensitive to the $i\leftrightarrow j$ symmetrization. The dominant result in the expression above is from the case with the $(j+2)$th charged scalar running in the loop. This is approximately the $\tilde{l}_j^-$ slepton. The symmetrization and the fact that $\lambda_{ijh}=-\lambda_{jih}$ suggest a perfect cancelation of the result in the limit of degenerate sleptons which correspond roughly to the $\tilde{l}_i^-$ and $\tilde{l}_j^-$ states. This has also been discussed in some detail in Ref. [5],

$$-\frac{y_{e_j}}{g_2}\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{(k+2)m}\mathcal{D}^{l*}_{(j+5)m} \simeq -\frac{y_{e_j}}{g_2}\frac{\lambda_{ikh}}{g_2}\frac{m_h\mu_h(\tilde{\mathcal{M}}^2_{RL})^*_{jk}}{M_s^3}, \tag{48}$$

where

$$(\tilde{\mathcal{M}}^2_{RL})_{jk}=[A_e^*-\mu_0\tan\beta]m_j\delta_{kj}+\frac{\sqrt{2}M_W\cos\beta}{g_2}\delta A^{E*}_{kj} - \frac{\sqrt{2}M_W\sin\beta}{g_2}(\mu_i\lambda^*_{ikj}) \tag{49}$$

gives the complete LR mixing of $\tilde{l}_j^+$ and $\tilde{l}_k^-$ states. The last part of the latter is a contribution beyond the well known MSSM parts.

$$\frac{\lambda_{ikh}}{g_2}\frac{\lambda_{jqp}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{(q+2)n}D^l_{(k+2)m}\mathcal{D}^{l*}_{(p+5)m} \simeq \frac{\lambda_{ikh}}{g_2}\frac{\lambda_{jhp}}{g_2}\frac{m_h(\tilde{\mathcal{M}}^2_{RL})^*_{pk}}{M_s^2}. \tag{50}$$

This is the most well known part of $\Pi^C_{\nu_{ij}}$ result discussed extensively in the literature. Note again the extra (last) term in the LR mixing $(\tilde{\mathcal{M}}^2_{RL})^*_{pk}$. Its contribution to neutrino masses in the case of $p\neq k$ may be particularly interesting.

For the $(\Pi_\xi \mathcal{M}_n^{-1}\xi^T)^C_{ij}$ part, we present the long list of terms in Appendix B. In the neutral loop counterpart above, we see that the class of indirect 1-loop result all involve the combination $B_i\mu_j\tan\beta$. Here for the charged loop results, we see the same parameter combination does give some important terms, but without the $\tan\beta$ factor. These are labeled as $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{3j}$—part 1 and $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 5 [with corresponding $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ part] inside the Appendix. In factor, these terms could easily dominate over the direct 1-loop terms from Π^C_ν over. They provide neutrino mass contributions of order $B_i\mu_j/M_s^2$.

Another type of interesting terms are given by those labeled as $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 4 [again with corresponding $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ part] and $\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 10 inside the Appendix. We have, roughly, the results $(\lambda_{ikh}/g_2)(\mu_j m_h/M_s^2)$ or $(\lambda_{ihk}/g_2)(\mu_j m_h/M_s^2)$.

Most of the other terms are actually not very interesting. They typically involve further suppression from factors such as $(m_i/M_s)(y_{e_i}/g_2)$. However, one should note that for the large $\tan\beta$ case, the $i=3$ part has an order 1 coupling (essentially the τ Yukawa) which renders the suppression not too strong. A careful numerical study will be necessary to check if there could be a scenario where such term could play a role.

The quark-squark loop results are much more simple as a class. In fact, parallel structure between the λ'_{ijk} coupling terms and the λ_{ijk} coupling terms can also be used to write down the results directly. In particular, for the indirect 1-loop part, we expected $(\lambda'_{ikh}/g_2)(\mu_j m_{d_h}/M_s^2)$ or $(\lambda'_{ihk}/g_2)(\mu_j m_{d_h}/M_s^2)$ to match the similar terms just discussed above. We list the details in Appendix C.

V. CONCLUDING REMARKS

We have listed and discussed the detailed results of all the neutrino mass terms within the GSSM, up to 1-loop order. Our approach gives expression for exact results, each to be obtained through a double summation over the fermion and scalar mass eigenstates running inside the loop. We further give approximate expressions of each of these terms through extracting the dominating pieces within the double summation and approximating the elements of the mass mixing matrices by perturbative diagonalization formulas. The validity of such perturbative diagonalizations are well founded on the experimental smallness of effects involving lepton flavor violation or R-parity violation. However, there are partial cancellations among pieces within the sum—a result of a GIM type unitary cancellation also pointed out in Refs. [17,18], rendering the approximate formulas agree only at order of magnitude level with the summed exact results. The latter is also cross-checked through numerical calculations, part of which is given in Appendix E for illustrative purposes. We most probably have given the results in more details than necessary. However, we emphasize that our ignorance about the nature of SUSY parameters, R-parity violating or otherwise, says that imposing much theoretical prejudice on the likely importance on some contribution over the others may be unwise. The detailed listing here is intended to provide a reference to later studies on any plausibly interesting scenario out the model. Numerical studies of the latter will be published independently.

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APPENDIX A: DETAILS OF $(\Pi_\xi \mathcal{M}_n^{-1} \xi^T)^N$ TERMS

$\Pi^N_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$,

$$-\mu_j \frac{\mu_0 M_1 M_2 - M_Z^2 \sin\beta\cos\beta(M_1\cos^2\theta_W + M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)} \frac{1}{4}[\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}]^2 M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{2m} - i\mathcal{D}^s_{7m}]$$

$$\simeq \frac{B_i\mu_j\tan\beta}{M_s^2}\left[\frac{\mu_0 M_1 M_2 - M_Z^2 \sin\beta\cos\beta(M_1\cos^2\theta_W + M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)} \frac{1}{4}(\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n})^2 M_{\chi_n^0}\right]. \tag{A1}$$

This is exactly expression (43) which we repeat.

$\Pi^N_{\nu_i \tilde{h}}(\mathcal{M}_n^{-1}\xi^T)_{3j}$,

$$\mu_j \frac{M_Z^2\cos^2\beta(M_1\cos^2\theta_W + M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)} \frac{1}{4}[\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}]^2 M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{1m} - i\mathcal{D}^s_{6m}]$$

$$\simeq \frac{-B_i\mu_j\tan\beta}{M_s^2}\left[\frac{M_Z^2\cos^2\beta(M_1\cos^2\theta_W + M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)} \frac{1}{4}(\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n})^2 M_{\chi_n^0}\right]. \tag{A2}$$

Again, the $\Pi^N_{\nu_i \tilde{h}}$ term has a structure similar to that of $\Pi^N_{\nu_i 0}$ (or $\Pi^N_{\nu_{ij}}$) with the replacement of $\tilde{h}_d(\equiv l_0)$ by $\tilde{h}_u^\dagger$.

$\Pi^N_{\nu_i \tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$—part 1,

$$\mu_j \frac{M_Z\cos\beta\mu_0 M_1\cos\theta_W}{\det(\mathcal{M}_n)} \frac{1}{4}\boldsymbol{X}_{3n}[\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}] M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{1m} + i\mathcal{D}^s_{6m}]$$

$$\simeq \frac{-B_i\mu_j\tan\beta}{M_s^2}\left[\frac{M_Z\cos\beta\mu_0 M_1\cos\theta_W}{\det(\mathcal{M}_n)} \frac{1}{4}\boldsymbol{X}_{3n}(\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}) M_{\chi_n^0}\right]. \tag{A3}$$

$\Pi^N_{\nu_i \tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$—part 2,

$$-\mu_j \frac{M_Z\cos\beta\mu_0 M_1\cos\theta_W}{\det(\mathcal{M}_n)} \frac{1}{4}\boldsymbol{X}_{(4+\alpha)n}[\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}] M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m} - i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{(2+\alpha)m} + i\mathcal{D}^s_{(7+\alpha)m}]$$

$$\simeq \frac{B_i\mu_j\tan\beta}{M_s^2}\left[\frac{M_Z\cos\beta\mu_0 M_1\cos\theta_W}{\det(\mathcal{M}_n)} \frac{1}{4}\boldsymbol{X}_{4n}(\tan\theta_W \boldsymbol{X}_{1n} - \boldsymbol{X}_{2n}) M_{\chi_n^0}\right]. \tag{A4}$$

Here, we have different terms for $\alpha=0-3$, among which we show only the $\alpha=0$ result. The $\alpha=1-3$ cases have obvious extra suppressions from the $\boldsymbol{X}_{(4+\alpha)n}$ matrix element and a smaller scalar mixing part. The former has an extra μ_j^*/M_s factor while the latter introduces a $(\tilde{m}^2_{L_{ij}}+\mu_i^*\mu_j)/M_s^2$ factor. The overall $\Pi^N_{\nu_i\tilde{W}}$ results are not too different from the previous ones above either.

$\Pi^N_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 1,

$$-\mu_j\frac{M_Z\cos\beta\mu_0M_2\sin\theta_W}{\det(\mathcal{M}_n)}\frac{1}{4}\tan\theta_W\boldsymbol{X}_{3n}[\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n}]M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m}-i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{1m}+i\mathcal{D}^s_{6m}]$$

$$\simeq\frac{B_i\mu_j\tan\beta}{M_s^2}\left[\frac{M_Z\cos\beta\mu_0M_2\sin\theta_W}{\det(\mathcal{M}_n)}\frac{1}{4}\tan\theta_W\boldsymbol{X}_{3n}(\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n})M_{\chi_n^0}\right]. \tag{A5}$$

$\Pi^N_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 2,

$$-\mu_j\frac{M_Z\cos\beta\mu_0M_2\sin\theta_W}{\det(\mathcal{M}_n)}\frac{1}{4}\tan\theta_W\boldsymbol{X}_{(4+\alpha)n}[\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n}]M_{\chi_n^0}[\mathcal{D}^s_{(i+2)m}-i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{(2+\alpha)m}+i\mathcal{D}^s_{(7+\alpha)m}]$$

$$\simeq\frac{B_i\mu_j\tan\beta}{M_s^2}\left[\frac{M_Z\cos\beta\mu_0M_2\sin\theta_W}{\det(\mathcal{M}_n)}\frac{1}{4}\tan\theta_W\boldsymbol{X}_{4n}(\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n})M_{\chi_n^0}\right]. \tag{A6}$$

If one rotates the B-ino and W-ino into a photino and a Z-ino, the photino would of course be decoupled from mass mixings with the neutral fermions. The closely related structures of $\Pi^N_{\nu_i\tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ and $\Pi^N_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$ reflect on that. One can certainly write the two part of the results together through a $\Pi^N_{\nu_i\tilde{Z}}$ term with diagrams involving the Z-ino part only. However, to the extent that photino and Z-ino are not mass eigenstates, there is really not much to gain.

APPENDIX B: DETAILS OF $(\Pi_\xi\mathcal{M}_n^{-1}\xi^T)^C$ TERMS

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$: Here, we introduce the order 1 constant,

$$C_4=\frac{\mu_0M_1M_2-M_Z^2\sin\beta\cos\beta(M_1\cos^2\theta_W+M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)}\times M_s \tag{B1}$$

to simplify the expressions, given as follows.

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 1,

$$\mu_j\frac{C_4}{M_s}\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{2m}D^{l*}_{2m}\simeq\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}C_4. \tag{B2}$$

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 2,

$$\mu_j\frac{C_4}{M_s}\frac{y_{e_i}}{g_2}\frac{y_{e_k}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{(k+2)n}D^l_{2m}D^{l*}_{(k+5)m}$$

$$\simeq\left(\frac{y_{e_i}}{g_2}\right)^2\frac{m_i^2\mu_i\mu_j\tan\beta}{M_s^3}C_4. \tag{B3}$$

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 3,

$$-\mu_j\frac{C_4}{M_s}\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{(k+2)m}D^{l*}_{2m}$$

$$\simeq-\frac{\lambda_{ikh}}{g_2}\frac{\mu_jm_h\mu_hB_k^*\tan\beta}{M_s^4}C_4. \tag{B4}$$

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 4,

$$-\mu_j\frac{C_4}{M_s}\frac{\lambda_{ikh}}{g_2}\frac{y_{e_p}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{(p+2)n}D^l_{(k+2)m}D^{l*}_{(p+5)m}$$

$$\simeq-\frac{\lambda_{ikh}}{g_2}\frac{y_{e_h}}{g_2}\frac{\mu_jm_h(\tilde{\mathcal{M}}^2_{RL})^*_{hk}}{M_s^3}C_4. \tag{B5}$$

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 5,

$$-\mu_j\frac{C_4}{M_s}\frac{y_{e_k}}{g_2}\boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{(k+2)m}D^{l*}_{(i+2)m}$$

$$\simeq-\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}C_4. \tag{B6}$$

$\Pi^C_{\nu_i0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 6,

$$\mu_j\frac{C_4}{M_s}\frac{y_{e_i}}{g_2}\frac{y_{e_k}}{g_2}\boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{(k+2)m}D^{l*}_{(i+5)m}$$

$$\simeq\frac{y_{e_i}}{g_2}\frac{y_{e_k}}{g_2}\frac{\mu_jm_k\mu_k(\tilde{\mathcal{M}}^2_{RL})^*_{ik}}{M_s^4}C_4. \tag{B7}$$

$\Pi^C_{\nu_i 0}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 7,

$$-\mu_j \frac{C_4}{M_s}\frac{\lambda_{ihk}}{g_2}\frac{y_{e_p}}{g_2}\boldsymbol{V}^*_{(p+2)n}M_{\chi_n^-}\boldsymbol{U}_{(h+2)n}D^l_{(p+2)m}D^{l*}_{(k+5)m}$$

$$\simeq -\frac{\lambda_{ihk}}{g_2}\frac{y_{e_h}}{g_2}\frac{\mu_j m_h(\tilde{\mathcal{M}}^2_{RL})^*_{kh}}{M_s^3}C_4. \quad \text{(B8)}$$

$\Pi^C_{\nu_i\tilde{h}}(\mathcal{M}_n^{-1}\xi^T)_{3j}$: Here, we need to use, in addition to above, expressions for the elements of the mixing matrix $D^l_{1(i+2)}\simeq B_i/M_s^2$ and $D^l_{1(i+5)}\simeq m_i\mu_i/M_s^2$; and introduce the order 1 constant

$$C_3=\frac{M_Z^2\cos^2\beta(M_1\cos^2\theta_W+M_2\sin^2\theta_W)}{\det(\mathcal{M}_n)}M_s \quad \text{(B9)}$$

to simplify the expressions. We also use M_c to denote a mass parameter of the (physical) chargino mass scale. The results are as follows.

$\Pi^C_{\nu_i\tilde{h}}(\mathcal{M}_n^{-1}\xi^T)_{3j}$—part 1,

$$\mu_j\frac{C_3}{M_s}\boldsymbol{V}^*_{1n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{1m}D^{l*}_{(i+2)m}\simeq\frac{B_i\mu_j}{M_s^2}C_3\frac{M_c}{M_s}. \quad \text{(B10)}$$

$\Pi^C_{\nu_i\tilde{h}}(\mathcal{M}_n^{-1}\xi^T)_{3j}$—part 2,

$$-\mu_j\frac{C_3}{M_s}\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{1n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{1m}D^{l*}_{(i+5)m}$$

$$\simeq-\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}C_3\frac{M_c}{M_s}. \quad \text{(B11)}$$

$\Pi^C_{\nu_i\tilde{h}}(\mathcal{M}_n^{-1}\xi^T)_{3j}$—part 3,

$$\mu_j\frac{C_3}{M_s}\frac{\lambda_{ihk}}{g_2}\boldsymbol{V}^*_{1n}M_{\chi_n^-}\boldsymbol{U}_{(h+2)n}D^l_{1m}D^{l*}_{(k+5)m}$$

$$\simeq\frac{\lambda_{ihk}}{g_2}\frac{\mu_j m_h\mu_h^* m_k\mu_k}{M_s^4}C_3. \quad \text{(B12)}$$

$\Pi^C_{\nu_i\tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ and $\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$: We have noted above in the case of Π^N_ξ the close similarity between $\Pi^N_{\nu_i\tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ and $\Pi^N_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$. The story in the same here, between $\Pi^C_{\nu_i\tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$ and $\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$, with some exception. Note that from comparing Eqs. (34) and (35), we can see that there is and extra term in $\mathcal{C}^R_{\tilde{B}nm}$ without the matching partner in $\mathcal{C}^R_{\tilde{W}nm}$. We list below all results of the B-ino case, namely, $\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$. Among the 10 parts listed below, 1–7 have the W-ino counterparts in $\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$, to be given with an extra factor $(-1/\tan\theta_W)(M_1/M_2)$, which we do not list explicitly. Parts 8–10 have no W-ino counterparts. We also introduce order 1 constants,

$$C_1=\frac{\tan\theta_W}{\sqrt{2}}\frac{M_Z\cos\beta\mu_0M_2\sin\theta_W}{\det(\mathcal{M}_n)}M_s \quad \text{(B13)}$$

to simplify the expressions.

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 1,

$$-\mu_j\frac{C_1}{M_s}\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{2m}D^{l*}_{2m}\simeq-\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}C_1. \quad \text{(B14)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 2,

$$-\mu_j\frac{C_1}{M_s}\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{(i+2)n}M_{\chi_n^-}\boldsymbol{U}_{(k+2)n}D^l_{2m}D^{l*}_{(k+2)m}$$

$$\simeq-\frac{y_{e_i}}{g_2}\frac{m_iB_i\mu_j\tan\beta}{M_s^3}C_1. \quad \text{(B15)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 3,

$$\mu_j\frac{C_1}{M_s}\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{(k+2)m}D^{l*}_{2m}$$

$$\simeq\frac{\lambda_{ikh}}{g_2}\frac{\mu_j m_h\mu_h B_k^*\tan\beta}{M_s^4}C_1. \quad \text{(B16)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 4,

$$\mu_j\frac{C_1}{M_s}\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}^*_{(h+2)n}M_{\chi_n^-}\boldsymbol{U}_{(p+2)n}D^l_{(k+2)m}D^{l*}_{(p+2)m}$$

$$\simeq\frac{\lambda_{ikh}}{g_2}\frac{\mu_j m_h(\tilde{m}^2_{L_{kh}}+\mu_k^*\mu_h)}{M_s^3}C_1. \quad \text{(B17)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 5,

$$-\mu_j\frac{C_1}{M_s}\boldsymbol{V}^*_{2n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{1m}D^{l*}_{(i+2)m}\simeq-\frac{B_i\mu_j}{M_s^2}C_1\frac{M_c}{M_s}. \quad \text{(B18)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 6,

$$\mu_j\frac{C_1}{M_s}\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{2n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{1m}D^{l*}_{(i+5)m}\simeq\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}C_1\frac{M_c}{M_s}. \quad \text{(B19)}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 7,

$$-\mu_j\frac{C_1}{M_s}\frac{\lambda_{ihk}}{g_2}\boldsymbol{V}^*_{2n}M_{\chi_n^-}\boldsymbol{U}_{(h+2)n}D^l_{1m}D^{l*}_{(k+5)m}$$

$$\simeq-\frac{\lambda_{ihk}}{g_2}\frac{\mu_j\mu_h^*\mu_k m_k}{M_s^3}C_1. \tag{B20}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 8,

$$-\mu_j\frac{C_1}{M_s}2\boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{(k+5)m}D^{l*}_{(i+2)m}$$

$$\simeq\frac{\mu_j m_k\mu_k(\tilde{\mathcal{M}}^2_{RL})^*_{ik}}{M_s^4}2C_1. \tag{B21}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 9,

$$\mu_j\frac{C_1}{M_s}2\frac{y_{e_i}}{g_2}\boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{2n}D^l_{(k+5)m}D^{l*}_{(i+5)m}$$

$$\simeq\frac{y_{e_i}}{g_2}\frac{m_i\mu_i\mu_j}{M_s^2}2C_1. \tag{B22}$$

$\Pi^C_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 10,

$$-\mu_j\frac{C_1}{M_s}2\frac{\lambda_{ihk}}{g_2}\boldsymbol{V}^*_{(p+2)n}M_{\chi_n^-}\boldsymbol{U}_{(h+2)n}D^l_{(p+5)m}D^{l*}_{(k+5)m}$$

$$\simeq-\frac{\lambda_{ihk}}{g_2}\frac{\mu_j m_k\tilde{m}^2_{E_{hk}}}{M_s^3}2C_1. \tag{B23}$$

APPENDIX C: DETAILS OF $(\Pi_\xi\mathcal{M}_n^{-1}\xi^T)^D$ TERMS

$\Pi^D_{\nu_{ij}}$: Note that the necessary symmetrization is not shown explicitly,

$$N_c\frac{\lambda'_{ikh}}{g_2}\frac{\lambda'_{jhp}}{g_2}m_{d_h}\mathcal{D}^d_{km}\mathcal{D}^{d*}_{(p+3)m}\simeq3\frac{\lambda'_{ikh}}{g_2}\frac{\lambda'_{jhp}}{g_2}\frac{m_{d_h}(\mathcal{M}^2_{RL})^*_{pk}}{M_s^2}, \tag{C1}$$

where

$$(\mathcal{M}^2_{RL})_{pk}=[A_d^*-\mu_0\tan\beta]m_{d_p}\delta_{kp}+\frac{\sqrt{2}M_W\cos\beta}{g_2}\delta A^{D*}_{kp}$$
$$-\frac{\sqrt{2}M_W\sin\beta}{g_2}(\mu_i\lambda'^*_{ikp}). \tag{C2}$$

$(\Pi_\xi\mathcal{M}_n^{-1}\xi^T)^D_{ij}$: $\Pi^D_{\nu_i\tilde{0}}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 1,

$$-N_c\mu_j\frac{C_4}{M_s}\frac{\lambda'_{ikh}}{g_2^2}\frac{y_{d_h}}{g_2^2}m_{d_h}\mathcal{D}^d_{km}\mathcal{D}^{d*}_{(h+3)m}$$

$$\simeq-\frac{\lambda'_{ikh}}{g_2}\frac{y_{d_h}}{g_2}\frac{\mu_j m_{d_h}(\mathcal{M}^2_{RL})^*_{hk}}{M_s^3}3C_4. \tag{C3}$$

$\Pi^D_{\nu_i\tilde{0}}(\mathcal{M}_n^{-1}\xi^T)_{4j}$—part 2,

$$-N_c\mu_j\frac{C_4}{M_s}\frac{\lambda'_{ihk}}{g_2}\frac{y_{d_h}}{g_2}m_{d_h}\mathcal{D}^d_{hm}\mathcal{D}^{d*}_{(k+3)m}$$

$$\simeq-\frac{\lambda'_{ihk}}{g_2}\frac{y_{d_h}}{g_2}\frac{\mu_j m_{d_n}(\mathcal{M}^2_{RL})^*_{kh}}{M_s^3}3C_4. \tag{C4}$$

$\Pi^D_{\nu_i\tilde{W}}(\mathcal{M}_n^{-1}\xi^T)_{2j}$,

$$N_c\mu_j\frac{C_1}{\tan\theta_W M_s}\frac{\lambda'_{ikh}}{g_2}m_{d_h}\mathcal{D}^d_{km}\mathcal{D}^{d*}_{hm}$$

$$\simeq\frac{\lambda'_{ikh}}{g_2}\frac{\mu_j m_{d_h}\tilde{m}^2_{Q_{kh}}}{M_s^3}\frac{3C_1}{\tan\theta_W}. \tag{C5}$$

$\Pi^D_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 1,

$$-N_c\mu_j\frac{C_1}{3M_s}\frac{\lambda'_{ikh}}{g_2}m_{d_h}\mathcal{D}^d_{km}\mathcal{D}^{d*}_{hm}\simeq-\frac{\lambda'_{ikh}}{g_2}\frac{\mu_j m_{d_h}\tilde{m}^2_{Q_{kh}}}{M_s^3}C_1. \tag{C6}$$

$\Pi^D_{\nu_i\tilde{B}}(\mathcal{M}_n^{-1}\xi^T)_{1j}$—part 2,

$$-N_c\mu_j\frac{2C_1}{3M_s}\frac{\lambda'_{ihk}}{g_2}m_{d_h}\mathcal{D}^d_{(h+3)m}\mathcal{D}^{d*}_{(k+3)m}$$

$$\simeq-\frac{\lambda'_{ihk}}{g_2}\frac{\mu_j m_{d_h}\tilde{m}^2_{D_{hk}}}{M_s^3}2C_1. \tag{C7}$$

APPENDIX D: THE Σ_ν RESULTS

$\Sigma^N_{\nu_{ij}}$: We have a simple result here, given as

$$\frac{1}{4}[\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n}]^2[\mathcal{D}^s_{(i+2)m}-i\mathcal{D}^s_{(i+7)m}][\mathcal{D}^s_{(j+2)m}$$
$$+i\mathcal{D}^s_{(j+7)m}]\simeq\frac{B_iB_j^*\tan^2\beta}{M_s^4}\left[\frac{1}{4}(\tan\theta_W\boldsymbol{X}_{1n}-\boldsymbol{X}_{2n})^2\right]$$
$$(n=1\text{–}4\ \ \text{dominate}). \tag{D1}$$

$\Sigma^C_{\nu_{ij}}$: We list all the individual terms below,

$$\boldsymbol{U}_{1n}\boldsymbol{U}^*_{1n}D^{l*}_{(i+2)m}D^{l}_{(j+2)m}\simeq\frac{(\tilde{m}^2_{L_{ji}}+\mu_i\mu_j^*)}{M_s^2},\tag{D2}$$

$$\frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\boldsymbol{U}_{2n}\boldsymbol{U}^*_{2n}D^{l*}_{(i+5)m}D^{l}_{(j+5)m}\simeq\frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\frac{\tilde{m}^2_{E_{ji}}}{M_s^2},\tag{D3}$$

$$\frac{\lambda_{ihk}}{g_2}\frac{\lambda^*_{jpq}}{g_2}\boldsymbol{U}_{(h+2)n}\boldsymbol{U}^*_{(p+2)n}D^{l*}_{(k+5)m}D^{l}_{(q+5)m}\simeq\frac{\lambda_{ihk}}{g_2}\frac{\lambda^*_{jhk}}{g_2},\tag{D4}$$

$$-\frac{y_{e_j}}{g_2}\boldsymbol{U}_{1n}\boldsymbol{U}^*_{2n}D^{l*}_{(i+2)m}D^{l}_{(j+5)m}\simeq-\frac{y_{e_j}}{g_2}\frac{(\tilde{M}^2_{RL})_{ji}}{M_s^2},\tag{D5}$$

$$-\frac{y_{e_j}}{g_2}\boldsymbol{U}_{2n}\boldsymbol{U}^*_{1n}D^{l*}_{(i+5)m}D^{l}_{(j+2)m}\simeq-\frac{y_{e_i}}{g_2}\frac{(\tilde{M}^2_{RL})^*_{ij}}{M_s^2},\tag{D6}$$

$$\frac{\lambda^*_{jhk}}{g_2}\boldsymbol{U}_{1n}\boldsymbol{U}^*_{(h+2)n}D^{l*}_{(i+2)m}D^{l}_{(k+5)m}\simeq\frac{\lambda^*_{jhk}}{g_2}\frac{\mu_h(\tilde{M}^2_{RL})_{ki}}{M_s^3},\tag{D7}$$

$$\frac{\lambda_{ihk}}{g_2}\boldsymbol{U}_{(h+2)n}\boldsymbol{U}^*_{1n}D^{l*}_{(k+5)m}D^{l}_{(j+2)m}\simeq\frac{\lambda_{ihk}}{g_2}\frac{\mu_h^*(\tilde{M}^2_{RL})^*_{kj}}{M_s^3},\tag{D8}$$

$$-\frac{y_{e_i}}{g_2}\frac{\lambda^*_{jhk}}{g_2}\boldsymbol{U}_{2n}\boldsymbol{U}^*_{(h+2)n}D^{l*}_{(i+5)m}D^{l}_{(k+5)m}\simeq\frac{y_{e_i}}{g_2}\frac{\lambda^*_{jhi}}{g_2}\frac{\mu_h}{M_s},\tag{D9}$$

$$-\frac{y_{e_j}}{g_2}\frac{\lambda_{ihk}}{g_2}\boldsymbol{U}_{(h+2)n}\boldsymbol{U}^*_{2n}D^{l*}_{(k+5)m}D^{l}_{(j+5)m}\simeq\frac{y_{e_j}}{g_2}\frac{\lambda_{ihj}}{g_2}\frac{\mu_h^*}{M_s},\tag{D10}$$

$$\frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\boldsymbol{V}^*_{(i+2)n}\boldsymbol{V}_{(j+2)n}D^{l}_{2m}D^{l*}_{2m}\simeq\frac{y_{e_i}}{g_2}\frac{y_{e_j}}{g_2}\delta_{ij},\tag{D11}$$

$$\frac{\lambda_{ihk}}{g_2}\frac{\lambda^*_{jpq}}{g_2}\boldsymbol{V}^*_{(h+2)n}\boldsymbol{V}_{(p+2)n}D^{l}_{(k+2)m}D^{l*}_{(q+2)m}\simeq\frac{\lambda_{ihk}}{g_2}\frac{\lambda^*_{jhq}}{g_2}\frac{\tilde{m}^2_{L_{kq}}+\mu_k^*\mu_q}{M_s^2},\tag{D12}$$

$$-\frac{y_{e_i}}{g_2}\frac{\lambda^*_{jkh}}{g_2}\boldsymbol{V}^*_{(i+2)n}\boldsymbol{V}_{(h+2)n}D^{l*}_{(k+2)m}D^{l}_{2m}\simeq-\frac{y_{e_i}}{g_2}\frac{\lambda^*_{jki}}{g_2}\frac{B_k\tan\beta}{M_s^2},\tag{D13}$$

$$-\frac{y_{e_j}}{g_2}\frac{\lambda_{ikh}}{g_2}\boldsymbol{V}_{(j+2)n}\boldsymbol{V}^*_{(h+2)n}D^{l}_{(k+2)m}D^{l*}_{2m}\simeq-\frac{y_{e_j}}{g_2}\frac{\lambda_{ikj}}{g_2}\frac{B_k^*\tan\beta}{M_s^2}.\tag{D14}$$

$\Sigma^D_{\nu_{ij}}$:

$$N_c\frac{\lambda'^*_{ink}}{g_2}\frac{\lambda'_{jnq}}{g_2}D^*_{d_k2m}D_{d_q2m}\simeq3\frac{\lambda'^*_{ink}}{g_2}\frac{\lambda'_{jnq}}{g_2},\tag{D15}$$

$$N_c\frac{\lambda'^*_{ikn}}{g_2}\frac{\lambda'_{jqn}}{g_2}D^*_{d_k1m}D_{d_q1m}\simeq3\frac{\lambda'^*_{ikn}}{g_2}\frac{\lambda'_{jqn}}{g_2}.\tag{D16}$$

APPENDIX E: SOME ILLUSTRATION ON THE VALIDITY OF THE APPROXIMATE FORMULAS THROUGH NUMERICAL CALCULATIONS

In order to see how well our approximated formulas of the 1-loop neutrino mass corrections work, we present here some of the numerical neutrino mass values from the approximated formulas and compare them verses those from the exact expressions of the corresponding neutrino mass terms. We have a disclaimer to pronounce first. What we do here are not numerical studies of phenomenological viable scenarios of neutrino masses generation within the model. We make no attempt to choose parameters to fit any neutrino oscillation data. Rather, we are choosing simple and quite arbitrary input parameters, only to check and give an idea on the validity of out analytical results. The practice also helps to illustrate some theoretical issues behind the formulas. We choose a set of convenient input parameters and compute and list results from nine of the long list of neutrino mass terms. While the results do give some idea on the relative strength of the various terms, the readers should be warned that this is only a consequence of a specific choice of inputs, which is in no sense generic or particularly phenomenologically interesting.

Our choice of input parameters is as follows. We take the SUSY mass as around the scale of 100 GeV. In the exact results calculations, however, we have to split the masses of different superpartners to avoid unwanted special cancellations. We will clarify on the latter issue below. We choose input values that turned up mass eigenvalues for the SUSY particles in the hundreds of GeV scale, details of which is really not interesting. The value of $\tan\beta$ is set at 3. The parameters responsible for the lepton number violating effect are simply taken to be the same numerically, at a value of 10^{-4}. Explicitly, $\lambda_{ijk}=\lambda'_{ijk}=10^{-4}$, $\mu_i=10^{-4}$ GeV, $B_i=10^{-4}$ GeV^2. The neutrino mass results are presented in Table I, in which we show only contributions to the (3,3) elements of the effective (SM) neutrino mass matrix. The upper part of the table corresponds to the results from the approximated formulas while the lower part to those from the corresponding exact expressions. In each part, the first line corresponds to terms in Eqs. (42),(43),(B2), the second one to Eqs. (B3),(B6),(B15) and the third one to Eqs. (B22),(48),(C1).

As one can see from the table, the difference between the two results for any specific contribution is within an order of magnitude. One could not really expect a better agreement than this. In fact, as discussed above and in some related earlier studies [17,18], the structure of the class of 1-loop diagrams are such that there is a GIM-type unitarity cancellation involved in the sum over mass eigenstates. Say, if all the mass eigenstate fermions of the same quantum number are degenerate, the sum over the set of fermion mass eigenstates in a neutrino mass term will be proportional to the mass matrix entry that a naive look at the Feynman diagram will suggest. In most cases, that is vanishing. Similarly, when the set of the mass eigenstate scalars involved in a certainly diagram is mass degenerate, the sum over the set of states gives a vanishing result due to unitarity of the diagonalizing matrix. Take Eq. (B6) as an illustrative explicit example, the exact expression of the neutrino mass contribution is proportional to

$$-\mu_j \frac{C_4}{M_s}\frac{y_{e_k}}{g_2}\boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}D^l_{(k+2)m}D^{l*}_{(i+2)m} \times B_0(p^2,M^2_{\chi_n^-},M^2_{\tilde{\ell}_m}).$$

In case of mass degeneracy, one can factor out a fermion summation

$$\sum_n \boldsymbol{V}^*_{(k+2)n}M_{\chi_n^-}\boldsymbol{U}_{1n}$$

and a scalar summation

$$\sum_m D^l_{(k+2)m}D^{l*}_{(i+2)m}.$$

TABLE I. Some numerical results from the chosen neutrino mass terms. (See text of Appendix E.)

Approximated formulas (eV)		
-3.85×10^{-9}	-9.35×10^{-9}	6.38×10^{-8}
1.78×10^{-10}	-4.38×10^{-8}	-2.47×10^{-9}
1.00×10^{-8}	3.31×10^{-7}	4.46×10^{-3}
Exact formulas (eV)		
-2.61×10^{-9}	-3.45×10^{-8}	1.45×10^{-8}
5.63×10^{-10}	-1.73×10^{-8}	-3.94×10^{-9}
-3.23×10^{-9}	6.90×10^{-7}	3.26×10^{-3}

The former is nothing but the vanishing $(k+2,1)$ entry of the charged fermion mass matrix, while the latter is zero by unitarity of the matrix D^l (for $i\neq k$). As discussed in Refs. [17,18], the lack of mass degeneracy leads to first order violation of such unitarity cancellations, which explains the nonvanishing results. It also explains the not better than order of magnitude agreement between our exact results, obtained really summing over all the contributions from the different mass eigenstates, and that from the approximate formulas, which only extract the analytical form of the largest term within such summations.

With the above explanation, we see that our approximate formulas do work as well as they are to be expected. We emphasize again that the approximate formulas mainly serve the purpose of illustrating the role of the lepton number violating parameters in each of the neutrino mass contribution term.

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