

## Gravitational field around a timelike current-carrying screwed cosmic string in scalar-tensor theories

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In this paper we obtain a space-time generated by a timelike current-carrying superconducting screwed cosmic string (TCSCS). This gravitational field is obtained in a modified scalar-tensor theory in the sense that torsion is taken into account. We show that this solution is compatible with a torsion field generated by the scalar field  $\phi$ . The analysis of the gravitational effects of a TCSCS shows up that the torsion effects that appear in the physical frame of Jordan-Fierz type can be described in a geometric form given by contorsion term plus a symmetric part which contains the scalar gradient. As an important application of this solution, we consider the linear perturbation method developed by Zel'dovich, investigate the accretion of cold dark matter due to the formation of wakes when a TCSCS moves with speed  $v$ , and discuss the role played by torsion. Our results are compared with those obtained for cosmic strings in the framework of scalar-tensor theories without taking torsion into account.

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### I. INTRODUCTION

Scalar-tensor theories of gravity represent the simplest and most natural generalization of Einstein's theory of general relativity. The earliest scalar-tensor theories considered a massless scalar field with a constant coupling to matter [1]. Later, scalar-tensor theories were generalized by having a scalar field self-interaction and dynamical coupling to matter [2]. More recently, these theories have been generalized further to the case of multiple scalar fields [3]. In these theories [4,5], the gravitational interaction is mediated by one or several long range fields in addition to the usual tensor field of Einstein's theory of general relativity. The principal consequence of these theories is the fact that at sufficiently high energy scales [6–8] they can be relevant. If gravity is essentially a scalar-tensor theory, there will be direct implications for cosmology and experimental tests of the gravitational interaction [9–11].

Thus, it seems natural to investigate a general theory of gravity which involves scalar-tensor fields, especially those aspects connected with the gravitational fields generated by topological defects, such as cosmic strings [12], and study their gravitational and cosmological consequences. The gravitational field of a cosmic string, in the context of the theory of general relativity is quite remarkable: a particle placed at rest around a straight, infinite, static cosmic string will not be attracted to it, there is no local gravity. Its space-time is locally flat but not globally. Thus, the external gravitational field due to a cosmic string may be approximately described by what is commonly called conical geometry. The nontrivial topology of this space-time leads to a number of interesting effects, such as, gravitational lensing [13], the

emission of radiation by a freely moving particle [14], the existence of an electrostatic self-force [15] on an electric charged particle at rest, and the so-called gravitational Aharonov-Bohm effect [16], among others. Therefore, the richness of effects produced by this defect as well as the new ideas this object brought to general relativity seems to justify the interest in the study of this structure, and specifically the possible role played by it in the framework of cosmology due to the fact that it carries a large energy density, especially in the context of scalar-tensor theories of gravity due to their relevance at high energies in which scale topological defects could be formed. In this context, some authors have studied solutions for cosmic strings in Brans-Dicke [17], in dilaton theory [18], and in situations with more general scalar-tensor couplings [19,20].

Torsion fields play an important role in the geometry of a cosmic string whose presence could have influenced the formation and evolution of structures in the Universe [21]. As an example of effects produced by torsion we can mention the one which corresponds to the contribution to neutrino oscillations [22]. The fact that torsion could influence some physical phenomena led several authors to argue that torsion may have been an important element in the early universe, when quantum effects of gravity were drastically important [23,24] and for this reason have to be taken into account. In the context of cosmology torsion is important and produces modifications of kinematic quantities, like shear, vorticity, acceleration and expansion [7,25–29].

Taking into account the arguments concerning the importance of torsion and that scalar-tensor theories of gravity could be important at least at sufficiently high energy in which scenario topological defects, such as a cosmic string could be formed, we investigated a modified scalar-tensor theory of gravity in the sense that a torsion field is present [30,31]. Based on this modified scalar-tensor theory we determined the space-time generated by a screwed supercon-

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ducting bosonic cosmic string and studied some of its features.

Following this approach, in this paper we will determine the space-time generated by a timelike current-carrying screwed cosmic string (TCSCS) and investigate some of its physical consequences. In this background, an interesting aspect to consider are the anisotropies of the cosmic microwave background radiation (CMBR). These anisotropies were analyzed using numerical simulations of high resolution which demonstrated the existence of scale densities and revealed the presence of significant small-scale structure on strings or “wiggleness” [32–35]. This substructure has important dynamical consequences and also causes loop formation on scales much smaller than the horizon. These results suggest that long strings are more important than loops in seeding density perturbations [36,37].

The assumption that a string network [38,39] presents wiggles, leads to the conclusion that wiggles cosmic strings accompanying strings with low velocities, present a significant peak in the CMBR and an enhancement in the matter power spectrum. The scale of the wiggles is much smaller than the characteristic length of the string. In fact it was argued that a distant observer will not be able to resolve the details of the wiggly structure, and therefore, in this case it is not possible to distinguish the wiggly cosmic string of those generated in the presence of torsion. For this reason it is important to investigate with details this framework with torsion and its possible effects.

The interesting effect which can be studied is the accretion of the cold dark matter by wakes when the cosmic string is moving in a space-time with torsion in the context of a scalar-tensor theory. This effect was studied taking into account the string power spectrum obtained in simulations, projecting this forward to the present day using linear theory of transfer functions for both cold and hot dark matter. This was an invaluable first step, but further developments are necessary because strings create nonlinear objects at early times and power spectrum provides an incomplete description of non-Gaussian perturbations. Our work assumes the same idea, but using torsion as an essential element. In the study of wiggles, the approach considered frequently is the one proposed by Zel’dovich [40]. Here, we will assume that this is the correct mechanism to study wiggles also when torsion is present. This assumption is justified by the fact that this mechanism considers nonlinear effects. We will determine the time-dependent metric in linearized gravity for arbitrary evolving string configurations. As we will see the use of the Zel’dovich approximation in this scenario induces perturbations grown in a cold dark matter universe. In doing so, we postulate that the small-scale structures existing in wiggles strings can be approximately scaled by the geometrical deformation that torsion produces [30].

In this work we study the implications on these phenomena when we have a TCSCS. Our purposes are to obtain the gravitational field surrounding a TCSCS and study some of its consequences, in particular, how the cosmological effects of long strings are affected by torsion and scalar fields as compared with the corresponding results in general relativity. One important effect which will be studied is the accretion of

the cold dark matter by wakes when a TCSCS is moving. Thus, in order to study this phenomenon we will analyze the formation and evolution of wakes in this space-time with special emphasis to the role played by torsion in the process of formation of wakes. Also we will consider a possible explanation to the anisotropies of the CMBR, which will be done by the analysis of perturbations in this background. This problem is most conveniently studied using the linear perturbation methods developed by Zel’dovich [40]. In this approach we consider the universe in the matter-dominated era  $t > t_{eq}$  with scale factor  $a(t) \sim t^{2/3}$ , and average density given by  $\rho_{av} = 1/6\pi Gt^2$ .

This paper is organized as follows. In Sec. II we present a short description of scalar-tensor theories with torsion fields with a discussion of geodesics based on energy conservation and in terms of contorsion. In Sec. III we obtain the solution that corresponds to a screwed cosmic string carrying a timelike current TCSCS by applying a method used by Linet [41] to solve the linearized Einstein’s equations. In Sec. IV, we study the scattering by a TCSCS, and in Sec. V, the Zel’dovich approximation in a theory with torsion is introduced. In Sec. VI, we study the accretion of cold dark matter by wakes. Finally, in Sec. VII we provide some closing remarks.

## II. SETTING UP SCALAR-TENSOR THEORIES OF GRAVITY WITH TORSION

In this section, we consider some basic features of scalar-tensor theories of gravity with the inclusion of torsion. An important aspect to point out in these theories is the information concerning the presence of torsion which comes out in the geodesic equations written in the Jordan-Fierz frame. It is important to call attention to the fact that usually test particles are affected by torsion if they are fermions. In our model, indeed, bosonic particles can also be affected by the torsion background. This effect appear in the geodesic equations which present contributions arising from torsion. If we only consider metric aspects, that is, if the metricity condition  $\nabla_{\mu} g_{\alpha\beta} = 0$  is assumed, we find that the connection of a Riemann-Cartan manifold  $U_4$ , is given by

$$\Gamma_{\lambda\nu}^{\alpha} = \{\overset{\alpha}{\lambda\nu}\}_{JF} + K_{\lambda\nu}^{\alpha}, \quad (2.1)$$

where  $\{\overset{\alpha}{\lambda\nu}\}_{JF}$  is the usual Christoffel symbol evaluated in the Jordan-Fierz frame with metric  $\tilde{g}_{\mu\nu}$ . The contorsion tensor  $K_{\lambda\nu}^{\alpha}$  reads

$$K_{\lambda\nu}^{\alpha} = -\frac{1}{2}(S_{\lambda\nu}^{\alpha} + S_{\nu\lambda}^{\alpha} - S_{\lambda\nu}^{\alpha}), \quad (2.2)$$

where  $S_{\lambda\nu}^{\alpha}$  is the torsion. As torsion can also exist in the absence of fermionic particles [8], let us assume that the dilaton can generate torsion which can be written, in terms of this field, as

$$S_{\mu\nu}^{\lambda} = (\delta_{\mu}^{\lambda} \partial_{\nu} \tilde{\phi} - \delta_{\nu}^{\lambda} \partial_{\mu} \tilde{\phi}) / 2\tilde{\phi}. \quad (2.3)$$

In this case the curvature tensor is defined as in a Riemannian space, using however the connections defined on

$U_4$ , instead of the Christoffel symbols. The action is the same of the Refs. [8,5,30] and takes the form

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{\phi} \tilde{R}(\{\}) - \frac{\kappa(\tilde{\phi})}{\tilde{\phi}} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} \right] + I_m(\tilde{g}_{\mu\nu}, \Psi_m), \quad (2.4)$$

where  $\kappa(\tilde{\phi}) = \omega(\tilde{\phi}) - \epsilon$ , with  $\omega(\tilde{\phi})$  being a general function of the scalar field and  $\epsilon$  is the torsion coupling constant [5]. The scalar curvature  $\tilde{R}(\{\})$  is evaluated in the Jordan-Fierz frame and the action corresponding to the matter fields is represented by  $I_m(\tilde{g}_{\mu\nu}, \Psi_m)$ .

In the physical frame of Jordan-Fierz type, the equations for the metric,  $\tilde{g}_{\mu\nu}$ , are

$$\tilde{G}_{\mu\nu} = \frac{\kappa(\tilde{\phi})}{\tilde{\phi}^2} \left( \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}^{\alpha\beta} \partial_\alpha \tilde{\phi} \partial_\beta \tilde{\phi} \right) + 8\pi G \tilde{T}_{\mu\nu}. \quad (2.5)$$

If we consider that the action of matter does not have fermionic fields, we find from Bianchi identities that

$$\partial_\nu (\sqrt{-\tilde{g}} \tilde{T}^{\mu\nu}) + \sqrt{-\tilde{g}} \{\}_{\nu\alpha}^{\mu} \tilde{T}^{\mu\nu} = 0. \quad (2.6)$$

This result permits us to write the geodesic equations in terms of the connection only and therefore, the contortion term does not appear. Then, in this case, the geodesic equations are

$$\frac{d^2 x^\mu}{d\tau^2} + \{\}_{\alpha\beta}^{\mu} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \quad (2.7)$$

This is the correct result corresponding to geodesic equations in the cases where spin is not present. It is worth calling attention to the fact that this result does not mean that torsion in our work does not contribute to geodesics. In fact, all contributions arising from torsion, in the case where spin is not present, are coded in the metric written in Jordan-Fierz frame and taking into account the contortion. The torsion contribution to geodesic equations becomes evident if we write these equations as a function of the dilaton field. The action proposed in Eq. (2.4), in Jordan-Fierz frame can be transformed to another frame called Einstein frame, using a field dependent conformal transformation [27] given by

$$\tilde{g}_{\mu\nu} = \Lambda^2(\phi) g_{\mu\nu}, \quad (2.8)$$

and defining the quantities  $\Lambda^2(\phi)$  and  $\alpha^2(\phi)$  as

$$G\Lambda^2(\phi) = \tilde{\phi}^{-1}$$

$$\alpha^2(\phi) \equiv \left( \frac{\partial \ln \Lambda(\phi)}{\partial \phi} \right)^2 = [2\omega(\tilde{\phi}) + 3]^{-1}. \quad (2.9)$$

Therefore, we can calculate all quantities in the Einstein frame and reexpress all of them in the physical frame of Jordan-Fierz type just by using the above conformal transformation.

Now, let us analyze Eq. (2.7) with the torsion written explicitly. To do this, it would be better to write Eq. (2.7) as a function of the contorsion. Thus, let us consider the Christoffel symbols in the Jordan-Fierz frame as a sum of the Christoffel symbols in the Einstein frame  $\{\}_{\alpha\beta}^{\mu}$ , the contorsion given by Eq. (2.2) and the dilaton  $\phi$ , as

$$\{\}_{\alpha\beta}^{\mu} \}_{JF} = \{\}_{\alpha\beta}^{\mu} \} + K_{(\alpha\beta)}{}^{\mu} + \frac{\alpha(\phi)}{2} (\delta_\alpha^\mu \partial_\beta \phi + \delta_\beta^\mu \partial_\alpha \phi), \quad (2.10)$$

where the contorsion  $K_{(\alpha\beta)}{}^{\mu}$  can be written, explicitly, as

$$K_{(\alpha\beta)}{}^{\mu} = \frac{\alpha(\phi)}{2} (\delta_\alpha^\mu \partial_\beta \phi + \delta_\beta^\mu \partial_\alpha \phi - 2g_{\alpha\beta} g^{\mu\nu} \partial_\nu \phi), \quad (2.11)$$

and the dilaton  $\phi$  is the solution of the equation of motion

$$\square_g \phi = -4\pi G \alpha(\phi) T, \quad (2.12)$$

where

$$\square_g \phi = \frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} \kappa(\phi) \partial^\mu \phi] \quad (2.13)$$

is the d'Alembertian in this background and  $\kappa(\phi)$  is defined as

$$\kappa(\phi) = 1 - 2\epsilon \alpha^2(\phi), \quad (2.14)$$

with  $\epsilon$  being the torsion coupling constant.

The important point to call attention here is the fact that the symmetric part of the contorsion appears in the Christoffel symbols in the Jordan-Fierz frame, which is the frame where the physical quantities are measurable. Notice that the torsion effects are taken into account in Eq. (2.10) and this is a peculiarity of the Jordan-Fierz frame. As we will see in Sec. IV, the expression for the Newtonian force on a test particle put this point into evidence.

### III. TIMELIKE CURRENT-CARRYING SCREWED COSMIC STRING IN SCALAR-TENSOR THEORIES

In this section we will consider the appropriate action for matter fields  $I_m$ , which can be used to obtain the solution of a timelike current-carrying screwed cosmic string in scalar-tensor theories. The model which we will consider here has been already discussed recently [30] in the magnetic case (spacelike current). In order to recall some features of this model we will include a brief discussion in this section, with the appropriate changes to be considered in the electric case.

Using the transformation given by Eq. (2.9), we can express action (2.4) in the Einstein frame in the following form:

$$\begin{aligned}
I = & \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R(\{\}) - 2\kappa(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi] \\
& + \int d^4x \sqrt{-g} [-\frac{1}{2}\Lambda^2\{D_\mu\Phi(D^\mu\Phi)^* - \frac{1}{2}D_\mu\Sigma(D^\mu\Sigma)^*\}] \\
& - \frac{1}{16\pi} [(F_{\mu\nu}F^{\mu\nu} + H_{\mu\nu}H^{\mu\nu}) - \Lambda^2V(|\Phi|,|\Sigma|)], \quad (3.1)
\end{aligned}$$

where  $D_\mu\Sigma = (\partial_\mu + ieA_\mu)\Sigma$  and  $D_\mu\Phi = (\partial_\mu + iqB_\mu)\Phi$  are the covariant derivatives, with  $A_\mu$  and  $B_\mu$  being the gauge fields and  $\Phi$  and  $\Sigma$  the scalar fields. The field strengths are defined as usual as  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ . Note that  $\kappa(\phi)$ , already defined, contains information coming from the scalar-tensor term  $\alpha^2(\phi)$  and from the torsion through the coupling constant  $\epsilon$ . In this work we will consider high orders in  $\kappa(\phi)$ , because we are interested in the torsion contribution at high-energy scales, when possibly cosmic strings were formed.

The vortex configuration associated with the fields  $(\Phi, B_\mu)$  is given by

$$\begin{aligned}
\Phi &= \varphi(r)e^{i\theta}, \\
B_\mu &= \frac{1}{q}[P(r)-1]\delta_\mu^\theta, \quad (3.2)
\end{aligned}$$

with  $(t, r, \theta, z)$  being the usual cylindrical coordinates with  $r \geq 0$  and  $0 \leq \theta < 2\pi$ . The fields  $\varphi(r)$  and  $P(r)$  obey the same boundary conditions as the ordinary cosmic strings [42], namely,  $\varphi(r) = \eta$  and  $P(r) = 0$  outside the string and  $\varphi(r) = 0$  and  $P(r) = 1$  in the core. The electromagnetic properties are represented by the fields  $(\Sigma, A_\mu)$  with the configurations

$$\begin{aligned}
\Sigma &= \sigma(r)e^{i\xi(z,t)}, \\
A_\mu &= \frac{1}{e} \left[ A_t(r) - \frac{\partial \xi(z,t)}{\partial t} \right] \delta_\mu^t. \quad (3.3)
\end{aligned}$$

In the string core, the  $\Sigma$  field acquires an expectation value and is responsible for the timelike current carried by the gauge field  $A_\mu$  which does not vanishes outside the string. The potential  $V(\varphi, \sigma)$  triggering the spontaneous symmetry breaking can be written in the most general case as

$$V(\varphi, \sigma) = \frac{\lambda_\varphi}{4}(\varphi^2 - \eta^2)^2 + f_{\varphi\sigma}\varphi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 - \frac{m_\sigma^2}{2}\sigma^2, \quad (3.5)$$

where  $\lambda_\varphi$ ,  $\lambda_\sigma$ ,  $f_{\varphi\sigma}$ , and  $m_\sigma$  are coupling constants. Considering the analogy with the ordinary cosmic string case, this potential possesses all the ingredients necessary to drive the formation of a screwed cosmic string with a timelike current.

Now, let us consider a cosmic string in a cylindrical coordinate system, in which situation we can write the metric for the electric case in the Einstein frame as [43,44]

$$ds^2 = e^{2(\gamma-\psi)}(dr^2 + dz^2) + \beta^2 e^{-2\psi}d\theta^2 - e^{2\psi}dt^2, \quad (3.6)$$

where  $\gamma$ ,  $\psi$ , and  $\beta$  depend only on  $r$ .

A straightforward calculation shows that the Einstein equations appropriately modified to take into account contributions coming from the scalar-tensor features and torsion, can be written, in the Einstein frame as

$$R_{\mu\nu} = 2\kappa(\phi)\partial_\mu\phi\partial_\nu\phi + 8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T), \quad (3.7)$$

$$\begin{aligned}
G_{\mu\nu} &= 2\kappa(\phi)\partial_\mu\phi\partial_\nu\phi - \kappa(\phi)g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi \\
&+ 8\pi GT_{\mu\nu}. \quad (3.8)
\end{aligned}$$

Thus, the Einstein equations in the space-time given by Eq. (3.6) reads

$$\begin{aligned}
\beta'' &= 8\pi G\beta(T_t^t + T_r^r)e^{2(\gamma-\psi)}, \\
(\beta\gamma')' &= 8\pi G\beta(T_r^r + T_\theta^\theta)e^{2(\gamma-\psi)}, \\
(\beta\psi')' &= 4\pi G\beta(T_t^t + T_r^r + T_\theta^\theta - T_z^z)e^{2(\gamma-\psi)}. \quad (3.9)
\end{aligned}$$

The equation describing the scalar field  $\phi$ , in this background, is given by

$$[(\beta\kappa(\phi)\phi)'] = 4\pi G\beta T\alpha(\phi)e^{2(\gamma-\psi)}. \quad (3.10)$$

In order to solve Eqs. (3.9) and (3.10), let us write explicitly the components of the energy-momentum tensor in this case, which are given as

$$\begin{aligned}
T_t^t &= -\frac{1}{2}\Lambda^2(\phi)\left\{e^{2(\psi-\gamma)}(\phi'^2 + \sigma'^2) + \frac{e^{2\psi}}{\beta^2}\phi^2P^2\right. \\
&+ e^{-2\psi}\sigma^2A_t^2 + \Lambda^{-2}(\phi)e^{-2\gamma}\left(\frac{A_t'^2}{4\pi e^2}\right) \\
&+ \left.\Lambda^{-2}(\phi)\frac{e^{2(2\psi-\gamma)}}{\beta^2}\left(\frac{P'^2}{4\pi q^2}\right) + 2\Lambda^2(\phi)V(\phi, \sigma)\right\}, \quad (3.11)
\end{aligned}$$

$$\begin{aligned}
T_z^z &= -\frac{1}{2}\Lambda^2(\phi)\left\{e^{2(\psi-\gamma)}(\phi'^2 + \sigma^2) + \frac{e^{2\psi}}{\beta^2}\phi^2P^2\right. \\
&- e^{-2\psi}\sigma^2A_t^2 - \Lambda^{-2}(\phi)e^{-2\gamma}\left(\frac{A_t'^2}{4\pi e^2}\right) \\
&+ \left.\Lambda^{-2}(\phi)\frac{e^{2(2\psi-\gamma)}}{\beta^2}\left(\frac{P'^2}{4\pi q^2}\right) + 2\Lambda^2(\phi)V(\phi, \sigma)\right\}, \quad (3.12)
\end{aligned}$$



$$\begin{aligned}
T_r^r = & \frac{1}{2} \Lambda^2(\phi) \left\{ e^{2(\psi-\gamma)}(\phi'^2 + \sigma'^2) - \frac{e^{2\psi}}{\beta^2} \phi^2 P^2 \right. \\
& + e^{-2\psi} \sigma^2 A_t^2 - \Lambda^{-2}(\phi) e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right) \\
& \left. + \Lambda^{-2}(\phi) \frac{e^{2(2\psi-\gamma)}}{\beta^2} \left( \frac{P'^2}{4\pi q^2} \right) - 2\Lambda^2(\phi) V(\phi, \sigma) \right\}, \quad (3.13)
\end{aligned}$$

$$\begin{aligned}
T_\theta^\theta = & -\frac{1}{2} \Lambda^2(\phi) \left\{ e^{2(\psi-\gamma)}(\phi'^2 + \sigma'^2) - \frac{e^{2\psi}}{\beta^2} \phi^2 P^2 \right. \\
& + e^{-2\psi} \sigma^2 A_t^2 - \Lambda^{-2}(\phi) e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right) \\
& \left. - \Lambda^{-2}(\phi) \frac{e^{2(2\psi-\gamma)}}{\beta^2} \left( \frac{P'^2}{4\pi q^2} \right) + 2\Lambda^2(\phi) V(\phi, \sigma) \right\}. \quad (3.14)
\end{aligned}$$

Note that in the present case of a timelike current, the  $T_r^r$  and  $T_\theta^\theta$  components have different sign in the electromagnetic part as compared to the spacelike case [30]. In this situation, because only the temporal component of the electromagnetic field is different from zero, the electric charge in the core of the string does not vanish. In next sections we will investigate the consequences of this fact, in the framework of the weak field approximation.

Now, we solve the previous set of equations given by Eq. (3.9), in the region outside the TCSCS, that is, for  $r_0 \leq r < \infty$ , where  $r_0$  is the radius of the string. In this region, the contribution to the energy-momentum tensor of the string reads

$$\begin{aligned}
T_t^t = & -\frac{1}{2} e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right), \quad T_z^z = \frac{1}{2} e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right), \\
T_r^r = & -\frac{1}{2} e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right), \quad T_t^t = \frac{1}{2} e^{-2\gamma} \left( \frac{A_t'^2}{4\pi e^2} \right). \quad (3.15)
\end{aligned}$$

If we consider the asymptotic conditions, we can conclude that only the field  $A_\mu$  does not vanish outside the string. In this case, the external solutions of Eq. (3.9) are formally the same of the scalar-tensor theory [20], but the  $\phi$  solution is different and comes from the equation

$$\phi' = \kappa^{-1}(\phi) \frac{\lambda}{r}. \quad (3.16)$$

The solutions for  $\beta(r)$  and  $\gamma(r)$  are the same obtained recently [20] and are written as

$$\beta = Br, \quad \gamma = m^2 \ln r/r_0, \quad (3.17)$$

where  $B$  and  $m$  are integration constants. If we use Brans-Dicke theory to estimate the order of magnitude of the correction induced by  $\kappa^{-1}(\phi)\lambda$ , considering particular values of the parameter  $\omega$  consistent with solar system experiments made by very large baseline interferometry [45], we conclude that the external solution in this theory for those values of  $\omega$  is the same as in the case of the superconducting cosmic string in scalar-tensor theory [20]. Thus, we will assume that the metric function  $\psi(r)$  has the same form obtained in the case without torsion [20], and then, it can be written as

$$\psi(r) = \frac{\left( \frac{r}{r_0} \right)^n (1+p)}{\left( \frac{r}{r_0} \right)^{2n} + p}, \quad (3.18)$$

where now the parameter  $n$  is such that the following relation holds:  $n^2 = \kappa^{-1}(\phi)\lambda^2 + m^2$ , which is the same result obtained in [30].

Therefore, the external metric for the TCSCS takes the form

$$\begin{aligned}
ds^2 = & \left( \frac{r}{r_0} \right)^{-2n} W^2(r) \left[ \left( \frac{r}{r_0} \right)^{2m^2} (dr^2 + dz^2) + B^2 r^2 d\theta^2 \right] \\
& - \left( \frac{r}{r_0} \right)^{2n} \frac{1}{W^2(r)} dt^2, \quad (3.19)
\end{aligned}$$

with  $W(r) = [(r/r_0)^{2n} + p]/[1+p]$ . In order to get all information concerning the current in the core of the string we will use the weak field approximation (for details of this procedure see Ref. [20]), in which case we can write the following relations:

$$\begin{aligned}
g_{\mu\nu} = & \eta_{\mu\nu} + h_{\mu\nu}, \\
\Lambda(\phi) = & \Lambda(\phi_0) + \Lambda'(\phi_0)\phi_{(1)}, \\
T_{\mu\nu} = & T_{(0)\mu\nu} + T_{(1)\mu\nu}, \\
\phi = & \phi_0 + \phi_{(1)}, \quad (3.20)
\end{aligned}$$

where  $\Lambda'(\phi_0) = \Lambda(\phi_0)\alpha(\phi_0)$ ,  $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$  is the Minkowski metric tensor, and  $\phi_0$  is a constant.

In this case, the energy-momentum tensor of the string source  $T_{(0)\mu\nu}$  (in Cartesian coordinates) has the following components:

$$\begin{aligned}
T_{(0)tt} = & U \delta(x) \delta(y) + \frac{Q^2}{4\pi} \nabla^2 \left( \ln \frac{r}{r_0} \right)^2, \\
T_{(0)zz} = & -\tau \delta(x) \delta(y) + \frac{Q^2}{4\pi} \nabla^2 \left( \ln \frac{r}{r_0} \right)^2, \quad (3.21)
\end{aligned}$$

$$T_{(0)ij} = -Q^2 \delta_{ij} \delta(x) \delta(y) + \frac{Q^2}{2\pi} \partial_i \partial_j \ln(r/r_0),$$

where the energy per unit length  $U$  and the tension per unit length  $\tau$ , are given, respectively, by

$$U = -2\pi \int_0^{r_0} T_{(0)t}^t r dr \quad (3.22)$$

and

$$\tau = -2\pi \int_0^{r_0} T_{(0)z}^z r dr, \quad (3.23)$$

and the charge in the core is given by

$$Q = -2\pi e \int_0^{r_0} r dr \sigma^2 A_t. \quad (3.24)$$

Now, let us find the matching conditions connecting the internal and external solutions. For this purpose, we shall use the linearized Einstein-Cartan equation in order to find the internal solution. These equations are given by

$$\nabla^2 h_{\mu\nu} = -16\pi G (T_{(0)\mu\nu} - \frac{1}{2} g_{(0)\mu\nu} T_{(0)}). \quad (3.25)$$

At this point let us use a method applied by Linet [41] to solve the linearized Einstein's equations using distribution functions. Doing this, we find that the internal solution of Eq. (3.25) with time-independent source is given by

$$\begin{aligned} h_{tt} &= -4\tilde{G}_0 [Q^2 \{\ln(r/r_0)\}^2 + (U - \tau - Q^2) \ln(r/r_0)], \\ h_{zz} &= -4\tilde{G}_0 [Q^2 \{\ln(r/r_0)\}^2 + (U - \tau + Q^2) \ln(r/r_0)], \end{aligned} \quad (3.26)$$

$$h_{ij} = 2\tilde{G}_0 [Q^2 r^2 \partial_i \partial_j - 2\delta_{ij} (U + \tau - Q^2) \ln(r/r_0)].$$

It is worth calling attention to the fact that torsion does not appear explicitly in the components of the metric. This is a consequence of the linearized approximation in the  $\phi$  field.

In this case, we can find the matching conditions using the fact that  $[\{\alpha_{\mu\nu}\}]_{r=r_0}^{(+)} = [\{\alpha_{\mu\nu}\}]_{r=r_0}^{(-)}$  and given by  $[g_{\alpha\rho} K_{(\mu\nu)}^\rho]_{r=r_0}^{(-)} = [g_{\alpha\rho} K_{(\mu\nu)}^\rho]_{r=r_0}^{(+)}$ , for contorsion [46,47] where  $(-)$  represents the internal region and  $(+)$  corresponds to the external region around  $r = r_0$ . Then the continuity conditions are

$$\begin{aligned} [g_{\mu\nu}]_{r=r_0}^{(-)} &= [g_{\mu\nu}]_{r=r_0}^{(+)}, \\ \left[ \frac{\partial g_{\mu\nu}}{\partial x^\alpha} \right]_{r=r_0}^{(-)} &= \left[ \frac{\partial g_{\mu\nu}}{\partial x^\alpha} \right]_{r=r_0}^{(+)}. \end{aligned} \quad (3.27)$$

Now, let us find the solution of the equation for the field  $\phi$ . It is given by

$$\square_g \phi_{(1)} = -4\pi\kappa^{-1} \tilde{G}_0 \alpha(\phi) T_{(0)}, \quad (3.28)$$

where  $T_{(0)} = -(U + \tau + Q^2) \delta(x) \delta(y)$ . Then, the solution of Eq. (3.28) in terms of the new coordinate,  $\rho = r[1 + \tilde{G}_0(4\tau - Q^2) - 4\tilde{G}_0\tau \ln(r/r_0) + 2\tilde{G}_0 Q^2 \ln^2(r/r_0)]$ , is given by

$$\phi_{(1)} = 2\tilde{G}_0 \kappa^{-1} (\phi) \alpha(\phi_0) (U + \tau + Q^2) \ln \frac{\rho}{r_0}. \quad (3.29)$$

Taking into account the linearized forms of the exterior and interior metrics, we obtain

$$\begin{aligned} m^2 &= 4\tilde{G}_0 Q^2, \\ B^2 &= 1 - 8\tilde{G}_0 \left( \tau - \frac{Q^2}{2} \right), \end{aligned}$$

$$\lambda = 2\tilde{G}_0 \alpha(\phi_0) (U + \tau + Q^2).$$

Finally, if we rewrite Eq. (3.26) in terms of the new coordinate  $\rho$  and use the result given by Eq. (3.29), the metric for a TCSCS, in the Einstein frame, can be written as

$$\begin{aligned} ds^2 &= \left\{ 1 - 4\tilde{G}_0 \left[ Q^2 \ln^2 \frac{\rho}{r_0} + (U - \tau + Q^2) \ln \frac{\rho}{r_0} \right] \right\} (d\rho^2 + dz^2) \\ &\quad - \left\{ 1 + 4\tilde{G}_0 \left[ Q^2 \ln^2 \frac{\rho}{r_0} + (U - \tau - Q^2) \ln \frac{\rho}{r_0} \right] \right\} dt^2 \\ &\quad + \rho^2 \left[ 1 - 8\tilde{G}_0 \left( \tau - \frac{Q^2}{2} \right) - 4\tilde{G}_0 (U - \tau - Q^2) \ln \frac{\rho}{r_0} \right. \\ &\quad \left. - 4\tilde{G}_0 Q^2 \ln^2 \frac{\rho}{r_0} \right] d\theta^2. \end{aligned} \quad (3.30)$$

Now, if we return to the Jordan-Fierz frame, the deficit angle associated with this space-time can be written in the linearized approximation as

$$\Delta\theta = 4\pi\tilde{G}_0 (U + \tau - 2Q^2). \quad (3.31)$$

This means that in this order of approximation there is no contribution arising from torsion. In fact, the contribution to the metric due to torsion in the Jordan-Fierz frame appears in the dilaton solution given by Eq. (3.29), but it is not preserved in the linearized expression given by Eq. (3.31), for the deficit angle. The reason for this absence of torsion in the deficit angle is that the contribution arising from torsion comes out only in second order in  $\tilde{G}_0$  and therefore it does not appear in the linearized solution we have considered. This result corresponds to the same one obtained in the time-like case of pure scalar-tensor theories of gravity [48].

#### IV. PARTICLE DEFLECTION NEAR A TCSCS

In the previous section we concluded that torsion does not contribute to the deficit angle, but some new physical effects appear associated with torsion in such a way that it plays a role as we shall see in what follows. In this section we study the geodesic equation in the space-time under consideration. To do this, we have to work with the metric given by Eq. (3.30) which was written in the Jordan-Fierz frame. For this we can use the Christoffel symbols in Jordan-Fierz frame as in Eq. (2.10). In this frame, the  $tt$  component of Eq. (2.10) is given by

$$\{^i_{tt}\}_{JF} = \{^i_{tt}\} + K_{(tt)}{}^i. \quad (4.1)$$

Let us consider the effect of the torsion on an uncharged particle moving around the defect, assuming that the particle has a speed  $|v| \leq 1$ . In this case the geodesic equations become

$$\frac{d^2 x^i}{dt^2} + \{^i_{tt}\}_{JF} = 0, \quad (4.2)$$

where  $i$  is the spatial coordinate index. We note here that the symmetric part of the dilaton gradient does not appear because the dilaton has no dependence on time. In the present case, in the linearized approximation, the Christoffel symbols in the Einstein frame are given by

$$\{^i_{tt}\} = -\frac{1}{2} \partial^i h_{tt}, \quad (4.3)$$

with  $g_{tt} = -1 + h_{tt}$ . According to the previous section, in the context of Einstein gravity the component  $h_{tt}$  reads

$$h_{tt} = -4\tilde{G}_0 \left\{ (U - \tau - Q^2) \ln\left(\frac{\rho}{r_0}\right) + Q^2 \ln\left(\frac{\rho}{r_0}\right)^2 \right\}. \quad (4.4)$$

In order to make our analysis more simple, let us consider this approach in the framework of Einstein gravity. In this case, it is necessary to compute the symmetric part of the contortion (2.11), which is given by

$$\begin{aligned} K_{(tt)}{}^r &= -\frac{\tilde{\phi}'}{2\tilde{\phi}} \sim \alpha(\phi_0) \phi'_{(1)} \\ &= 2\frac{1}{\rho} \tilde{G}_0 \kappa^{-1}(\phi) \alpha^2(\phi_0) (U + \tau + Q^2), \end{aligned} \quad (4.5)$$

and the geodesic equations can be written as

$$\frac{d^2 x^i}{dt^2} - \frac{1}{2} \partial^i h_{tt} + K_{(tt)}{}^i = 0, \quad (4.6)$$

where  $h_{tt}$  does not contain a contribution arising from torsion as we can see from Eq. (4.4). We can see that this expression is compatible with the Christoffel symbols in Jordan-Fierz frame calculated with the general form (2.10). Then, in this framework the contribution arising from torsion is contained entirely in the symmetric part of the contorsion  $K_{(tt)}{}^i$  given by Eq. (4.5), evaluated in the Jordan-Fierz frame. This form of the geodesic equations is more suitable to an analysis of the new contribution arising from the dilaton-torsion aspects.

Taking into account previous considerations we can conclude that the gravitational acceleration induced by the string around it, is given by

$$a = \nabla h_{tt} - 2 \frac{\tilde{G}_0 \kappa^{-1}(\phi) \alpha^2(\phi_0) (U + \tau + Q^2)}{\rho}, \quad (4.7)$$

and as a consequence, the torsion contribution to the force reads

$$f_{tors} = -\frac{2m}{\rho} \tilde{G}_0 \kappa^{-1}(\phi) \alpha^2(\phi_0) (U + \tau + Q^2). \quad (4.8)$$

Therefore, the total force on a test particle due to a TCSCS can be written as

$$\begin{aligned} f = & -\frac{4\tilde{G}_0 m}{\rho} \left[ Q^2 \left( \frac{U - \tau}{Q^2} - 1 + 2\ln(\rho/r_0) \right) \right. \\ & \left. + \frac{1}{2} \kappa^{-1}(\phi) \alpha^2(\phi_0) Q^2 \left( 1 + \frac{U + \tau}{Q^2} \right) \right]. \end{aligned} \quad (4.9)$$

In what follows, let us consider the deflection of particles moving past the string. Assuming for simplicity that the direction of propagation is perpendicular to the string, we can write the metric, in terms of Minkowskian coordinates, as

$$ds^2 = (1 - \tilde{h}_{tt})[-dt^2 + dx^2 + dy^2], \quad (4.10)$$

where  $\tilde{h}_{tt}$  is given by

$$\begin{aligned} \tilde{h}_{tt} = & -4\tilde{G}_0 \left\{ Q^2 \left[ \ln\left(\frac{\rho}{r_0}\right) \right] + U - \tau - Q^2 + \frac{\alpha^2 \kappa^{-1}(\phi)}{2} (U + \tau \right. \\ & \left. + Q^2) \right\} \ln\left(\frac{\rho}{r_0}\right). \end{aligned} \quad (4.11)$$

Note that in this case, we can conclude that there is a change in the geodesics due to the presence of the contortion (4.6). In order to investigate the formation of a wake moving behind a TCSCS, we will first consider the rest frame of the string with a velocity  $v$  in the  $x$  direction. Thus, in this situation, the geodesic equations (4.6) can be written in the linearized version as

$$2\ddot{x} = -(1 - \dot{x}^2 - \dot{y}^2) \partial_x h_{tt} + (1 - \dot{y}^2) \alpha(\phi_0) \partial_x \phi_{(1)}, \quad (4.12)$$

$$2\ddot{y} = -(1 - \dot{x}^2 - \dot{y}^2) \partial_y h_{tt} + (1 - \dot{x}^2) \alpha(\phi_0) \partial_y \phi_{(1)}, \quad (4.13)$$

where  $h_{tt}$  is given by Eq. (4.4) and the overdot denotes derivative with respect to  $t$ . Now, let us concentrate our attention on the last terms of Eqs. (4.12) and (4.13) due to the fact that they contain information concerning the roles of the scalar field and torsion. For our analysis it is enough to consider only terms of first order in  $\tilde{G}_0$ , in which case (4.13) can be integrated over the unperturbed trajectory  $x = vt$ ,  $y = y_0$ . Doing this procedure and going to the frame in which the string has a velocity  $v$ , we find that particles entering the wake have a transverse velocity given by

$$v_t = 4\pi\tilde{G}_0(U + \tau - 2Q^2)v\gamma + \frac{4\pi\tilde{G}_0\left\{Q^2\left[\ln\left(\frac{\rho}{r_0}\right)\right] + U - \tau - Q^2 + \frac{1}{2}\alpha^2\kappa^{-1}(\phi)(U + \tau + Q^2)\right\}}{v\gamma}. \quad (4.14)$$

This result tell us that the first term contains the usual contribution of the deficit angle to the velocity of the particles. The second term contains the contributions arising from torsion and electromagnetic field. A quick glance at this equation allows us to understand the essential role played by torsion in the context of the present formalism. For example, if torsion is present, even in the case in which the string has no current, an attractive gravitational force comes out. In the context of the TCSCS, torsion enhances the force that a test particle feels outside the string. This peculiar fact may have meaningful astrophysical as well as cosmological implications, as for example, contributing to the process of formation of structures.

**V. ZEL'DOVICH APPROXIMATION IN TORSION SPACE-TIME**

In this section we study the linear perturbation method developed by Zel'dovich [40] and apply it to the case of a space-time with torsion. We will assume that dark matter particles interact very weakly, so that all forces on the particles of nongravitational origin can be ignored. In the case of the ordinary cosmic strings this approach has been tested against exact solution and  $N$ -body simulations with satisfactory results [49]. An important advantage of the Zel'dovich approach is the fact that it can be used in the case where the evolution is strongly nonlinear. In this paper we consider a linear evolution. We analyze the perturbation in the dark matter background when the cosmic string is formed considering an unperturbed universe. We also investigate the accretion of cold dark matter by straight strings in a universe in the matter-dominated era  $t > t_{eq}$ , with scale factor  $a(t) \sim t^{2/3}$ , and average density given by

$$\rho_{av} = \frac{1}{6\pi G t^2}. \quad (5.1)$$

Let us write the trajectories of a cold dark matter particle as

$$\vec{r}(x, t) = a(t)[\vec{x} + \Psi(x, t)], \quad (5.2)$$

where  $\vec{x}$  is the unperturbed comoving position of a particle and  $\Psi$  is the comoving displacement measured from the position of the particle.

Now, let us analyze the equation of motion for this particle in the presence of torsion. We find a Newton's-like equation in Cartesian coordinates in the background corresponding to the metric (4.10), which reads as

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}_r\Lambda, \quad (5.3)$$

where the gravitational potential  $\Lambda$  is given by

$$\Lambda = \Lambda_{CDM} + \Lambda_{TCSCS}, \quad (5.4)$$

with  $\Lambda_{CDM}$  being the gravitational potential due to the cold dark matter. In this work we will consider that the presence of the cold dark matter does not affect the cosmic string configuration, but on the other hand, the cosmic string perturbs the dark matter trajectories. In this case, the cosmic string gravitational potential  $\Lambda_{TCSCS}$  can be written as a function of the linearized cosmic string metric given by

$$\Lambda_{TCSCS} = -2\tilde{G}_0\left\{Q^2\left[\ln\left(\frac{\rho}{r_0}\right)\right] + U - \tau - Q^2 + \frac{\alpha^2\kappa^{-1}(\phi)}{2}(U + \tau + Q^2)\right\}\ln\left(\frac{\rho}{r_0}\right), \quad (5.5)$$

and therefore, the trajectories of the cold dark matter particles are perturbed by the TCSCS space-time.

The gravitational potential  $\Lambda(x, t)$  satisfies the Poisson equation

$$\nabla_r^2\Lambda = 4\pi G(\rho + \rho_{TCSCS}), \quad (5.6)$$

where  $\rho$  is the cold dark matter density and  $\rho_{TCSCS}$  is the perturbation due to the string. Mass conservation implies that

$$\rho(\vec{r}, t) = \frac{\rho_{av}(t)a^3(t)}{|\det(\partial\vec{r}/\partial\vec{x})|}. \quad (5.7)$$

Using Eqs. (5.2) and (4.6), thus we obtain

$$\frac{\partial\vec{r}}{\partial\vec{x}} = a(t)[1 + \nabla_x \cdot \Psi(\vec{x}, t)], \quad (5.8)$$

and then, in the approximation we are considering, we get

$$\rho(\vec{r}, t) = \rho_{av}(t)[1 - \nabla_x \cdot \Psi(\vec{x}, t)]. \quad (5.9)$$

From Eq. (5.2) for the trajectories of cold dark matter particles, we can obtain the relations

$$\ddot{r} = \ddot{a}\Psi + 2\dot{a}\dot{\Psi} + a\ddot{\Psi}. \quad (5.10)$$

Now, consider the fact that  $a \sim t^{2/3}$ . Thus, we have

$$\ddot{\Psi} + \frac{4}{3t}\dot{\Psi} - \frac{2}{3t^2}\Psi = -\frac{v_t}{a^3}. \quad (5.11)$$



In order to solve Eq. (5.11), let us consider an idealized situation in which the cosmic string is formed in the time  $t_i > t_{eq}$ , in an initially unperturbed universe. The perturbation caused by the cosmic string in the time  $t > t_i$  can be found by solving Eq. (5.11) with the following initial conditions:

$$\Psi(\vec{x}, t_i) = \dot{\Psi}(\vec{x}, t_i) = 0. \quad (5.12)$$

The solution of Eq. (5.11), with the initial conditions given above, can be written as

$$\Psi(\vec{x}, t) = 3v_t \left[ 1 - \frac{2}{5} \frac{t_i}{t} - \frac{3}{5} \left( \frac{t}{t_i} \right)^{(2/3)} \right], \quad (5.13)$$

with  $v_t$  given by Eq. (4.14). This result shows the influence of the parameters contained in the expression for  $v_t$  given in Eq. (4.14), which determines the TCSCS space-time. It represents the time-dependent accretion of the cold dark matter by a TCSCS. As we saw in the last section the transversal velocity of the accretion by wake,  $v_t$ , depends on the gravitational effects arising from torsion. We can identify torsion through the presence of the parameter  $\kappa(\phi)$ . Using Eq. (2.14) we conclude that this contribution is given by  $\alpha^2(1 - 2\epsilon\alpha^2)^{-1}[U + \tau + Q^2]/v\gamma$ , and therefore it enhances the attraction between the cold dark matter particles in the wake and introduces new parameters that can be adjusted by simulation in order to describe the observational spectrum. This contribution comes in the contortion part given by Eq. (4.5). The factor  $\epsilon$ , in the second term, contains torsion signature, which we are considering as an arbitrary parameter. Nowadays, this parameter is taken to be very small, but in cosmic string scale formation or in high-energy scales [50], torsion effects probably give us an interesting contribution. In the case where  $\epsilon$  can be neglected, the effect of torsion is to enhance dilaton effect. Then, torsion effects are important and if analyzed when  $t \sim t_i$ , it may be relevant and thus, cannot be neglected. As we are working in the linearized approximation, the results obtained give us only an approximate idea of the effects of torsion.

It is worth calling attention to the fact that in the present case, the time-dependent solution has the same form of the ordinary cosmic string, but the accretion velocity has an interesting dependence on the dilaton solution that contains the current-carrying and torsion terms.

## VI. WAKE EVOLUTION IN A BACKGROUND WITH TORSION

In this section we study the formation of sheetlike wakes when a TCSCS moves fastly. In the previous section we investigated the effect of cosmic string formation in the cold dark matter universe, considering an unperturbed universe. Now, we will make a quantitative description of accretion onto wakes using the Zel'dovich approximation developed in last section, but now considering the presence of torsion and assuming that in the linearized solution, torsion has some contribution. To investigate this problem, we have to solve the following differential equation:

$$\ddot{\Psi} + \frac{4}{3t}\dot{\Psi} - \frac{2}{3t^2}\Psi = -\frac{u_p}{a^3}, \quad (6.1)$$

where  $u_p$  is the linearized contribution coming from the dilaton torsion in the background under consideration and is given by

$$u_p = \frac{2\pi\tilde{G}_0\kappa^{-1}\alpha^2(\phi_0)(U + \tau + Q^2)}{v\gamma}. \quad (6.2)$$

In this case, differently from the last section, the wakes begin to move with a velocity due to the perturbation, with magnitude given by Eq. (6.2). We will consider that the wakes have a dissipation only induced by torsion effects in the geodesic equation and that the appropriate initial conditions are

$$\Psi(t_i) = 0, \quad \dot{\Psi}(t_i) = -v_t, \quad (6.3)$$

where  $v_t$  contains the contribution of the torsion. The solution in a time immediately after  $t_i$ , where the torsion effects are present, is given by

$$\Psi(x, t) = \frac{3}{2} \left[ u_p - \frac{1}{5}(u_p - v_t t_i) \frac{t_i}{t} - \frac{3}{5} \left( u_p - \frac{2}{3} v_t t_i \right) \left( \frac{t}{t_1} \right)^{2/3} \right]. \quad (6.4)$$

Note that, when  $t \gg t_i$ , the torsion perturbation  $u_p$  can be neglected. In this situation, there is a contribution coming from the torsion contained in  $v_t$ , which is small, but does not vanish nowadays. Thus, taking into account this fact, Eq. (6.4) turns into

$$\Psi(x, t) = \frac{3}{5} v_t \left[ \frac{t_i^2}{t} - t_i \left( \frac{t}{t_i} \right)^{2/3} \right]. \quad (6.5)$$

The turn around surfaces, where particles stop expanding with the Hubble flow in the  $x$  direction and begin falling back towards the wake, can be found from the condition  $\dot{r}_x = 0$  or, equivalently, from  $x + 2\Psi(x, t) = 0$ . This yields

$$x(t) = 3v_t \left[ \frac{u_p}{v_t} - \frac{1}{5} \left( \frac{u_p}{v_t} - t_i \right) \frac{t_i}{t} - \frac{3}{5} \left( \frac{u_p}{v_t} - \frac{2}{3} t_i \right) \left( \frac{t}{t_i} \right)^{2/3} \right]. \quad (6.6)$$

The wake thickness  $d(t)$  and the surface mass density  $\sigma(t)$  of the wake are given, respectively, by

$$d(t) = 2x(t) \left( \frac{t}{t_i} \right)^{2/3} \sim 6v_t \left[ \frac{u_p}{v_t} - \frac{1}{5} \left( \frac{u_p}{v_t} - t_i \right) \frac{t_i}{t} - \frac{3}{5} \left( \frac{u_p}{v_t} - \frac{2}{3} t_i \right) \right] \times \left( \frac{t}{t_i} \right)^{2/3} \left( \frac{t}{t_i} \right)^{2/3} \quad (6.7)$$

and

$$\sigma(t) = \rho_t d(t) \sim \frac{v_t}{\pi \tilde{G}_0 t^2} \left[ \frac{u_p}{v_t} - \frac{1}{5} \left( \frac{u_p}{v_t} - t_i \right) \frac{t_i}{t} - \frac{3}{5} \left( \frac{u_p}{v_t} - \frac{2}{3} t_i \right) \times \left( \frac{t}{t_i} \right)^{2/3} \right] \left( \frac{t}{t_i} \right)^{2/3}. \quad (6.8)$$

We investigated the wake evolution in a background with torsion using  $\rho(t) = 1/6\pi\tilde{G}_0 t^2$ . This is the same expression as in the case of the flat universe, and constitutes a reasonable approximation in the case of a linearized solution in terms of torsion in the matter-dominated era with the wake formed at  $t_i \sim t_{eq}$ . The most important new feature of the results here presented is to consider the wake in a background with torsion.

In the wake evolution case the contribution of the background to the wake vanishes and the dilaton-torsion contribution only appears in the transversal velocity of the accretion  $v_t$ . In the case where the torsion parameter  $\epsilon$  is small, this contribution can be neglected as compared with the electromagnetic effects and the only effect of the torsion is to amplify the dilaton interactions. But if we analyze the result when  $t \sim t_i$ , we conclude that the terms which contain contributions arising from torsion can be relevant and the  $\epsilon$  parameter can be adjusted in order to correspond to the early era. In this scenario, torsion could dominate the accretion of matter. Other interesting analyses can be done when the electromagnetic current vanishes, in which case the contribution is due only to dilaton-torsion effects, in which case  $\Psi(x, t)$  is given by  $\Psi(x, t) = \frac{3}{5} u_p [t_i^2/t - t_i(t/t_i)^{2/3}]$ . In this scenario, at high-energy scale, these effects can be measurable [50], in principle.

## VII. CONCLUSION

We have obtained the solution that corresponds to a time-like current-carrying screwed cosmic string (TCSCS). Screwed cosmic strings are stable topological defects and has been obtained in the framework of a general scalar-tensor theory including torsion. In the model in which spin vanishes, torsion is a  $\phi$  gradient and propagates outside the string. In fact, torsion is small but gives a non-negligible contribution to the geodesic equations obtained from the contortion term and from the scalar fields. The motivation to consider this scenario comes from the fact that scalar-tensor gravitational fields are important for a consistent description of gravity, at least at sufficiently high energy scales. On the other hand, torsion can induces some physical effects and could be important at some energy scale, as for example, in the low-energy limit of a string theory.

The analysis of the metric and contortion help us to understand the consequences of the gravitational interaction due to a TCSCS at a cosmological level. One important consequence is related with the gravitational field surrounding a TCSCS, which is divergent for the state parameter approaching the mass of the current carrier, and thus the gravitational effects seem unbounded. However, it is important to call attention to the fact that this divergence is strongly connected with another divergence, namely, that is associated with the

string tension: as  $U$  increases, the tension decreases to zero, and eventually becomes negative so that the corresponding state is absolutely unstable against the transverse perturbations and should go into a stable state. Therefore, the gravitational effects of such strings are indeed limited even at classical level.

In the space-time generated by a TCSCS, massless particles (such as photons) will be deflected by an angle  $\Delta\theta = 4\pi\tilde{G}_0(U + \tau - 2Q^2)$ . From the observational point of view, it would be impossible to distinguish a screwed string from its general relativity partner, just by considering effects based on deflection of light, as for instance, double image effect. On the other hand, trajectories of massive particles will be affected by torsion coupling, which is generated by a space-time with torsion [51,52].

We have shown that wakes produced by the string in one Hubble time can have important effects due to torsion. If the string is moving with normal velocity  $v$  through matter, a transversal velocity appears. It is worth calling attention to the fact that there exists, in this case, a new contribution to the transversal velocity given by  $v_t = 2\pi\tilde{G}_0\kappa^{-1}\alpha^2(\phi_0)(U + \tau + Q^2)/v\gamma$ , which is associated with aspects of the scalar-tensor theories which includes torsion.

We also have shown that the propagation of photons is unaffected by a TCSCS and it is only affected by the angular deficit. This result shows us that the effect of torsion on massive particles is qualitatively different from its effect on radiation. This aspect becomes especially relevant when calculating CMBR anisotropy and the power spectrum as wiggly cosmic strings.

The investigations concerning the formation and evolution of wakes in the space-time of TCSCS shows that there is an effect arising from torsion on the process of wakes formation. Using the Zel'dovich approximation we analyzed the linear perturbations in this space-time. The accretion of cold dark matter in the isolated strength cosmic string is studied and the effect of torsion was pointed out. Assuming the validity of the linear perturbation methods developed by Zel'dovich in this background and that dark matter particles interact very weakly, in such a way that all forces different from the gravitational one can be ignored, it was shown that the accretion of matter by wakes formation when a TCSCS moves with speed  $v$  depends on the features of the scalar field and torsion.

Therefore, assuming that torsion have had a physically relevant role during the early stages of the Universe's evolution, we can say that torsion fields may be potential sources of dynamical stresses which, when coupled to other fundamental fields (i.e., the gravitational and scalar fields), might have performed an important action during the phase transitions leading to formation of topological defects such as the TCSCS we have considered. Therefore, it seems an important issue to investigate basic models and scenarios involving cosmic defects within the context of scalar-tensor theories with torsion and one of the reasons for this is the fact that torsion would be relevant in  $t \sim t_i$ , that is, in the early stages of our Universe.

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