

Effective potential and vacuum stability within universal extra dimensions

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The one-loop effective potential calculated for a generic model that originates from 5-dimensional theory reduced down to 4 dimensions is considered. The cutoff and dimensional regularization schemes are discussed and compared. It is demonstrated that the prescriptions are consistent with each other and lead to the same physical consequences. The stability of the ground state is discussed for a U(1) model that is supposed to mimic the standard model extended to 5 dimensions. It has been shown that fermionic Kaluza-Klein modes can dramatically influence the shape of the effective potential shifting the instability scale even by several orders of magnitude.

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I. INTRODUCTION

For some time there has been increased interest in the possible extensions of the standard model (SM) that allow for fields living in extra dimensions. One possible scenario, referred to as the universal extra dimensions (UED) model [1] assumes that all the SM degrees of freedom propagate in compactified extra dimension of the size of $R \sim \text{TeV}^{-1}$.¹ It has been shown that in fact R^{-1} as low as $\sim 0.3 \text{ TeV}$ is allowed by the precision electroweak observables [1]. Constraints from flavor changing processes have been carefully analyzed in Refs. [3,4] while the anomalous magnetic moment has been studied in Ref. [5]. All the analysis concludes that even $R^{-1} \sim 0.3 \text{ TeV}$ is consistent with the existing experimental data. The main reason for the suppression of extra contributions to the above observables is the momentum conservation in the fifth dimension. In the equivalent 4D theory this implies that an emission of a single non-zero Kaluza-Klein (KK) mode is forbidden. Consequently there is no tree-level contribution to the electroweak observables, and therefore KK effects are suppressed. However, the large size of R could lead to exciting phenomenology at the next generation of colliders [6].

Constraints from the precision electroweak observables on the Higgs physics have been analyzed in Refs. [1] and [7]. In particular Ref. [7] shows the allowed region for the Higgs-boson mass m_h and the compactification radius R in the 5D UED compactified on S^1/Z_2 . It turns out that for $m_h \sim 0.9 \text{ TeV}$ even $R^{-1} \sim 0.25 \text{ TeV}$ is allowed. Since effects of KK modes appear at the 1-loop therefore one could expect their relevance for processes that emerge at the 1-loop level in the SM, an illustration of that reasoning could be found in Refs. [3,4] and [5]. Here we will consider influence of extra dimensional physics on the stability of the ground state. It is well known that within the SM model [8] and variety of its extensions [9] contributions from fermionic degrees of freedom could lead to an effective potential that is unbounded

from below, provided the Higgs boson mass is small enough [10]. That implies a lower bound on m_h as a function of the cutoff scale below which the theory is supposed to be stable. Since the compactification of the 5D theory leads to existence of an infinite tower of 4D fermions, therefore it is natural to expect that the SM picture of the effective potential will be modified.² Indeed, as we have found the influence of fermionic KK modes on the scale of stability is dramatic, *the scale could be shifted by many orders of magnitude*

The paper is organized as follows. In Sec. II, we discuss generic properties of the effective potential both in the cutoff and the dimensional regularization. Section III presents details of the 5D model considered here and also analytical results for the effective potential. In Sec. IV, we discuss numerical results. Concluding remarks are given in Sec. V.

II. THE GENERIC EFFECTIVE POTENTIAL

Here we will present results for a contribution to the one-loop effective potential coming from an infinite tower of virtual KK modes (numbered by an integer n). The following generic formula is applicable both for fermions and bosons circulating³ in loops:

$$V(\varphi) = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \ln[l^2 E^2 + (n + \omega)^2 \pi^2], \quad (1)$$

where ω is a constant shift, $E^2 \equiv p^2 + m^2(\varphi)$, $m^2(\varphi)$ is the background field dependent mass squared of virtual KK modes, the momentum p is defined in the Euclidean space ($p^2 = p_0^2 + (\vec{p})^2$), the field independent factor $l \equiv \pi R$ was introduced for dimensional reasons and all unnecessary constant terms have been dropped.

A. Divergences

There are two sources of possible divergences appearing in the effective potential (1): (i) the momentum integration,

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¹The first studies of possible effects of SM fields living in TeV-scale extra dimensions were performed by Antoniadis [2].

²For earlier discussion of the instability within extra dimensional theories, see Ref. [11].

³For vector bosons the Landau gauge should be adopted, while for fermions extra minus sign must be added.

and (ii) the infinite sum over KK modes. The integral could be regularized either by the dimensional method or by the cutoff, while for the sum one can, for instance, use the method adopted by Delgado, Pomarol and Quirós (DPQ) in Ref. [12], the ζ regularization (see e.g. [13]) or just truncation of the series (for the discussion see Refs. [14,15]).

There is a comment here in order. Since both the integration and the summation are not convergent therefore the interchange of their ordering seems to be a nontrivial issue. This question was already addressed in Ref. [13] in the framework of 5D SUSY model compactified on the orbifold $S^1/(Z_2 \times Z'_2)$. The authors computed the effective potential performing first the integration with dimensional regularization and then adopting the ζ regularization for the KK sum. It has been shown that when dimensional regularization is adopted⁴ then both orderings lead to the same ultraviolet finite result separately for scalars and fermions. So, the “KK regularization” used both in Ref. [12] and in Ref. [13] leads to the same result. However this regularization seems to suffer from certain drawbacks.

Since the 5D theory is nonrenormalizable therefore there must exist certain physical cutoff Λ_5 , related to the scale of more fundamental high-energy physics, e.g., string theory. Therefore performing loop expansion in 5D it would be natural to cut all loop integrals d^5p at the scale Λ_5 . From the 4D perspective the summation over KK modes corresponds to the integration over the fifth momentum component, so it seems to be appropriate to limit the sum to $n \leq \Lambda_5 R$, what would roughly guarantee that we sum all modes that are lighter than the cutoff. In contrast to this strategy the KK regularization requires summation over all the modes, therefore its physical meaning seems to be rather unclear.⁵

Reference [13] shows that for the KK regularization the resulting effective potential in the limit $R \rightarrow 0$ is different when we decompactify ($R \rightarrow 0$) before the regularization (assuming that all nonzero KK modes decouple in this case one recovers the 4D effective potential generated just by the zero mode) and after the regularization (the KK-regularized effective potential diverges in this limit).

In this paper we are going to discuss vacuum stability, so for a given mass of the Higgs boson zero mode we will determine the scale below which the model makes sense (the vacuum is stable). Therefore it seems to be meaningful to restrict the mass spectrum of the KK modes to those which are lighter than the cutoff, so in the following we will also consider truncation of series over KK modes to those $n < n_{max} \equiv \Lambda_5 R$. From the 5D perspective, this will correspond to a cutoff for the integration over the fifth momentum com-

ponent. Then, of course, the sum is finite and therefore question of ordering for the summation and integration becomes meaningless. Concerning the regularization of the d^4p integral the analogous approach would be to adopt a cutoff regulator. We will illustrate this strategy below.

Even though the cutoff regularization seems to be the most natural one, there exist also arguments against it. The standard objections are the following.

Because of the compactification on the circle, the shift along the extra direction, $y \rightarrow y + 2\pi R$, should leave the theory unchanged. Therefore the fifth component of momentum is quantized to be elements of Z/R . A consequence of that is the “integer shift” symmetry, i.e. a symmetry under an integer shift of KK modes. Obviously, cutting the series breaks the symmetry, as there would be no modes to go.

Another drawback of the regularization through a limited number of modes is the fact that 5D gauge invariance is broken in that case. Namely, limiting the number of KK modes we impose a condition on the 5D gauge transformation parameter $\theta(x,y)$ that has the following general expansion:

$$\theta(x,y) = \frac{1}{\sqrt{2\pi R}} \left[\theta_0(x) + \sqrt{2} \sum_{n=1}^{\infty} \theta_n(x) \cos(m_n y) \right]. \quad (2)$$

Therefore, if we had summed up to n_{max} , then obviously, the series would not be able to reproduce all possible 5D gauge parameter functions $\theta(x,y)$.

So, it is essential to look for a regularization prescription that would be consistent with all the symmetries that are present. The dimensional regularization is the standard option that satisfy the requirement. An interesting and natural generalization of dimensional regularization for sums over KK modes was developed in Refs. [16,17]. The strategy is in its spirit similar to the method adopted earlier by DPQ in Ref. [12], namely the sum could be traded for a one-dimensional contour integral that one can regularize by analytic continuation in the number of dimensions. The great advantage of this approach is that both the gauge and also the “integer shift” symmetries are preserved.

Therefore for completeness and comparison we will consider in the following sections the effective potential found adopting both the cutoff regularization with limited KK summation and the KK regularization [12] proposed by DPQ.⁶

B. Limited KK summation and cutoff regularization

In this section we will discuss an effective potential within a 5D theory of a scalar field assuming that only a zero mode (in KK expansion) of the scalar can acquire a vacuum expectation value. Because of later applications we will restrict ourself to the sum over non-negative n and $\omega = 0$ in the effective potential (1). Then for a limited number of KK modes with the 4D cutoff (Λ) regularization the effective potential reads

⁴The effective potential found in Ref. [12] was ultraviolet divergent, however note that the cutoff regularization was adopted there. It is easy to see that for the dimensional regularization the result would be finite.

⁵An interesting observation has been made in Refs. [14,15], where the authors showed that the vanishing of quadratic divergences that happens separately for bosons and fermions is a consequence of cancellation between contributions of states of mass larger than the cutoff Λ_5 and light states laying below the cutoff.

⁶As it will be discussed shortly the KK regularization leads to the same result as the dimensional regularization of the sum over KK modes and of the integral along the line proposed in Refs. [16,17].

$$V_{eff\ bare}^{1-loop} = \frac{1}{32\pi^2} \sum_{n=0}^{n_{max}} \left\{ \Lambda^2 m^2(\varphi) + \frac{1}{2} [m^2(\varphi) + m_n^2]^2 \right. \\ \left. \times \left[\ln \left(\frac{m^2(\varphi) + m_n^2}{\Lambda^2} \right) - \frac{1}{2} \right] \right\}, \quad (3)$$

where φ is the external classical field (the zero mode), $m_n^2 \equiv (n/R)^2$ and $n_{max} \equiv \Lambda_5 R$ for Λ_5 being the 5D cutoff of the dp_5 integration. Therefore imposing such a limit on the number of modes is roughly equivalent to 5D cutoff regularization of dp_5 integration. The terms that are divergent in the limit $\Lambda \rightarrow \infty$ are the following:

$$V_{eff}^{1-loop}|_{div} = \frac{(n_{max} + 1)}{32\pi^2} \left\{ m^2(\varphi) \left[\Lambda^2 - \frac{n_{max}}{3R^2} \right] \right. \\ \left. \times (1 + 2n_{max}) \ln(R\Lambda) - m^4(\varphi) \ln(R\Lambda) \right\}. \quad (4)$$

There is a comment here in order. In a case of mixing between virtual degrees of freedom, nondiagonal mass matrices may appear and the eigenvalues are in general nonpolynomial functions of φ [see, for example, the (A_5, χ_n) system for the model discussed in Sec. III]. At first sight this seems to jeopardize the process of renormalization since only φ^2 and φ^4 counter-terms are at our disposal (within a ϕ^4 theory), while the divergent contributions appear to be non-polynomial functions of φ . However, for a general mass matrix we should replace $m^2(\varphi)$ and $m^4(\varphi)$ that appear in $V_{eff}^{1-loop}|_{div}$ by $\text{Tr}[m^2(\varphi)]$ and $\text{Tr}[m^4(\varphi)]$, respectively. Since $\text{Tr}[\dots]$ is invariant under diagonalization, one may use the nondiagonal basis here, then, because all elements of the initial nondiagonal mass matrix squared are in general quadratic in φ , therefore the counterterms at hand turn out to be sufficient to remove all the divergences.

Let us now specify the theory as a real ϕ^4 theory in 5D defined by the following potential:

$$V_{tree} = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda_5 \phi^4. \quad (5)$$

Note that λ_5 is dimensionful and ϕ has dimension of mass^{3/2}. After reducing to 4D the tree level bare potential for the classical field φ is the following:

$$V_{tree} = \frac{1}{2} \mu^2 \varphi^2 + \frac{1}{4} \lambda \varphi^4 \quad (6)$$

where now λ is dimensionless and φ has dimension of mass. In order to remove the divergent contributions one has to adopt appropriate counterterms. The renormalization conditions that we will choose are the following:

$$\left. \frac{d^2 V_{eff}}{d\varphi^2} \right|_{\varphi=0} = \mu_r^2, \quad \left. \frac{d^4 V_{eff}}{d\varphi^4} \right|_{\varphi=0} = 3! \lambda_r \quad (7)$$

for the 4D tree-level potential shown in Eq. (5). The bare parameters μ^2 and λ are related to the renormalized ones and to the counterterms through

$$\mu^2 = \mu_r^2 + \delta\mu^2, \quad \lambda = \lambda_r + \delta\lambda. \quad (8)$$

In the case of the potential (5) we have the following form of $m^2(\varphi)$:

$$m^2(\varphi) = \frac{1}{2} \left. \frac{d^2 m^2(\varphi)}{d\varphi^2} \right|_{\varphi=0} \varphi^2 + m^2(0). \quad (9)$$

It is straightforward to prove that the conditions (7) lead to the following counterterms:

$$\delta\mu^2 = - \left. \frac{d^2 V_{eff}^{1-loop}}{d\varphi^2} \right|_{\varphi=0} \\ = - \frac{1}{32\pi^2} \sum_{n=0}^{n_{max}} \left. \frac{d^2 m^2(\varphi)}{d\varphi^2} \right|_{\varphi=0} \\ \times \left[\Lambda^2 + (m^2(0) + m_n^2) \ln \left(\frac{m^2(0) + m_n^2}{\Lambda^2} \right) \right], \quad (10)$$

$$\delta\lambda = - \frac{1}{3!} \left. \frac{d^4 V_{eff}^{1-loop}}{d\varphi^4} \right|_{\varphi=0} \\ = - \frac{1}{64\pi^2} \sum_{n=0}^{n_{max}} \left(\left. \frac{d^2 m^2(\varphi)}{d\varphi^2} \right|_{\varphi=0} \right)^2 \\ \times \left[1 + \ln \left(\frac{m^2(0) + m_n^2}{\Lambda^2} \right) \right]. \quad (11)$$

It could be easily verified that the above counterterms do cancel the divergences in $V_{eff}^{1-loop}|_{div}$; note that the form of $m^2(\varphi)$ given in Eq. (9) is essential for the cancellation. Eventually, the renormalized 1-loop contribution to the effective potential reads

$$V_{eff\ ren\ 1}^{1-loop} = \frac{1}{32\pi^2} \sum_{n=0}^{n_{max}} \left\{ \frac{1}{2} (m^2(\varphi) + m_n^2)^2 \ln \left(\frac{m^2(\varphi) + m_n^2}{m^2(0) + m_n^2} \right) \right. \\ \left. - \frac{3}{4} m^4(\varphi) + m^2(\varphi) \left[m^2(0) - \frac{1}{2} m_n^2 \right] \right\}. \quad (12)$$

As it was already mentioned for general nondiagonal mass matrices the condition (9) does not hold. Nevertheless, as we have already discussed above the renormalization procedure could be successfully performed. Then it would be convenient to split the counterterms into divergent and finite parts. Since the divergent contributions to the effective potential are linear functions of $m^2(\varphi)$ and $m^4(\varphi)$ only, they can therefore be replaced by $\text{Tr}[m^2(\varphi)]$ and $\text{Tr}[m^4(\varphi)]$, respectively and for them (in the nondiagonal basis) the form (9) holds. However, for finite parts the renormalization condi-

tions (7) turn out to be very inconvenient as they lead to quite complicated expressions for the renormalized effective potential; therefore one can modify the above renormalization prescription such that one will only keep the divergent parts of δm^2 and $\delta \lambda$. However, we will not discuss this renormalization prescription hereafter.

C. Dimensional regularization

It will be useful to repeat the derivation of the effective potential proposed by DPQ [12] and compare with the dimensional regularization of the KK sum adopted in Ref. [16]. In order to find $V(\varphi)$ defined in Eq. (1) we first define

$$W = \frac{1}{2} \sum_{n=-\infty}^{\infty} \ln[(lE)^2 + (n + \omega)^2 \pi^2]. \quad (13)$$

Instead of W we calculate

$$\frac{\partial W}{\partial E} = l^2 E \sum_{n=-\infty}^{\infty} \frac{1}{(lE)^2 + (n + \omega)^2 \pi^2} \quad (14)$$

that is already convergent. By that procedure, an infinite, but constant (E -independent) term was dropped. This is, of course, legal, since the constant is φ independent and therefore its elimination corresponds to the renormalization of the cosmological constant. Then replacing the infinite sum in Eq. (14) by an integral in the complex plane and applying the residues theorem to perform the integral leads to the following result:

$$W = lE + \frac{1}{2} \{ \ln(1 - r e^{-2lE}) + \ln(1 - r^{-1} e^{-2lE}) \}, \quad (15)$$

where $r \equiv e^{-2i\omega\pi}$. The first term in Eq. (15), that is the limit of the full W when $R \rightarrow \infty$, leads to the effective potential for the uncompactified 5D:

$$V^{(\infty)} = l \int \frac{d^4 p}{(2\pi)^4} \sqrt{p^2 + m^2(\varphi)}. \quad (16)$$

The integral over $d^4 p$ is obviously divergent; let us adopt regularization by a cutoff (as it was done in Ref. [12]) and for comparison also the dimensional regularization:

$$V^{(\infty)} = \frac{R}{60\pi} \begin{cases} m^5(\varphi) + \frac{1}{2} \sqrt{\Lambda^2 + m^2(\varphi)} \\ \times [3\Lambda^4 + \Lambda^2 m^2(\varphi) - 2m^4(\varphi)] & \text{cutoff,} \\ m^5(\varphi) & \text{dim.} \end{cases} \quad (17)$$

It is seen that $V^{(\infty)}$ is finite when the dimensional regularization is adopted.

As we have already mentioned there are two sources of divergences: the sum and the $d^4 p$ integral. In Ref. [12] the sum was regularized-renormalized through the differentiation and then integration with respect to E , while for the divergent integral the result is shown in Eq. (17) as the cutoff

option. It turns out that the dimensional regularization of both the sum and the integral proposed in Ref. [16] leads to the same result as the one presented above provided the integral is dimensionally regularized. It will be instructive to compare both methods in order to understand the puzzling agreement. In Ref. [16] the sum is regularized by the following replacement [see Eq. (11) of Ref. [16]]:

$$I = \int d^4 p_4 \sum_{n \geq 0} f\left(p_4, \frac{n}{R}\right) \rightarrow \frac{1}{2\pi i} \int d^{D_4} p_4 \int_{\oplus} d^{D_5} p_5 \mathcal{P}^+(p_5) f(p_4, p_5), \quad (18)$$

where the notation of Ref. [16] was adopted. Then the author concludes that in fact it would be enough to regularize the integral since the divergent part appears to be a function of $D_4 + D_5$ only. At first sight this statement looks confusing since we might have started with a divergent sum on the left-hand side (LHS) of Eq. (18). The sum is replaced by the integral over $d^{D_5} p_5$ and it looks as if this regularization of the sum is needed. The solution of this illusive puzzle seems to be the following. Note that for the effective potential calculation, the function $f(p_4, n/R)$ depends on the background field dependent mass $m(\varphi)$ only through $p_4^2 + m^2(\varphi)$. Therefore a constant that is p_4 independent on the LHS of Eq. (18) does not depend on $m(\varphi)$ as well. Since the divergence of the sum was dropped in the DPQ approach by the differentiation and then integration with respect to E therefore we know that it was $p_4^2 + m^2(\varphi)$ -independent constant. Let us now locate this divergence in the dimensional approach. It turns out that it is hidden (and then erased) in Eq. (18), namely *the dimensional regularization of the integral over $d^4 p_4$ at the same time regularizes the integral and also removes the constant (p_4 -independent) contribution to the sum.* This happens because of the following peculiar property of the dimensional regularization:

$$\int d^{D_4} p_4 (\text{const}) = 0. \quad (19)$$

Therefore, no wonder that in fact it is not necessary to regularize the sum if the dimensional regularization is adopted for the $d^4 p_4$. The dimensional regularization takes care of both the divergent integral and the divergent constant contribution to the sum. So, it is clear now why both the method adopted by DPQ [12] and the one developed in Ref. [16] lead to the same result.⁷

In the remaining part of this paper we will apply methods developed in this section to the 5D U(1) model of universal extra dimensions. Then, expressions for the effective potential will either contain sums that start at a zero mode

⁷At most they may differ by $m(\varphi)$ -independent constant. We have confirmed that by explicit calculation. The results are identical separately for boson and fermion contributions to the effective potential.

($n=0$) or at $n=1$.⁸ Therefore the final result (for $\omega=0$) for both cases in dimensional regularization of the d^4p integral is the following:

$$V(m^2) = \frac{1}{2}(V^{(\infty)}(m^2) + V^{(R)}(m^2) \pm V_0(m^2)), \quad (20)$$

where $+$ or $-$ corresponds to the zero mode included or excluded in the sum, respectively. The contributions to the effective potential read

$$\begin{aligned} V^{(\infty)}(m^2) &= \frac{R}{60\pi} m^5(\varphi), \\ V_0(m^2) &= \frac{1}{64\pi^2} m^4(\varphi) \left\{ -C_{UV} + \ln\left(\frac{m^2(\varphi)}{\kappa^2}\right) - \frac{3}{2} \right\}, \\ V^{(R)}(m^2) &= -\frac{1}{64\pi^6} \frac{1}{R^4} \{x^2 \text{Li}_3(e^{-x}) \\ &\quad + 3x \text{Li}_4(e^{-x}) + 3 \text{Li}_5(e^{-x})\}, \end{aligned} \quad (21)$$

where $x \equiv 2\pi R \sqrt{m^2(\varphi)}$, $C_{UV} = 2/(4-n) - \gamma_E + \ln(4\pi)$ ($\gamma_E = 0.5772\dots$ is the Euler-Mascheroni constant), κ is the regularization scale and $V^{(\infty)}$ corresponds to the decompactification limit ($R \rightarrow \infty$), $V^{(R)}$ is the contribution from all the KK modes (summed from $-\infty$ to $+\infty$) and V_0 is the zero mode effective potential. The polylogarithm $\text{Li}_n(x)$ is defined by

$$\text{Li}_n(x) = \sum_{s=1}^{\infty} \frac{x^s}{s^n}. \quad (22)$$

Note that $V^{(\infty)}$, V_0 and $V^{(R)}$ contributions correspond to the three terms separated in Ref. [16] and denoted by I_{5D} , I_{4D} and I_{finite} , respectively. It is amazing that the divergence from the zero mode is still there, while in the DPQ approach with dimensional regularization it was gone (note that there the KK summation started at $n = -\infty$). This means that the singular contribution from the zero mode must be canceled by the sum over $n \neq 0$ in the DPQ method. The explanation of this is the presence of the zero mode in the above consideration.

In order to get rid of the singularities present in V_0 we will adopt the $\overline{\text{MS}}$ renormalization, then the 1-loop contribution to the effective potential reads

$$V_{eff ren}^{1-loop} \overline{\text{MS}} = \frac{1}{2}(V^{(\infty)}(m^2) + V^{(R)}(m^2) \pm V_{0 finite}(m^2)), \quad (23)$$

where $V_{0 finite}$ is V_0 with the term $\propto C_{UV}$ subtracted.

⁸Note that in Ref. [12] the summation is performed from $n = -\infty$ to $n = +\infty$, while here we have considered separately the zero-mode contribution and the remaining KK modes from $n=1$ to $n = +\infty$; that explains the factor 1/2 in Eq. (20).

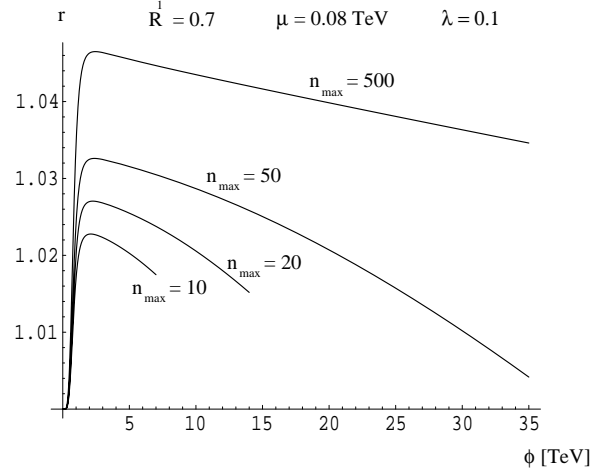


FIG. 1. The ratio defined by Eq. (27) for the φ^4 theory for $\kappa = n_{max}/(2R)$, $R=1/0.7$, $\mu=0.08$ TeV, $\lambda=0.1$, and the shift parameter $V_0=0.01$ TeV⁴. The curves from the left to the right correspond to increasing cutoffs: $n_{max}=10, 20, 50$ and 500 .

D. Decoupling of heavy KK modes

In Sec. II B we have discussed the one-loop effective potential for 5D φ^4 theory described by the tree-level potential specified in Eq. (5). Using the cutoff regularization of the 4D integral and adopting the on-shell renormalization conditions (7) we have found in Eq. (12) the renormalized effective potential that originates from the first n_{max} KK modes that can be written as

$$V_{eff ren}^{1-loop \Lambda} = \sum_{n=0}^{n_{max}} V_n(\varphi) \quad (24)$$

for

$$\begin{aligned} V_n(\varphi) &= \frac{1}{32\pi^2} \left\{ \frac{1}{2} (m^2(\varphi) + m_n^2)^2 \ln\left(\frac{m^2(\varphi) + m_n^2}{m^2(0) + m_n^2}\right) \right. \\ &\quad \left. - \frac{3}{4} m^4(\varphi) + m^2(\varphi) \left[m^2(0) - \frac{1}{2} m_n^2 \right] \right\}, \end{aligned} \quad (25)$$

where $m^2(\varphi)$ was defined in Eq. (9). In order to investigate the decoupling of heavy KK modes (corresponding to large n) in the model it is useful to expand $V_n(\varphi)$ in the limit of $n \rightarrow \infty$ and then sum over n :

$$V_{eff ren}^{1-loop \Lambda} = \frac{1}{32\pi^2} \sum_{n=0}^{n_{max}} \left[-\frac{1}{2} m^2(0) m_n^2 + \frac{1}{4} m^4(0) + \mathcal{O}\left(\frac{1}{n^2}\right) \right]. \quad (26)$$

As it is seen, only leading ($\sim m_n^2$) and subleading (m_n -independent) terms are divergent when the summation over n is performed in the limit $n_{max} \rightarrow \infty$. The key observation is that those terms are ϕ independent. Even though the above sum is divergent, the divergence is a constant, ϕ -independent contribution to the effective potential and therefore will be irrelevant. That happens because there is no

couplings that could grow with n .⁹ Of course, the remaining, finite part of the effective potential [denoted in Eq. (26) by $\mathcal{O}(1/n^2)$] depends on φ and leads to the genuine effective potential.¹⁰ In other words, the decoupling of heavy KK modes takes place as a consequence of renormalization of the cosmological constant.

In order to discuss the decoupling more quantitatively, it is worth comparing the effective potential obtained within the cutoff regularization (12) with the one for the minimal subtraction (23). One could wish to plot the simple ratio: $V_{eff\ ren\ 1}^{1-loop\ \Lambda}(\varphi)/V_{eff\ ren\ \overline{MS}}^{1-loop}(\varphi)$. However, it turns out that in the vicinity of $\varphi=0$ the MS-renormalized 1-loop contribution to the effective potential has a zero and the plot of the ratio is very unstable. Fortunately, the value of 1-loop contribution both to $V_{eff\ ren\ 1}^{1-loop\ \Lambda}(\varphi)$ and $V_{eff\ ren\ \overline{MS}}^{1-loop}(\varphi)$ is in this region by far negligible comparing to the tree level contribution. Therefore we will modify the naive ratio as follows:

Since the tree-level potential is the reference point for 1-loop corrections therefore we will add $V_{tree}(\varphi)$ both in the denominator and the numerator.

To eliminate the unwanted irrelevant constant contributions¹¹ to the effective potentials we will subtract $V_{eff}(0)$ contributions both in the denominator and the numerator.

The effective potentials obtained according to the above prescription have zeros in the vicinity $\varphi=0$ that are slightly misplaced in the denominator and the numerator, therefore we introduce a constant shift V_0 in order to screen the instability caused by the zero of the denominator.

So, we will adopt the following ratio to compare the cutoff and dimensional regularization:

$$r(\kappa, n_{max}; \varphi) \equiv \frac{V_{tree}(\varphi) + [V_{eff\ ren\ 1}^{1-loop\ \Lambda}(\varphi) - V_{eff\ ren\ 1}^{1-loop\ \Lambda}(0)] + V_0}{V_{tree}(\varphi) + [V_{eff\ ren\ \overline{MS}}^{1-loop}(\varphi) - V_{eff\ ren\ \overline{MS}}^{1-loop}(0)] + V_0}. \quad (27)$$

The ratio $r=r(\kappa, n_{max}; \varphi)$ is, of course, a function of the cutoff ($\Lambda_5=n_{max}/R$) and the regularization scale (κ). In

⁹In the next section we will discuss in detail the 5D model based on U(1) gauge symmetry. We will observe there that mass matrix for the (A_5, χ_n) system is nondiagonal and in fact the off-diagonal entries are of the form $n\varphi/R$, suggesting that there exist coupling constants growing with n . However as it will be seen, the determinant and the trace of the mass matrix grows as n^4/R^4 and n^2/R^2 ; therefore even in that case in the limit of large n we shall anticipate decoupling of heavy modes. The explicit calculations confirm this expectation.

¹⁰The corresponding analogous phenomena could also be found in the method of DPQ [12]; as it was discussed earlier, an infinite φ -independent term was dropped there through differentiation and subsequent integration over E .

¹¹It is especially important in light of the proceeding discussion of the decoupling in the case of the cutoff regularization.

Fig. 1 we plot $r=r(n_{max}/(2R), n_{max}; \varphi)$ for $n_{max}=10, 20, 50$ and 500, which corresponds to the choice¹² of the regularization scale $\kappa=\Lambda/2$. For a given n_{max} , the ratio $r=r(n_{max}/(2R), n_{max}; \varphi)$ is plotted against φ varying from 0 up to the appropriate cutoff $\Lambda=n_{max}/R$. Note, however, that the cutoff corresponding to $n_{max}=500$, $\Lambda=350$ TeV, is not shown for the sake of clarity of the figure. However, it has been checked that even in this case r remains within the 5% distance from 1. For the purpose of Fig. 1, we have used the mass parameter $\mu=0.08$ TeV, the quartic coupling constant $\lambda=0.1$, and the shift $V_0=0.01$ TeV⁴.¹³ It has been checked that for $0 \leq V_0 \leq 1$ TeV⁴ the ratio r remains below 1.05 for $\varphi \geq 1$ TeV even though the shape in the region $1 \leq \varphi \leq 5$ TeV is influenced by the choice of V_0 . However, it should be emphasized that for $\varphi \geq 5$ TeV (for the stability we will discuss the effective potential for field strength $\varphi \geq 1$ TeV) the curves are almost insensitive to V_0 .

As it is seen from the plot, even though for small φ , $r(\kappa, n_{max}; \varphi)$ is a monotonically rising function of n_{max} (moving from the bottom to the top of the figure the subsequent curves correspond to increasing n_{max}), nevertheless, eventually for larger φ , r approaches 1 closer for curves corresponding to larger n_{max} . In fact, this is what we should expect if the effective potential calculated in the cutoff and MS schemes were close.

Conclusion that can be drawn from this picture is that the cutoff and the minimal subtraction schemes are consistent and the dependence on the cutoff is very weak. One should however remember that we have adopted two different renormalization schemes and therefore the agreement is never expected to be perfect.

III. U(1) MODEL

In this section we will construct a simple 5D model that could successfully mimic the SM as far as the shape of the effective potential is concerned. For a gauge group we choose U(1). In order to break spontaneously the symmetry, we will introduce a complex scalar ϕ . To have a zero-mode massive fermion (the analog of the top quark) we will have to introduce two 5D fermions: ψ and λ . The model is defined by the Lagrangian density

$$\mathcal{L}(x, y) = -\frac{1}{4} F^{MN} F_{MN} + (D_M \phi)^\dagger (D^M \phi) - V^{(5)}(\phi) + \mathcal{L}_{GF}^{(5)} + \mathcal{L}_f^{(5)}, \quad (28)$$

where

$$F_{MN}(x, y) \equiv \partial_M A_N(x, y) - \partial_N A_M(x, y) \\ D_M \equiv \partial_M + i e_5 A_M(x, y),$$

¹²Other possible choices of κ , e.g. $\kappa=\varphi$, do not change results for r substantially. Note that here we have decided to adopt the same cutoff for 4D and 5D: $\Lambda_5=\Lambda$.

¹³If we plotted r for $\varphi \geq 1$ TeV (that is large enough to pass the zero of the denominator) we would not need to introduce V_0 .

$$V^{(5)}(\phi) \equiv \mu^2 |\phi|^2 + \lambda_5 |\phi|^4,$$

$$\phi(x, y) = \frac{1}{\sqrt{2}} [h(x, y) + i\chi(x, y)],$$

$$y \equiv x^4,$$

where A_M is a gauge field and D_M is a covariant derivative. We will assume that the tree-level potential is stable, so $\lambda_5 > 0$.

Hereafter we will adopt the following form of the gauge fixing Lagrangian:¹⁴

$$\mathcal{L}_{GF}^{(5)} = -\frac{1}{2\xi} \left[\partial_\mu A^\mu - \xi \left(\partial_5 A_5 + e_5 \frac{v\chi}{\sqrt{2\pi R}} \right) \right]^2, \quad (29)$$

where $v = \langle h_0 \rangle$ is the vacuum expectation value of the zero mode of the scalar $h(x, y)$.

In order to generate massive zero modes for fermions we will introduce here two fermion fields, one charged [$\psi(x, y)$] and one neutral [$\lambda(x, y)$] under U(1):

$$\begin{aligned} \mathcal{L}_f^{(5)} = & \bar{\psi}(x, y) \gamma^M [i\partial_M + e_5 A_M] \psi(x, y) + \bar{\lambda}(x, y) \gamma^M i\partial_M \\ & \times \lambda(x, y) - [g_5 \bar{\psi}(x, y) \phi(x, y) \lambda(x, y) + \text{H.c.}]. \end{aligned} \quad (30)$$

The action of the U(1) local symmetry is defined by

$$\begin{aligned} \phi(x, y) & \rightarrow e^{-ie_5 \theta(x, y)} \phi(x, y) \\ \psi(x, y) & \rightarrow e^{-ie_5 \theta(x, y)} \psi(x, y) \\ \lambda(x, y) & \rightarrow \lambda(x, y) \\ A_M(x, y) & \rightarrow A_M(x, y) + \partial_M \theta(x, y). \end{aligned} \quad (31)$$

The compactification of the extra dimension is specified by the following S^1/Z_2 orbifold conditions.

All the fields and the gauge function $\theta(x, y)$ remain unchanged under a shift $y \rightarrow y + 2\pi R$,

$$\begin{aligned} A_\mu(x, y) & = A_\mu(x, -y), \quad A_5(x, y) = -A_5(x, -y), \\ \phi(x, y) & = \phi(x, -y), \\ \psi_R(x, y) & = \psi_R(x, -y), \quad \psi_L(x, y) = -\psi_L(x, -y), \\ \lambda_L(x, y) & = \lambda_L(x, -y), \quad \lambda_R(x, y) = -\lambda_R(x, -y), \\ \theta(x, y) & = \theta(x, -y). \end{aligned} \quad (32)$$

KK expansions read

¹⁴For discussion of the Lorentz noncovariant R_ξ gauges, see Refs. [19,20].

$$\begin{aligned} A^\mu(x, y) & = \frac{1}{\sqrt{2\pi R}} \left[A_0^\mu(x) + \sqrt{2} \sum_{n=1}^{\infty} A_n^\mu(x) \cos(m_n y) \right], \\ A^5(x, y) & = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} A_n^5(x) \sin(m_n y), \\ \phi(x, y) & = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_n(x) \cos(m_n y), \\ \psi(x, y) & = \frac{1}{\sqrt{2\pi R}} \left\{ \psi_{R0}(x) + \sqrt{2} \sum_{n=1}^{\infty} [\psi_{Rn}(x) \cos(m_n y) \right. \\ & \quad \left. + \psi_{Ln}(x) \sin(m_n y)] \right\}, \\ \lambda(x, y) & = \frac{1}{\sqrt{2\pi R}} \left\{ \lambda_{L0}(x) + \sqrt{2} \sum_{n=1}^{\infty} [\lambda_{Ln}(x) \cos(m_n y) \right. \\ & \quad \left. + \lambda_{Rn}(x) \sin(m_n y)] \right\}, \\ \theta(x, y) & = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \theta_n(x) \cos(m_n y), \end{aligned} \quad (33)$$

where $m_n \equiv n/R$, subscripts R and L are referring to 4D chiral fields and it is assumed that $A_{50} = 0$. In the following we will adopt the following notation for the real and imaginary parts of $\phi_n(x)$:

$$\phi_0 = \frac{1}{2} (h_0 + i\chi_0), \quad \phi_{n \neq 0} = \frac{1}{\sqrt{2}} (h_n + i\chi_n). \quad (34)$$

It is worth noticing that after compactification the 4D Lagrangian expressed in terms of KK modes is still gauge invariant and the U(1) transformations of the gauge fields read

$$A_{n\mu}(x) \rightarrow \begin{cases} A_{0\mu}(x) + \sqrt{2} \partial_\mu \theta_0(x) & \text{for } n=0, \\ A_{n\mu}(x) + \partial_\mu \theta_n(x) & \text{for } n \neq 0, \end{cases} \quad (35)$$

$$A_{n5}(x) \rightarrow A_{n5}(x) - \frac{n}{R} \theta_n(x). \quad (36)$$

The corresponding infinitesimal transformation for $\phi_n(x)$ is the following:

$$\begin{aligned} \phi_0(x) & \rightarrow \phi_0(x) - \frac{ie}{\sqrt{2}} \left(2\theta_0(x) \phi_0(x) + \sum_{m=1}^{\infty} \theta_m(x) \phi_m(x) \right), \\ \phi_n(x) & \rightarrow \phi_n(x) - \frac{ie}{\sqrt{2}} \sum_{m,l=0}^{\infty} A_{nm} \theta_m(x) \phi_l(x), \end{aligned} \quad (37)$$

where A_{nm} is defined in Appendix A and $e \equiv e_5 / \sqrt{2\pi R}$.

The goal of this paper is to investigate stability of the ground state of the model. Therefore first we have to deter-

mine the tree level potential; the next step will be to calculate the effective potential at the 1-loop level. Expanding in KK modes and integrating over y yields the following 4D potential:

$$\begin{aligned}
V^{(4)} = & \sum_{n=0}^{\infty} [m_n^2 + \mu^2] \phi_n^* \phi_n + \mu^2 \phi_0^* \phi_0 \\
& + \frac{\lambda}{2} \sum_{n,m,k,l=0}^{\infty} B_{nmkl} \phi_n^* \phi_m \phi_k^* \phi_l \\
& + \frac{e^2}{2} \sum_{n,m,k,l=0}^{\infty} D_{nmkl} A_{5n} A_{5m} \phi_k^* \phi_l \\
& - \frac{ie}{\sqrt{2}} \sum_{n,m,k=0}^{\infty} C_{nmk} m_n A_{5m} (\phi_n^* \phi_k - \phi_k^* \phi_n) \\
& + \frac{\xi}{2} \sum_{n=0}^{\infty} (m_n A_{5n} + v e \chi_n)^2, \tag{38}
\end{aligned}$$

where $\lambda \equiv \lambda_5 / (2\pi R)$ and the coefficients B_{nmkl}, D_{nmkl} and C_{nmk} are defined in Appendix A.

In spite of the fact that the potential looks complicated, it is easy to see that for $\lambda_5 > 0$ the potential is positive definite in the limit of $|\phi_n|^2 \rightarrow \infty$ and therefore the ground state is stable. The 4D potential emerges from the 5D potential, the Higgs-boson kinetic term and the gauge fixing term:

$$\begin{aligned}
V^{(4)} = & \int_0^{2\pi R} dy \left[V_5(x, y) + (D_5 \phi)^* (D_5 \phi) \right. \\
& \left. + \frac{\xi}{2} \left(\partial_5 A_5 + e_5 \frac{v \chi}{\sqrt{2\pi R}} \right)^2 \right], \tag{39}
\end{aligned}$$

where $D_5 \phi$ is the fifth component of the covariant derivative of the Higgs field and the last term emerges from the gauge fixing term. So, it is clear that the 4D potential must be positive definite as it is an integral over a positive function. In the following we will investigate 1-loop corrections to the effective potential.

We will consider the case $\mu^2 < 0$; then it is easy to see that if $-\mu^2 \leq 1/R^2$ only the zero mode $h_0(x)$ can develop a nonzero vacuum expectation value, and at the tree level we get

$$\langle h_0(x) \rangle \equiv v = \sqrt{\frac{-\mu^2}{\lambda}}. \tag{40}$$

We will calculate the effective potential in the direction of the tree level vacuum: $\chi_0 = h_n = \chi_n = A_{5n} = 0$ and $h_0 \neq 0$. The Landau gauge defined here by $\xi = 0$ will be adopted hereafter.

We will expand the 4D Lagrangian around $\chi_0 = h_n = \chi_n = A_{5n} = 0$ and $h_0 \rightarrow h_0 + \varphi$, where φ is the classical constant (in 4D) external background field for the calculation of one-

loop Green's functions that are necessary for the effective potential. Then in the Landau gauge the following mass terms are obtained:

$$\begin{aligned}
m_{h_0}^2 & \equiv \frac{\partial^2 V}{\partial h_0^2} = \mu^2 + 3\lambda \varphi^2, \\
m_{\chi_0}^2 & \equiv \frac{\partial^2 V}{\partial \chi_0^2} = \mu^2 + \lambda \varphi^2, \\
m_{h_n h_m}^2 & \equiv \frac{\partial^2 V}{\partial h_n \partial h_m} = (m_n^2 + \mu^2 + 3\lambda \varphi^2) \delta_{nm} \equiv m_{h_n}^2 \delta_{nm}, \\
& \left[\begin{array}{cc} \frac{\partial^2 V}{\partial A_{5n} \partial A_{5m}} & \frac{\partial^2 V}{\partial A_{5n} \partial \chi_m} \\ \frac{\partial^2 V}{\partial A_{5m} \partial \chi_n} & \frac{\partial^2 V}{\partial \chi_n \partial \chi_m} \end{array} \right] \\
& = \left[\begin{array}{cc} e^2 \varphi^2 & -em_n \varphi \\ -em_n \varphi & (m_n^2 + \mu^2 + \lambda \varphi^2) \end{array} \right] \delta_{nm}.
\end{aligned}$$

In the following part of this section we will show separate contributions to the effective potential calculated in the MS scheme in dimensional regularization.

Let us start with the (A_{5n}, χ_m) system. The mixing in the mass matrix for A_{5n} and χ_m causes some technical difficulties that are described in Appendix B. The final result for the (A_{5n}, χ_m) system is the following:

$$V_{eff}^{(A_5, \chi)} = \frac{1}{2} (V_{mix}^{(\infty)} + V_{mix}^{(R)} - V_{0\ finite}^{(A_0)} - V_{0\ finite}^{(\chi_0)}), \tag{41}$$

where $V_{mix}^{(\infty)}$ and $V_{mix}^{(R)}$ are the analogs of the ‘‘divergent’’ and finite contributions to the effective potential (21) in the case of mixing:

$$V_{mix}^{(\infty)} = -\frac{y^{1/2}(y^2-1)x^5}{2^{12}\sqrt{2}\pi^5 R^4} F\left(-\frac{1}{4}, \frac{7}{4}; 2; 1 - \frac{1}{y^2}\right), \tag{42}$$

$$V_{mix}^{(R)} = -\frac{y^{3/2}(1+y)^{1/4} x^{7/2}}{2^9 \pi^5 \sqrt{\pi} R^4} \text{Li}_{3/2}(e^{-x\sqrt{1+y}}), \tag{43}$$

where $F(a, b; c; z)$ is the hypergeometric function,

$$x \equiv 2\pi R \sqrt{a} \quad \text{and} \quad y \equiv \frac{2\sqrt{b}}{a} \tag{44}$$

for a and b defined in Eq. (B7). $V_{0\ finite}^{(A_0)}$ and $V_{0\ finite}^{(\chi_0)}$ are the finite parts of scalar contributions [see Eq. (21)] to the effective potential calculated for the zero mode vector boson mass ($m_{A_0}^2 = e^2 \varphi^2$) and Goldstone boson ($m_{\chi_0}^2 = \mu^2 + \lambda \varphi^2$), respectively.

All neutral scalar modes contribute to the effective potential as follows:

$$V_{eff}^{(s)}(\varphi) = \frac{1}{2} [V^{(\infty)}(m_{h_0}^2) + V^{(R)}(m_{h_0}^2) + V_{0\ finite}(m_{h_0}^2)] \\ + V_{0\ finite}(m_{\chi_0}^2) + V_{eff}^{(A_5, \chi)}(\varphi). \quad (45)$$

For the vector boson contribution we get

$$V_{eff}^{(v)}(\varphi) = \frac{3}{2} [V^{(\infty)}(m_{A_0}^2) + V^{(R)}(m_{A_0}^2) + V_{0\ finite}^v(m_{A_0}^2)], \quad (46)$$

where the zero-mode vector contribution reads

$$V_0^v(m^2) = \frac{1}{64\pi^2} m^4(\varphi) \left\{ -C_{UV} + \ln \left(\frac{m^2(\varphi)}{\kappa^2} \right) - \frac{5}{6} \right\}, \quad (47)$$

and $m_{A_n}^2 = m_n^2 + e^2 \varphi^2$.

After KK expansion and integration over y the 4D fermionic Lagrangian reads (see Ref. [18] for a similar construction)

$$\mathcal{L}_f^{(4)} = \bar{f}_0(i\gamma^\mu \partial_\mu - m_{f_0})f_0 + \sum_{n=1}^{\infty} [\bar{\xi}_n i\gamma^\mu \partial_\mu \xi_n - \bar{\xi}_n M_n \xi_n], \quad (48)$$

where $m_{f_0} = g\varphi/\sqrt{2}$, $g \equiv g_5/\sqrt{2\pi R}$, $m_n = n/R$ and

$$f_0 = \psi_{R_0} + \lambda_{L_0}, \quad \xi_n = \begin{pmatrix} \psi_{Rn} + \psi_{Ln} \\ \lambda_{Rn} + \lambda_{Ln} \end{pmatrix}, \\ M_n = \begin{pmatrix} -m_n & m_{f_0} \\ m_{f_0} & +m_n \end{pmatrix}. \quad (49)$$

After diagonalization the fermionic mass matrix reads

$$\mathcal{M} = \begin{pmatrix} -(m_n^2 + m_{f_0}^2)^{1/2} & 0 \\ 0 & (m_n^2 + m_{f_0}^2)^{1/2} \end{pmatrix}. \quad (50)$$

So, we have two fermions degenerate in masses (the minus in front of the upper component mass can be removed through a chiral rotation).

Fermions (no color degrees of freedom included) contribute to the effective potential as follows:

$$V_{eff}^{(f)}(\varphi) = -4V_{0\ finite}(m_{f_0}^2) - \frac{8}{2} [V^{(\infty)}(m_{f_0}^2) \\ + V^{(R)}(m_{f_0}^2) - V_{0\ finite}(m_{f_0}^2)], \quad (51)$$

for $m_{f_n}^2 = m_n^2 + m_{f_0}^2$.

Eventually, the total 1-loop effective potential is given by the following formula:

$$V_{eff}^{1-loop} = V_{tree} + V_{eff}^{(s)} + V_{eff}^{(v)} + V_{eff}^{(f)}, \quad (52)$$

where

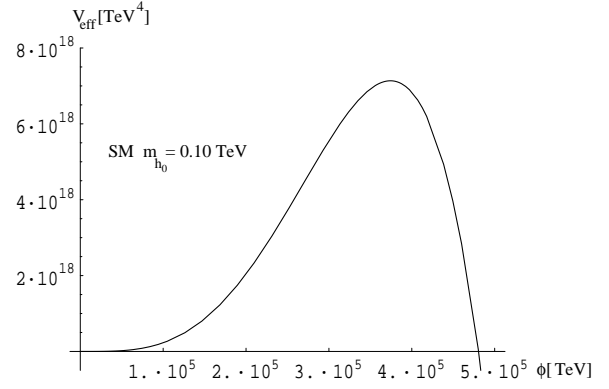


FIG. 2. The zero-mode (SM-like) 1-loop effective potential for $m_{h_0} = 0.10$ TeV in the dimensional regularization.

$$V_{tree}(\varphi) = \frac{\mu^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4. \quad (53)$$

IV. RESULTS

In order to mimic the SM we have adopted the following parameters for the plots: $e = \sqrt{4\pi/137}$, $v = 0.246$ TeV, the fermion zero-mode mass $m_{f_0} = 0.150$ TeV and the renormalization scale $\kappa = 0.1$ TeV. We will adopt the asymptotic formula for $V_{mix}^{(R)}$ given in Eq. (B13), however it should be emphasized that it provides an excellent approximation in the whole parameter range that is of interest here.

It is seen from the plots that effects of nonzero KK modes are very dramatic (see Figs. 2,3 and 4,5). For instance, for $m_{h_0} = 0.10$ TeV and $R^{-1} = 0.3$ TeV the instability scale is shifted down from 4.8×10^5 TeV to 3.6 TeV. The model is much less stable as a consequence of the presence of the KK modes. Closer inspection shows that the result is triggered by the fermionic contribution to the 4D effective potential and the leading contribution emerges from $V^{(\infty)}$. Note that since we wished to construct a model that would possess a zero-mode massive fermion therefore it was necessary to introduce the extra 5D fermion. As a consequence the model con-

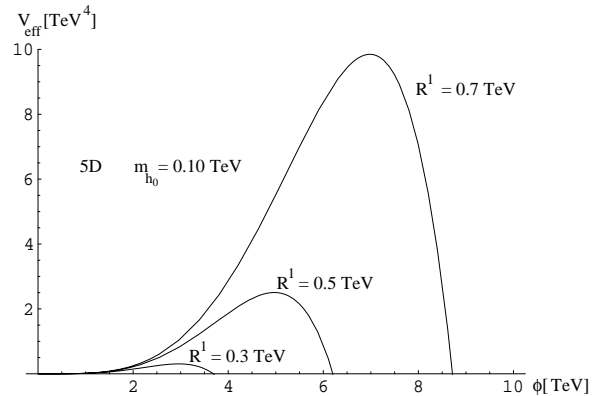


FIG. 3. The full 1-loop effective potential in the dimensional regularization for $m_{h_0} = 0.10$ TeV. The compactification radius $R^{-1} = 0.3, 0.5, 0.7$ TeV was adopted (higher curves correspond to smaller R). All other parameters are specified in the text.

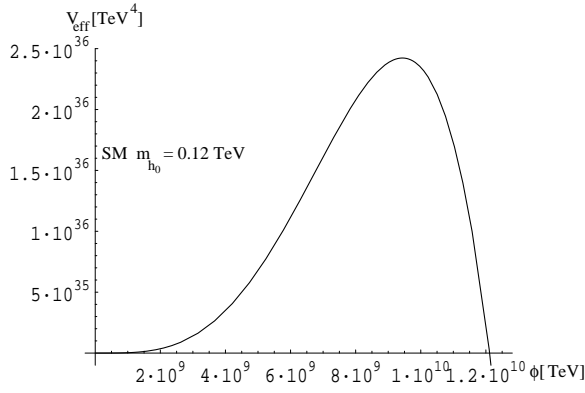


FIG. 4. The zero-mode (SM-like) 1-loop effective potential for $m_{h_0} = 0.12$ TeV in the dimensional regularization.

tains after reduction to 4D doubly degenerated Dirac fermions for each KK mode that enhances the fermionic contributions and is the source of the extra factor of 2 in front of the second term in Eq. (51). If the factor 2 is removed (just to test the effect of fermion doubling) the result changes and for instance for $m_{h_0} = 0.10$ TeV and $R^{-1} = 0.3$ TeV the instability appears at 6.5 TeV instead of 3.6 TeV, obviously the model would be more stable. It turns out that for our model (with full spectrum of fermions) the fermionic KK contribution is by a factor of 2.5–5 larger (for $\varphi \approx 0.5\text{--}3.5$ TeV at $m_{h_0} = 0.10$ TeV and $R^{-1} = 0.3$ TeV) than the zero-mode contribution. As a consequence the tree level potential bends down more rapidly for much lower field strengths than for the zero mode only.

V. CONCLUSIONS

We have discussed the effective potential in 4-dimensional models that originate from 5-dimensional ones reduced down to 4 dimensions. The cutoff and the dimensional regularization schemes were discussed and compared. It was shown that the prescriptions are consistent with each other and lead to the same physical consequences (see Fig. 1). It turned out that when the number of KK modes included (n_{max}) varies between 10 and 500, the effective potential calculated within the cutoff regularization accompanied by the on-shell renormalization is never farther than 5% away from the potential found in the dimensional regularization with \overline{MS} .

In order to take into account nondiagonal mass matrices we have generalized the standard technique for the calculation of KK contributions to the effective potential developed by Delgado, Pomarol and Quirós in Ref. [12]. We have constructed a simple U(1) 5-dimensional model containing gauge boson, a complex scalar and two fermions. The model parameters were adjusted, so that the model should mimic 5-dimensional extension of the standard model. The one-loop effective potential for the model was calculated adopting the dimensional regularization with the \overline{MS} renormalization. As in the standard model the effective potential turned out to be unbounded from below as a consequence of fermionic contributions. It has been found that the presence of the tower of

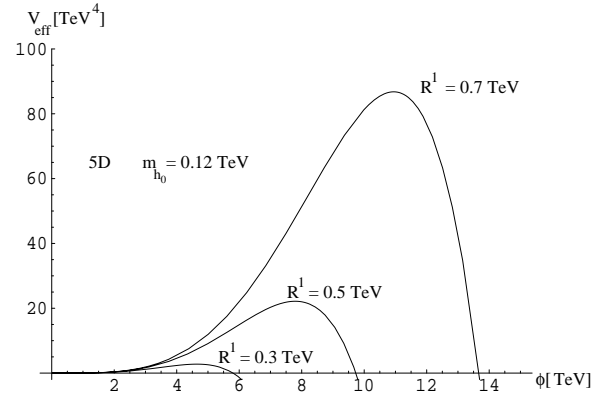


FIG. 5. The full 1-loop effective potential in the dimensional regularization for $m_{h_0} = 0.12$ TeV. The compactification radius $R^{-1} = 0.3, 0.5, 0.7$ TeV was adopted (higher curves correspond to smaller R). All other parameters are specified in the text.

fermionic KK modes leads to a major modification of the effective potential and in particular could substantially lower the scale of instability. For instance, for $m_{h_0} = 0.10$ TeV and $R^{-1} = 0.3$ TeV the instability scale is shifted down from the standard model value 4.8×10^5 TeV to 3.6 TeV. The model is much less stable as a consequence of the presence of the KK modes. The same qualitative behavior of the effective potential is expected for the true 5-dimensional extension of the standard model. The order of magnitude for the instability scale should not differ very much from the results presented here, however for a definite prediction for the instability scale as a function of the Higgs-boson mass a dedicated study is necessary [21].

ACKNOWLEDGMENTS

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APPENDIX A

The integrals used in the text:

$$\int_0^{2\pi R} \cos(m_n y) dy = (2\pi R) \delta_{n,0},$$

$$\int_0^{2\pi R} \sin(m_n y) dy = 0,$$

$$\int_0^{2\pi R} \cos(m_n y) \cos(m_m y) dy = \begin{cases} (\pi R) \delta_{n,m} & \text{for } n, m \neq 0, \\ 2\pi R & \text{for } n, m = 0, \end{cases}$$

$$\int_0^{2\pi R} \sin(m_n y) \sin(m_m y) dy = \begin{cases} (\pi R) \delta_{n,m} & \text{for } n, m \neq 0, \\ 0 & \text{for } n, m = 0, \end{cases}$$

$$\int_0^{2\pi R} \sin(m_n y) \cos(m_m y) dy = 0,$$

$$\int_0^{2\pi R} \cos(m_n y) \cos(m_m y) \cos(m_l y) dy = \frac{\pi R}{2} A_{nml},$$

$$\int_0^{2\pi R} \sin(m_n y) \sin(m_m y) \cos(m_l y) dy = \frac{\pi R}{2} C_{nml},$$

$$\int_0^{2\pi R} \cos(m_n y) \cos(m_m y) \sin(m_l y) dy = 0,$$

$$\int_0^{2\pi R} \cos(m_n y) \cos(m_m y) \cos(m_k y) \cos(m_l y) dy = \frac{\pi R}{4} B_{nmkl},$$

$$\int_0^{2\pi R} \sin(m_n y) \sin(m_m y) \cos(m_k y) \cos(m_l y) dy = \frac{\pi R}{4} D_{nmkl}, \quad (\text{A1})$$

where

$$\begin{aligned} A_{nml} &\equiv \delta_{l,n+m} + \delta_{l,n-m} + \delta_{l,-n+m} + \delta_{l,-n-m}, \\ C_{nml} &\equiv -\delta_{l,n+m} + \delta_{l,n-m} + \delta_{l,-n+m} - \delta_{l,-n-m}, \\ B_{nmkl} &\equiv \delta_{l,-n-m+k} + \delta_{l,n+m-k} + \delta_{l,-n+m+k} \\ &\quad + \delta_{l,n-m-k} + \delta_{l,n-m+k} + \delta_{l,-n+m-k} \\ &\quad + \delta_{l,n+m+k} + \delta_{l,-n-m-k}, \\ D_{nmkl} &\equiv \delta_{l,-n+m-k} + \delta_{l,-n+m+k} + \delta_{l,n-m+k} \\ &\quad + \delta_{l,n-m-k} - \delta_{l,n+m-k} - \delta_{l,n+m+k} \\ &\quad - \delta_{l,-n-m+k} - \delta_{l,-n-m-k}. \end{aligned} \quad (\text{A2})$$

APPENDIX B

Since in the case of mixing between KK modes the standard technique developed in Ref. [12] for a calculation of the effective potential cannot be applied directly, we present here some details of the derivation that leads to the result shown in Eq. (41). In a case of nondiagonal mass matrix M^2 we have to consider the following form of the effective potential in Euclidean space:

$$V(\varphi) = \frac{1}{2} \text{Tr} \left\{ \int \frac{d^4 p}{(2\pi)^4} \sum_{n=-\infty}^{\infty} \ln[l^2(p^2 + M^2)] \right\}, \quad (\text{B1})$$

where M is in general a nondiagonal mass matrix for KK modes and we have restricted ourselves to the no-shift case: $\omega = 0$. For the (A_5, χ_n) system we have

$$M^2 = \begin{pmatrix} e^2 \varphi^2 & -e \varphi m_n \\ -e \varphi m_n & \mu^2 + \lambda \varphi^2 + m_n^2 \end{pmatrix}. \quad (\text{B2})$$

Going to the diagonal form of M^2 it is easy to see that

$$\text{Tr}\{\ln[l^2(p^2 + M^2)]\} = \ln[l^4(p^4 + p^2 \text{Tr} M^2 + \text{Det} M^2)]. \quad (\text{B3})$$

Since

$$\text{Tr} M^2 = e^2 \varphi^2 + \mu^2 + \lambda \varphi^2 + m_n^2$$

and

$$\text{Det} M^2 = e^2 \varphi^2 (\mu^2 + \lambda \varphi^2) \quad (\text{B4})$$

we obtain eventually

$$\text{Tr}\{\ln[l^2(p^2 + M^2)]\} = \ln[l^2 E^2 + n^2 \pi^2], \quad (\text{B5})$$

where irrelevant constant terms have been dropped and

$$E^2 = p^2 + a + \frac{b}{p^2}, \quad (\text{B6})$$

with

$$a = e^2 \varphi^2 + \mu^2 + \lambda \varphi^2 \quad \text{and} \quad b = e^2 \varphi^2 (\mu^2 + \lambda \varphi^2). \quad (\text{B7})$$

Following the method adopted for diagonal mass matrices, one needs to differentiate $W \equiv \frac{1}{2} \sum_{n=-\infty}^{\infty} \ln[(lE)^2 + n^2 \pi^2]$ with respect to E , then trade the summation for a contour integral and eventually integrate over E . The result is

$$W = lE + \ln(1 - e^{-2lE}) + \text{const.} \quad (\text{B8})$$

The term that is ultraviolet divergent for a cutoff regularization emerges from the integral of the first term in Eq. (B8):

$$V_{mix}^{(\infty)} = l \int \frac{d^4 p}{(2\pi)^4} \sqrt{p^2 + a + \frac{b}{p^2}}. \quad (\text{B9})$$

The compactification radius dependent contribution consists of the integral of the second term in Eq. (B8):

$$V_{mix}^{(R)} = \int \frac{d^4 p}{(2\pi)^4} \ln(1 - e^{-2lE}). \quad (\text{B10})$$

The following formula will be adopted:

$$\begin{aligned} \int_0^{\infty} \frac{x^{\alpha-1} dx}{(ax^2 + 2bx + c)^{\rho}} &= a^{-\alpha/2} c^{\alpha/2 - \rho} \mathbf{B}(\alpha, 2\rho - \alpha) \\ &\quad \times \mathbf{F}\left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2}; \rho + \frac{1}{2}; 1 - \frac{b^2}{ac}\right), \end{aligned} \quad (\text{B11})$$

where $B(x,y)$ and $F(a,b;c;z)$ are the Euler beta function and hypergeometric function, respectively. Using the above result one can show that for the dimensional regularization the integral in Eq. (B9) is finite in the limit $n \rightarrow 4$ and the corresponding potential reads¹⁵

$$V_{mix}^{(\infty)} = -\frac{y^{1/2}(y^2-1)x^5}{2^{12}\sqrt{2}\pi^5 R^4} F\left(-\frac{1}{4}, \frac{7}{4}; 2; 1 - \frac{1}{y^2}\right), \quad (\text{B12})$$

where x and y are defined in the main text, see Eq. (44).

The integral $V_{mix}^{(R)} = \int [d^4 p / (2\pi)^4] \ln(1 - e^{-2IE})$ is more difficult to perform, so we will adopt an asymptotic expansion

¹⁵It could be verified that the following result reproduces the formula (17) in the limit $b \rightarrow 0$.

in the limit $2\pi R\varphi \rightarrow \infty$ that is an excellent approximation in the region of our interest.¹⁶ The result reads

$$V_{mix}^{(R)} \simeq -\frac{y^{3/2}(1+y)^{1/4}x^{7/2}}{2^9\pi^5\sqrt{\pi}R^4} \text{Li}_{3/2}(e^{-x\sqrt{1+y}}). \quad (\text{B13})$$

Eventually, the contribution to the effective potential from the (A_{5n}, χ_n) system is the following:

$$V_{eff}^{(A_5, \chi)} = \frac{1}{2} (V_{mix}^{(\infty)} + V_{mix}^{(R)} - V_0^{(A_0)} - V_0^{(\chi_0)}), \quad (\text{B14})$$

where $V_0^{(A_0)}$ and $V_0^{(\chi_0)}$ are the finite parts of scalar contributions [see Eq. (21)] to the effective potential calculated for the zero mode vector boson mass ($m_{A_0}^2 = e^2\varphi^2$) and Goldstone boson ($m_{\chi_0}^2 = \mu^2 + \lambda\varphi^2$), respectively.

¹⁶Since we are interested in the stability of the vacuum, it is therefore enough to know the shape of the effective potential for $\varphi \sim \text{few TeV}$, which turns out to be sufficient for the application of the asymptotic expansion of the integral.

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