

Primeval corrections to the CMB anisotropies

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We show that deviations of the quantum state of the inflaton from the thermal vacuum of inflation may leave an imprint in the CMB anisotropies. The quantum dynamics of the inflaton in such a state produces corrections to the inflationary fluctuations, which may be observable. Because these effects originate from IR physics below the Planck scale, they will dominate over any trans-Planckian imprints in any theory which obeys decoupling. Inflation sweeps away these initial deviations and forces its quantum state closer to the thermal vacuum. We view this as the quantum version of the cosmic no-hair theorem. Such imprints in the CMB may be a useful, independent test of the duration of inflation or of significant features in the inflaton potential about 60 e -folds before inflation ended, instead of an unlikely discovery of the signatures of quantum gravity. The absence of any such substructure would suggest that inflation lasted uninterrupted much longer than $O(100)$ e -folds.

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Generic models of inflation produce a lot of accelerated expansion. They far surpass the minimum needed to solve the horizon and flatness problems, of the order of $N \approx 60$. For example, in the case of chaotic inflation driven by a power-law potential $\lambda \phi^n/n$, one finds that typically $N \gtrsim 10^{24/n} \gg O(100)$. This yields perhaps the most robust of all inflationary predictions that the Universe should be spatially flat, with $\Omega_{\text{matter}} + \Omega_{\text{DE}} = 1$ [1]. More generally, this is usually taken to mean that inflation acts as a powerful amnesia tonic, efficiently relieving the Universe of the memory of its initial state. Having fewer e -folds or producing significant changes in the inflaton sector midway through inflation requires fine-tunings of the initial conditions and/or the inflaton sector beyond those which are deemed acceptable by the current lore [2]. A more optimistic stance could be that any signs of short inflation or new dynamics during it is an indication of some as yet unknown new physics, which the inflaton is sensitive to. Although such phenomena may appear like fine-tuning by the current lore, one could hope to identify the underlying physics with a better understanding of inflation. Hence either short inflation or changes of the inflaton dynamics ~ 60 e -folds before the exit at this point cannot be taken as a robust prediction¹ but as an indication of something special about inflationary dynamics and/or the initial conditions.

On the other hand, we can take a bottom-up approach to conceptual cosmology, and simply ask if we can measure for how long the final stage of inflation went on uninterrupted. For example, the current cosmological observations are indirectly sensitive to short inflation, because it could leave a nonvanishing spatial curvature of the Universe. At present the observations limit the spatial curvature to be at most a few percent of the total Ω , and the bounds are a little bit weaker if the curvature is positive [3–5]. The bounds will be

improved some in the future [6]. Thus it would be interesting to consider alternative probes of the length of inflation or of significant features in the inflaton dynamics.

In this note we show that substructure in the CMB anisotropies could provide us with another probe of inflation some ~ 60 e -folds before the end. If inflation was interrupted ~ 60 e -folds before the exit by environmental conditions, induced either by a non-inflationary stage or by a change of slow roll parameters, the quantum state of the inflaton during the generation of the inflationary fluctuations was not the usual thermal vacuum, but included some deviations from it. These effects may resemble classical inhomogeneities, in that they can be viewed as lumps of energy on top of the ground state, and can be represented as coherent state excitations of the thermal vacuum. They may also be intrinsically quantum, encoding initial phase correlations arranged by quantum effects before inflation or by the dynamics which may have intervened at the onset of the last 60 e -folds. The latter effects can be represented as squeezed states, which have been prepared by primordial quantum effects preceding inflation.²

We will demonstrate explicitly how such a squeezed state arises from the kinks in the inflaton slow roll parameters. During the onset of the final stage of inflation the frequencies of the inflaton eigenmodes change in time slightly non-adiabatically. This induces a Bogoliubov transformation between the modes before and after the transition, and therefore between their corresponding annihilation and creation operators. We will take the initial inflaton state to be the ground state of the theory just before the transition, because the transition takes only $O(1)$ Hubble times to complete. Therefore it is basically a sudden transition for modes with horizon-size wavelengths, so that the system remains in the state it occupied before the transition. Because of the Bogoliubov transformation, this state is a squeezed state on top of the thermal vacuum defined by the theory after the onset of the final stage of inflation.

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¹Possibly excepting anthropic arguments, about which we are agnostic at the moment.²We thank J. D. Bjorken for a very useful discussion of this issue.

The intuitive picture of this dynamics is akin to the quantum-mechanical system in a deep potential well, whose depth is suddenly increased. Prior to the change the system settles in its ground state, given by the minimum energy state in the well. Because the transition is fast, the system remains trapped in this state. However, after the transition, this state will not be the minimum energy state any more, since the depth of the well increased. Hence the system will be in an excited state on top of the new ground state. When the inflaton is quantized in such a state, its deviations from the thermal vacuum will correct the standard thermal vacuum result. They may be stronger than the imprints from new physics computable by effective field theory [7]. We find that such effects contribute a factor $[1 + \Delta(\eta_H - \epsilon_H)(H/p)\sin(2p/H) + \dots]$ to the thermal vacuum result, where \dots stand for additional slow roll and adiabatic corrections. Here H is the Hubble scale during inflation, $p \gtrsim H$ is the physical momentum of the fluctuation at the moment of transition, $\epsilon_H = \dot{\phi}^2/[2m_p^2 H^2]$ and $\eta_H = -\ddot{\phi}/[H\dot{\phi}]$ are the slow roll parameters, $\Delta(\eta_H - \epsilon_H)$ is the change of their difference at the transition between two stages, ϕ is the inflaton vacuum expectation value, overdot is a derivative with respect to the comoving time t , and $m_p^2 = 8\pi/G_N$ is the reduced Planck mass. We note that in long inflation with very kinky structure in the inflaton potential a large change in ϵ_H would have even more dramatic consequences already in the leading order density contrast [8,9], rendering the subleading corrections irrelevant. This can be seen by rewriting $\delta\rho/\rho$ as $\delta\rho/\rho \propto H/[\sqrt{\epsilon_H} m_p]$, and noting that it would change a lot if ϵ_H jumped while H stayed fixed. Since we are interested in the subleading corrections on top of the standard result, we will ignore long inflation which had such a strong jump in ϵ_H midcourse. However our treatment gives a way of distinguishing the models with a milder variation of ϵ_H , but a large change in η_H , that would not affect so strongly the leading order result. We will also comment on the possibility of softer features in the inflaton potential which could mimic the effects we consider already at the leading order.

Our signal might remind one of the effects recently claimed to arise from the trans-Planckian physics during inflation (for various approaches and discussions see, e.g., Refs. [10–18], and references therein, and for other ways to get similar signals see [19,20]). A simple framework for the formulation of such effects is provided by the α -vacua [16–18]. However the short-distance behavior of quantum field theory in α -vacua forces one to abandon locality and decoupling in order to regulate the theory in the UV, once interactions are included [21–24]. This means that the theory cannot be kept under full calculational control as an interacting quantum field theory,³ conflicting with the usual notion of decoupling. Further problems arise from considering the diffuse gamma-ray background measured by EGRET which al-

ready excludes the possibility of detectable imprints of α -vacua in the CMB [25]. A proposal for avoiding this was offered in [17], but it mandates changing α , and so the vacuum, in time as a function of the dominant source of energy density in the Universe, which is again in conflict with decoupling. Thus it is difficult to regard the results of a naive perturbation theory around α -vacua [17,18] as predictions for signatures of quantum gravity.

Our result differs in crucial ways. We obtain it by computing the fluctuations in an excited state on top of the usual thermal vacuum, generated by the intervening low energy evolution. The momentum p in it is not some fixed trans-Planckian scale, but the physical momentum of the fluctuations expelled out of the horizon, evaluated at the transition, and so the imprint decreases with their wavelength. Further there is the $\Delta(\eta_H - \epsilon_H)$ suppression. Thus our imprints originate completely from the IR physics below the Planck scale. As long as quantum gravity yields the usual effective quantum field theory below the Planck scale obeying decoupling, which we assume here, these IR effects provide the dominant influence on the CMB, irrespective of the details of short distance dynamics. Inflation pushes the Universe into the thermal vacuum; the longer the inflationary stage, the closer to the thermal vacuum were the state during which the observable fluctuations in the CMB background were produced. This is the quantum-mechanical version of the *cosmic no-hair* theorem. It implies that the signatures of quantum deviations from the thermal vacuum could be very sensitive to the duration of inflation. Our results provide a very simple, yet general, demonstration of this, complementing [19].

The main implication of our analysis for observations is that if for any reason $\eta_H - \epsilon_H$ changed significantly ~ 60 e -folds before the end of inflation, the effects of such a change in the CMB may be visible in the horizon-scale fluctuations today. If inflation were short, and the Universe had a spatial curvature close to observability, with Ω_k of few percent, the effects we consider should be within observer's reach. We specifically note that such effects may lead to the reduction of power in the low ℓ CMB multipoles in some models of inflation. Conversely, the absence of any such substructure would strongly suggest that inflation lasted uninterrupted much more than the minimum 60 e -folds. A long inflation would wipe out any spatial curvature and produce $\Omega_{\text{matter}} + \Omega_{\text{DE}} = 1$. The memory of the quantum correlations in the initial state of the Universe would be deeply buried in very small effects which would be extremely efficiently obscured by post-inflationary nonlinear dynamics.

We now turn to our setup. We will use a very simple toy model, where we imagine that the Universe can be described for the most part by a (spatially flat) Friedman-Robertson-Walker (FRW) line element, perturbed by initial inhomogeneities and later by inflationary fluctuations. We confine our analysis to the regime where the perturbations are small, $\delta\rho/\rho \ll 1$. We further imagine that some agent altered inflationary dynamics ~ 60 e -folds before the end. Either inflation was short, following an epoch of decelerated expansion, such as, e.g., a brief radiation era after the primordial singularity, or the inflaton went over a potential bump which changed the

³The cosmological constant problem is, in our opinion, still too much of a mystery to be taken as a clear-cut directive for abandoning decoupling in hope that this would simultaneously fix all the problems with α -vacua.

slow roll parameters. The differences in the quantum dynamics of the inflaton before and after this transition are captured by the time variation of the frequencies of the inflaton eigenmodes. Before and after inflation, their form is controlled by different backgrounds. The rapid change of the background environment during the transition will induce a slightly non-adiabatic contribution to the frequencies. Thus the eigenmodes will also be modified non-adiabatically. This will change the Hilbert space⁴ of the theory by inducing a Bogoliubov rotation of the annihilation and creation operators. The quick onset of the last stage of inflation enables us to treat the transition at the quantum level as basically a sudden transition, where the quantum system remains in the state which it occupied just before the transition. If we take the quantum state of the inflaton just before the transition to be the vacuum of the theory then, this state will be a squeezed state on top of the thermal vacuum after the transition. Therefore the inflaton state from which the inflationary fluctuations originate is populated with long-wavelength quanta of the inflaton with quantum correlations prearranged by the preceding evolution. This is of course an idealized choice; in reality the inflaton state at this instant may be an excitation of the vacuum, which may contain inhomogeneities etc. Such a general state can be viewed as a coherent state on top of our initial vacuum. Since our purpose here is to explore the quantum correlations in the initial vacuum itself, we will ignore the excitations of this state because they behave like localized inhomogeneities in the inflating patch, which will get inflated away as usual. This simplified analysis is sufficient to illustrate our main points. We expect that a more detailed analysis will share the qualitative features of our main results as long as inflation is short.

The effective field theory description of the quantum fluctuations of the inflaton, which we rely on, will break down at the cutoff, say at the string scale, but because of decoupling this does not produce significant effects on the horizon scales, where the initial correlations dominate. This is reflected in our calculation by the fact that the correlations are suppressed by a power of the momentum, and so drop off at short distances. This in turn means that the initial state is regulated in the UV in the usual way, and is not subject to the maladies plaguing α -vacua discussed in Refs. [21–24]. In the limit of eternal de Sitter space, these correlations would completely disappear and the initial state becomes precisely the usual thermal vacuum, instead of one of the α -vacua. A consequence of this is that as inflation proceeds, shorter wavelength modes are expelled out of the apparent horizon. These modes encode progressively less information about the initial quantum deviations. Hence the state of the inflationary Universe when these fluctuations are frozen appears closer to the thermal vacuum. We view this as a quantum-mechanical version of the cosmic no-hair theorem.

We first briefly review the gauge-invariant perturbation theory for inflation [30–33]. Much of the useful formalism of the evolution of fluctuations is also exhibited in [34–36].

We restrict our attention to the scalar perturbations in longitudinal gauge which is sufficient for our purposes. Once the results are expressed in terms of the gauge-invariant variables, one can change the gauge at will anyway. Thus the line element is

$$ds^2 = a^2(\eta) [-\{1 + 2\Psi(\eta, \vec{x})\} d\eta^2 + \{1 + 2\Phi(\eta, \vec{x})\} d\vec{x}^2]. \tag{1}$$

The conformal time η is related to the usual comoving FRW time t by $dt = a d\eta$. In the longitudinal gauge, the two metric perturbations Ψ, Φ coincide with gauge-invariant potentials for the perturbations. In general, they are not independent. However, their detailed relationship depends on the matter contents of the Universe that sets the background of Eq. (1).

During inflation, the dominant source of the stress energy in the Einstein’s equations is the inflaton field. The inflaton field sector can be written as

$$\phi(\eta, \vec{x}) = \phi(\eta) + \delta\phi(\eta, \vec{x}). \tag{2}$$

The independent background equations are, using conformal time variables,

$$3m_p^2 \mathcal{H}^2 = \frac{(\phi')^2}{2} + a^2 V(\phi),$$

$$\phi'' + 2\mathcal{H}\phi' + a^2 \frac{\partial V}{\partial \phi} = 0, \tag{3}$$

where the prime denotes the derivative with respect to the conformal time and $\mathcal{H} = a'/a$. A detailed analysis of the perturbations yields $\Psi = -\Phi$. Moreover, the potential Φ and the inflaton perturbation $\delta\phi$ are related by momentum conservation as

$$\phi' \delta\phi = -2m_p^2 (\Phi' + \mathcal{H}\Phi). \tag{4}$$

Hence during inflation only one of the perturbations $\Phi, \delta\phi$ is independent. One chooses it such that it has the canonical commutation relations. The quantum mechanical calculation then links Φ to the properties of the inflaton effective action and the quantum state of the inflaton during the period of inflation when the fluctuations are produced [31,32].

To determine the effect of the perturbations in an all-inclusive way, summing all the contributions to the ripples in the spacetime caused by the inflaton fluctuations, one performs an infinitesimal diffeomorphism $\delta\eta = \delta\rho/\rho' = \delta\phi/\phi'$ [37], because during inflation $\rho = V(\phi)$, $\delta\rho = \partial_\phi V \delta\phi$, and $\rho' = \partial_\phi V \phi'$ to the leading order in the slow roll parameters. In this new gauge, the curvature perturbation is the total perturbation of the $\eta = \text{const}$ hypersurfaces, and it is given in terms of another gauge-invariant potential $\Theta = \Phi - (\mathcal{H}/\phi') \delta\phi$ as

$$\frac{\delta\mathcal{R}_3}{R} = -\frac{1}{3a^2 H^2} \vec{\nabla}^2 \Theta(\eta, \vec{x}), \tag{5}$$

⁴Here we will not dwell on the conceptual problems involved in defining the Hilbert space of the inflaton in the first place [26–29].

where $H = \dot{a}/a = \mathcal{H}/a$ is the comoving Hubble parameter and $\vec{\nabla}$ denotes derivatives with respect to the spatial coordinates \vec{x} . The canonically normalized scalar field corresponding to this perturbation, which is to be promoted into the quantum inflaton field, is

$$\varphi = a \delta\phi - \frac{a \phi'}{\mathcal{H}} \Phi = -\mathcal{Z}\Theta, \quad (6)$$

which is clearly gauge invariant, being defined in terms of Θ . Following the common practice we have defined $\mathcal{Z} = a \phi' / \mathcal{H} = a \dot{\phi} / H$ [31,33]. Because the unperturbed background is spatially flat, we can expand all the fields in Fourier modes, $f_{\vec{k}}(\eta) = \int d^3\vec{x} / (2\pi)^{3/2} f(\eta, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}$. Then $(\delta\mathcal{R}_3/R)(k) = (k^2/3a^2H^2)\Theta_{\vec{k}}(\eta)$, and the definition of the power spectrum $\mathcal{P}(k) \delta^{(3)}(\vec{k}-\vec{q}) = (k^3/2\pi^2) \langle \Theta_{\vec{k}}(\eta) \Theta_{\vec{q}}^\dagger(\eta) \rangle$ gives

$$\mathcal{P}(k) \delta^{(3)}(\vec{k}-\vec{q}) = \frac{k^3}{2\pi^2} \left(\frac{H}{\dot{\phi}} \right)^2 \left\langle \frac{\varphi_{\vec{k}}}{a} \frac{\varphi_{\vec{q}}^\dagger}{a} \right\rangle, \quad (7)$$

where $\langle \mathcal{O} \rangle$ stands for the quantum expectation value of the 2-point operator \mathcal{O} in the quantum state of inflation.

The scalar field (6) is the properly defined, gauge-invariant small fluctuation of the inflaton. In perturbation theory its dynamics is governed by the quadratic action

$$S_\varphi = \frac{1}{2} \int d\eta d^3\vec{x} \left((\varphi')^2 - (\vec{\nabla}\varphi)^2 + \frac{\mathcal{Z}''}{\mathcal{Z}} \varphi^2 \right). \quad (8)$$

To quantize the theory, we use the field and its conjugate momentum in the momentum picture:

$$\begin{aligned} \varphi_{\vec{k}}(\eta) &= \int \frac{d^3\vec{x}}{(2\pi)^{3/2}} \varphi(\eta, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}, \\ \pi_{\vec{k}}(\eta) &= \int \frac{d^3\vec{x}}{(2\pi)^{3/2}} \pi(\eta, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}. \end{aligned} \quad (9)$$

Note that $\varphi_{\vec{k}}^\dagger = \varphi_{-\vec{k}}$, $\pi_{\vec{k}}^\dagger = \pi_{-\vec{k}}$. From Eq. (8) we have $\pi = \varphi'$, which using Eq. (9) translates to $\pi_{\vec{k}}(\eta) = \varphi_{\vec{k}}'(\eta)$ for the Fourier transforms. One can check that the canonical commutation relations $[\varphi(\eta, \vec{x}), \pi(\eta, \vec{y})] = i \delta^{(3)}(\vec{x}-\vec{y})$ imply $[\varphi_{\vec{k}}(\eta), \pi_{\vec{q}}^\dagger(\eta)] = i \delta^{(3)}(\vec{k}-\vec{q})$. The Hamiltonian is

$$H_\varphi = \frac{1}{2} \int d^3\vec{k} \left(\pi_{\vec{k}}^\dagger \pi_{\vec{k}} + \left(k^2 - \frac{\mathcal{Z}''}{\mathcal{Z}} \right) \varphi_{\vec{k}}^\dagger \varphi_{\vec{k}} \right). \quad (10)$$

The Fourier modes of φ obey the field equation⁵ [31,33] (where $k = |\vec{k}|$)

$$\varphi_{\vec{k}}'' + \left(k^2 - \frac{\mathcal{Z}''}{\mathcal{Z}} \right) \varphi_{\vec{k}} = 0. \quad (11)$$

The mode expansion of the field $\varphi(\eta, \vec{x})$ is

$$\varphi(\eta, \vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} [b(\vec{k}) u_{\vec{k}}(\eta) e^{i\vec{k}\cdot\vec{x}} + b^\dagger(\vec{k}) u_{\vec{k}}^*(\eta) e^{-i\vec{k}\cdot\vec{x}}], \quad (12)$$

where the annihilation and creation operators $b(\vec{k}), b^\dagger(\vec{q})$ satisfy the usual operator algebra

$$\begin{aligned} [b(\vec{k}), b^\dagger(\vec{q})] &= \delta^{(3)}(\vec{k}-\vec{q}), \\ [b(\vec{k}), b(\vec{q})] &= [b^\dagger(\vec{k}), b^\dagger(\vec{q})] = 0. \end{aligned} \quad (13)$$

The orthogonal eigenmodes $u_{\vec{k}}, u_{\vec{k}}^*$ of Eq. (11) are easy to construct in the slow roll approximation, when $\epsilon_H = \dot{\phi}^2/2m_p^2H^2$ and $\eta_H = -\dot{\phi}/H\dot{\phi}$ are small, $\epsilon_H, \eta_H \ll 1$. During slow roll inflation, to $\mathcal{O}(\epsilon_H, \eta_H)$, we have $(1-\epsilon_H)\eta = -1/aH$ and so $\mathcal{Z}''/\mathcal{Z} = (2-3\eta_H+6\epsilon_H)/\eta^2$. Then the mode equation (11) becomes [39]

$$u_k'' + \left(k^2 - \frac{2-3\eta_H+6\epsilon_H}{\eta^2} \right) u_k = 0. \quad (14)$$

The standard choice [31,32] of the eigenmodes $u_{\vec{k}}, u_{\vec{k}}^*$ is to take

$$\begin{aligned} u_{\vec{k}}(\eta) &= -\frac{\sqrt{\pi\eta}}{2} H_\nu^{(-)}(k\eta), \\ u_{\vec{k}}^*(\eta) &= -\frac{\sqrt{\pi\eta}}{2} H_\nu^{(+)}(k\eta), \end{aligned} \quad (15)$$

⁵It is easy to find the asymptotic behavior of the solutions of Eq. (11) in the general case. To the leading order, in the short wavelength limit, $k^2 \gg \mathcal{Z}''/\mathcal{Z}$, one finds $\varphi_{\vec{k}} \rightarrow A_{\vec{k}} \cos(k\eta + \theta_{\vec{k}})$, while in the limit of long wavelengths, where $k^2 \ll \mathcal{Z}''/\mathcal{Z}$, the result is $\varphi_{\vec{k}} \rightarrow B_{\vec{k}} \mathcal{Z} + C_{\vec{k}} \mathcal{Z} \int d\eta / \mathcal{Z}^2$. From Eq. (6) the curvature perturbation is [31]

$$\Theta_{\vec{k}} = - \begin{cases} \left(\frac{H}{\dot{\phi}} \right) \frac{A_{\vec{k}}}{a} \cos(k\eta + \theta_{\vec{k}}) & \text{if } k^2 \eta^2 \gg 2 \\ B_{\vec{k}} + C_{\vec{k}} \int \frac{d\eta}{\mathcal{Z}^2} & \text{if } k^2 \eta^2 \ll 2. \end{cases}$$

The $\propto C_{\vec{k}}$ mode in the latter case is the decaying superhorizon mode. From this it is clear that any short wavelength, oscillatory mode excited inside the apparent horizon H^{-1} will end up expelled out of it by inflationary stretching, where it will freeze-out retaining a nearly constant amplitude $B_{\vec{k}}$, producing a nearly scale invariant spectrum of perturbations [31,32]. Thermodynamics of this process of modes leaking out of the apparent horizon during inflation, which resembles a leaky can, has been discussed in [38].

as the positive and negative frequency modes, respectively, where $\nu = 3/2 - \eta_H + 2\epsilon_H$ [39]. The normalization of u_k is chosen such that Eq. (14) follows from the canonical commutation relations $[\varphi(\eta, \vec{x}), \pi(\eta, \vec{y})] = i\delta^3(\vec{x} - \vec{y})$. Substituting u_k, u_k^* in Eq. (12) amounts to choosing the thermal vacuum $|0\rangle$ as the ground state of the theory, because it is annihilated by the operators corresponding to the positive frequency modes in Eq. (15):

$$b(\vec{q})|0\rangle = 0. \quad (16)$$

Using $|0\rangle$ as the state of the inflaton during inflation and ignoring the slow roll corrections, in which case the eigenmodes (15) reduce to

$$u_k = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) e^{-ik\eta},$$

$$u_k^* = \frac{1}{\sqrt{2k}} \left(1 + \frac{i}{k\eta} \right) e^{ik\eta}, \quad (17)$$

leads to $\langle 0 | (\varphi_{\vec{k}}/a)(\varphi_{\vec{q}}^\dagger/a) | 0 \rangle = (2\pi^2/k^3)(H/2\pi)^2 \delta^{(3)}(\vec{k} - \vec{q})$, yielding the standard result for the power spectrum

$$\mathcal{P}(k) = \left(\frac{H}{\dot{\phi}} \right)^2 \left(\frac{H}{2\pi} \right)^2. \quad (18)$$

The corrections from slow roll effects, new physics and initial quantum correlations come in the form of multiplicative factors which can be calculated within a given theory.

In what follows we will focus on the corrections from quantum correlations and slow roll effects. In short inflation the evolution did not have enough time to prepare the system in the thermal vacuum. One could instead take the instantaneous vacuum defined by the Heisenberg operators $b(\vec{k}, \eta_0), b^\dagger(\vec{k}, \eta_0)$ at the time when inflation began [33]. This would reproduce the thermal vacuum in the limit $\eta_0 \rightarrow -\infty$. However as we have explained above, there are additional effects coming from the background evolution. During the transition the function $\mathcal{Z}''/\mathcal{Z}$ in the mode equation (11) changes rapidly, inducing modifications of the frequencies of the inflaton eigenmodes. This results in a Bogoliubov transformation in the Hilbert space which also includes non-adiabatic contributions. The net effect is that because the transition is quick the inflaton is trapped in the state it occupied just before the onset of the last stage of inflation, which is now a squeezed state on top of the inflaton thermal vacuum because of the transition-induced Bogoliubov transformation. Below we will compute the corrections from the quantum correlations in this state. We will also include the slow roll corrections, computed in [39], and the corrections from the adiabatic evolution of the vacuum, because they may be numerically significant. We will ignore possible contributions from the initial inhomogeneities, because while they will typically be important at shorter scales, they are quickly inflated away. Thus they should not affect the fluctuations at horizon crossing after a few e -folds.

Let us now define the initial state of the inflaton. As we said above, we take the initial state of the last ~ 60 e -folds of inflation to be the instantaneous vacuum just before the onset of this stage. This state is annihilated by the Heisenberg picture annihilation operator at the time η_0^- , which denotes the instant just before the transition. The Heisenberg picture annihilation and creation operators obey the canonical commutation relations $[b(\vec{k}, \eta), b^\dagger(\vec{q}, \eta)] = \delta^{(3)}(\vec{k} - \vec{q})$, $[b(\vec{k}, \eta), b(\vec{q}, \eta)] = [b^\dagger(\vec{k}, \eta), b^\dagger(\vec{q}, \eta)] = 0$, and can be defined in terms of the fields and their conjugate momenta. They are [35,36]

$$b(\vec{k}, \eta) = \frac{1}{\sqrt{2}} \left(\sqrt{k} \varphi_{\vec{k}}(\eta) + \frac{i}{\sqrt{k}} \pi_{\vec{k}}(\eta) \right),$$

$$b^\dagger(\vec{k}, \eta) = \frac{1}{\sqrt{2}} \left(\sqrt{k} \varphi_{\vec{k}}^\dagger(\eta) - \frac{i}{\sqrt{k}} \pi_{\vec{k}}^\dagger(\eta) \right). \quad (19)$$

Thus the instantaneous vacuum $|\bar{0}\rangle$ of the theory just before the last stage of inflation obeys

$$b(\vec{k}, \eta_0^-) |\bar{0}\rangle = 0. \quad (20)$$

We note here that we are interested only in the leading order contributions from the non-thermal effects induced by the sharp transition. We will see that the slow roll corrections, the contributions from initial quantum correlations, and the corrections from adiabatic evolution come with their own small parameters: powers of ϵ_H, η_H ; $\Delta(\eta_H - \epsilon_H)H/p$; and $(H/p)^2$, respectively. Therefore in computing the leading order form of each one we can ignore the others. This allows us to use the massless eigenmodes in place of the exact Hankel functions in Eq. (15) when computing the corrections from adiabatic evolution and initial correlations, and only use the slow roll-improved eigenmodes (15) when considering the slow roll corrections. We will include these slow roll corrections because they may be numerically significant by simply incorporating the known result for the slow roll corrections from [39]. A more general calculation accounting for interference terms may be interesting in order to get a more suitable framework for data fits, but is beyond the scope of the present work.

To determine the effects of the transition on the Hilbert space, and specifically on the state $|\bar{0}\rangle$ (20), we consider the field equation (11) in a general environment. One can show that in general

$$\frac{\mathcal{Z}''}{\mathcal{Z}} = \frac{a''}{a} + \frac{1}{2} \left(\frac{\epsilon'_H}{\epsilon_H} \right)' + \frac{\epsilon'_H}{\epsilon_H} \mathcal{H} + \frac{1}{4} \left(\frac{\epsilon'_H}{\epsilon_H} \right)^2. \quad (21)$$

To find the effects of the sudden transition on the Hilbert space, we need to evolve the Heisenberg operators of the theory through the transition using the field equation (11) with the general form of $\mathcal{Z}''/\mathcal{Z}$ included. Indeed, from inspecting Eq. (21), it is clear that the contribution to $\mathcal{Z}''/\mathcal{Z}$ coming from $\frac{1}{2}(\epsilon'_H/\epsilon_H)'$ experiences a jump. Because ϵ'_H/ϵ_H is linear in ϵ_H, η_H , the difference of ϵ'_H/ϵ_H before and after

the transition will be of the same order as the quantity itself. This is the source of the non-adiabatic evolution of the operators. Basically, the environment pumps some energy into them during the transition. To determine the transformation, we can treat the problem as a Schrödinger problem with a piecewise smooth potential:

$$\varphi_{\vec{k}}'' + [k^2 - V(\eta)]\varphi_{\vec{k}} = 0, \quad (22)$$

where

$$V(\eta) = \frac{a''}{a} + \frac{1}{2} \left(\frac{\epsilon_H'}{\epsilon_H} \right)' + \frac{\epsilon_H'}{\epsilon_H} \mathcal{H} + \frac{1}{4} \left(\frac{\epsilon_H'}{\epsilon_H} \right)^2. \quad (23)$$

The problem of matching the operators is completely analogous to the quantum mechanical problem of a particle scattering on a potential bump. Rather than looking at the specifics of an explicit construction of the modes for any given environment before the last ~ 60 e -folds of inflation, we take the shortcut and find the effect of the transition directly on the field operators. To do this, we impose the continuity of $\varphi_{\vec{k}}$ across the transition in the usual way, and determine the jump of $\varphi_{\vec{k}}'$ as dictated by $\frac{1}{2}(\epsilon_H'/\epsilon_H)'$ in the Gaussian pillbox integration of Eq. (22). Denoting the quantities slightly before the transition by the argument η_0^- and those slightly after by the argument η_0^+ , the integration yields

$$\varphi_{\vec{k}}'(\eta_0^+) = \varphi_{\vec{k}}'(\eta_0^-) + \frac{\epsilon_H'^+}{2\epsilon_H^+} \varphi_{\vec{k}}(\eta_0) - \frac{\epsilon_H'^-}{2\epsilon_H^-} \varphi_{\vec{k}}(\eta_0). \quad (24)$$

The slow roll parameter ϵ_H obeys $\epsilon_H'/\epsilon_H = -2\mathcal{H}(\eta_H - \epsilon_H)$. This gives the jump conditions

$$\begin{aligned} \varphi_{\vec{k}}(\eta_0^+) &= \varphi_{\vec{k}}(\eta_0^-), \\ \varphi_{\vec{k}}'(\eta_0^+) &= \varphi_{\vec{k}}'(\eta_0^-) - \Delta(\eta_H - \epsilon_H) \mathcal{H}_0 \varphi_{\vec{k}}(\eta_0), \end{aligned} \quad (25)$$

where ϵ_H and η_H are evaluated during inflation. Here $\Delta(\eta_H - \epsilon_H) = (\eta_H^+ - \epsilon_H^+) - (\eta_H^- - \epsilon_H^-)$ is the change of the differences of the slow roll parameters after and before the transition. We note that away from the transition, by the form of the action (8), we always have $\pi_{\vec{k}} = \varphi_{\vec{k}}'$. Thus we can rewrite the jump conditions (25) as the matching conditions for the fields and the momenta at the transition:

$$\begin{aligned} \varphi_{\vec{k}}(\eta_0^+) &= \varphi_{\vec{k}}(\eta_0^-), \\ \pi_{\vec{k}}(\eta_0^+) &= \pi_{\vec{k}}(\eta_0^-) - \Delta(\eta_H - \epsilon_H) \mathcal{H}_0 \varphi_{\vec{k}}(\eta_0). \end{aligned} \quad (26)$$

This makes the effect of the sudden transition very clear: it enforces a canonical transformation on the variables $\varphi_{\vec{k}}, \pi_{\vec{k}}$ describing the inflaton dynamics after the evolution has begun. This in turn induces a Bogoliubov transformation between the annihilation and creation operators. The Bogoliubov transformation is proportional to the change in $\eta_H - \epsilon_H$ during the transition. If inflation was short, such a change arises because of the very nature of the slow roll regime. Namely, because during inflation the curvature of the inflaton

potential is small compared to the Hubble scale, $\partial_\phi^2 V \ll H^2$, and the inflaton rolls slowly, if $\epsilon_H = \dot{\phi}^2/2m_p^2 H^2 < 1, \eta_H = -\ddot{\phi}/H\dot{\phi} < 1$ during inflation, they will satisfy $\epsilon_H \ll 1, \eta_H \ll 1$ before it. This is because $\dot{\phi}^2, \ddot{\phi}$ hardly changed at all, and $H^2 = \rho/3m_p^2$ was bigger before. In the case of long inflation such a change can arise from bumps in the inflaton potential or enhanced interactions with matter sector for special values of the inflaton. However we bear in mind that ϵ_H should not have changed by too much since that would have produced a large variation of the leading order result, that would render the subleading corrections which we are considering essentially irrelevant.

Using Eqs. (19) and (26) we can write the Bogoliubov transformation of the annihilation and creation operators induced by the transition. Denoting those just before the transition by their argument η_0^- and those after by η_0^+ , the Bogoliubov transformation is

$$\begin{aligned} & \begin{pmatrix} b(\vec{k}, \eta_0^-) \\ b^\dagger(-\vec{k}, \eta_0^-) \end{pmatrix} \\ &= \begin{pmatrix} 1 + i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k} & i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k} \\ -i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k} & 1 - i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k} \end{pmatrix} \\ & \times \begin{pmatrix} b(\vec{k}, \eta_0^+) \\ b^\dagger(-\vec{k}, \eta_0^+) \end{pmatrix}. \end{aligned} \quad (27)$$

We can now rewrite our initial inflaton state (20) as

$$b(\vec{k}, \eta_0^+) |\tilde{0}\rangle = -i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k} b^\dagger(-\vec{k}, \eta_0^+) |\tilde{0}\rangle \quad (28)$$

retaining only the terms of the order $\mathcal{O}(\Delta(\eta_H - \epsilon_H) \mathcal{H}_0/k, 1/k^2 \eta_0^2)$, in accordance with our approximations. We stress the key properties of this state. The state (28) is a direct and inevitable consequence of evolution. It contains the contributions both from non-adiabatic effects during the transition to the last stage of inflation and from the adiabatic dynamics during it. It is different from the thermal vacuum, albeit by terms which vanish in either of the limits $\epsilon_H \rightarrow 0, \eta_0 \rightarrow -\infty$ and $k \rightarrow \infty$. This means that in the limit of pure de Sitter space and also at very short distances the quantum correlations in $|\tilde{0}\rangle$ rapidly disappear. Hence the theory defined by Eqs. (27) and (28) is consistent with decoupling. Although this comes from the suppressions by $1/k^2 = -1/\bar{V}^2$, which resembles a non-local term, it is automatically induced by the backreaction, and is perfectly well behaved at short distances, where the theory may be cutoff in the usual way in order to regulate its UV behavior.

We can now compute the imprints of $|\tilde{0}\rangle$ on the inflationary fluctuations. Using $\pi_{\vec{k}} = \varphi_{\vec{k}}', \varphi_{\vec{k}}(\eta) = u_{\vec{k}}(\eta) b(\vec{k}) + u_{\vec{k}}^*(\eta) b^\dagger(-\vec{k})$ and the eigenmodes (15) we can solve for the evolution of the Heisenberg operators $b(\vec{k}, \eta), b^\dagger(\vec{k}, \eta)$

from the transition onwards. In terms of the Schrödinger operators defined in Eq. (12), they are

$$\begin{aligned}
b(\vec{k}, \eta) &= f_k(\eta)b(\vec{k}) + g_k(\eta)b^\dagger(-\vec{k}), \\
b^\dagger(-\vec{k}, \eta) &= f_k^*(\eta)b^\dagger(-\vec{k}) + g_k^*(\eta)b(\vec{k}), \\
f_k(\eta) &= \sqrt{\frac{k}{2}}u_k(\eta) + \frac{i}{\sqrt{2k}}u_k'(\eta), \\
g_k(\eta) &= \sqrt{\frac{k}{2}}u_k^*(\eta) + \frac{i}{\sqrt{2k}}u_k^{*'}(\eta). \quad (29)
\end{aligned}$$

The functions f_k, g_k obey $f_k^*f_k - g_k^*g_k = 1$ by virtue of the Wronskian relation of the eigenmodes $u_k u_k^{*'} - u_k' u_k^* = i$, and so Eq. (29) in fact is the evolution-induced adiabatic Bogoliubov rotation between $b(\vec{k}), b^\dagger(\vec{k})$ and $b(\vec{k}, \eta), b^\dagger(\vec{k}, \eta)$. Using this general form of the solutions, it is straightforward to obtain the evolution of the Heisenberg operators from the time η_0^+ to η . One finds (dropping the superscript ‘+’ on η_0 for notational simplicity)

$$\begin{aligned}
b(\vec{k}, \eta) &= U_k(\eta, \eta_0)b(\vec{k}, \eta_0) + V_k(\eta, \eta_0)b^\dagger(-\vec{k}, \eta_0), \\
b^\dagger(-\vec{k}, \eta) &= U_k^*(\eta, \eta_0)b^\dagger(-\vec{k}, \eta_0) + V_k^*(\eta, \eta_0)b(\vec{k}, \eta_0), \quad (30)
\end{aligned}$$

where

$$\begin{aligned}
U_k(\eta, \eta_0) &= f_k(\eta)f_k^*(\eta_0) - g_k(\eta)g_k^*(\eta_0), \\
V_k(\eta, \eta_0) &= g_k(\eta)f_k(\eta_0) - f_k(\eta)g_k(\eta_0). \quad (31)
\end{aligned}$$

These functions satisfy $U_k^*(\eta, \eta_0)U_k(\eta, \eta_0) - V_k^*(\eta, \eta_0)V_k(\eta, \eta_0) = 1$ because $f_k^*f_k - g_k^*g_k = 1$, and so they indeed also comprise a time-dependent Bogoliubov transformation. Now using Eq. (30) we can finally write down the solution for the field modes $\varphi_{\vec{k}}(\eta) = (1/\sqrt{2k}) \times [b_2(\vec{k}, \eta) + b_2^\dagger(-\vec{k}, \eta)]$ in terms of the Heisenberg operators $b_2(\vec{k}, \eta_0), b_2^\dagger(\vec{k}, \eta_0)$,

$$\begin{aligned}
\varphi_{\vec{k}}(\eta) &= \frac{1}{\sqrt{2k}}[U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)]b_2(\vec{k}, \eta_0) \\
&+ \frac{1}{\sqrt{2k}}[U_k^*(\eta, \eta_0) + V_k(\eta, \eta_0)]b_2^\dagger(-\vec{k}, \eta_0). \quad (32)
\end{aligned}$$

It is now straightforward albeit tedious to compute the 2-point function of the operator (32) in the initial state $|\bar{0}\rangle$ defined in Eq. (28). Expanding the initial state to the first order in $\Delta(\eta_H - \epsilon_H)$ as given in Eq. (28), the result is

$$\begin{aligned}
\langle \bar{0} | \frac{\varphi_{\vec{k}}(\eta)}{a} \frac{\varphi_{\vec{q}}^\dagger(\eta)}{a} | \bar{0} \rangle &= \frac{1}{2ka^2} \delta^{(3)}(\vec{k} - \vec{q}) \{ |U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)|^2 \\
&+ \mathcal{A}_k [U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)]^2 \\
&+ \mathcal{A}_k^* [U_k^*(\eta, \eta_0) + V_k(\eta, \eta_0)]^2 \}, \quad (33)
\end{aligned}$$

where

$$\mathcal{A}_k = -i\Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{2k}. \quad (34)$$

These expressions however still contain higher powers of the slow roll parameters and $(k\eta_0)^{-1}$ than is allowed by our approximations. Therefore we need to organize the result (33) as a consistent perturbative expansion. Our organizing principle is to view the result (33) as the standard 2-point function of the inflaton in the thermal vacuum, plus small corrections coming from slow roll corrections and from the initial correlations encoded in the definition of the initial state of inflation (28). This is a direct consequence of our assumption that the inflaton state in which the fluctuations are produced was the adiabatic vacuum just before the transition. The symmetries of the approximate de Sitter space used to define this vacuum then guarantee that the effects of the transition can be organized as a perturbation series. The small dimensionless numbers which characterize the corrections are the following: the slow roll parameters ϵ_H, η_H alone, which account for the fact that the apparent horizon is slowly growing during inflation, the even powers of $\mathcal{H}_0/k = -1/k\eta_0$ controlling the adiabatic evolution and $\Delta(\eta_H - \epsilon_H)\mathcal{H}_0/k$ controlling the magnitude of the initial quantum correlations in $|\bar{0}\rangle$ in Eq. (28). Because we are interested here only in comparing these effects in the leading order, we will ignore the interference between them. By our choice of the inflaton state, which implicitly rests on the validity of the slow roll approximation, these terms will be subleading. If the slow roll conditions are strongly violated, the interference terms may become larger, but the leading order effects will be even more important. Having assumed the validity of perturbation theory, we can ignore this regime altogether.

To get the slow roll corrections on top of the thermal vacuum result accounted for, we can take the limit of the 2-point function (33) where the state (28) reduces to the thermal vacuum. This amounts to taking $\mathcal{O}(\epsilon_H)$ terms in Eqs. (33) to zero, and taking $f_k(\eta_0) \rightarrow 1, g_k(\eta_0) \rightarrow 0$. Using Eq. (29) and (31) it is easy to verify that in this limit $U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0) \rightarrow \sqrt{2k}u_k(\eta)$. Hence the 2-point function indeed reduces to the thermal vacuum result,

$$\begin{aligned}
\langle \bar{0} | \frac{\varphi_{\vec{k}}(\eta)}{a} \frac{\varphi_{\vec{q}}^\dagger(\eta)}{a} | \bar{0} \rangle &\rightarrow \langle 0 | \frac{\varphi_{\vec{k}}(\eta)}{a} \frac{\varphi_{\vec{q}}^\dagger(\eta)}{a} | 0 \rangle \\
&= \frac{|u_k(\eta)|^2}{a^2} \delta^{(3)}(\vec{k} - \vec{q}). \quad (35)
\end{aligned}$$

The slow roll corrections to the leading order have been computed in [39] by expanding the Hankel functions (15) to the leading order in the slow roll parameters at horizon crossing. We can simply take their result, which with our conventions is

$$\begin{aligned} \langle 0 | \frac{\varphi_{\vec{k}}(\eta)}{a} \frac{\varphi_{\vec{q}}^\dagger(\eta)}{a} | 0 \rangle &= \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2 [1 + 2(2 - \ln 2 - \gamma_{em}) \\ &\quad \times (2\epsilon_H - \eta_H) - 2\epsilon_H] \delta^{(3)}(\vec{k} - \vec{q}), \end{aligned} \quad (36)$$

where $\gamma_{em} = 0.5772 \dots$ is the Euler-Mascheroni constant.

Now we turn to the effects arising from the deviation of the state (28) from the thermal vacuum. From the discussion above, it is clear that they are encased in the factor

$$\begin{aligned} \mathcal{F} &= \frac{1}{2k|u_k(\eta)|^2} \{ [U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)]^2 \\ &\quad + \mathcal{A}_k [U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)]^2 \\ &\quad + \mathcal{A}_k^* [U_k^*(\eta, \eta_0) + V_k(\eta, \eta_0)]^2 \}, \end{aligned} \quad (37)$$

which should be evaluated in the limit $\eta \rightarrow 0$ using the relativistic modes (17) in the definitions of f_k , g_k , U_k and V_k above. Hence

$$f_k(\eta) = \left(1 - \frac{i}{k\eta} - \frac{1}{2k^2\eta^2} \right) e^{-ik\eta}, \quad g_k(\eta) = \frac{1}{2k^2\eta^2} e^{ik\eta}. \quad (38)$$

To simplify the calculation, note that $1/\eta_0 = -\mathcal{H}_0$. After the evicition from the horizon, $u_k \rightarrow -(i/\sqrt{2kk\eta})e^{-ik\eta}$. Thus, $f_k + g_k^* = \sqrt{2ku_k} \rightarrow -(i/k\eta)e^{-ik\eta}$. Therefore

$$\begin{aligned} [U_k(\eta, \eta_0) + V_k^*(\eta, \eta_0)]_{|k\eta| \ll 1} \\ = -\frac{i}{k\eta} \left[\left(1 - i\frac{\mathcal{H}_0}{k} - \frac{\mathcal{H}_0^2}{2k^2} \right) e^{-ik/\mathcal{H}_0} + \frac{\mathcal{H}_0^2}{2k^2} e^{ik/\mathcal{H}_0} \right]. \end{aligned} \quad (39)$$

Substituting this in \mathcal{F} in Eq. (37), we find [keeping only the terms up to $\mathcal{O}(\Delta(\eta_H - \epsilon_H)\mathcal{H}_0/k, \mathcal{H}_0^2/k^2)$]

$$\mathcal{F} = 1 + \frac{\mathcal{H}_0^2}{k^2} \cos\left(\frac{2k}{\mathcal{H}_0}\right) + \Delta(\eta_H - \epsilon_H) \frac{\mathcal{H}_0}{k} \sin\left(\frac{2k}{\mathcal{H}_0}\right). \quad (40)$$

In this expression, $\mathcal{H}_0/k = H/p \ll 1$, where we have defined the physical momentum of the mode $p = k/a_0$ at the moment of the transition to the last ~ 60 e -folds of inflation. This is simply the statement that the fluctuations are produced inside the inflating Hubble patch, meaning that their wavelength is originally inside the apparent horizon. We can rewrite the form factor \mathcal{F} in terms of these variables as

$$\mathcal{F} = 1 + \frac{H^2}{p^2} \cos\left(\frac{2p}{H}\right) + \Delta(\eta_H - \epsilon_H) \frac{H}{p} \sin\left(\frac{2p}{H}\right). \quad (41)$$

In this parameterization, the unity corresponds to the thermal vacuum result, the second term [proportional to $\mathcal{O}((H/p)^2)$] to the adiabatic evolution of the vacuum, and the third term [proportional to $\mathcal{O}(\Delta(\eta_H - \epsilon_H)(H/p))$] to the effects of the quantum correlations encoded in the state $|\bar{0}\rangle$ (28) by the transition to the final stage of inflation.

To see the total effect on the density fluctuations, we fold Eq. (41) with the slow roll-corrected 2-point function in the thermal vacuum. Again expanding to the linear order in all corrections, we obtain the full 2-point function at horizon crossing:

$$\begin{aligned} \langle \bar{0} | \frac{\varphi_{\vec{k}}(\eta)}{a} \frac{\varphi_{\vec{q}}^\dagger(\eta)}{a} | \bar{0} \rangle \\ = \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2 [1 + \mathcal{D}(p, H, \epsilon_H, \eta_H)] \delta^{(3)}(\vec{k} - \vec{q}), \end{aligned} \quad (42)$$

where

$$\begin{aligned} \mathcal{D}(p, H, \epsilon_H, \eta_H) &= 2(2 - \ln 2 - \gamma_{em})(2\epsilon_H - \eta_H) - 2\epsilon_H \\ &\quad + \frac{H^2}{p^2} \cos\left(\frac{2p}{H}\right) + \Delta(\eta_H - \epsilon_H) \frac{H}{p} \sin\left(\frac{2p}{H}\right). \end{aligned} \quad (43)$$

Substituting this into the formula for the power spectrum (7) yields for modes with $k\eta_0 = p/H > 1$, which were expelled out of the horizon,

$$\mathcal{P}(k) = \left(\frac{H}{\phi} \right)^2 \left(\frac{H}{2\pi} \right)^2 [1 + \mathcal{D}(p, H, \epsilon_H, \eta_H)], \quad (44)$$

which is our main result.

This shows that if there is a sudden change in $\eta_H - \epsilon_H$, either because inflation is short, lasting not much more than the minimum 60 e -folds, or because η_H jumps ~ 60 e -folds before the end of long inflation while ϵ_H remains roughly constant, which would keep the leading order result unchanged and confine the interesting effects in the corrections we derive the imprints of this change in the inflationary fluctuations could be at a level that may affect the observations. The perturbations from the modes which are expelled out of the horizon just after this transition may receive important contributions from the quantum correlations inside the inflating patch, which would be comparable with the slow roll and adiabatic effects. As inflation progresses and shorter and shorter wavelength modes are expelled out of the horizon, these corrections will rapidly diminish below the observable level. That is clear from Eqs. (43) and (44) because the contributions from these correlations are suppressed by H/p as the momentum increases. We interpret this as the quantum version of the cosmic no-hair theorem: as inflation proceeds the quantum state of the inflaton, out of which the fluctuations emerge, is less and less different from the thermal vacuum. We stress again that while the effect in Eqs. (43),

(44) might be vaguely reminiscent of the corrections claimed to arise in the α -vacua [17], they really are completely different. Our signal explicitly depends on the physical momentum p or the wavelength of the perturbations $\lambda = 1/p$, rather than on some fixed trans-Planckian cutoff. Further, we find that there is a suppression by the change of the difference of the slow roll parameters $\Delta(\eta_H - \epsilon_H)$. Thus our effects only appear in the long wavelength perturbations, and rapidly vanish in the UV. In this way, our results are fully consistent with the conventional lore of effective field theory, because decoupling of the UV physics is guaranteed.

Nevertheless, the result (43), (44) has interesting implications for observational cosmology. At present, the case for inflation is growing stronger as more data are accumulated [3–5]. However it is difficult to use observations to place bounds on the duration of inflation or the properties of the inflationary potential. Our result may serve as an additional probe of the inflationary dynamics. While somewhat model dependent, our result suggests that in the case of either short inflation or longer inflation with a large $\Delta(\eta_H - \epsilon_H)$ the density spectrum, and therefore the CMB, may retain some information about the initial quantum correlations at the instant when this stage began. There may be models where such terms could be at the level of few percents, and therefore observable. Because the effects in Eqs. (43), (44) come with a distinct trigonometric modulation at the largest scales, this might help in the search for them. Note that at shorter scales, as the momentum increases, the modulation essentially disappears: the statistical sampling of the data tells us that we must average the trigonometric functions over several periods. This would render the modulation at short scales impossible to detect, and therefore completely irrelevant.

In some models, the effects leading to Eqs. (43) and (44) may even suppress power on large scales, reducing the low ℓ multipoles in the CMB anisotropy. These multipoles are sensitive to scales of the order of the horizon today and larger, and so in short inflation they could be affected by the superhorizon modes at the onset of inflation, obeying $k\eta_0 = p/H < 1$. Although one does not have firm control over the fluctuations on scales $p/H \ll 1$ because they would be strongly affected by any initial inhomogeneities outside the inflating patch, we can at least estimate how much power would be transferred to these modes by inflationary dynamics if the initial inhomogeneities were negligible. In this regime we can neglect slow roll corrections and the $\mathcal{O}(\Delta(\eta_H - \epsilon_H))$ term encoding initial quantum correlations. Then using the result for the power spectrum including adiabatic corrections, valid on all scales, $\mathcal{P}(k) = (H/2\pi)^2 (H/\dot{\phi})^2 \{1 + H^2/2p^2 + (H^2/p^2 - H^4/2p^4)\cos(2p/H) - (H/p)^3\sin(2p/H)\}$, we find the leading order power spectrum for the superhorizon modes at the onset of inflation by taking the limit $p/H < 1$,

$$\mathcal{P}(k) = \frac{4}{9} \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \left(\frac{p}{H}\right)^2. \quad (45)$$

This shows that the deposit of power in superhorizon modes during inflation is strongly suppressed, as expected. Thus for

short inflation, the reduced amplitude of the low ℓ multipoles in the CMB anisotropy arises from combining (44) for $p > H$ and (45) for $p < H$.

One may correctly warn that the trigonometric modulation present in Eqs. (43), (44) need not be an unambiguous indication of the presence of short inflation or long inflation with a jump in $\eta_H - \epsilon_H$. For example, one may try to redefine the background of the theory by redefining the inflationary potential $V(\phi) \rightarrow W(\phi)$ by solving the differential equation

$$\frac{\partial_\phi W}{W^{3/2}} = \frac{\partial_\phi V}{V^{3/2}} [1 + \mathcal{D}(p, H, \epsilon_H, \eta_H)]^{-1/2}. \quad (46)$$

Thus our effects might be mimicked by a different potential, where they would be confined in the leading order result. However if the effects we are discussing are quantitatively significant, this redefinition of the potential would not *remove* the question; it would merely *change* it. One would still be forced to ask “What produced such features in the inflaton potential ~ 60 e -folds before the end of inflation, which gave rise to such signals?” regardless of the root cause of the signal itself. Hence the presence of such effects would indicate interesting physics either way. Their detection would be a win-win situation. On the other hand, if no such effects are ever seen, it would be natural to argue that inflation went on uninterrupted for significantly more than the bare minimum of ~ 60 e -folds. This would bury any information in the inflationary perturbations about the initial state below the discernible level. However, in such an instance one could plausibly argue that the curvature of the spatial sections is very tiny, and therefore that the density of dark energy plus dark matter is practically indistinguishable from unity.

In summary, we have shown how quantum correlations in the quantum state of the inflaton affect the density perturbations and the CMB. Our calculations are in full agreement with the usual effective field theory and decoupling, and are performed in the controllable regime of perturbation theory, where the Universe can be treated as a weakly perturbed FRW cosmology. The effects of these correlations are suppressed by a power of the momentum, and vanish in the UV. We find that if inflation did not last much longer than the necessary minimum of 60 or so e -folds, or if $\eta_H - \epsilon_H$ changed significantly at that time, the initial correlations may yield observable imprints. Thus the amplitude of the corrections from these quantum correlations may be a sensitive probe of the inflationary dynamics at ~ 60 e -folds before the end. Viewing the issue of the inflationary dynamics during the final stage as a purely observational matter, we feel that the prospective searches for such effects would be a worthy enterprise, since they could shed light on the darkness from which our Universe emerged.

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- [1] A. Guth, Phys. Rev. D **23**, 347 (1981); A. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
- [2] G.F. Ellis, D.H. Lyth, and M.B. Mijic, Phys. Lett. B **271**, 52 (1991); J.P. Uzan, U. Kirchner, and G.F. Ellis, Mon. Not. R. Astron. Soc. **344**, L65 (2003); G. Efstathiou, *ibid.* **343**, L95 (2003); A. Linde, J. Cosmol. Astropart. Phys. **05**, 002 (2003); C.R. Contaldi, M. Peloso, L. Kofman, and A. Linde, *ibid.* **07**, 002 (2003).
- [3] Boomerang Collaboration, A.H. Jaffe *et al.*, Phys. Rev. Lett. **86**, 3475 (2001); Boomerang Collaboration, A.E. Lange *et al.*, Phys. Rev. D **63**, 042001 (2001); Boomerang Collaboration, C.B. Netterfield *et al.*, Astrophys. J. **571**, 604 (2002).
- [4] S. Hanany *et al.*, Astrophys. J. Lett. **545**, L5 (2000); A. Balbi *et al.*, *ibid.* **545**, L1 (2000); **558**, L145(E) (2001); A.T. Lee *et al.*, *ibid.* **561**, L1 (2001).
- [5] D.N. Spergel *et al.*, Astrophys. J., Suppl. Ser. **148**, 175 (2003).
- [6] D.J. Eisenstein, W. Hu, and M. Tegmark, Astrophys. J. **518**, 2 (1998).
- [7] N. Kaloper, M. Kleban, A. Lawrence, and S. Shenker, Phys. Rev. D **66**, 123510 (2002).
- [8] A.A. Starobinsky, JETP Lett. **55**, 489 (1992).
- [9] J. Adams, B. Cresswell, and R. Easther, Phys. Rev. D **64**, 123514 (2001).
- [10] R.H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001); J. Martin and R.H. Brandenberger, Phys. Rev. D **63**, 123501 (2001).
- [11] J.C. Niemeyer, Phys. Rev. D **63**, 123502 (2001); A. Kempf, *ibid.* **63**, 083514 (2001); A. Kempf and J.C. Niemeyer, *ibid.* **64**, 103501 (2001).
- [12] T. Tanaka, astro-ph/0012431.
- [13] R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, Phys. Rev. D **64**, 103502 (2001); **67**, 063508 (2003).
- [14] A.A. Starobinsky, JETP Lett. **73**, 371 (2001).
- [15] L. Hui and W.H. Kinney, Phys. Rev. D **65**, 103507 (2002).
- [16] N.A. Chernikov and E.A. Tagirov, Ann. Poincare Phys. Theor. A **9**, 109 (1968); E.A. Tagirov, Ann. Phys. **76**, 561 (1973); J. G eh eniau and C. Schomblond, Acad. R. Belg. Bull. Cl. Sci. **54**, 1147 (1968); C. Schomblond and P. Spindel, Ann. Inst. Henri Poincare, Sect. A **25**, 67 (1976); E. Mottola, Phys. Rev. D **31**, 754 (1985); B. Allen, *ibid.* **32**, 3136 (1985); R. Bousso, A. Maloney, and A. Strominger, *ibid.* **65**, 104039 (2002).
- [17] U.H. Danielsson, Phys. Rev. D **66**, 023511 (2002); J. High Energy Phys. **07**, 040 (2002); L. Bergstrom and U.H. Danielsson, *ibid.* **12**, 038 (2002).
- [18] K. Goldstein and D.A. Lowe, Phys. Rev. D **67**, 063502 (2003); Nucl. Phys. **B669**, 325 (2003).
- [19] C.P. Burgess, J.M. Cline, F. Lemieux, and R. Holman, J. High Energy Phys. **02**, 048 (2003); C.P. Burgess, J. Cline, and R. Holman, J. Cosmol. Astropart. Phys. **10**, 004 (2003).
- [20] D.J. Chung, A. Notari, and A. Riotto, J. Cosmol. Astropart. Phys. **10**, 012 (2003).
- [21] T. Banks and L. Mannelli, Phys. Rev. D **67**, 065009 (2003).
- [22] M.B. Einhorn and F. Larsen, Phys. Rev. D **67**, 024001 (2003); **68**, 064002 (2003).
- [23] N. Kaloper, M. Kleban, A. Lawrence, S. Shenker, and L. Susskind, J. High Energy Phys. **11**, 037 (2002).
- [24] H. Collins, R. Holman, and M.R. Martin, Phys. Rev. D **68**, 124012 (2003).
- [25] A.A. Starobinsky and I.I. Tkachev, JETP Lett. **76**, 235 (2002).
- [26] T. Banks, hep-th/0007146; T. Banks and W. Fischler, hep-th/0102077; hep-th/0111142; T. Banks, W. Fischler, and S. Paban, J. High Energy Phys. **12**, 062 (2002).
- [27] S. Hellerman, N. Kaloper, and L. Susskind, J. High Energy Phys. **06**, 003 (2001); W. Fischler, A. Kashani-Poor, R. McNees, and S. Paban, *ibid.* **07**, 003 (2001).
- [28] E. Witten, hep-th/0106109.
- [29] L. Dyson, M. Kleban, and L. Susskind, J. High Energy Phys. **10**, 011 (2002); N. Goheer, M. Kleban, and L. Susskind, *ibid.* **07**, 056 (2003).
- [30] J.M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
- [31] V.F. Mukhanov and G.V. Chibisov, JETP Lett. **33**, 532 (1981); V.F. Mukhanov, *ibid.* **41**, 493 (1985).
- [32] A.A. Starobinsky, Phys. Lett. **117B**, 175 (1982); S.W. Hawking, *ibid.* **115B**, 295 (1982); A.H. Guth and S.Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J.M. Bardeen, P.J. Steinhardt, and M.S. Turner, Phys. Rev. D **28**, 679 (1983); A.D. Linde, Phys. Lett. **116B**, 335 (1982).
- [33] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys. Rep. **215**, 203 (1992).
- [34] L.P. Grishchuk and Y.V. Sidorov, Phys. Rev. D **42**, 3413 (1990).
- [35] A. Albrecht, P. Ferreira, M. Joyce, and T. Prokopec, Phys. Rev. D **50**, 4807 (1994).
- [36] D. Polarski and A.A. Starobinsky, Class. Quantum Grav. **13**, 377 (1996).
- [37] D.H. Lyth, Phys. Rev. D **31**, 1792 (1985); J. Martin and D.J. Schwarz, *ibid.* **57**, 3302 (1998); K.A. Malik and D. Wands, gr-qc/9804046.
- [38] A. Albrecht, N. Kaloper, and Y.S. Song, hep-th/0211221.
- [39] E.D. Stewart and D.H. Lyth, Phys. Lett. B **302**, 171 (1993).