Spacetime-varying couplings and Lorentz violation

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Spacetime-varying coupling constants can be associated with violations of local Lorentz invariance and *CPT* symmetry. An analytical supergravity cosmology with a time-varying fine-structure constant provides an explicit example. Estimates are made for some experimental constraints.

Since Dirac's large-number hypothesis [1], spacetime-varying couplings have remained the subject of various theoretical and experimental studies. Such couplings are natural in many unified theories [2], and current claims of observational evidence for a time-varying electromagnetic coupling [3] have sparked a revival of this idea [4].

In this work, we investigate the role of Lorentz symmetry in the subject, showing that spacetime-varying couplings can be associated with Lorentz and *CPT* violation [5]. This result is intuitively reasonable because translation invariance is broken in a theory with spacetime-varying couplings, while translations and Lorentz transformations are intertwined in the Poincaré group. The vacuum then behaves as a spacetime-varying medium so Lorentz isotropy can be lost in local inertial frames.

As an illustration, consider a spacetime-varying coupling ξ associated with a term containing derivatives in a Lagrangian $\mathcal{L}$. A simple example involving a scalar ϕ is a term $\mathcal{L} \supset \xi \partial_\mu \phi^* \partial^\mu \phi$, which implies $\mathcal{L} \supset -\frac{1}{2}\partial_\mu \xi(\phi^* \partial^\mu \phi + \text{H.c.})$ upon integration by parts. If ξ varies smoothly, $\partial_\mu \xi$ has a piece that behaves in a local inertial frame as a coefficient k_μ for Lorentz and *CPT* violation. More generally, nonscalar fields can play a role, and the effects can arise through subsidiary conditions involving coefficients like k_μ appearing in the equations of motion.

All possible Lorentz-violating Lagrangian terms are given by the Lorentz- and *CPT*-violating standard-model extension [6], and many have been bounded experimentally in precision experiments with hadrons [7,8], protons and neutrons [9], electrons [10,11], photons [12,13], and muons [14]. The theory contains all observer Lorentz scalars formed by combining operators and coefficients having Lorentz indices. Terms of this type arise, for example, from spontaneous Lorentz violation [15] and in realistic noncommutative field theories [16]. The presence of translation violations induced by spacetime-varying couplings complicates theoretical and experimental analyses. Here, we focus on showing that spacetime-varying couplings and apparent Lorentz violation can arise naturally, even when the dynamics of the underlying theory is Lorentz invariant and involves only constant couplings.

Our analysis is performed in the context of $N=4$ supergravity in four dimensions. This theory is a limit of the $N=1$ supergravity in 11 spacetime dimensions and hence also of M theory. It is sufficiently simple to permit analytical calculation involving the graviton, photon, dilaton, and axion fields, while retaining generic features of a more realistic fundamental theory. We show that smoothly varying couplings can naturally be obtained from a simple cosmological solution. In particular, in electrodynamics the fine structure constant $\alpha = e^2/4\pi$ and the θ angle acquire related spacetime dependences, driving the Lorentz violation.

The spectrum of the $N=4$ supergravity in four spacetime dimensions consists of the graviton, represented by the metric $g_{\mu\nu}$, four gravitinos, six Abelian graviphotons A_μ^{jk} four fermions, and a complex scalar Z that contains an axion and a dilaton. The latin indices $j,k,...$ denote vector indices in the SO(4) internal symmetry, and the graviphotons lie in the adjoint representation. The bosonic part $\mathcal{L}$ of the Lagrangian can be written [17]

$$\mathcal{L} = -\frac{1}{2}\sqrt{g}R - \frac{1}{4}\sqrt{g}M_{jklm}F_{\mu\nu}^{jk}F^{lm\mu\nu}$$

$$-\frac{1}{8}\sqrt{g}N_{jklm}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{jk}F_{\rho\sigma}^{lm} + \sqrt{g}\frac{\partial_\mu Z \partial^\mu \bar{Z}}{(1-Z\bar{Z})^2}, \quad (1)$$

where Planck units are adopted. The generalized electromagnetic coupling constant M_{jklm} and the θ-term coupling N_{jklm} are both real and determined by the complex scalar Z according to

$$M_{jklm} + iN_{jklm} = \frac{\delta_{[j|l|}\delta_{k]m}(1-Z^2) - i\varepsilon_{jklm}Z}{(1+Z^2)}. \quad (2)$$

For present purposes, it is convenient to apply the Cayley map $W = -i(Z-1)/(Z+1)$ taking the unit disk into the upper half plane. Writing $W = A + iB$, the scalar kinetic term becomes $\mathcal{L}_b = \sqrt{g}(\partial_\mu A \partial^\mu A + \partial_\mu B \partial^\mu B)/4B^2$, and M and N undergo corresponding transformations. Then, B can be identified with the string-theory dilaton.

We consider the case in which only one graviphoton, $F_{\mu\nu}^{12} \equiv F_{\mu\nu}$, is excited. The bosonic Lagrangian then becomes

$$\mathcal{L} = -\frac{1}{2}\sqrt{g}R - \frac{1}{4}\sqrt{g}MF_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\sqrt{g}NF_{\mu\nu}\widetilde{F}^{\mu\nu}$$
$$+ \sqrt{g}(\partial_\mu A\partial^\mu A + \partial_\mu B\partial^\mu B)/4B^2, \quad (3)$$

with $\widetilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}/2$ and

$$M = \frac{B(A^2+B^2+1)}{(1+A^2+B^2)^2-4A^2},$$

$$N = \frac{A(A^2+B^2-1)}{(1+A^2+B^2)^2-4A^2}. \quad (4)$$

Consider a cosmology in this theory involving a flat ($k=0$) Friedmann-Robertson-Walker (FRW) model. The line element for the associated spacetime is

$$ds^2 = dt^2 - a^2(t)(dx^2+dy^2+dz^2), \quad (5)$$

where t is the comoving time and $a(t)$ is the cosmological scale factor. The usual assumptions of homogeneity and isotropy imply that A and B are also functions only of t. Solving the Einstein equations with just the scalar field as a source of energy and momentum yields $a(t) \sim t^{1/3}$, which is an expansion rate far slower than seen in our Universe. A standard approach to obtain a more realistic theory adds an energy-momentum tensor $T_{\mu\nu} = \rho u_\mu u_\nu$ describing galaxies and other matter, where u^μ is a unit timelike vector orthogonal to spatial surfaces and $\rho(t)$ is the energy density of the matter. In our supergravity model, an energy momentum tensor of this form arises from the fermionic sector because the fermion kinetic terms are uncoupled from the scalar field W, and so $T_{\mu\nu}$ is independent of W.

Ignoring the graviphoton for the moment, the Einstein equations for the supergravity cosmology in the presence of the fermion matter are

$$G_{\mu\nu} = T_{\mu\nu} + \frac{1}{2B^2}(\partial_\mu A\partial_\nu A + \partial_\mu B\partial_\nu B)$$
$$- \frac{1}{4B^2}g_{\mu\nu}(\partial_\lambda A\partial^\lambda A + \partial_\lambda B\partial^\lambda B). \quad (6)$$

For the $k=0$ FRW model, this expression contains only two independent equations:

$$-3\frac{\ddot{a}}{a} = \frac{1}{2}\rho + \frac{1}{2B^2}(\dot{A}^2+\dot{B}^2), \quad \frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2} = \frac{1}{2}\rho, \quad (7)$$

where a dot indicates a time derivative. The system is also governed by the equations of motion for A and B:

$$\frac{d}{dt}\left(\frac{a^3\dot{A}}{B^2}\right) = 0, \quad \frac{d}{dt}\left(\frac{a^3\dot{B}}{B^2}\right) + \frac{a^3}{B^3}(\dot{A}^2+\dot{B}^2) = 0. \quad (8)$$

The final equation determining the time evolution, $d(\rho a^3)/dt = 0$, follows from conservation of energy.

It turns out these five equations can be integrated analytically. Suppose that at the present time t_n the Universe has matter density ρ_n and scale size $a_n = a(t_n)$. Energy conservation yields $\rho(t) = c_n/a^3(t)$, where $c_n = \rho_n a_n^3$. Integration of one Einstein equation then gives

$$a(t) = \left[\frac{3}{4}c_n(t+t_0)^2 - c_1\right]^{1/3}. \quad (9)$$

Here, c_1 is an integration constant describing the amount of energy in the scalar fields. Also, t_0 is another integration constant, chosen here as $t_0 = \sqrt{4c_1/3c_n}$ to fix the time origin $t=0$ at the moment of the initial singularity when $a(t)=0$. Note that for $t \gg t_0$ we find $a(t) \sim t^{2/3}$, as expected for a $k=0$ matter-dominated Universe.

The equation of motion for A can be integrated once to give $\dot{A} = c_2 B^2/a^3$, where c_2 is an integration constant. The remaining equations can be solved to yield a functional form for A and B in terms of a parameter time τ. This leaves two equations, related through the Bianchi identities. After some algebra, we find

$$A = \pm\lambda\tanh\left(\frac{1}{\tau}+c_3\right) + A_0, \quad B = \lambda\,\text{sech}\left(\frac{1}{\tau}+c_3\right), \quad (10)$$

where $\lambda \equiv \mp 4c_1/\sqrt{3}c_2 t_0$, and c_3, A_0 are integration constants. The cosmological time t is given in terms of the parametric time τ by $t = t_0[\coth(\sqrt{3}/4\tau)-1]$, so $t=0$ when $\tau=0$ and t increases when τ increases. In what follows, it suffices to adopt the simplifying choice $c_3=0$. At late times $t \gg t_0$, we then find $\tau \approx \sqrt{3}t/4t_0$, $A \approx \pm 4\lambda t_0/\sqrt{3}t + A_0$, and $B \approx \lambda(1-8t_0^2/3t^2)$. This means both A and B tend to constant values at late times on a time scale set by t_0. The value of the string-theory dilaton therefore tends to a constant in this supergravity cosmology, despite the absence of a dilaton potential.

We next consider excitations of $F_{\mu\nu}$ in the axion-dilaton background (10). For the moment, we restrict attention to localized excitations in spacetime regions that are small on a cosmological scale. This corresponds to most experimental situations, and it is therefore appropriate to work in a local inertial frame.

With a θ angle, the conventional electrodynamics Lagrangian in a local inertial frame can be written

$$\mathcal{L}_{\text{em}} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{\theta}{16\pi^2}F_{\mu\nu}\widetilde{F}^{\mu\nu}. \quad (11)$$

The graviphoton in the axion-dilaton background can be regarded as a model for the photon in cosmologically varying scalar fields, so we take $e^2 \equiv 1/M$, $\theta \equiv 4\pi^2 N$. Since M, N are functions of the background fields A, B, it follows that e, θ acquire spacetime dependence in an arbitrary local inertial frame.

The equations of motion in the presence of charged matter described by a 4-current j^ν are

$$\frac{1}{e^2}\partial_\mu F^{\mu\nu} - \frac{2}{e^3}(\partial_\mu e)F^{\mu\nu} + \frac{1}{4\pi^2}(\partial_\mu\theta)\widetilde{F}^{\mu\nu} = j^\nu. \quad (12)$$

In a trivial background, the last two terms on the left-hand side of this equation would vanish and the usual Maxwell equations would emerge. Here, however, the extra two terms lead to apparent Lorentz-violating effects despite being coordinate invariant. On small cosmological scales, $\partial_\mu M$ and $\partial_\mu N$ are approximately constant, and they therefore select a preferred direction in the local inertial frame. This means that particle Lorentz symmetry, as defined in the first paper of Ref. [6], is broken.

Note that the expansion in a textbook FRW cosmology without scalar couplings lacks this violation because a local Lorentz-symmetric inertial frame always exists, whereas in the present case the variation of M and N implies particle Lorentz violation in any local inertial frame. Indeed, the above cosmology-induced Lorentz violation is independent of the details of the $N=4$ supergravity model. Any similarly implemented smooth spacetime variation of the electromagnetic couplings on cosmological scales leads to such effects. This suggests particle Lorentz violation could be a common feature of models with spacetime-dependent couplings.

In the local inertial frame, we can write

$$\mathcal{L}'_{\rm em} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{8\pi^2}(\partial_\mu\theta)A_\nu\widetilde{F}^{\mu\nu}. \quad (13)$$

A nonzero constant contribution from $\partial_\mu\theta$ demonstrates explicitly the violations of particle Lorentz invariance and *CPT* symmetry. To facilitate contact with the conventional notation in the Lorentz-violating standard-model extension, we can identify $(k_{AF})_\mu \equiv e^2\partial_\mu\theta/8\pi^2$. In our supergravity model, $(k_{AF})_\mu$ is timelike.

The special case of constant e and constant $(k_{AF})_\mu$ has been discussed extensively in the literature [12,6,18]. Under these conditions, the Lagrangian (13) is invariant under spacetime translations, but the associated conserved energy fails to be positive definite and so leads to instabilities. It is natural to ask how this difficulty is circumvented in the present model, which arises from a positive-definite supergravity theory [19].

A key difference is that, instead of being nondynamical and constant, $(k_{AF})_\mu$ depends in the present model on the dynamical degrees of freedom A, B. Excitations with $F_{\mu\nu} \neq 0$ therefore cause perturbations δA, δB away from the cosmological solutions (10), so that $A \to A+\delta A$ and $B \to B+\delta B$. It follows that $\theta \to \theta+\delta\theta$ and that the energy-momentum tensor $(T^{\rm b})^{\mu\nu}$ of the background receives an additional contribution, $(T^{\rm b})^{\mu\nu} \to (T^{\rm b}_F)^{\mu\nu} = (T^{\rm b})^{\mu\nu} + \delta(T^{\rm b})^{\mu\nu}$. This contribution can compensate for negative-energy ones from the $(k_{AF})_\mu$ term.

The compensation mechanism can be illustrated explicitly at the classical level in the Lagrangian $\mathcal{L} = \mathcal{L}'_{\rm em} + \mathcal{L}_{\rm b}$ [20]. The relevant feature for present purposes is the A and B dependence of θ, so for simplicity e can be taken as constant. We begin by splitting the total conserved energy-momentum tensor into two pieces, $(T^t_F)^{\mu\nu} = (T^{\rm em})^{\mu\nu} + (T^{\rm b}_F)^{\mu\nu}$, where

$$(T^{\rm em})^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu A^\lambda)}\partial^\nu A^\lambda - \eta^{\mu\nu}\mathcal{L}'_{\rm em},$$

$$(T^{\rm b}_F)^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu A)}\partial^\nu A + \frac{\partial\mathcal{L}}{\partial(\partial_\mu B)}\partial^\nu B - \eta^{\mu\nu}\mathcal{L}_{\rm b}. \quad (14)$$

Explicitly, we find

$$(T^{\rm em})^{\mu\nu} = \frac{1}{e^2}F^\mu_\lambda F^{\lambda\nu} + \frac{1}{4e^2}\eta^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} + \frac{1}{8\pi^2}(\partial^\nu\theta)A_\lambda\widetilde{F}^{\lambda\mu}. \quad (15)$$

Negative-energy contributions can arise only from the last term. Similarly, we obtain

$$(T^{\rm b}_F)^{\mu\nu} = \frac{\partial^\mu A\partial^\nu A}{2B^2} - \frac{\eta^{\mu\nu}}{4B^2}(\partial_\lambda A\partial^\lambda A + \partial_\lambda B\partial^\lambda B)$$
$$+ \frac{\partial^\nu B\partial^\nu B}{2B^2} - \frac{1}{8\pi^2}(\partial^\nu\theta)A_\lambda\widetilde{F}^{\lambda\mu}, \quad (16)$$

where again only the last term can lead to negative-energy contributions. Combining the two equations shows that the total conserved energy is positive definite, even when a nonzero $(k_{AF})_\mu$ is generated. The apparent paradox arises only because the two pieces $(T^{\rm em}_F)^{\mu\nu}$ and $(T^{\rm b}_F)^{\mu\nu}$, each with positivity difficulties, are separately conserved when $\partial^\nu\theta$ is constant [21].

Another interesting issue concerns the limits from existing experiments on the induced Lorentz-violating and time-varying couplings. Consider again the theory (13) in the supergravity background (10) with the choice $c_3 = 0$. The phenomenological constraint $e^2(t\to\infty) \simeq 4\pi/137$ implies $|A_0| \simeq 1$ and $\lambda \lesssim 2\pi/137$. Within this parameter space, choose $\lambda = 2\pi/137$ and $A_0 = \sqrt{1-\lambda^2}$, which further simplifies the analysis because it leads to a vanishing θ at late times, $\theta(t\to\infty) = 0$. In fact, the estimates below remain valid or improve for other choices in more than 98% of the allowed parameter space.

The comoving time t and the time coordinate in comoving local inertial frames agree to first order. Assuming late times $t \gg t_0$, we find $e^2 \sim 2\lambda \mp 8\lambda^2 t_0/\sqrt{3}t$ and hence $\dot\alpha/\alpha \sim \pm 4\lambda t_0/\sqrt{3}t^2$. Current observational bounds on $\dot\alpha/\alpha$ at late times, i.e., at relatively small redshifts, are obtained from the Oklo fossil reactor as $|\dot\alpha/\alpha| \lesssim 10^{-16}$ yr^{-1} [22]. Taking $t_n \simeq 10^{10}$ yr for the present age of the Universe then yields the estimate $t_0 \lesssim 10^6$ yr, consistent with the late-times assumption.

The coefficient $(k_{AF})_\mu$ for Lorentz and *CPT* violation is also constrained by the Oklo data, and indeed constraints on axion-photon couplings of the form (13) have previously been studied in the context of axion and quintessence models [23] and *CPT* baryogenesis [24]. In the present supergravity cosmology, we have $\dot N \sim \mp 2t_0/\sqrt{3}\lambda t^2$ at late times, giving $|(k_{AF})_0| \lesssim 10^{-46}$ GeV. Although model dependent, this estimate compares favorably with the direct observational limit $(k_{AF})_0 \lesssim 10^{-42}$ GeV in Ref. [12]. Inverting the reasoning, the latter can be used to bound the variation of α. We find $|\dot\alpha/\alpha| \lesssim 10^{-12}$ yr^{-1}, consistent with the Oklo data [22].
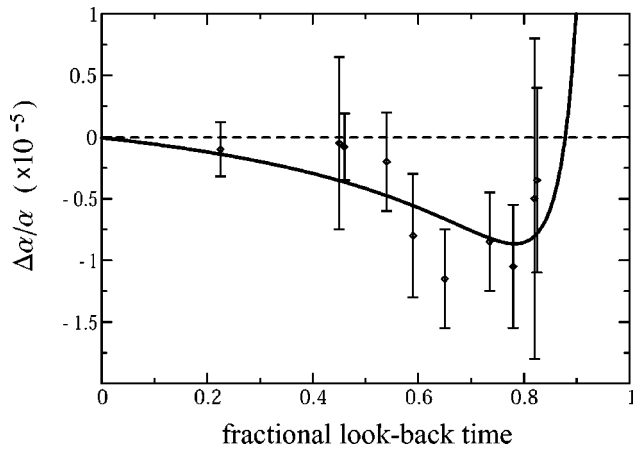
FIG. 1. Sample relative variation of the fine-structure constant with fractional look-back time $1-t/t_n$.

In the supergravity cosmology, the dependence of α on time can be relatively complicated. As an example, the solid line in Fig. 1 displays the relative variation of α for the case $t_n/t_0 = 2000$, as a function of the fractional look-back time

$1-t/t_n$ to the big bang. The parameters λ, A_0 have been changed fractionally by parts in 10^4 relative to the choices $2\pi/137$, $\sqrt{1-\lambda^2}$. This provides an approximate match to the recently reported data for $\dot\alpha$, also plotted in Fig. 1, obtained from measurements of high-redshift spectra over periods of approximately $0.6t_n$ to $0.8t_n$ assuming $H_0 = 65$ km/s/Mpc, $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$ [3]. The parameter choices lie within the constraints on $(k_{AF})^0$, but have no overlap with the Oklo data set and yield a nonasymptotic present-day value of the fine-structure constant. The solid line reflects both nonlinear features and a sign change for $\dot\alpha$.

In summary, we have established that local Lorentz and *CPT* violation can be associated with spacetime-varying couplings. The effect is generic in theories with derivative couplings to cosmological fields. Despite the simplicity of the underlying mechanism, the resulting time variation can be complicated and offers an interesting avenue for phenomenological exploration.

[1] P. A. M. Dirac, Nature (London) **139**, 323 (1937).

[2] See, e.g., E. Cremmer and J. Scherk, Nucl. Phys. **B118**, 61 (1977); A. Chodos and S. Detweiler, Phys. Rev. D **21**, 2167 (1980); W. J. Marciano, Phys. Rev. Lett. **52**, 489 (1984); T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994).

[3] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).

[4] For a review, see, e.g., J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).

[5] For overviews of *CPT* and Lorentz violation see, e.g., *CPT and Lorentz Symmetry II*, edited by V. A. Kostelecký (World Scientific, Singapore, 2002).

[6] V. A. Kostelecký and R. Potting, Phys. Rev. D **51**, 3923 (1995); D. Colladay and V. A. Kostelecký, *ibid.* **55**, 6760 (1997); **58**, 116002 (1998); V. A. Kostelecký and R. Lehnert, *ibid.* **63**, 065008 (2001).

[7] KTeV Collaboration, H. Nguyen, in Ref. [5]; OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C **76**, 401 (1997); DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (1997); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001); FOCUS Collaboration, J. M. Link *et al.*, Phys. Lett. B **556**, 7 (2003).

[8] D. Colladay and V. A. Kostelecký, Phys. Lett. B **344**, 259 (1995); Phys. Rev. D **52**, 6224 (1995); Phys. Lett. B **511**, 209 (2001); V. A. Kostelecký and R. Van Kooten, Phys. Rev. D **54**, 5585 (1996); V. A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998); Phys. Rev. D **61**, 016002 (2000); **64**, 076001 (2001); N. Isgur *et al.*, Phys. Lett. B **515**, 333 (2001).

[9] L. R. Hunter *et al.*, in *CPT and Lorentz Symmetry*, edited by V. A. Kostelecký (World Scientific, Singapore, 1999); D. Bear *et al.*, Phys. Rev. Lett. **85**, 5038 (2000); D. F. Phillips *et al.*, Phys. Rev. D **63**, 111101(R) (2001); M. A. Humphrey *et al.*, Phys. Rev. A **62**, 063405 (2000); V. A. Kostelecký and C. D. Lane, Phys. Rev. D **60**, 116010 (1999); J. Math. Phys. **40**, 6245

(1999); R. Bluhm *et al.*, Phys. Rev. Lett. **88**, 090801 (2002).

[10] H. Dehmelt *et al.*, Phys. Rev. Lett. **83**, 4694 (1999); R. Mittleman *et al.*, *ibid.* **83**, 2116 (1999); G. Gabrielse *et al.*, *ibid.* **82**, 3198 (1999); R. Bluhm *et al.*, *ibid.* **82**, 2254 (1999); **79**, 1432 (1997); Phys. Rev. D **57**, 3932 (1998).

[11] B. Heckel, in Ref. [5]; R. Bluhm and V. A. Kostelecký, Phys. Rev. Lett. **84**, 1381 (2000).

[12] S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[13] V. A. Kostelecký and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001); Phys. Rev. D **66**, 056005 (2002); V. A. Kostelecký and A. G. M. Pickering, Phys. Rev. Lett. **91**, 031801 (2003); J. Lipa *et al.*, Phys. Rev. Lett. **90**, 060403 (2003).

[14] V. W. Hughes *et al.*, Phys. Rev. Lett. **87**, 111804 (2001); R. Bluhm *et al.*, *ibid.* **84**, 1098 (2000).

[15] V. A. Kostelecký and S. Samuel, Phys. Rev. D **39**, 683 (1989); **40**, 1886 (1989); Phys. Rev. Lett. **63**, 224 (1989); **66**, 1811 (1991); V. A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); Phys. Rev. D **63**, 046007 (2001); V. A. Kostelecký, M. Perry, and R. Potting, Phys. Rev. Lett. **84**, 4541 (2000); M. S. Berger and V. A. Kostelecký, Phys. Rev. D **65**, 091701(R) (2002).

[16] S. M. Carroll *et al.*, Phys. Rev. Lett. **87**, 141601 (2001); Z. Guralnik *et al.*, Phys. Lett. B **517**, 450 (2001); A. Anisimov *et al.*, Phys. Rev. D **65**, 085032 (2002); C. E. Carlson *et al.*, Phys. Lett. B **518**, 201 (2001).

[17] E. Cremmer and B. Julia, Nucl. Phys. **B159**, 141 (1979).

[18] R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999); F. R. Klinkhamer, Nucl. Phys. **B578**, 277 (2000); C. Adam and F. R. Klinkhamer, *ibid.* **B607**, 247 (2001), and references therein.

[19] The conserved symmetric energy-momentum tensor for the Lagrangian (3) acquires no contribution from the N term be-

cause the latter is independent of the metric. The other terms are positive definite.

[20] At the quantum level, radiative corrections mix these terms. See V. A. Kostelecký, C. D. Lane, and A. G. M. Pickering, Phys. Rev. D **65**, 056006 (2002).

[21] A constant timelike $(k_{AF})_\mu$ violates microcausality [6,18]. The supergravity cosmology may avoid this, but a complete analysis of this is outside our present scope.

[22] T. Damour and F. Dyson, Nucl. Phys. **B480**, 37 (1996); Y. Fujii *et al.*, *ibid.* **B573**, 377 (2000); K. Olive *et al.*, Phys. Rev. D **66**, 045022 (2002).

[23] See, e.g., D. Harari and P. Sikivie, Phys. Lett. B **289**, 67 (1992); S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).

[24] M. Li and X. Zhang, hep-ph/0209093; see also O. Betolami *et al.*, Phys. Lett. B **395**, 178 (1997).