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Reheating stage after inflation

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We point out that inflaton decay products acquire plasma masses during the reheating phase following inflation. The plasma masses may render inflaton decay kinematically forbidden, causing the temperature to remain frozen for a period at a plateau value. We show that the final reheating temperature may be uniquely determined by the inflaton mass, and may not depend on its coupling. Our findings have important implications for the thermal production of dangerous relics during reheating (e.g., gravitinos), for extracting bounds on particle physics models of inflation from cosmic microwave background anisotropy data, for the production of massive dark matter candidates during reheating, and for models of baryogenesis or leptogenesis where massive particles are produced during reheating.

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I. INTRODUCTION

At the end of inflation [1] the energy density of the Universe is locked up in a combination of kinetic energy and potential energy of the inflaton field ϕ, with the bulk of the inflaton energy density in the zero-momentum mode of the field. Thus, the Universe at the end of inflation is in a cold, low-entropy state with few degrees of freedom, very much unlike the present hot, high-entropy Universe. After inflation the frozen inflaton-dominated Universe must somehow be defrosted and become a high-entropy, radiation-dominated Universe.

One path to defrosting the Universe after inflation is known as ''reheating'' [2]. The simplest way to envision the reheating process is if the comoving energy density in the zero mode of the inflaton decays into normal particles in a perturbative way. The decay products then scatter and thermalize to form a thermal background.[1]

Of particular interest is a quantity known as the reheat temperature, denoted as T_{RH}. The reheat temperature is properly thought of as the maximum temperature of the radiation-dominated Universe. It is not necessarily the maximum temperature obtained by the Universe after inflation [2,4–6].

[1]We do not consider here the possible role of nonlinear dynamics leading to explosive particle production known as ''preheating'' [3]. In this paper we are concerned with the case in which the inflaton field decays through perturbative processes. This happens anytime the resonant parameter q [3], which depends upon the coupling constant of the inflaton field to light fields and on the initial conditions of the inflaton field after inflation, is smaller than unity. Notice, however, that the final reheating temperature after a preheating stage might well be determined by a perturbative decay of the residual inflaton oscillations. If so, our study applies in this case also.

The reheat temperature is defined by assuming an instantaneous conversion of the energy density in the inflaton field into radiation when the decay width of the inflaton energy, Γ_ϕ, is equal to H, the expansion rate of the Universe. The reheat temperature is calculated quite easily [2]. After inflation the inflaton field executes coherent oscillations about the minimum of the potential. Averaged over several oscillations, the coherent oscillation energy density redshifts as matter: $\rho_\phi \propto a^{-3}$, where a is the Robertson-Walker scale factor. If we denote as ρ_I and a_I the total inflaton energy density and the scale factor at the onset of coherent oscillations immediately after the end of inflation, then the Hubble expansion rate as a function of a is (M_{Pl} is the Planck mass)

$$H(a)=\sqrt{\frac{8\pi}{3}\frac{\rho_I}{M_{Pl}^2}\left(\frac{a_I}{a}\right)^3}. \tag{1}$$

Equating $H(a)$ and Γ_ϕ leads to an expression for a_I/a. Now if we assume that all available coherent energy density is instantaneously converted into radiation at this value of a_I/a, we can define the reheat temperature by setting the coherent energy density, $\rho_\phi=\rho_I(a_I/a)^3$, equal to the radiation energy density, $\rho_R=(\pi^2/30)g_* T_{RH}^4$, where g_* is the effective number of relativistic degrees of freedom at temperature T_{RH}. The result is

$$\begin{aligned} T_{RH}&=\left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\alpha_\phi^{1/2}\sqrt{M_\phi M_{Pl}} \\ &=0.2\left(\frac{100}{g_*}\right)^{1/4}\alpha_\phi^{1/2}\sqrt{M_\phi M_{Pl}}, \end{aligned} \tag{2}$$

where we have expressed the inflaton decay width as $\Gamma_\phi=\alpha_\phi M_\phi$.

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There are various reasons to suspect that the reheating temperature is small. For instance, in local supersymmetric theories [7] gravitinos (and other dangerous relics such as moduli fields) are produced during reheating. Unless reheating is delayed, gravitinos will be overproduced, leading to a large undesired entropy production when they decay after big-bang nucleosynthesis [8]. The limit from gravitino overproduction is $T_{RH} \lesssim 10^9 - 10^{10}$ GeV, or even stronger [9].

Again, we emphasize that the reheat temperature is best regarded as the temperature below which the Universe expands as a radiation-dominated Universe, with the scale factor decreasing as $g_*^{-1/3} T^{-1}$. In this regard it has a limited meaning [2,4]. As the scalar field decays into light states, the decay products rapidly thermalize forming a plasma with temperature T. The latter grows until it reaches a maximum value T_{max} and then decreases as $T \propto a^{-3/8}$ down to the temperature T_{RH}, which *should not* be used as the maximum temperature obtained by the Universe during reheating. The maximum temperature is, in fact, much larger than T_{RH} and it is incorrect to assume that the maximum abundance of a massive particle species produced after inflation is suppressed by a factor of $\exp(-M/T_{RH})$. This has important implications for the idea of superheavy dark matter [5], supersymmetric dark matter [6,10] and baryogenesis [11].

The goal of this paper is to present a simple, but relevant observation that changes the usual picture of the temperature evolution during reheating. During the process of reheating the inflaton decay products scatter and thermalize to form a thermal background. A thermalized particle species produced during the first stages of reheating acquires a plasma mass $m_p(T)$ of the order of gT, where g is the typical (gauge) coupling governing the particle interactions [12]. This happens because forward scatterings of fermions do not change the distribution functions of particles, but modify their free-dispersion relations, producing a plasma mass. The dispersion relation can be well approximated for both scalars and fermions by $\omega^2 = k^2 + m_p^2(T)$, where ω and k are the energy and the three-momentum of the particle in the thermal background, respectively. The presence of thermal masses implies that the inflaton zero-mode cannot decay into light states if its mass M_ϕ is smaller than about gT. The decay process is simply kinematically forbidden.[2]

Our observation is that during the reheating stage, the inflaton starts decaying and the temperature of the plasma rises. If the maximum temperature obtained by the Universe during reheating, T_{max}, is larger than about $g^{-1} M_\phi$, the inflaton decay channel into light states become inaccessible and the decay process stops as soon as the temperature has reached a value of the order of $g^{-1} M_\phi$. Subsequently, expansion cools the plasma, lowering the temperature and the corresponding plasma masses of the light states. The inflaton is then free to decay. However, as soon as this happens, the temperature of the plasma rises and the inflaton decay process becomes kinematically forbidden again. As a result, one expects a prolonged period during which the temperature of the plasma is frozen to a plateau value of the order of $g^{-1} M_\phi$.

Our observation has various implications. First of all, let us notice that we do not know the mass of the inflaton field around the minimum of its potential during the reheating stage. Indeed, from the recent Wilkinson Microwave Anisotropy Probe (WMAP) cosmic microwave background (CMB) anisotropy data [14] we only have limited information about that portion of the inflaton potential experienced by the inflaton field during inflation; we know that it has to be quite flat in order to allow a sufficiently long period of exponential growth of the scale factor [1,15–17]. However, we know nothing about the inflaton mass during reheating, since this depends upon a portion of the inflaton potential which is not accessible to any observations. This amounts to saying that M_ϕ should be regarded as a free parameter. Even more, in many inflationary scenarios, e.g., hybrid models [1], the reheating dynamics may be determined by a scalar field χ different from the inflaton field. (In the following, the terminology "mass of the inflaton" will be therefore used in a loose way.)

Suppose that the reheating temperature T_{RH} defined in Eq. (2), is larger than $g^{-1} M_\phi$. This means that when the inflaton decay lifetime is of the order of the age of the Universe, the inflaton field would like to decay, but is not allowed to because the plasma masses of the light decay products are too large. Only when the energy density stored in the inflaton field becomes smaller than about $\rho_\phi \sim (g^{-1} M_\phi)^4$ will the particles in the plasma have a mass smaller than M_ϕ and inflaton can promptly decay. Under these circumstances the reheating temperature of the Universe should be

$$T_{RH} \simeq \frac{M_\phi}{g}, \tag{3}$$

which is directly related to the inflaton mass and independent of the inflaton decay rate.

Before concluding the Introduction, we note that our effect is applicable in situations other than reheating after inflation. It would apply, for instance, if the Universe is ever dominated by a decaying nonrelativistic particle.

The rest of the paper is organized as follows. In Sec. II we analyze in detail the behavior of the temperature during the reheating stage and in particular we characterize the plateau stage both analytically and numerically. Section III is devoted to the study of some applications of our findings. We focus on the production of gravitinos during reheating, on the evaluation of the number of e-folds after inflation which has recently acquired particular relevance in order to restrict models of inflation from the CMB anisotropy data, and on the production of massive particles. Finally, in Sec. IV we present our conclusions.

II. REHEATING WITH THERMAL MASSES

We now discuss the reheating process, assuming that the decay products of the inflaton field rapidly thermalize and acquire "plasma" masses $m_p(T)$ of the order of gT, where g

[2]This observation was made first in the context of the Affleck-Dine baryogenesis scenario [13].

is the coupling constant for a particle in the plasma.[3]

There are two assumptions that deserve elaboration. The first aspect is the assumption of "rapid" thermalization. The time scale for thermalization of the inflaton decay products is $(n\sigma)^{-1}$ where σ is a cross section for the scattering of the decay products and n is the number density of scatterers. The thermalization is rapid if this time scale is short compared to the time scale for energy extraction from the inflaton, assumed to be equal to the lifetime of the inflaton, Γ_ϕ^{-1}. As studied in Refs. [18,19], thermalization is dominated by $2\to3$ scatterings, which cause thermalization within a Hubble time if the typical energy of the decay products is smaller than $\alpha^3 M_{Pl}\sim10^{15}$ GeV, with $\alpha=g^2/4\pi$. Since the initial energy of the decay products is of the order of the inflaton mass, we conclude that thermalization will rapidly occur for inflaton masses below 10^{15} GeV.

The second important aspect of the assumption is that the inflaton decay products have a thermal mass of the order of gT, where $g\sim0.5$ is a typical gauge coupling constant. One might imagine that the inflaton decays into some weakly interacting particles which then subsequently decay into "thermal" particles with gauge interactions. But in any case, eventually the decay sequence must include particles with gauge interactions for which there will be a thermal mass.

To model the effect of plasma masses, let us consider, for the moment, a model Universe with two components: inflaton field energy ρ_ϕ and radiation energy density ρ_R which contains all the light degrees of freedom produced after decay. For simplicity, we can think that all the produced particles in the radiation component have couplings of the same strength.[4] Also, we consider the simplest type of decay, that is, the decay of the inflaton into scalars. In the case of decay into scalars, the only effect of the masses is to modify the phase space of the products, while the case of fermions is slightly different, since the scattering amplitude also depends on the masses.

The presence of thermal masses implies that the decay width of the inflaton is no longer the zero-temperature result $\Gamma_\phi=\alpha_\phi M_\phi$, but becomes

$$\Gamma_\phi(T)=\alpha_\phi M_\phi\sqrt{1-4\frac{m_p^2(T)}{M_\phi^2}}=\alpha_\phi M_\phi\sqrt{1-4\frac{g^2T^2}{M_\phi^2}}. \tag{4}$$

The consequence of this simple fact is that the dynamics of reheating drastically changes when the temperature of the plasma is such that $m_p(T)$ becomes as large as M_ϕ. When $m_p(T)\ll M_\phi$, the effect is negligible, while the decays stop when $m_p(T)\approx M_\phi$ since the phase space factor goes to zero as $T=M_\phi/2g\approx M_\phi$.

[3]We expect $g\lesssim1$, and will assume $g=1/2$ for numerical estimates.

[4]Since the energies are well above the weak scale, even neutrinos will interact with gauge-coupling strength.

With the above assumptions, the Boltzmann equations describing the redshift and interchange in the energy density among the different components are

$$\dot\rho_\phi+3H\rho_\phi+\Gamma_\phi(T)\rho_\phi=0$$

$$\dot\rho_R+4H\rho_R-\Gamma_\phi(T)\rho_\phi=0, \tag{5}$$

where dot denotes time derivative.

It is clear that the system behaves in such a way that T never becomes larger than $M_\phi/2g$, otherwise the factor $\Gamma_\phi(T)$ would become imaginary. In other words, when T reaches this value we have a phase with approximately constant T during which the decays are suppressed for kinematic reasons. During this phase ρ_R stays constant, while ρ_ϕ decreases like a^{-3}. We recall that without plasma masses, the behavior of T is very different: immediately after inflation ends it grows rapidly to T_{max}, and then decreases like $a^{-3/8}$ until it reaches T_{RH}. At this point the ϕ field decays completely and the Universe becomes radiation dominated.

Taking into account the effect of plasma masses, we may have three possibilities:

$$\text{Case I}\quad T_{max}<M_\phi$$

$$\text{Case II}\quad T_{RH}<M_\phi<T_{max}$$

$$\text{Case III}\quad M_\phi<T_{RH}.$$

In case I, the effect of the plasma mass is negligible. This is the case, for instance, in which the decay rate of the inflaton is suppressed by powers of the Planck scale. In case II, after a very short time T grows to M_ϕ, then stays approximately constant for a while, then decreases as $a^{-3/8}$ until reheating and the radiation dominated phase begins. In case III, again after a very short time, T grows to M_ϕ and after a long phase of constant T, the Universe directly enters the radiation-dominated phase after the time of reheating, which would be determined by ignoring plasma effects.

We want now to discriminate, in terms of the fundamental parameters of the inflaton field, the applicable case (I, II, or III), and the duration of the constant-T phase. First, recall that the maximum temperature obtained after inflation is given by [6]

$$T_{max}=\left(\frac{3}{8}\right)^{2/5}\left(\frac{15}{2\pi^3}\right)^{1/4}\alpha_\phi^{1/4}\left(\frac{M_{Pl}^2H_I}{g_*M_\phi^3}\right)^{1/4}M_\phi$$

$$=0.6\alpha_\phi^{1/4}\left(\frac{M_{Pl}V^{1/2}}{g_*M_\phi^3}\right)^{1/4}M_\phi, \tag{6}$$

where V is the value of the inflaton potential at the end of inflation. The reheating temperature was defined in Eq. (2). We may now determine the conditions that determine the operative case in terms of the value of the decay constant α_ϕ:

$$\text{Case I} \qquad \alpha_\phi \lesssim g_* \frac{M_\phi^3}{g^4 M_{Pl} V^{1/2}}$$

$$\text{Case II} \quad g_* \frac{M_\phi^3}{g^4 M_{Pl} V^{1/2}} \lesssim \alpha_\phi \lesssim g_*^{1/2} \frac{M_\phi}{g^2 M_{Pl}}$$

$$\text{Case III} \quad g_*^{1/2} \frac{M_\phi}{g^2 M_{Pl}} \lesssim \alpha_\phi .$$

If we put $V^{1/4} \approx 10^{13}$ GeV, $M_\phi \approx 10^8$ GeV and $g_* \approx 10^2$, we obtain

$$\text{Case I} \quad \alpha_\phi \lesssim 10^{-18}$$

$$\text{Case II} \quad 10^{-18} \lesssim \alpha_\phi \lesssim 3 \times 10^{-10}$$

$$\text{Case III} \quad 3 \times 10^{-10} \lesssim \alpha_\phi .$$

Next, we may estimate the duration of the constant-T phase in cases II and III. We will denote by a_I the value of the scale factor at the beginning of the reheating phase and by a_F its value at the end of the constant-T phase.

In case II, a_F may be estimated by assuming the usual scaling of the temperature ignoring plasma mass effects during reheating, $T \propto a^{-3/8}$, and finding the value of a when T drops below the value $M_\phi/2g$. The behavior of T is

$$\frac{T}{M_\phi} \simeq \left(\frac{54}{\pi^5}\right)^{1/8} \alpha_\phi^{1/4} \left(\frac{M_{Pl} V^{1/2}}{g_* M_\phi^3}\right)^{1/4} \left(\frac{a}{a_I}\right)^{-3/8}. \tag{7}$$

Imposing the condition $T/M_\phi = 1/2g$ to define a_F we find

$$\frac{a_F}{a_I} = (2g)^{8/3} \left(\frac{54}{\pi^5}\right)^{1/3} \alpha_\phi^{2/3} \frac{M_{Pl}^{2/3} V^{1/3}}{M_\phi^2}. \tag{8}$$

In terms of number of e-folds, imposing $V^{1/4} \approx 10^{14}$ GeV, $M_\phi \approx 10^9$ GeV) we obtain $N \approx 30 + 2/3 \ln(\alpha_\phi)$.

In case III, the situation is much different from the case ignoring plasma effects. In the usual case (without plasma masses) the system would enter the radiation-dominated era at the time of ϕ decay ($\Gamma_\phi = H$):

$$\frac{a_{RH}}{a_I} = \left(\frac{8\pi}{3}\right)^{1/3} \frac{V^{1/3}}{M_\phi^{2/3} M_{Pl}^{2/3} \alpha_\phi^{2/3}}. \tag{9}$$

In our case, though, decays are not possible so long as T is larger than $M_\phi/2g$. So, the ϕ energy density continues evolving approximately like a^{-3} until ρ_ϕ becomes smaller than ρ_R, at which time ϕ can decay without enhancing the temperature (and so closing the phase space for the decay). So the condition is simply for case III is

$$V \left(\frac{a_I}{a_F}\right)^3 \lesssim \frac{\pi^2}{30} g_* \left(\frac{M_\phi}{2g}\right)^4, \tag{10}$$

which implies

$$\frac{a_F}{a_I} \approx 4 g^{4/3} \left(\frac{V}{g_* M_\phi^4}\right)^{1/3} \approx \left(\frac{V}{M_\phi^4}\right)^{1/3}. \tag{11}$$

In terms of number of e-folds, imposing again realistic values for this case, $V^{1/4} \approx 8 \times 10^{11}$ GeV, $M_\phi \approx 2 \times 10^7$ GeV, we obtain $N \approx 14$. The two cases reduce to the same value in the intermediate case (i.e., the case in which $T_{RH} \simeq M_\phi$).

Now we want to analyze in detail what happens to the system in cases II and III by numerically solving the Boltzmann equations. In order to do this it is more convenient to express the Boltzmann equations in terms of dimensionless quantities that can absorb the effect of expansion of the Universe. This may be accomplished with the definitions

$$\Phi \equiv \rho_\phi M_\phi^{-1} a^3; \quad R \equiv \rho_R a^4. \tag{12}$$

It is also convenient to use the scale factor, rather than time, as the independent variable, so we define a variable $x = a M_\phi$. With this choice the system of equations can be written as (prime denotes d/dx)

$$\Phi' = -\sqrt{\frac{3}{8\pi}} \frac{M_{Pl}}{M_\phi} \alpha_\phi \sqrt{1 - 4\frac{g^2 T^2(x)}{M_\phi^2}} \frac{x}{\sqrt{\Phi x + R}} \Phi$$

$$R' = \sqrt{\frac{3}{8\pi}} \frac{M_{Pl}}{M_\phi} \alpha_\phi \sqrt{1 - 4\frac{g^2 T^2(x)}{M_\phi^2}} \frac{x^2}{\sqrt{\Phi x + R}} \Phi, \tag{13}$$

where the temperature $T(x)$ depends upon R and g_*, the effective number of degrees of freedom in the radiation:

$$\frac{T(x)}{M_\phi} = \left(\frac{30}{g_* \pi^2}\right)^{1/4} \frac{R^{1/4}}{x}. \tag{14}$$

It is straightforward to solve the system of equations in Eq. (13) with initial conditions at $x = x_I$ of $R(x_I) = X(x_I) = 0$ and $\Phi(x_I) = \Phi_I$. It is convenient to express $\rho_\phi(x = x_I)$ in terms of the expansion rate at x_I, which leads to

$$\Phi_I = \frac{3}{8\pi} \frac{M_{Pl}^2}{M_\phi^2} \frac{H_I^2}{M_\phi^2} x_I^3 . \tag{15}$$

The numerical value of x_I is irrelevant.

We show in Figs. 1 and 2 the solution of the system respectively in cases II and III. They follow the qualitative behavior we described, with the prominent constant-T phase.

III. APPLICATIONS

A. Thermal production of gravitinos

The first question we want to address is the production of gravitinos during reheating, taking into account the effect of thermal masses.[5] It is known that the overproduction of grav-

[5]Here we consider gravitinos, but the results are easily generalized to other dangerous light relics.

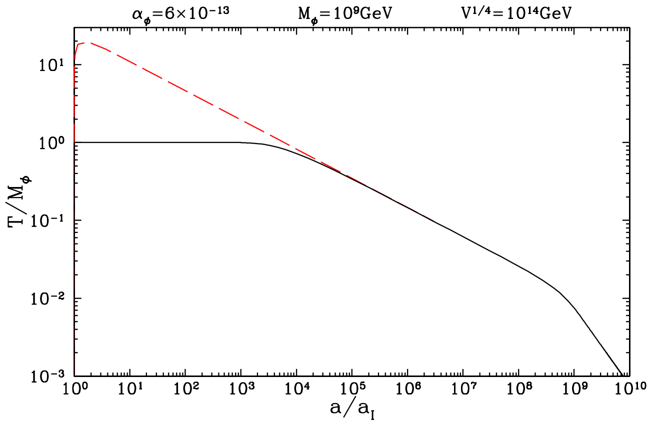

FIG. 1. The behavior of the temperature during reheating, without (dashed line) and with (solid line) plasma mass effects, for case II: $T_{RH}<M_\phi$.

itinos represents a major obstacle in constructing cosmological models based on supergravity [7]. Gravitinos decay very late and, if they are copiously produced during the evolution of the early Universe, their energetic decay products destroy ^{4}He and D by photodissociation, thus jeopardizing the successful nucleosynthesis predictions [8,9]. As a consequence, the ratio of the number density of gravitinos $n_{3/2}$ to the entropy density s should be smaller than about

$$\frac{n_{3/2}}{s}\lesssim 10^{-12} \tag{16}$$

for gravitinos with mass of the order of 100 GeV.

Gravitinos could be produced in the early Universe because of thermal scatterings in the plasma during the stage of reheating after inflation. Usually, to avoid the overproduction of gravitinos, one has to require that the reheating temperature T_{RH} after inflation is not larger than about 10^8–10^9 GeV [8]. In our case, the relevant parameter is no longer T_{RH}, since the temperature is cutoff by the effect of thermal masses. We present here an analysis of the thermal generation of gravitinos during reheating with a phase of constant temperature.

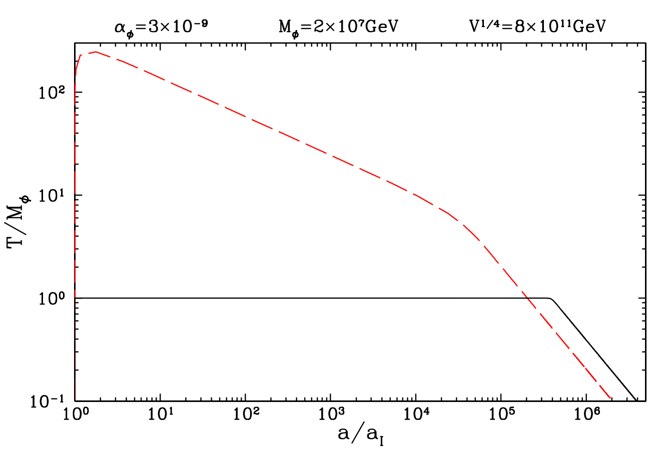

FIG. 2. The behavior of the temperature during reheating, without (dashed line) and with (solid line) plasma mass effects, for case III: $T_{RH}>M_\phi$.

Recall the salient aspects of the calculation of the gravitino abundance without thermal masses. The gravitino abundance is determined by the Boltzmann equation

$$\frac{dn_{3/2}}{dt}+3Hn_{3/2}=-\langle\sigma_A v\rangle[n_{3/2}^2-(n_{3/2}^2)_{eq}], \tag{17}$$

where $\langle\sigma_A v\rangle\propto 1/M_{Pl}^2$ is the thermal average of the gravitino annihilation cross section times the Møller velocity. Assuming the actual gravitino density is much less than its equilibrium value $(n_{3/2})_{eq}=3g_{3/2}\zeta(3)T^3/4\pi^2$ ($g_{3/2}$ is the number of degrees of freedom of the gravitino), the evolution of the comoving gravitino number density ($N=a^3n_{3/2}$) is quite simple:

$$\frac{dN_{3/2}}{da}=\frac{ca^2T^6}{HM_{Pl}^2}, \tag{18}$$

where $c=(3g_{3/2}\zeta(3)/4\pi^2)^2$.

In the radiation-dominated phase $H\propto a^{-2}$ and $T\propto a^{-1}$, so that the dominant contribution to $N_{3/2}$ comes from small a, corresponding to large T. During reheating $H\propto a^{-3/2}$. If plasma effects are not important $T\propto a^{-3/8}$ during reheating, while if plasma effects are important $T\propto$const during reheating. In either case, the dominant contribution to $N_{3/2}$ comes from large a, corresponding to the end of reheating. Therefore we can calculate $N_{3/2}$ at the end of the reheating era (the beginning of the radiation-dominated era) and compare it to the comoving entropy density $N_s=a^3T^32\pi^2g_*/45$. The result is

$$\frac{N_{3/2}}{N_s}=\frac{n_{3/2}}{s}\approx\begin{cases}10^{-2}\dfrac{T_{RH}}{M_{Pl}} & (T_{RH}<M_\phi\ \text{ cases I, II})\\[2ex] 10^{-2}\dfrac{M_\phi}{M_{Pl}} & (T_{RH}>M_\phi\ \text{ case III}).\end{cases} \tag{19}$$

Comparing Eqs. (16) and (19), one obtains the bounds

$$(10^8-10^9)\ \text{GeV}\gtrsim\begin{cases}T_{RH} & (T_{RH}<M_\phi\ \text{ cases I, II})\\ M_\phi & (T_{RH}>M_\phi\ \text{ case III}).\end{cases} \tag{20}$$

This calculation illustrates the point that in case III, the reheat temperature T_{RH} has no meaning.

B. Number of *e*-folds after inflation

The quality and quantity of observational data have reached the point where it is possible to start placing meaningful constraints on inflationary models [14–16]. In the phenomenology of extracting predictions from even simple inflation models, one of the significant uncertainties is the location of the inflaton corresponding to when scales of observational interest crossed the Hubble radius during inflation. Recent studies of this issue [17,20] have pointed out that a significant factor is the uncertainty in the duration of the reheating phase. Lack of knowledge of the duration of the reheating results in an uncertainty in the number of e-folds of expansion after inflation ends [2]. The uncertainty is usually parametrized in terms of the reheat temperature, with the uncertainty in the number of e-folds of inflation depending on $\ln T_{RH}^{1/3}$.

As we have stressed, in case III the reheat temperature has no meaning; the radiation-dominated era commences with $T=M_\phi$. If case III is obtained, then previous formulas for the number of e-folds should depend on

$$\Delta N=\frac{1}{3}\ln\frac{M_\phi}{V^{1/4}}, \tag{21}$$

instead of the traditional formula used for ΔN [2], $\Delta N=\frac{1}{3}\ln T_{RH}/V^{1/4}$, i.e., $T_{RH}\sim\sqrt{\Gamma_\phi M_{Pl}}$ should be replaced by M_ϕ. This means that if case III is attained, the number of e-folds corresponding to scales of observational interest is smaller than in the usually adopted case by a factor $\frac{1}{3}\ln\sqrt{\alpha_\phi M_{Pl}/M_\phi}$.

Proper calculation of the number of e-folds after inflation is crucial in determining the viability of inflation models. The change in the number of e-folds in case III may be crucial.

C. Production of massive particles

Our findings may be relevant for the production of massive particles during the reheating stage and, in particular, for the production of superheavy dark matter (WIMPZILLAS, where WIMP stands for weakly interacting massive particle) [5,21] and leptogenesis [22].

There are many reasons to believe that the present mass density of the Universe is dominated by a WIMP, a fossil relic of the early Universe. Theoretical ideas and experimental efforts have focused mostly on production and detection of thermal relics, with mass typically in the range a few GeV to a hundred GeV. However, during the transition from the end of inflation to the beginning of the radiation phase, superheavy and nonthermal particles may be generated. If they are stable they may provide a significant contribution to the total dark matter density of the Universe.

Let us consider a superheavy particle X with mass M_X. In this section we will restrict our attention to case III for which the final reheating temperature is fixed by the inflaton mass, and we consider the case in which $M_X>M_\phi$.[6] We suppose that the X particles are produced in pairs during the reheating stage by annihilation of light states. The corresponding Boltzmann equation for the number density n_X reads

[6]In the opposite case, the superheavy dark matter may be generated directly by the inflaton decay and its abundance will depend on the coupling between the inflaton and the X particles. In such a case, the final abundance of the superheavy dark matter will be model dependent.

$$\frac{dn_X}{dt}+3Hn_X=-\langle\sigma_A v\rangle[n_X^2-(n_X^2)_{eq}], \tag{22}$$

where $\langle\sigma_A v\rangle\simeq\alpha_X/M_X^2$ is the thermal average of the annihilation cross section times the Møller velocity. Assuming the actual density n_X is much less than its equilibrium value $(n_X)_{eq}=g_X(M_X T/2\pi)^{3/2}e^{-M_X/T}$ (g_X is the number of degrees of freedom of the X particles) and remembering that dominant contribution to the production comes from end of reheating when the temperature is of the order of M_ϕ, we can estimate the ratio between the number density of X particles and the entropy density at the end of reheating to be

$$\frac{n_X}{s}\simeq 10^{-2}\frac{g_X^2}{g_*^{3/2}}M_{Pl}M_X\langle\sigma_A v\rangle\left(\frac{M_X}{M_\phi}\right)^2 e^{-2M_X/M_\phi}, \tag{23}$$

corresponding to a present-day abundance of

$$\Omega_X h^2\simeq 10^{22}g_X^2 M_X^2\langle\sigma_A v\rangle\left(\frac{M_X}{M_\phi}\right)^2 e^{-2M_X/M_\phi}. \tag{24}$$

Taking $M_X^2\langle\sigma_A v\rangle\sim 1$, a moderate hierarchy between the inflaton mass and the superheavy dark matter particle M_X, $M_X/M_\phi\sim 30$, may explain the observed value for the dark matter abundance of about 30%. Equation (24) is much different than previous results [5].

Our findings also have important implications for the conjecture that ultra-high energy cosmic rays, above the Greisen-Zatsepin-Kuzmin cutoff of the cosmic ray spectrum, may be produced in decays of superheavy long-living particles [23–25]. In order to produce cosmic rays of energies larger than about 10^{13} GeV, the mass of the X particles must be very large, $M_X\gtrsim 10^{13}$ GeV, and their lifetime τ_X cannot be much smaller than the age of the Universe, $\tau_X\gtrsim 10^{10}$ yr. With the smallest value of the lifetime, the observed flux of ultra-high energy cosmic rays will be reproduced with a rather low density of X particles, $\Omega_X\sim 10^{-12}$. The expression, Eq. (24), suggests that the X particles can be produced in the right amount by collisions taking place during the reheating stage after inflation, if the inflaton mass is about a factor of 40 smaller than M_X.

Let us now discuss the consequences of our results for the leptogenesis scenario [22] (even though our findings can be easily generalized to any out-of-equilibrium scenario for the production the baryon asymmetry) where the lepton asymmetry L is reprocessed into baryon number by the anomalous sphaleron transitions [26]. Again we will assume case III for which the final reheating temperature is fixed by the inflaton mass.

In the simplest leptogenesis scenario, the lepton asymmetry is generated by the out-of-equilibrium decay of a massive right-handed Majorana neutrino, whose addition to the standard model spectrum breaks $B-L$.

Let us indicate by n_N the number density per comoving volume of the lightest right-handed neutrino N, the one whose final decay (into left-handed leptons and Higgs bosons) is responsible for the generation of the lepton asymmetry. We can approximate the Boltzmann equation for N as

$$\frac{dn_N}{dt}+3Hn_N=-\Gamma_N[n_N-(n_N)_{eq}], \tag{25}$$

where Γ_N is the decay rate of N for the processes $N \to H^\dagger \ell_L, H\overline{\ell}_L$. Assume again that $M_\phi < M_N$ and that the actual density n_N is much less than its equilibrium value $(n_N)_{eq}=2(M_N T/2\pi)^{3/2}e^{-M_N/T}$. Since the dominant contribution to the production of right-handed neutrinos will come from end of reheating when the temperature is of the order of M_ϕ, we can estimate the ratio between the number density of N particles and the entropy density at the end of reheating to be

$$\frac{n_N}{s}\simeq\frac{10^{-1}}{g_*^{3/2}}\left(\frac{\Gamma_N M_{Pl}}{M_\phi^2}\right)\left(\frac{M_N}{M_\phi}\right)^{3/2}e^{-M_N/M_\phi}$$
$$\lesssim\frac{10^{-1}}{g_*}\left(\frac{M_N}{M_\phi}\right)^{3/2}e^{-M_N/M_\phi}, \tag{26}$$

where in the last expression we have imposed that when right-handed neutrinos are produced, their direct decay is inefficient, i.e.,

$$K=\left.\frac{\Gamma_N}{H}\right|_{T\simeq M_\phi}\simeq\frac{\Gamma_N M_{Pl}}{g_*^{1/2}M_\phi^2}\lesssim 1. \tag{27}$$

The limiting case $K \sim 1$ would mean that the right-handed neutrinos enter into chemical equilibrium as soon as they are generated.

The ratio in Eq. (26) remains constant until the right-handed neutrinos decay generating a lepton asymmetry $L=\epsilon(n_N/s)$, where ϵ is the small parameter containing the information about the CP-violating phases and the loop factors. The corresponding baryon asymmetry is $B=(28/79)L$, assuming only standard model degrees of freedom, and therefore the final baryon asymmetry is bounded to be smaller than

$$B=10^{-4}\epsilon\left(\frac{M_N}{M_\phi}\right)^{3/2}e^{-M_N/M_\phi}. \tag{28}$$

For a hierarchical spectrum of right-handed neutrinos, it has been shown that there is a model independent upper bound on the CP asymmetry produced in the right-handed neutrino decays, $\epsilon \lesssim 3m_{\nu_3}M_N/(8\pi v^2)$, where m_{ν_3} is the mass of the heaviest of the left-handed neutrinos and v is the standard model Higgs vacuum expectation value [27]. Therefore, the maximum value of the baryon asymmetry in Eq. (28) is further bounded from above by (taking $m_{\nu_3}\sim 0.07$ eV, the atmospheric neutrino mass scale)

$$B\lesssim 10^{-6}\left(\frac{M_N}{10^{10}\text{ GeV}}\right)\left(\frac{M_N}{M_\phi}\right)^{3/2}e^{-M_N/M_\phi}. \tag{29}$$

The requirement that B is larger than 2×10^{-11} implies that the ratio M_N/M_ϕ cannot be larger than about 15.

IV. CONCLUSIONS

Reheating after inflation occurs due to particle production by the oscillating inflaton field, and its dynamics is very rich. In this paper we have observed that the inflaton decay products acquire plasma masses during the reheating phase. The plasma masses may render inflaton decay kinematically forbidden, causing the temperature to remain frozen for a period at a plateau value. This happens in any models where the decay rate of the inflaton field Γ_ϕ is larger than about M_ϕ^2/M_{Pl}. This condition does not seem to be very restrictive. If the condition is met, the final reheating temperature is uniquely determined by the inflaton mass, and not by its coupling. If the reheating dynamics is mainly dominated by a scalar field χ different from the inflaton, then the final reheating temperature may be determined in terms of the mass of the χ field. An example is if reheating takes place along a flat supersymmetric direction whose mass is the soft supersymmetry breaking scale $\widetilde{m}\sim 10^2$ GeV and whose couplings to ordinary matter are of the order of unity. In such a case, the effects of plasma blocking are crucial for determining the final reheating temperature to be $T_{RH}\sim\widetilde{m}$.

We have shown that our results are relevant for the thermal production of dangerous relics during reheating, for extracting bounds on particle physics models of inflation from cosmic microwave background anisotropy data, for the production of massive dark matter candidates during reheating, and for models of baryogenesis or leptogenesis where massive particles are produced during reheating.

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