Self-energy-part resummed quark and gluon propagators in a spin-polarized quark matter and generalized Boltzmann equations

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We construct perturbative frameworks for studying nonequilibrium spin-polarized quark matter. We employ the closed-time-path formalism and use the gradient approximation in derivative expansion. After constructing self-energy-part resummed quark and gluon propagators, we formulate two kinds of mutually equivalent perturbative frameworks: The first one is formulated on the basis of the initial-particle distribution function, and the second one is formulated on the basis of a ''physical'' particle distribution function. In the course of the construction of the second framework, the generalized Boltzmann equations and their relatives *directly* come out, which describe the evolution of the system. The frameworks are relevant to the study of a magnetic character of quark matter, e.g., possible quark stars.

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I. INTRODUCTION

The possible recent discovery of a quark star [1,2] has renewed our interest in the study of quark matter. The possibility of the existence of a quark liquid in a ferro-magnetic phase has been pointed out [3]. For analyzing the magnetic property of quark matter in a consistent manner [4–6], it is necessary to construct self-energy-part resummed quark and gluon propagators in spin-polarized quark matter, and thereby frame a perturbation theory.

The spin-polarized quark matter is, in general, out of equilibrium. For dealing with such systems, we employ the closed-time-path formalism [4,5]. In this formalism, propagators, vertices, and self-energy parts enjoy (2×2)-matrix forms, denoted '' $\hat{\ }$.'' Let $\hat{G}(x,y)$ be a generic two-point function. Fourier transforming with respect to $x-y$ (Wigner transformation), we have $\hat{G}(P,X)$ with $X=(x+y)/2$. We assume that $\hat{G}(P,X)$ depends weakly on X. Then, as usual, employing a derivative expansion (DEX), we use the gradient approximation

$$\hat{G}(P;X)\simeq\hat{G}(P;Y)+(X-Y)^{\mu}\partial_{Y\mu}\hat{G}(P;Y).$$

We refer to the first term on the right-hand side (RHS) as the leading part (term) and to the second term as the gradient part (term). Throughout this paper, we assume that the density matrix is a color singlet, so that the quark and gluon propagators are diagonal in color space and independent of the color index. Then, we drop the color index throughout.

The plan of the paper is as follows. In Sec. II, the leading term in the DEX of the self-energy-part resummed (SEPR) quark propagator is constructed. In Sec. III, we construct the leading term of the SEPR gluon propagator in a Coulomb gauge. In Secs. II and III, the argument X is dropped throughout. In Sec. IV, we present the gradient terms of the quark and gluon propagators. Then, we frame two mutually equivalent perturbative frameworks. One framework is constructed in terms of the ''bare'' number-density function (and its relative), and the other, which we call physical-N scheme, is constructed in terms of the ''renormalized'' number-density function (and its relative). The latter scheme accompanies the generalized Boltzmann equation for the renormalized number-density function and its relatives. The form for the leading part of the SEPR gluon propagator in a covariant gauge is given in Appendix D.

II. QUARK PROPAGATOR

A. Preliminaries

1. Spin-polarization vector

We define a spin-polarization vector $\mathcal{S}(P)$ as follows. For a timelike $(P^2=p_0^2-\vec{p}^{\,2}>0)$ mode, we choose $\mathcal{S}^\mu=(0,\vec{\zeta})$ $(\equiv\zeta^\mu)$ $[\vec{\zeta}^{\,2}=1]$ in the rest frame, where $P^\mu=(\epsilon(p_0)\sqrt{P^2},\vec{0})$. Similarly, for a spacelike $(P^2<0)$ mode, we choose $\mathcal{S}^\mu=(0,\vec{\zeta})$ in the ''$p_0=0$ frame,'' where $P^\mu=(0,\sqrt{-P^2}\,\vec{\xi})$ $(\equiv\sqrt{-P^2}\xi^\mu)$ $[\vec{\xi}^{\,2}=1,\vec{\xi}\cdot\vec{\zeta}=0]$. $\mathcal{S}(P)$ in any frame, where $P^\mu=(p_0,\vec{p})$, is obtained through a Lorentz transformation:

$$\mathcal{S}^\mu(P)=\theta(P^2)\left\{\frac{\vec{p}\cdot\vec{\zeta}[P^\mu+\epsilon(p_0)\sqrt{P^2}\,n^\mu]}{\sqrt{P^2}[\sqrt{P^2}+|p_0|]}+\zeta^\mu\right\}+\theta(-P^2)\left\{-\frac{\vec{p}\cdot\vec{\zeta}[P^\mu+\epsilon(\vec{p}\cdot\vec{\xi})\sqrt{-P^2}\xi^\mu]}{\sqrt{-P^2}[\sqrt{-P^2}+|\vec{p}\cdot\vec{\xi}|]}+\zeta^\mu\right\},\tag{2.1}$$

$$\mathcal{S}\cdot P=0,\quad \mathcal{S}^2=-1,\tag{2.2}$$

$$n^\mu=(1,\vec{0}).$$

When a magnetic field is applied along the $\vec{\zeta}$ direction, $p_0>0$ modes with positive (negative) charge go to the state $\mathcal{S}^\mu(P)$ $[-\mathcal{S}^\mu(P)]$, while their ''antiparticle'' counterparts ($p_0<0$ modes) go to the state $-\mathcal{S}^\mu(P)$ $[\mathcal{S}^\mu(P)]$. In what follows, the concrete form (2.1) is not used, but only the properties (2.2) will be used.

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The projection operators $\mathcal{P}_\pm(P)$ onto the states of definite polarization $(\pm)$ read

$$\mathcal{P}_\rho(P)=\frac{1+\rho\epsilon(p_0)\gamma_5\not{\mathcal{S}}(P)}{2}.$$

2. Orthogonal basis in Minkowski space and the standard form

As an orthogonal basis in Minkowski space, we choose

$$P^\mu,\quad \mathcal{S}^\mu,$$

$$N^\mu=n^\mu-\frac{p_0}{P^2}P^\mu+\mathcal{S}_0\mathcal{S}^\mu \quad (N^2=\mathcal{S}_0^2-\vec{p}^{\,2}/P^2),$$

$$e_\perp^\mu=i\epsilon^{\mu\nu\rho\sigma}P_\nu N_\rho\mathcal{S}_\sigma \quad (e_\perp^2=-P^2N^2).$$

A generic (4×4)-matrix function $A(P,N,\mathcal{S},e_\perp)$ in a Dirac-matrix space is written in the form,

$$A=A_1'+A_2'\gamma_5+A_3'\not{P}+A_4'\not{N}+A_5'\not{\mathcal{S}}+A_6'\not{e}_\perp+A_7'\gamma_5\not{P}+A_8'\gamma_5\not{N}+A_9'\gamma_5\not{\mathcal{S}}+A_{10}'\gamma_5\not{e}_\perp+A_{11}'\not{P}\not{N}+A_{12}'\not{P}\not{\mathcal{S}}+A_{13}'\not{P}\not{e}_\perp+A_{14}'\not{N}\not{\mathcal{S}}+A_{15}'\not{N}\not{e}_\perp+A_{16}'\not{\mathcal{S}}\not{e}_\perp. \tag{2.3}$$

We decompose A into four parts,

$$A=\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho A\mathcal{P}_\sigma\equiv\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho A^{\rho\sigma}\mathcal{P}_\sigma, \tag{2.4}$$

and write $A^{\rho\sigma}$ in the form

$$A^{\rho\rho}=A_1^{\rho\rho}+A_2^{\rho\rho}\not{P}+A_3^{\rho\rho}\not{N}+A_4^{\rho\rho}\not{P}\not{N},$$

$$A^{\rho-\rho}=\gamma_5[A_1^{\rho-\rho}+A_2^{\rho-\rho}\not{P}+A_3^{\rho-\rho}\not{N}+A_4^{\rho-\rho}\not{P}\not{N}]. \tag{2.5}$$

It is a straightforward task to obtain

$$A_1^{\rho\rho}=A_1'+\rho\epsilon(p_0)A_9',\quad A_2^{\rho\rho}=A_3'-\rho\epsilon(p_0)N^2A_{15}',$$

$$A_3^{\rho\rho}=A_4'+\rho\epsilon(p_0)P^2A_{13}',\quad A_4^{\rho\rho}=A_{11}'+\rho\epsilon(p_0)A_6',$$

$$A_1^{\rho-\rho}=A_2'-\rho\epsilon(p_0)A_5',\quad A_2^{\rho-\rho}=A_7'+\rho\epsilon(p_0)A_{12}',$$

$$A_3^{\rho-\rho}=A_8'+\rho\epsilon(p_0)A_{14}',\quad A_4^{\rho-\rho}=A_{16}'-\rho\epsilon(p_0)A_{10}'. \tag{2.6}$$

We refer to Eq. (2.5) as the standard form (SF) and $A^{\rho\sigma}$ or $A_j^{\rho\sigma}$ as a SF element of A. It is to be understood that the (bare and self-energy-part resummed) propagators and the self-energy part, which appear in the following, are to be written in the SF.

B. Bare propagator

First of all, we note that the bare propagator matrix $\hat{S}(P)$ and the self-energy-part resummed propagator matrix $\hat{G}(P)$ enjoy the symmetry property,

$$\hat{S}^\dagger(P)=-\hat{\tau}_1\gamma^0\,{}^t\hat{S}(P)\gamma^0\hat{\tau}_1,\quad \hat{G}^\dagger(P)=-\hat{\tau}_1\gamma^0\,{}^t\hat{G}(P)\gamma^0\hat{\tau}_1, \tag{2.7}$$

which results from the Hermiticity of the density matrix. Here, $\hat{\tau}_1$ is the first Pauli matrix, $\dagger$ acts on Dirac gamma matrix function, e.g., $(A\not{P})^\dagger=A^*P^\mu\gamma_\mu^\dagger$, and ${}^t\hat{S}(P)$ denotes the transpose of the (2×2)-matrix function $\hat{S}(P)$, etc.

The bare propagator $\hat{S}(P)$ is an inverse of $\hat{S}^{-1}(P)=(\not{P}-m)\hat{\tau}_3$. A general solution to $\hat{S}^{-1}\hat{S}=\hat{S}\hat{S}^{-1}=1$ is

$$\hat{S}(P)=\hat{S}^{(0)}(P)+S_K(P)\hat{M}_+, \tag{2.8}$$

$$\hat{S}^{(0)}(P)=\sum_{\rho=\pm}\mathcal{P}_\rho[\hat{S}_{RA}(P)-f_\rho(S_R-S_A)\hat{M}_+], \tag{2.9}$$

$$S_K(P)=-\sum_{\rho=\pm}C_{\rho-\rho}(P)[\Delta_R(P)-\Delta_A(P)]\times\mathcal{P}_\rho\gamma_5(\not{P}-m)\not{N}\mathcal{P}_{-\rho}, \tag{2.10}$$

where the suffix "K" stands for the "Keldish component" and

$$\hat{S}_{RA}(P)=\begin{pmatrix} S_R & 0\\ S_R-S_A & -S_A\end{pmatrix},$$

$$\hat{M}_\pm=\begin{pmatrix} 1 & \pm1\\ \pm1 & 1\end{pmatrix},$$

$$S_{R(A)}=(\not{P}+m)\Delta_{R(A)}(P)=\frac{\not{P}+m}{P^2-m^2\pm ip_00^+}, \tag{2.11}$$

$$f_\rho(P)=\theta(p_0)N_\rho(|p_0|,\vec{p})+\theta(-p_0)[1-\overline{N}_\rho(|p_0|,-\vec{p})]. \tag{2.12}$$

Here $S_{R(A)}$ is the retarded (advanced) propagator, and $N_\rho(|p_0|,\vec{p})$ $[\overline{N}_\rho(|p_0|,-\vec{p})]$ $(\rho=\pm)$ is the bare number-density function of a quark [an antiquark] with polarization $\rho\mathcal{S}(P)$, energy $|p_0|$ $(=\sqrt{\vec{p}^{\,2}+m^2})$, and momentum $\vec{p}$ $[-\vec{p}]$. S_K in Eq. (2.10) connects opposite polarization states. From Eqs. (2.7), (2.8), and (2.10), we have

$$(C_{+-}(P))^*=C_{-+}(P).$$

The derivative expansion is an efficient device for dealing with quasiuniform systems near equilibrium or nonequilibrium quasistationary systems. For such systems, S_K is small when compared to $\hat{S}^{(0)}$.

C. Dyson equation

The self-energy-part $(\hat{\Sigma})$ resummed propagator $\hat{G}$ obeys the Dyson equation

$$\hat{G}(P)=\hat{S}(P)[1+\hat{\Sigma}(P)\hat{G}(P)]=[1+\hat{G}(P)\hat{\Sigma}(P)]\hat{S}(P). \tag{2.13}$$

We write $\hat{G}$ and $\hat{\Sigma}$ in SF's,

$$\hat{G}=\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho\hat{G}^{\rho\sigma}\mathcal{P}_\sigma,\quad \hat{\Sigma}=\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho\hat{\Sigma}^{\rho\sigma}\mathcal{P}_\sigma.$$

It is worth mentioning that, for the system that enjoys an azimuthal symmetry around the $\vec{\zeta}$ direction, $\hat{\Sigma}$, and then also $\hat{G}$, are independent of $E_\perp^\mu$, provided that we choose $\vec{\xi}=\vec{p}\times\vec{\zeta}/|\vec{p}\times\vec{\zeta}|$. Then, from Eqs. (2.3)–(2.6), we have

$$\hat{\Sigma}_4^{\rho-\rho}=0\quad(\rho=\pm),$$

$$\hat{\Sigma}_j^{++}=\hat{\Sigma}_j^{--}\quad(j=2,3,4).$$

Same relations hold for $\hat{G}$'s.

Substituting the SF's for $\hat{S}$, $\hat{\Sigma}$, and $\hat{G}$ in Eq. (2.13), we obtain coupled equations

$$\hat{G}^{\rho\sigma}=\hat{S}^{\rho\sigma}+(\hat{S}\hat{\Sigma}\hat{G})^{\rho\sigma}=\hat{S}^{\rho\sigma}+(\hat{G}\hat{\Sigma}\hat{S})^{\rho\sigma}\quad(\rho,\sigma=\pm), \tag{2.14}$$

where $(\hat{S}\hat{\Sigma}\hat{G})^{\rho\sigma}\equiv\sum_{\xi,\zeta=\pm}\hat{S}^{\rho\xi}\hat{\Sigma}^{\xi\zeta}\hat{G}^{\zeta\sigma}$, etc. The relation that involves $(\cdots)^{\rho\sigma}$ is to be understood to hold when sandwiched between projection operators $\mathcal{P}_\rho\cdots\mathcal{P}_\sigma$. We write Eq. (2.14), with obvious notation, as

$$\hat{\mathbf{G}}=\hat{\mathbf{S}}+\hat{\mathbf{S}}\hat{\mathbf{\Sigma}}\hat{\mathbf{G}}=\hat{\mathbf{S}}+\hat{\mathbf{G}}\hat{\mathbf{\Sigma}}\hat{\mathbf{S}}, \tag{2.15}$$

where boldface letters denote (2×2) matrix in a "polarization space."

From Eq. (2.7), we obtain the symmetry relations for the SF elements of $\hat{G}^{\rho\sigma}$ [cf. Eqs. (2.4) and (2.5)],

$$(\hat{G}_j^{\rho\sigma}(P))^*=-\sigma_j^{\rho\sigma}\hat{\tau}_1\,{}^t\hat{G}_j^{\sigma\rho}(P)\hat{\tau}_1, \tag{2.16}$$

where

$$\sigma_j^{\rho\sigma}=\begin{cases}+\quad\text{for }(\rho\sigma,j) &= (\rho\rho,1),(\rho\rho,2),(\rho\rho,3),\\ & (\rho-\rho,2),(\rho-\rho,3),\\ & (\rho-\rho,4)\\ -\quad\text{for }(\rho\sigma,j) &= (\rho\rho,4),(\rho-\rho,1).\end{cases}$$

Similar relations hold for $\hat{\Sigma}_j^{\rho\sigma}$'s.

Let us introduce (2×2)-matrix function $\mathbf{f}$ in the polarization space,

$$\mathbf{f}=\mathrm{diag}(f_+,f_-).$$

Then $\hat{\mathbf{S}}$ is written as

$$\hat{\mathbf{S}}=\hat{\mathbf{S}}^{(0)}+\mathbf{S}_K\hat{M}_+, \tag{2.17}$$

$$\hat{\mathbf{S}}^{(0)}=\hat{S}_{RA}\mathbf{1}-\mathbf{f}(S_R-S_A)\hat{M}_+$$

$$\mathbf{S}_K=[S_R(P)-S_A(P)]\gamma_5\not{N}\mathbf{C}(P), \tag{2.18}$$

$$\mathbf{C}(P)=\begin{pmatrix}0 & C_{+-}(P)\\ C_{-+}(P) & 0\end{pmatrix}.$$

Among the components of $\hat{\mathbf{\Sigma}}$ is a relation

$$\mathbf{\Sigma}_{11}+\mathbf{\Sigma}_{12}+\mathbf{\Sigma}_{21}+\mathbf{\Sigma}_{22}=0. \tag{2.19}$$

Then, $\hat{\mathbf{\Sigma}}$ is written as

$$\hat{\mathbf{\Sigma}}=\hat{\mathbf{\Sigma}}^{(0)}-\mathbf{\Sigma}_K\hat{M}_-, \tag{2.20}$$

$$\hat{\mathbf{\Sigma}}^{(0)}=\begin{pmatrix}\mathbf{\Sigma}_R & 0\\ -\mathbf{\Sigma}_R+\mathbf{\Sigma}_A & -\mathbf{\Sigma}_A\end{pmatrix}-(\mathbf{\Sigma}_R\mathbf{f}-\mathbf{f}\,\mathbf{\Sigma}_A)\hat{M}_-, \tag{2.21}$$

$$\mathbf{\Sigma}_R=\mathbf{\Sigma}_{11}+\mathbf{\Sigma}_{12}=-\mathbf{\Sigma}_{22}-\mathbf{\Sigma}_{21}, \tag{2.22}$$

$$\mathbf{\Sigma}_A=\mathbf{\Sigma}_{11}+\mathbf{\Sigma}_{21}=-\mathbf{\Sigma}_{22}-\mathbf{\Sigma}_{12}, \tag{2.23}$$

$$\mathbf{\Sigma}_K=\mathbf{f}\,\mathbf{\Sigma}_{11}-\mathbf{\Sigma}_{11}\mathbf{f}+\mathbf{f}\,\mathbf{\Sigma}_{21}+\mathbf{\Sigma}_{12}(1-\mathbf{f}). \tag{2.24}$$

From Eq. (2.16) with $\hat{\Sigma}_j^{\rho\sigma}$ for $\hat{G}_j^{\rho\sigma}$, we obtain the symmetry relations

$$(\Sigma_{Rj}^{\rho\sigma}(P))^*=\sigma_j^{\rho\sigma}\Sigma_{Aj}^{\sigma\rho}(P),$$

$$(\Sigma_{Kj}^{\rho\sigma}(P))^*=-\sigma_j^{\rho\sigma}\Sigma_{Kj}^{\sigma\rho}(P).$$

Among the components of $\hat{\mathbf{G}}$ is a relation

$$\mathbf{G}_{11}+\mathbf{G}_{22}=\mathbf{G}_{12}+\mathbf{G}_{21}. \tag{2.25}$$

Then, $\hat{\mathbf{G}}$ is written as

$$\hat{\mathbf{G}}=\hat{\mathbf{G}}^{(0)}+\mathbf{G}_K\hat{M}_+, \tag{2.26}$$

$$\hat{\mathbf{G}}^{(0)}=\begin{pmatrix}\mathbf{G}_R & 0\\ \mathbf{G}_R-\mathbf{G}_A & -\mathbf{G}_A\end{pmatrix}-(\mathbf{G}_R\mathbf{f}-\mathbf{f}\mathbf{G}_A)\hat{M}_+, \tag{2.27}$$

$$\mathbf{G}_R=\mathbf{G}_{11}-\mathbf{G}_{12}=-\mathbf{G}_{22}+\mathbf{G}_{21}, \tag{2.28}$$

$$\mathbf{G}_A=\mathbf{G}_{11}-\mathbf{G}_{21}=-\mathbf{G}_{22}+\mathbf{G}_{12}, \tag{2.29}$$

$$\mathbf{G}_K=\mathbf{G}_{11}\mathbf{f}-\mathbf{f}\mathbf{G}_{11}+\mathbf{f}\mathbf{G}_{21}+\mathbf{G}_{12}(1-\mathbf{f}). \tag{2.30}$$

It is worth mentioning that, for equilibrium systems, $\mathbf{\Sigma}_K=\mathbf{G}_K=0$. From Eq. (2.16) follow the symmetry relations

$$(G_{Rj}^{\rho\sigma}(P))^*=\sigma_j^{\rho\sigma}G_{Aj}^{\sigma\rho}(P),$$

$$(G_{Kj}^{\rho\sigma}(P))^*=-\sigma_j^{\rho\sigma}G_{Kj}^{\sigma\rho}(P). \tag{2.31}$$

Substitution of Eqs. (2.17), (2.20), and (2.26) into Eq. (2.15) yields

$$\hat{\mathbf{G}}^{(0)}=\hat{\mathbf{S}}^{(0)}+\hat{\mathbf{S}}^{(0)}\hat{\boldsymbol{\Sigma}}^{(0)}\hat{\mathbf{G}}^{(0)}=\hat{\mathbf{S}}^{(0)}+\hat{\mathbf{G}}^{(0)}\hat{\boldsymbol{\Sigma}}^{(0)}\hat{\mathbf{S}}^{(0)}, \tag{2.32}$$

$$\mathbf{G}_K=\mathbf{S}_K+S_R\boldsymbol{\Sigma}_R\mathbf{G}_K+\mathbf{S}_K\boldsymbol{\Sigma}_A\mathbf{G}_A-S_R\boldsymbol{\Sigma}_K\mathbf{G}_A$$
$$=\mathbf{S}_K+\mathbf{G}_R\boldsymbol{\Sigma}_R\mathbf{S}_K+\mathbf{G}_K\boldsymbol{\Sigma}_A S_A-\mathbf{G}_R\boldsymbol{\Sigma}_K S_A. \tag{2.33}$$

From Eq. (2.32), we obtain

$$\mathbf{G}_{R(A)}=[\not P-m-\boldsymbol{\Sigma}_{R(A)}]^{-1}, \tag{2.34}$$

where use has been made of $(\hat{S}^{(0)-1})^{\rho\sigma}=\delta^{\rho\sigma}(\not P-m)\hat{\tau}_3$. We get from Eq. (2.34), after some manipulation,

$$G_R^{\rho\rho}=[\not P-m-\Sigma_R^{\rho\rho}-\Sigma_R^{\rho-\rho}G_R^{(\mathrm{pre})-\rho-\rho}\Sigma_R^{-\rho\rho}]^{-1}, \tag{2.35}$$

$$G_R^{\rho-\rho}=G_R^{(\mathrm{pre})\rho\rho}\Sigma_R^{\rho-\rho}G_R^{-\rho-\rho}=G_R^{\rho\rho}\Sigma_R^{\rho-\rho}G_R^{(\mathrm{pre})-\rho-\rho}, \tag{2.36}$$

where

$$G_R^{(\mathrm{pre})\rho\rho}=[\not P-m-\Sigma_R^{\rho\rho}]^{-1}. \tag{2.37}$$

As has been remarked above after Eq. (2.14), Eq. (2.37) is to be understood to mean

$$\mathcal{P}_\rho G_R^{(\mathrm{pre})\rho\rho}[\not P-m-\Sigma_R^{\rho\rho}]\mathcal{P}_\rho$$
$$=\mathcal{P}_\rho[\not P-m-\Sigma_R^{\rho\rho}]G_R^{(\mathrm{pre})\rho\rho}\mathcal{P}_\rho=\mathcal{P}_\rho 1\mathcal{P}_\rho=\mathcal{P}_\rho. \tag{2.38}$$

Such an understanding also applies to Eq. (2.35). Concrete form for $\mathbf{G}_{R(A)}$ will be given in the next section.

As for $\mathbf{G}_K$, Eq. (2.33), we show in Appendix A that

$$\mathbf{G}_K=\mathbf{G}_K^{(1)}+\mathbf{G}_K^{(2)}+\mathbf{G}_K^{(3)}, \tag{2.39}$$

$$\mathbf{G}_K^{(1)}=-\mathbf{G}_R\boldsymbol{\Sigma}_K\mathbf{G}_A, \tag{2.40}$$

$$\mathbf{G}_K^{(2)}=\mathbf{G}_R[\gamma_5\not N\mathbf{C}(P)\boldsymbol{\Sigma}_A-\boldsymbol{\Sigma}_R\gamma_5\not N\mathbf{C}(P)]\mathbf{G}_A\equiv\mathbf{G}_R\mathbf{H}_l\mathbf{G}_A, \tag{2.41}$$

$$\mathbf{G}_K^{(3)}=\mathbf{G}_R\gamma_5\not N\mathbf{C}(P)-\gamma_5\not N\mathbf{C}(P)\mathbf{G}_A. \tag{2.42}$$

"l" of $\mathbf{H}_l$ in Eq. (2.41) stands for the "leading part" of the DEX. As mentioned above at the end of Sec. II B, for quasi-uniform systems near equilibrium or nonequilibrium quasistationary systems, $\mathbf{G}_K^{(2)}$ and $\mathbf{G}_K^{(3)}$ are much smaller than $\mathbf{G}_K^{(1)}$. The SF for $\mathbf{G}_K^{(1)}$ will be given in the next section. The standard forms for $\mathbf{H}_l$ in Eq. (2.41) and $\gamma_5\not N\mathbf{C}(P)$ in Eq. (2.42) are also given in the next section. The SF's for $\mathbf{G}_K^{(2)}$ and $\mathbf{G}_K^{(3)}$ are obtained by repeatedly using the formulas in Appendix B.

D. Self-energy-part resummed propagator $\hat{\mathbf{G}}$

It is convenient to introduce

$$\hat{\hat{G}}^{\rho\rho}=\begin{pmatrix} G_R^{\rho\rho} & G_K^{(1)\rho\rho} \\ 0 & -G_A^{\rho\rho}\end{pmatrix},\quad \hat{\hat{\Sigma}}^{\rho\sigma}=\begin{pmatrix}\Sigma_R^{\rho\sigma} & \Sigma_K^{\rho\sigma} \\ 0 & -\Sigma_A^{\rho\sigma}\end{pmatrix}.$$

We observe that Eq. (2.35) and $G_K^{(1)\rho\rho}$ in Eq. (2.40) are unified to a matrix equation

$$\hat{\hat{G}}^{\rho\rho}=[(\not P-m)\hat{\tau}_3-\hat{\hat{\Sigma}}^{\rho\rho}-\hat{\hat{\Sigma}}^{\rho-\rho}\hat{\hat{G}}^{(\mathrm{pre})-\rho-\rho}\hat{\hat{\Sigma}}^{-\rho\rho}]^{-1}, \tag{2.43}$$

where

$$\hat{\hat{G}}^{(\mathrm{pre})\rho\rho}=\begin{pmatrix} G_R^{(\mathrm{pre})\rho\rho} & G_K^{(\mathrm{pre})\rho\rho} \\ 0 & -G_A^{(\mathrm{pre})\rho\rho}\end{pmatrix}=[(\not P-m)\hat{\tau}_3-\hat{\hat{\Sigma}}^{\rho\rho}]^{-1}. \tag{2.44}$$

1. Forms for $G_{R(A)}^{(pre)\rho\rho}$ and $G_K^{(pre)\rho\rho}$

The SF for $(\not P-m)\hat{\tau}_3-\hat{\hat{\Sigma}}^{\rho\rho}$ reads

$$(\not P-m)\hat{\tau}_3-\hat{\hat{\Sigma}}^{\rho\rho}(P)=-[m\hat{\tau}_3+\hat{\hat{\Sigma}}_1^{\rho\rho}(P)]+[\hat{\tau}_3-\hat{\hat{\Sigma}}_2^{\rho\rho}(P)]\not P$$
$$-\hat{\hat{\Sigma}}_3^{\rho\rho}(P)\not N-\hat{\hat{\Sigma}}_4^{\rho\rho}(P)\not P\not N. \tag{2.45}$$

One obtains the expressions for the SF elements $G_{Tj}^{(\mathrm{pre})\rho\rho}$ ($T=R,A,K$ and $j=1$–4) through straightforward but tedious manipulation of Eq. (2.44), which includes Eq. (2.38). Writing $\Sigma_j\equiv\Sigma_{Rj}^{\rho\rho}$ for short, we have

$$G_{Rj}^{(\mathrm{pre})\rho\rho}=\sigma_j^{\rho\rho}(G_{Aj}^{(\mathrm{pre})\rho\rho})^* \quad (j=1\text{–}4),$$

$$G_{R1}^{(\mathrm{pre})\rho\rho}=\frac{m+\Sigma_1}{\mathcal{D}_{\mathrm{pre}}^{\rho\rho}},\quad G_{R2}^{(\mathrm{pre})\rho\rho}=\frac{1-\Sigma_2}{\mathcal{D}_{\mathrm{pre}}^{\rho\rho}},$$

$$G_{Rl}^{(\mathrm{pre})\rho\rho}=-\frac{\Sigma_l}{\mathcal{D}_{\mathrm{pre}}^{\rho\rho}} \quad (l=3,4),$$

$$G_{Kj}^{(\mathrm{pre})\rho\rho}=\frac{\sum_{l=1}^{4}\mathcal{N}_j^{(l)}\Sigma_{Kl}^{\rho\rho}}{\mathrm{Im}\{[(m+\Sigma_1)^2-N^2(\Sigma_3)^2][(\Sigma_2^*-1)^2-N^2(\Sigma_4^*)^2]\}} \quad (j=1\text{–}4),$$

where

$$\mathcal{D}^{\rho\rho}_{\rm pre}=[(1-\Sigma_2)^2-N^2(\Sigma_4)^2]P^2-(m+\Sigma_1)^2+N^2(\Sigma_3)^2 \qquad (2.46)$$

and

$$\mathcal{N}_1^{(1)}=-[E_{13}^{(+)}F_{24}+E_{24}^{(+)}F_{13}],\quad \mathcal{N}_1^{(2)}=-2F_{13}{\rm Re}F_{1234}^{(-)},$$

$$\mathcal{N}_1^{(3)}=2N^2{\rm Re}[F_{13}H_{24}+F_{24}H_{13}],$$

$$\mathcal{N}_1^{(4)}=2iN^2F_{13}{\rm Im}F_{1423}^{(+)},$$

$$\mathcal{N}_2^{(1)}=-2F_{24}{\rm Re}F_{1234}^{(+)},\quad \mathcal{N}_2^{(2)}=-[E_{24}^{(-)}F_{13}+E_{13}^{(-)}F_{24}],$$

$$\mathcal{N}_2^{(3)}=2N^2F_{24}{\rm Re}F_{1423}^{(+)},$$

$$\mathcal{N}_2^{(4)}=2iN^2{\rm Im}[F_{13}H_{24}+F_{24}H_{13}],$$

$$\mathcal{N}_3^{(1)}=-2{\rm Re}[F_{13}H_{24}-F_{24}H_{13}],$$

$$\mathcal{N}_3^{(2)}=-2F_{13}{\rm Re}F_{1423}^{(-)},$$

$$\mathcal{N}_3^{(3)}=-[E_{13}^{(+)}F_{24}-E_{24}^{(+)}F_{13}],\quad \mathcal{N}_3^{(4)}=2iF_{13}{\rm Im}F_{1234}^{(-)},$$

$$\mathcal{N}_4^{(1)}=-2iF_{24}{\rm Im}F_{1423}^{(-)},$$

$$\mathcal{N}_4^{(2)}=2i{\rm Im}[-F_{13}H_{24}+F_{24}H_{13}],$$

$$\mathcal{N}_4^{(3)}=2iF_{24}{\rm Im}F_{1234}^{(+)},\quad \mathcal{N}_4^{(4)}=-[E_{13}^{(-)}F_{24}-E_{24}^{(-)}F_{13}],$$

with

$$E_{13}^{(\pm)}=|m+\Sigma_1|^2\pm N^2|\Sigma_3|^2,\quad E_{24}^{(\pm)}=|1-\Sigma_2|^2\pm N^2|\Sigma_4|^2,$$

$$F_{24}={\rm Im}\frac{(1-\Sigma_2)^2-N^2(\Sigma_4)^2}{\mathcal{D}^{\rho\rho}_{\rm pre}},$$

$$F_{13}={\rm Im}\frac{(m+\Sigma_1)^2-N^2(\Sigma_3)^2}{\mathcal{D}^{\rho\rho}_{\rm pre}},$$

$$F_{1234}^{(\pm)}=(m+\Sigma_1)(1-\Sigma_2^*)\pm N^2\Sigma_3\Sigma_4^*,$$

$$F_{1423}^{(\pm)}=(m+\Sigma_1)\Sigma_4^*\pm(1-\Sigma_2)\Sigma_3^*,$$

$$H_{13}=(m+\Sigma_1)\Sigma_3^*,\quad H_{24}=(1-\Sigma_2)\Sigma_4^*.$$

2. Form for $G^{\rho\rho}_{R(A)}$ and $G^{(1)\rho\rho}_{K}$, Eqs. (2.35) and (2.40)

Using the definition (B1) in Appendix B, one can write the quantity in the square brackets in Eq. (2.43) as

$$(\not P-m)\hat{\tau}_3-\hat{\hat{\Sigma}}^{\rho\rho}-[[\hat{\hat{\Sigma}}\otimes\hat{\hat{G}}^{(\rm pre)}]\otimes\hat{\hat{\Sigma}}]^{\rho\rho}. \qquad (2.47)$$

The SF for this is obtained by the repeated use of the formulas in Appendix B:

$$\begin{aligned}\text{Eq. (2.47)}=&-[m\hat{\tau}_3+\hat{\hat{\Sigma}}_1^{\rho\rho}+\hat{A}_1^{\rho-\rho}\hat{\hat{\Sigma}}_1^{-\rho\rho}-P^2\hat{A}_2^{\rho-\rho}\hat{\hat{\Sigma}}_2^{-\rho\rho}-N^2\hat{A}_3^{\rho-\rho}\hat{\hat{\Sigma}}_3^{-\rho\rho}-P^2N^2\hat{A}_4^{\rho-\rho}\hat{\hat{\Sigma}}_4^{-\rho\rho}]+[\hat{\tau}_3-\hat{\hat{\Sigma}}_2^{\rho\rho}-\hat{A}_1^{\rho-\rho}\hat{\hat{\Sigma}}_2^{-\rho\rho}\\&+\hat{A}_2^{\rho-\rho}\hat{\hat{\Sigma}}_1^{-\rho\rho}-N^2\hat{A}_3^{\rho-\rho}\hat{\hat{\Sigma}}_4^{-\rho\rho}-N^2\hat{A}_4^{\rho-\rho}\hat{\hat{\Sigma}}_3^{-\rho\rho}]\not P-[\hat{\hat{\Sigma}}_3^{\rho\rho}+\hat{A}_1^{\rho-\rho}\hat{\hat{\Sigma}}_3^{-\rho\rho}-P^2\hat{A}_2^{\rho-\rho}\hat{\hat{\Sigma}}_4^{-\rho\rho}-\hat{A}_3^{\rho-\rho}\hat{\hat{\Sigma}}_1^{-\rho\rho}\\&-P^2\hat{A}_4^{\rho-\rho}\hat{\hat{\Sigma}}_2^{-\rho\rho}]\not N-[\hat{\hat{\Sigma}}_4^{\rho\rho}+\hat{A}_1^{\rho-\rho}\hat{\hat{\Sigma}}_4^{-\rho\rho}-\hat{A}_2^{\rho-\rho}\hat{\hat{\Sigma}}_3^{-\rho\rho}+\hat{A}_3^{\rho-\rho}\hat{\hat{\Sigma}}_2^{-\rho\rho}+\hat{A}_4^{\rho-\rho}\hat{\hat{\Sigma}}_1^{-\rho\rho}]\not P\not N,\end{aligned} \qquad (2.48)$$

where

$$\begin{aligned}\hat{A}_1^{\rho-\rho}=&\hat{\hat{\Sigma}}_1^{\rho-\rho}\hat{\hat{G}}_1^{(\rm pre)-\rho-\rho}+P^2\hat{\hat{\Sigma}}_2^{\rho-\rho}\hat{\hat{G}}_2^{(\rm pre)-\rho-\rho}\\&+N^2\hat{\hat{\Sigma}}_3^{\rho-\rho}\hat{\hat{G}}_3^{(\rm pre)-\rho-\rho}-P^2N^2\hat{\hat{\Sigma}}_4^{\rho-\rho}\hat{\hat{G}}_4^{(\rm pre)-\rho-\rho},\end{aligned}$$

$$\begin{aligned}\hat{A}_2^{\rho-\rho}=&\hat{\hat{\Sigma}}_1^{\rho-\rho}\hat{\hat{G}}_2^{(\rm pre)-\rho-\rho}+\hat{\hat{\Sigma}}_2^{\rho-\rho}\hat{\hat{G}}_1^{(\rm pre)-\rho-\rho}\\&-N^2\hat{\hat{\Sigma}}_3^{\rho-\rho}\hat{\hat{G}}_4^{(\rm pre)-\rho-\rho}+N^2\hat{\hat{\Sigma}}_4^{\rho-\rho}\hat{\hat{G}}_3^{(\rm pre)-\rho-\rho},\end{aligned}$$

$$\begin{aligned}\hat{A}_3^{\rho-\rho}=&\hat{\hat{\Sigma}}_1^{\rho-\rho}\hat{\hat{G}}_3^{(\rm pre)-\rho-\rho}+P^2\hat{\hat{\Sigma}}_2^{\rho-\rho}\hat{\hat{G}}_4^{(\rm pre)-\rho-\rho}\\&+\hat{\hat{\Sigma}}_3^{\rho-\rho}\hat{\hat{G}}_1^{(\rm pre)-\rho-\rho}-P^2\hat{\hat{\Sigma}}_4^{\rho-\rho}\hat{\hat{G}}_2^{(\rm pre)-\rho-\rho},\end{aligned}$$

$$\begin{aligned}\hat{A}_4^{\rho-\rho}=&\hat{\hat{\Sigma}}_1^{\rho-\rho}\hat{\hat{G}}_4^{(\rm pre)-\rho-\rho}+\hat{\hat{\Sigma}}_2^{\rho-\rho}\hat{\hat{G}}_3^{(\rm pre)-\rho-\rho}\\&-\hat{\hat{\Sigma}}_3^{\rho-\rho}\hat{\hat{G}}_2^{(\rm pre)-\rho-\rho}+\hat{\hat{\Sigma}}_4^{\rho-\rho}\hat{\hat{G}}_1^{(\rm pre)-\rho-\rho}.\end{aligned} \qquad (2.49)$$

We observe that Eq. (2.48) with Eq. (2.49) is obtained from Eq. (2.45) through the following substitutions (T stands for R, A, or K):

$$\begin{aligned}\Sigma_{T1}^{\rho\rho}\to&\Sigma_{T1}^{\rho\rho}+T_{111}^{\rho\rho}+P^2T_{221}^{\rho\rho}+N^2T_{331}^{\rho\rho}\\&-P^2N^2T_{441}^{\rho\rho}-P^2[T_{122}^{\rho\rho}+T_{212}^{\rho\rho}-N^2T_{342}^{\rho\rho}+N^2T_{432}^{\rho\rho}]\\&-N^2[T_{133}^{\rho\rho}+P^2T_{243}^{\rho\rho}+T_{313}^{\rho\rho}-P^2T_{423}^{\rho\rho}]\\&-P^2N^2[T_{144}^{\rho\rho}+T_{234}^{\rho\rho}-T_{324}^{\rho\rho}+T_{414}^{\rho\rho}],\end{aligned}$$

$$\begin{aligned}\Sigma_{T2}^{\rho\rho}\to&\Sigma_{T2}^{\rho\rho}+T_{112}^{\rho\rho}+P^2T_{222}^{\rho\rho}+N^2T_{332}^{\rho\rho}\\&-P^2N^2T_{442}^{\rho\rho}-T_{121}^{\rho\rho}-T_{211}^{\rho\rho}+N^2T_{341}^{\rho\rho}-N^2T_{431}^{\rho\rho}\\&+N^2[T_{134}^{\rho\rho}+P^2T_{244}^{\rho\rho}+T_{314}^{\rho\rho}-P^2T_{424}^{\rho\rho}]\\&+N^2[T_{143}^{\rho\rho}+T_{233}^{\rho\rho}-T_{323}^{\rho\rho}+T_{413}^{\rho\rho}],\end{aligned}$$

$$\Sigma_{T3}^{\rho\rho}\to\Sigma_{T3}^{\rho\rho}+T_{113}^{\rho\rho}+P^2T_{223}^{\rho\rho}+N^2T_{333}^{\rho\rho}$$
$$-P^2N^2T_{443}^{\rho\rho}-P^2[T_{124}^{\rho\rho}+T_{214}^{\rho\rho}-N^2T_{344}^{\rho\rho}+N^2T_{434}^{\rho\rho}]$$
$$-[T_{131}^{\rho\rho}+P^2T_{241}^{\rho\rho}+T_{311}^{\rho\rho}-P^2T_{421}^{\rho\rho}]$$
$$-P^2[T_{142}^{\rho\rho}+T_{232}^{\rho\rho}-T_{322}^{\rho\rho}+T_{412}^{\rho\rho}],$$

$$\Sigma_{T4}^{\rho\rho}\to\Sigma_{T4}^{\rho\rho}+T_{114}^{\rho\rho}+P^2T_{224}^{\rho\rho}+N^2T_{334}^{\rho\rho}$$
$$-P^2N^2T_{444}^{\rho\rho}-[T_{123}^{\rho\rho}+T_{213}^{\rho\rho}-N^2T_{343}^{\rho\rho}+N^2T_{433}^{\rho\rho}]$$
$$+[T_{132}^{\rho\rho}+P^2T_{242}^{\rho\rho}+T_{312}^{\rho\rho}-P^2T_{422}^{\rho\rho}]$$
$$+[T_{141}^{\rho\rho}+T_{231}^{\rho\rho}-T_{321}^{\rho\rho}+T_{411}^{\rho\rho}]. \tag{2.50}$$

Here for $T=R$ and A,

$$R_{ijl}^{\rho\rho}=\Sigma_{Ri}^{\rho-\rho}G_{Rj}^{(\mathrm{pre})-\rho-\rho}\Sigma_{Rl}^{-\rho\rho}$$

and

$$A_{ijl}^{\rho\rho}=\Sigma_{Ai}^{\rho-\rho}G_{Aj}^{(\mathrm{pre})-\rho-\rho}\Sigma_{Al}^{-\rho\rho},$$

respectively, and, for $T=K$,

$$K_{ijl}^{\rho\rho}=\Sigma_{Ri}^{\rho-\rho}[G_{Rj}^{(\mathrm{pre})-\rho-\rho}\Sigma_{Kl}^{-\rho\rho}-G_{Kj}^{(\mathrm{pre})-\rho-\rho}\Sigma_{Al}^{-\rho\rho}]$$
$$+\Sigma_{Ki}^{\rho-\rho}G_{Aj}^{(\mathrm{pre})-\rho-\rho}\Sigma_{Al}^{-\rho\rho}.$$

Then, the expressions for $G_{Rj}^{\rho\rho}$, $G_{Aj}^{\rho\rho}$, and $G_{Kj}^{(1)\rho\rho}$ $(j=1$–$4)$ are obtained from those of their counterparts, in respective order, $G_{Rj}^{(\mathrm{pre})\rho\rho}$, $G_{Aj}^{(\mathrm{pre})\rho\rho}$, and $G_{Kj}^{(\mathrm{pre})\rho\rho}$ with the above substitutions.

3. The forms for $H_l^{\rho\sigma}$ in $\mathbf{G}_K^{(2)\rho\sigma}$, Eq. (2.41), and for $\gamma_5 \not{N} C^{\rho\sigma}(P)$ in $\mathbf{G}_K^{(3)}$, Eq. (2.42)

The form for $H_l^{\rho\sigma}$ is obtained by using the formulas in Appendix B:

$$H_l^{\rho\rho}=-C_{\rho-\rho}(N^2\Sigma_{A3}^{-\rho\rho}-N^2\Sigma_{A4}^{-\rho\rho}\not{P}+\Sigma_{A1}^{-\rho\rho}\not{N}-\Sigma_{A2}^{-\rho\rho}\not{P}\not{N})$$
$$+(N^2\Sigma_{R3}^{\rho-\rho}-N^2\Sigma_{R4}^{\rho-\rho}\not{P}-\Sigma_{R1}^{\rho-\rho}\not{N}+\Sigma_{R2}^{\rho-\rho}\not{P}\not{N})C_{-\rho\rho},$$

$$H_l^{\rho-\rho}=\gamma_5[C_{\rho-\rho}(N^2\Sigma_{A3}^{-\rho-\rho}-N^2\Sigma_{A4}^{-\rho-\rho}\not{P}$$
$$+\Sigma_{A1}^{-\rho-\rho}\not{N}-\Sigma_{A2}^{-\rho-\rho}\not{P}\not{N})+(N^2\Sigma_{R3}^{\rho\rho}-N^2\Sigma_{R4}^{\rho\rho}\not{P}$$
$$-\Sigma_{R1}^{\rho\rho}\not{N}+\Sigma_{R2}^{\rho\rho}\not{P}\not{N})C_{\rho-\rho}]. \tag{2.51}$$

The form for $\gamma_5\not{N}\mathbf{C}^{\rho\sigma}(P)$ is given by Eq. (2.51) with

$$\Sigma_{A1}^{\rho\sigma}\to\delta^{\rho\sigma},\quad \Sigma_{Aj}^{\rho\sigma}\to 0\quad (j=2-4),$$

$$\Sigma_{Rj}^{\rho\sigma}\to 0\quad (j=1\text{–}4).$$

4. The form for $\hat{G}^{\rho-\rho}$

Having obtained the expression for $\hat{G}^{\rho\rho}$, we can get the expression for $\hat{G}^{\rho-\rho}$ from Eqs. (2.36), (2.39)–(2.42) by repeatedly using the formulas in Appendix B.

III. GLUON PROPAGATOR

A. Preliminary

We adopt a Coulomb gauge. The result for a covariant gauge is summarized in Appendix D.

As an orthogonal basis in Minkowski space, we choose

$$\tilde{P}^\mu\equiv P^\mu-p_0n^\mu=(0,\vec{p}),\quad \tilde{\zeta}^\mu=(0,\vec{\zeta}-(\vec{\zeta}\cdot\vec{p})\vec{p}/\vec{p}^{\,2}),$$

$$n^\mu=(1,\vec{0}),\quad E_\perp^\mu=\epsilon^{\mu\nu\rho\sigma}\tilde{P}_\nu\tilde{\zeta}_\rho n_\sigma. \tag{3.1}$$

These vectors are orthogonal with each other and their norms are

$$\tilde{P}^2=-\vec{p}^{\,2},\quad \tilde{\zeta}^2=-1+(\vec{\zeta}\cdot\vec{p})^2/\vec{p}^{\,2},$$

$$n^2=1,\quad E_\perp^2=\vec{p}^{\,2}\tilde{\zeta}^2.$$

Incidentally, $\epsilon^{\mu\nu\rho\sigma}\tilde{P}_\rho\tilde{\zeta}_\sigma$, $\epsilon^{\mu\nu\rho\sigma}\tilde{P}_\rho n_\sigma$, and $\epsilon^{\mu\nu\rho\sigma}\tilde{\zeta}_\rho n_\sigma$ are not independent but are constructed out of the above four vectors, e.g., $\epsilon^{\mu\nu\rho\sigma}\tilde{P}_\rho\tilde{\zeta}_\sigma=(E_\perp^\mu n^\nu-n^\mu E_\perp^\nu)$, etc.

We define the projection operators

$$\mathcal{P}_T^{\mu\nu}(P)=g^{\mu\nu}-\frac{n^\mu n^\nu}{n^2}-\frac{\tilde{P}^\mu\tilde{P}^\nu}{\tilde{P}^2}, \tag{3.2}$$

$$\mathcal{P}_L^{\mu\nu}(P)=\frac{n^\mu n^\nu}{n^2}, \tag{3.3}$$

$$\mathcal{P}_G^{\mu\nu}(P)=\frac{\tilde{P}^\mu\tilde{P}^\nu}{\tilde{P}^2}. \tag{3.4}$$

Although, $n^2=1$, we have written n^2 explicitly for later convenience. In the above definitions, "T," "L," and "G" stand, in respective order, for transverse, longitudinal, and gauge fixing. [Following tradition, we call $n^\mu n^\nu/n^2$ in Eq. (3.3) the "longitudinal projection operator."] From Eqs. (3.1)–(3.4), one can show that

$$\tilde{P}_\mu\mathcal{P}_U^{\mu\nu}=\mathcal{P}_U^{\nu\mu}\tilde{P}_\mu=\delta_{UG}\tilde{P}^\nu,$$

$$n_\mu\mathcal{P}_U^{\mu\nu}=\mathcal{P}_U^{\nu\mu}n_\mu=\delta_{UL}n^\nu,$$

$$\tilde{\zeta}_\mu\mathcal{P}_U^{\mu\nu}=\mathcal{P}_U^{\nu\mu}\tilde{\zeta}_\mu=\delta_{UT}\tilde{\zeta}^\nu,$$

$$(E_\perp)_\mu\mathcal{P}_U^{\mu\nu}=\mathcal{P}_U^{\nu\mu}(E_\perp)_\mu=\delta_{UT}E_\perp^\nu. \tag{3.5}$$

Let $\mathbf{A}$ be a generic second-rank tensor in Minkowski space, whose $(\mu\nu)$ component is $(\mathbf{A})^{\mu\nu}=A^{\mu\nu}$. $A^{\mu\nu}$ is decomposed as

$$A^{\mu\nu}(P)=\sum_{U,V=T,L,G}\mathcal{P}_U^{\mu\rho}(A_{UV})_{\rho\sigma}\mathcal{P}_V^{\sigma\nu}$$

$$\equiv\sum_{U,V=T,L,G}(\mathcal{P}_U\cdot A_{UV}\cdot\mathcal{P}_V)^{\mu\nu}, \tag{3.6}$$

$$A_{TT}^{\mu\nu}=A_1^{TT}\mathcal{P}_T^{\mu\nu}+A_2^{TT}\tilde{\zeta}^\mu\tilde{\zeta}^\nu-A_3^{TT'}\tilde{\zeta}^\mu E_\perp^\nu+A_3^{T'T}E_\perp^\mu\tilde{\zeta}^\nu,$$

$$A_{LL}^{\mu\nu}=A_1^{LL}\mathcal{P}_L^{\mu\nu},$$

$$A_{GG}^{\mu\nu}=A_1^{GG}\mathcal{P}_G^{\mu\nu},$$

$$A_{TL}^{\mu\nu}=A_1^{TL}\tilde{\zeta}^\mu n^\nu+A_2^{TL}E_\perp^\mu n^\nu,$$

$$A_{LT}^{\mu\nu}=A_1^{LT}n^\mu\tilde{\zeta}^\nu-A_2^{LT}n^\mu E_\perp^\nu,$$

$$A_{TG}^{\mu\nu}=A_1^{TG}E_\perp^\mu\tilde{P}^\nu+A_2^{TG}\tilde{\zeta}^\mu\tilde{P}^\nu,$$

$$A_{GT}^{\mu\nu}=A_1^{GT}\tilde{P}^\mu E_\perp^\nu-A_2^{GT}\tilde{P}^\mu\tilde{\zeta}^\nu,$$

$$A_{LG}^{\mu\nu}=A_1^{LG}n^\mu\tilde{P}^\nu,$$

$$A_{GL}^{\mu\nu}=-A_1^{GL}\tilde{P}^\mu n^\nu. \tag{3.7}$$

From Eq. (3.5) follows $(\mathcal{P}_U\cdot A_{UV}\cdot\mathcal{P}_V)^{\mu\nu}=A_{UV}^{\mu\nu}$ $(U,V=T,L,G)$. We call Eqs. (3.6) and (3.7) the SF's and refer $A_{UV}^{\mu\nu}$ $(U,V=T,L,G)$ or A_j^{UV} $(U,V=T,T',L,G)$ to as a SF element of $A^{\mu\nu}$. It is to be understood that the (bare and self-energy-part resummed) propagators and the self-energy part, which appear in the following, are to be written in the SF.

B. Bare propagators

1. Bare gluon propagator

First of all, we note that the bare propagator matrix $\hat{\mathbf{D}}(P)$ and the self-energy-part resummed propagator matrix $\hat{\mathbf{G}}(P)$ enjoy the "symmetry" property,

$$(\hat{D}^{\mu\nu}(P))^*=-\hat{\tau}_1{}^t\hat{D}^{\nu\mu}(P)\hat{\tau}_1,$$

$$(\hat{G}^{\mu\nu}(P))^*=-\hat{\tau}_1{}^t\hat{G}^{\nu\mu}(P)\hat{\tau}_1,$$

$$\hat{D}^{\mu\nu}(P)={}^t\hat{D}^{\nu\mu}(-P),$$

$$\hat{G}^{\mu\nu}(P)={}^t\hat{G}^{\nu\mu}(-P). \tag{3.8}$$

The first two equations result from the Hermiticity of the density matrix.

$\hat{D}^{\mu\nu}(P)$ is an inverse of

$$(\hat{D}^{-1}(P))^{\mu\nu}=-\left[P^2g^{\mu\nu}-P^\mu P^\nu+\frac{1}{\lambda}\tilde{P}^\mu\tilde{P}^\nu\right]\hat{\tau}_3 \tag{3.9}$$

with λ a gauge parameter. A general solution to $(\mathbf{D}^{-1}\mathbf{D})^{\mu\nu}=g^{\mu\nu}$ is written as

$$\hat{\mathbf{D}}=\hat{\mathbf{D}}^{(0)}+\mathbf{D}_K\hat{M}_+, \tag{3.10}$$

$$\hat{\mathbf{D}}^{(0)}=\hat{\mathbf{D}}_{RA}+\tilde{f}(\mathbf{D}_R-\mathbf{D}_A)\hat{M}_+, \tag{3.11}$$

$$\hat{\mathbf{D}}_{RA}=\begin{pmatrix}\mathbf{D}_R & 0\\ \mathbf{D}_R-\mathbf{D}_A & -\mathbf{D}_A\end{pmatrix}, \tag{3.12}$$

$$D_K^{\mu\nu}=(D_K)_{TT}^{\mu\nu}=-\tilde{C}^{\mu\nu}[\Delta_R(P)-\Delta_A(P)], \tag{3.13}$$

$$\tilde{C}^{\mu\nu}=C_2^{TT}(P)\tilde{\zeta}^\mu\tilde{\zeta}^\nu-C_3^{TT'}(P)\tilde{\zeta}^\mu E_\perp^\nu+C_3^{T'T}(P)E_\perp^\mu\tilde{\zeta}^\nu, \tag{3.14}$$

where

$$\tilde{f}(P)=\theta(p_0)N(|p_0|,\vec{p})-\theta(-p_0)[1+N(|p_0|,-\vec{p})],$$

$$D_R^{\mu\nu}=(D_A^{\mu\nu})^*=-\Delta_R\mathcal{P}_T^{\mu\nu}-\frac{1}{\tilde{P}^2}\left[1+\lambda\frac{p_0^2}{\tilde{P}^2}\right]\mathcal{P}_L^{\mu\nu}$$
$$-\frac{\lambda}{\tilde{P}^2}\mathcal{P}_G^{\mu\nu}-\lambda\frac{p_0}{\tilde{P}^4}(\tilde{P}^\mu n^\nu+n^\mu\tilde{P}^\nu). \tag{3.15}$$

Here N is the number density of the transverse gluon and $\Delta_{R(A)}$ is as in Eq. (2.11). From Eqs. (3.8), (3.10), and (3.13), we have

$$(C_2^{TT})^*=C_2^{TT},\quad (C_3^{TT'})^*=-C_3^{T'T}. \tag{3.16}$$

Note that, for the quasiuniform systems near equilibrium, C_2^{TT}, $C_3^{TT'}$, and $C_3^{T'T}$ are small when compared to $\tilde{f}$.

2. Bare ghost propagator

A bare Fadeev-Popov (FP) ghost propagator $\hat{\tilde{D}}$ is

$$\hat{\tilde{D}}=\frac{1}{\tilde{P}^2}\hat{\tau}_3. \tag{3.17}$$

C. Dyson equation

1. Gluon sector

The self-energy-part $(\hat{\mathbf{\Pi}})$ resummed propagator $\hat{\mathbf{G}}$ obeys

$$\hat{\mathbf{G}}(P)=\hat{\mathbf{D}}(P)-\hat{\mathbf{D}}(P)\hat{\mathbf{\Pi}}(P)\hat{\mathbf{G}}(P). \tag{3.18}$$

From Eq. (3.8), we obtain the symmetry relations, for the SF elements of $\hat{G}^{\mu\nu}$,

$$(\hat{G}_j^{UV}(P))^*=-\sigma_j^{UV}\hat{\tau}_1{}^t\hat{G}_j^{VU}(P)\hat{\tau}_1,$$

$$\hat{G}_j^{UV}(P)={}^t\hat{G}_j^{VU}(-P)\quad(U,V=T,T',L,G), \tag{3.19}$$

where $\sigma_j^{UV}=\sigma_j^{VU}$ with $\sigma_j^{UU}=+$ $(U=T,L,G)$ and

$$\sigma_j^{UV}=\begin{cases}+ & \text{for } (UV,j)=(TL,1),(TG,1)\\ - & \text{for } (UV,j)=(TT',3),(TL,2),\\ & \qquad (TG,2),(LG,1).\end{cases}$$

Similar relations hold for $(\hat{\Pi}_j)_{UV}$'s.

Components of $\hat{\mathbf{\Pi}}$ follow the same relation as Eq. (2.19) and then $\hat{\mathbf{\Pi}}$ is written as

$$\hat{\mathbf{\Pi}}=\hat{\mathbf{\Pi}}^{(0)}-\mathbf{\Pi}_K\hat{M}_-, \tag{3.20}$$

$$\hat{\mathbf{\Pi}}^{(0)}=\begin{pmatrix}\mathbf{\Pi}_R & 0\\ -\mathbf{\Pi}_R+\mathbf{\Pi}_A & -\mathbf{\Pi}_A\end{pmatrix}+\tilde{f}(\mathbf{\Pi}_R-\mathbf{\Pi}_A)\hat{M}_-, \tag{3.21}$$

$$\mathbf{\Pi}_R=\mathbf{\Pi}_{11}+\mathbf{\Pi}_{12}, \tag{3.22}$$

$$\mathbf{\Pi}_A=\mathbf{\Pi}_{11}+\mathbf{\Pi}_{21}, \tag{3.23}$$

$$\mathbf{\Pi}_K=(1+\tilde{f})\mathbf{\Pi}_{12}-\tilde{f}\mathbf{\Pi}_{21}. \tag{3.24}$$

From Eq. (3.19) with $\hat{\Pi}_j$ for $\hat{G}_j$, we obtain the symmetry relations $(U,V,=T,T',L,G)$

$$(\Pi_{Aj}^{UV}(P))^*=\sigma_j^{UV}\Pi_{Rj}^{VU}(P), \tag{3.25}$$

$$(\Pi_{Kj}^{UV}(P))^*=-\sigma_j^{UV}\Pi_{Kj}^{VU}(P), \tag{3.26}$$

$$\Pi_{Rj}^{UV}(P)=\Pi_{Aj}^{VU}(-P)=\sigma_j^{UV}(\Pi_{Rj}^{UV}(-P))^*,$$

$$\Pi_{Kj}^{UV}(P)=\Pi_{Kj}^{VU}(-P)-\epsilon(p_0)[N(|p_0|,\vec{p})-N(|p_0|,-\vec{p})][\Pi_{Rj}^{VU}(-P)-\Pi_{Aj}^{VU}(-P)].$$

Components of $\mathbf{G}$ follow the same relation as Eq. (2.25) and then $\hat{\mathbf{G}}$ is written as

$$\hat{\mathbf{G}}=\hat{\mathbf{G}}^{(0)}+\mathbf{G}_K\hat{M}_+, \tag{3.27}$$

$$\hat{\mathbf{G}}^{(0)}=\begin{pmatrix}\mathbf{G}_R & 0\\ \mathbf{G}_R-\mathbf{G}_A & -\mathbf{G}_A\end{pmatrix}+\tilde{f}(\mathbf{G}_R-\mathbf{G}_A)\hat{M}_+, \tag{3.28}$$

$$\mathbf{G}_R=\mathbf{G}_{11}-\mathbf{G}_{12}, \tag{3.29}$$

$$\mathbf{G}_A=\mathbf{G}_{11}-\mathbf{G}_{21}, \tag{3.30}$$

$$\mathbf{G}_K=(1+\tilde{f})\mathbf{G}_{12}-\tilde{f}\mathbf{G}_{21}. \tag{3.31}$$

For equilibrium systems, $\mathbf{\Pi}_K=\mathbf{G}_K=0$. From Eq. (3.19), follows the symmetry relations $(U,V,=T,T',L,G)$

$$(G_{Aj}^{UV}(P))^*=\sigma_j^{UV}G_{Rj}^{VU}(P), \tag{3.32}$$

$$(G_{Kj}^{UV}(P))^*=-\sigma_j^{UV}G_{Kj}^{VU}(P), \tag{3.33}$$

$$G_{Rj}^{UV}(P)=G_{Aj}^{VU}(-P)=\sigma_j^{UV}(G_{Rj}^{UV}(-P))^*, \tag{3.34}$$

$$G_{Kj}^{UV}(P)=G_{Kj}^{VU}(-P)+\epsilon(p_0)[N(|p_0|,\vec{p})-N(|p_0|,-\vec{p})][G_{Rj}^{VU}(-P)-G_{Aj}^{VU}(-P)]. \tag{3.35}$$

Substitution of Eqs. (3.10), (3.20), and (3.27) into Eq. (3.18) yields

$$\hat{\mathbf{G}}^{(0)}=\hat{\mathbf{D}}^{(0)}-\hat{\mathbf{D}}^{(0)}\hat{\mathbf{\Pi}}\hat{\mathbf{G}}^{(0)}, \tag{3.36}$$

$$\mathbf{G}_K=\mathbf{D}_K-\mathbf{D}_R\mathbf{\Pi}_R\mathbf{G}_K+\mathbf{D}_R\mathbf{\Pi}_K\mathbf{G}_A-\mathbf{D}_K\mathbf{\Pi}_A\mathbf{G}_A. \tag{3.37}$$

Equation (3.36) is formally solved to give

$$\mathbf{G}_{R(A)}=[\mathbf{D}^{-1}+\mathbf{\Pi}_{R(A)}]^{-1}. \tag{3.38}$$

For later convenience, we rewrite Eq. (3.38) as [cf. Eq. (3.9)]

$$\mathbf{G}_R=[\mathbf{D}_0^{-1}+\mathbf{\Pi}'_R]^{-1}, \tag{3.39}$$

$$(D_0^{-1})^{\mu\nu}=-\tilde{P}^2\left[\mathcal{P}_T^{\mu\nu}+\mathcal{P}_L^{\mu\nu}+\frac{1}{\lambda}\mathcal{P}_G^{\mu\nu}\right], \tag{3.40}$$

$$\Pi_R'^{\mu\nu}=\Pi_R^{\mu\nu}-p_0^2(\mathcal{P}_T^{\mu\nu}+\mathcal{P}_G^{\mu\nu})+p_0(n^\mu\tilde{P}^\nu+\tilde{P}^\mu n^\nu). \tag{3.41}$$

The SF for $\mathbf{G}_{R(A)}$ will be given in the next section.

As for $\mathbf{G}_K$, Eq. (3.37), through similar procedure as in the quark case (cf. Appendix A), we obtain

$$\mathbf{G}_K=\mathbf{G}_K^{(1)}+\mathbf{G}_K^{(2)}+\mathbf{G}_K^{(3)},$$

$$\mathbf{G}_K^{(1)}=\mathbf{G}_R\mathbf{\Pi}_K\mathbf{G}_A,$$

$$\mathbf{G}_K^{(2)}=-\mathbf{G}_R[\tilde{\mathbf{C}}\mathbf{\Pi}_A-\mathbf{\Pi}_R\tilde{\mathbf{C}}]\mathbf{G}_A\equiv-\mathbf{G}_R\tilde{\mathbf{H}}_l\mathbf{G}_A, \tag{3.42}$$

$$\mathbf{G}_K^{(3)}=\mathbf{G}_R\tilde{\mathbf{C}}-\tilde{\mathbf{C}}\mathbf{G}_A,$$

where $\tilde{\mathbf{C}}$ is as in Eq. (3.14). SF elements of $\tilde{\mathbf{H}}_l$ are given in the next section. The SF elements of $\mathbf{G}_K$ are obtained by repeatedly using the formulas in Appendix C.

2. Ghost sector

The self-energy-part $(\hat{\tilde{\Pi}})$ resummed propagator $\hat{\tilde{G}}$ obeys

$$\hat{\tilde{G}}(P)=\hat{\tilde{D}}(P)[1+\hat{\tilde{\Pi}}(P)\hat{\tilde{G}}(P)]=[1+\hat{\tilde{G}}(P)\hat{\tilde{\Pi}}(P)]\hat{\tilde{D}}. \tag{3.43}$$

Since $\hat{\tilde{D}}$, Eq. (3.17), is a diagonal (2×2)-matrix, $\hat{\tilde{\Pi}}$ is also diagonal. Then, from Eq. (3.43), $\hat{\tilde{G}}$ is diagonal also. Among the components of $\hat{\tilde{\Pi}}$ $(\hat{\tilde{G}})$, there is the same relation as Eq. (2.19) [Eq. (2.25)]. Then we have

$$\hat{\tilde{\Pi}}=\tilde{\Pi}\,\hat{\tau}_3,\quad \hat{\tilde{G}}=\tilde{G}\,\hat{\tau}_3 \tag{3.44}$$

with $\tilde{\Pi}$ and $\tilde{G}$ real, and

$$\tilde{G}(P)=\frac{1}{\tilde{P}^2-\tilde{\Pi}(P)}=-\frac{1}{\vec{p}^2+\tilde{\Pi}(P)}. \tag{3.45}$$

D. Self-energy-part resummed gluon propagator

1. Form for $(G_{R(A)})_{UV}$

Through a Slavnov-Taylor identity, $(\hat{G}_1)_{UG}$ ($U=G,T,L$) and $(\hat{G}_2)_{TG}$ [and then also $(\hat{G}_1)_{GU}$ and $(\hat{G}_2)_{GT}$ via Eq. (3.19)] are related to the self-energy-part resummed FP-ghost propagator $\hat{\tilde{G}}$ and the FP-ghost "pre-self-energy part"[1] $\hat{\tilde{\Pi}}_\mu$.

The Slavnov-Taylor identity reads [7]:

$$\hat{G}_{\mu\nu}\tilde{P}^\nu=\lambda[\hat{\tau}_3\hat{\tilde{\Pi}}_\mu-P_\mu]\hat{\tilde{G}}=\lambda[\hat{\tau}_3(\hat{\tilde{\Pi}}_\mu-p_0n_\mu\hat{\tau}_3)-\tilde{P}_\mu]\hat{\tilde{G}}. \tag{3.46}$$

Here $\hat{\tilde{G}}(P)$ is as in Eq. (3.44) with Eq. (3.45). As in Eq. (3.44), $\hat{\tilde{\Pi}}_\mu$ is a diagonal (2×2) matrix,

$$\hat{\tilde{\Pi}}_\mu=\tilde{\Pi}_\mu\hat{\tau}_3 \quad (\tilde{\Pi}_\mu^*=\tilde{\Pi}_\mu). \tag{3.47}$$

Substitution of the SF for $\hat{G}^{\mu\nu}$ into Eq. (3.46) yields

$$G_{R1}^{GG}(P)=G_{A1}^{GG}(P)=\lambda\frac{1}{\tilde{P}^2}[\tilde{\Pi}(P)-\tilde{P}^2]\tilde{G}(P)=-\lambda\frac{1}{\tilde{P}^2}, \tag{3.48}$$

$$G_{R1}^{TG}(P)=G_{A1}^{TG}(P)=\lambda\frac{1}{\tilde{P}^2E_\perp^2}[E_\perp^\mu\tilde{\Pi}_\mu(P)]\tilde{G}(P), \tag{3.49}$$

$$G_{R2}^{TG}(P)=G_{A2}^{TG}(P)=\lambda\frac{1}{\tilde{P}^2\tilde{\zeta}^2}[\tilde{\zeta}^\mu\tilde{\Pi}_\mu(P)]\tilde{G}(P), \tag{3.50}$$

$$G_{R1}^{LG}(P)=G_{A1}^{LG}(P)=\lambda\frac{1}{\tilde{P}^2n^2}[n^\mu\tilde{\Pi}_\mu(P)-p_0]\tilde{G}(P), \tag{3.51}$$

$$G_{K1}^{GG}=G_{K1}^{TG}=G_{K2}^{TG}=G_{K1}^{LG}=0. \tag{3.52}$$

All the above quantities are real. In deriving Eq. (3.48), Eq. (3.45) has been used. Substituting Eqs. (3.48)–(3.52) into Eq. (3.35), we obtain $G_{Kj}^{GT}=G_{K1}^{GL}=0$. From the above formulas, we see that $\hat{G}_{UG}^{\mu\nu}$ ($U=T,L,G$) and $\hat{G}_{GU}^{\mu\nu}$ ($U=T,L$) vanish in the strict Coulomb gauge ($\lambda=0$).

We are now in a position to obtain $(\mathbf{G}_R)_{UV}$ ($U\neq G,V\neq G$) from Eq. (3.39). We divide $(\mathbf{G}_R)_{UV}$ into two pieces, $(\mathbf{G}_R)_{UV}=(\mathbf{G}_R^{(\lambda=0)})_{UV}+(\mathbf{G}_R^{(\lambda)})_{UV}$, the latter of which vanishes in the strict Coulomb gauge ($\lambda=0$).

Straightforward manipulation of Eq. (3.39) using the formulas in Appendix C yields, for the SF elements of $\mathbf{G}^{(\lambda=0)}$ ($\equiv\mathbf{G}_R^{(\lambda=0)}$) ($\Pi_j^{UV}\equiv\Pi_{Rj}^{UV}$),

$$\mathcal{D}G_1^{(\lambda=0)TT}=-(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})(\tilde{P}^2-\Pi_1^{LL})+\tilde{\zeta}^2n^2\Pi_1^{TL}\Pi_1^{LT},$$

$$\mathcal{D}G_2^{(\lambda=0)TT}=-(\tilde{P}^2-\Pi_1^{LL})\Pi_2^{TT}+\tilde{P}^2n^4\Pi_2^{TL}\Pi_2^{LT}+n^2\Pi_1^{TL}\Pi_1^{LT},$$

$$\mathcal{D}G_3^{(\lambda=0)T'T}=-(\tilde{P}^2-\Pi_1^{LL})\Pi_3^{T'T}-n^2\Pi_1^{LT}\Pi_2^{TL},$$

$$\mathcal{D}G_1^{(\lambda=0)LL}=-(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})(\tilde{P}^2-\Pi_1'^{TT})+\tilde{P}^2\tilde{\zeta}^4n^2\Pi_3^{TT'}\Pi_3^{T'T},$$

$$\mathcal{D}G_2^{(\lambda=0)TL}=-(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})\Pi_2^{TL}-\tilde{\zeta}^2\Pi_1^{TL}\Pi_3^{T'T},$$

$$\mathcal{D}G_1^{(\lambda=0)LT}=-(\tilde{P}^2-\Pi_1'^{TT})\Pi_1^{LT}-\tilde{P}^2\tilde{\zeta}^2n^2\Pi_2^{LT}\Pi_3^{T'T}, \tag{3.53}$$

where

$$\begin{aligned}\mathcal{D}=&[(\tilde{P}^2-\Pi_1'^{TT})(\tilde{P}^2-\Pi_1^{LL})-\tilde{P}^2\tilde{\zeta}^2n^4\Pi_2^{LT}\Pi_2^{TL}]\\&\times(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})\\&-\tilde{\zeta}^2n^2\Pi_1^{LT}[(\tilde{P}^2-\Pi_1'^{TT})\Pi_1^{TL}+\tilde{P}^2\tilde{\zeta}^2n^2\Pi_3^{TT'}\Pi_2^{TL}]\\&+\vec{p}^2\tilde{\zeta}^4\Pi_3^{T'T}[(\tilde{P}^2-\Pi_1^{LL})\Pi_3^{TT'}+n^2\Pi_1^{TL}\Pi_2^{LT}].\end{aligned}$$

Here we note that, from Eq. (3.41), $\tilde{P}^2-\Pi_1'^{TT}=P^2-\Pi_1^{TT}$ holds. The SF elements of the gauge-parameter dependent part $\mathbf{G}^{(\lambda)}\equiv\mathbf{G}_R^{(\lambda)}$ read

[1] $\hat{\tilde{\Pi}}_\mu$ is evaluated by replacing the vertex factor $gC_{abc}\tilde{P}^\mu$ at the "end vertex" with gC_{abc}. Here the end vertex is the vertex from which the outgoing ghost comes out of the diagram. Then, the ghost self-energy part $\hat{\tilde{\Pi}}$ is related to $\hat{\tilde{\Pi}}_\mu$ through $\hat{\tilde{\Pi}}=\tilde{P}^\mu\hat{\tilde{\Pi}}_\mu$.

$$\mathcal{D}G_1^{(\lambda)TT} = -\tilde{P}^4\tilde{\zeta}^2 n^2[(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})\{(\tilde{P}^2-\Pi_1^{LL})\Pi_1^{GT}+n^2\Pi_1'^{GL}\Pi_2^{LT}\}$$
$$+\tilde{\zeta}^2\Pi_2^{GT}\{(\tilde{P}^2-\Pi_1^{LL})\Pi_3^{TT'}+n^2\Pi_1^{TL}\Pi_2^{LT}\}-\tilde{\zeta}^2 n^2\Pi_1^{LT}\{\Pi_1^{GT}\Pi_1^{TL}-\Pi_1'^{GL}\Pi_3^{TT'}\}]G_1^{TG},$$

$$\mathcal{D}G_2^{(\lambda)TT} = -\frac{\tilde{P}^2}{\tilde{\zeta}^2\Pi_3^{T'T}}[\{(\tilde{P}^2-\Pi_1^{LL})\Pi_2^{GT}+n^2\Pi_1'^{GL}\Pi_1^{LT}\}(\tilde{P}^2-\Pi_1'^{TT})^2+\tilde{P}^2\tilde{\zeta}^2 n^4\Pi_2^{TL}(\tilde{P}^2-\Pi_1'^{TT})\{\Pi_1^{LT}\Pi_1^{GT}-\Pi_2^{LT}\Pi_2^{GT}\}$$
$$-\tilde{P}^2\tilde{\zeta}^4 n^2\Pi_3^{T'T}\{(\tilde{P}^2-\Pi_1^{LL})(\Pi_2^{GT}\Pi_3^{TT'}-\Pi_1^{GT}\Pi_2^{TT})+n^2\Pi_1'^{GL}(\Pi_1^{LT}\Pi_3^{TT'}-\Pi_2^{TT}\Pi_2^{LT})+n^2\Pi_1^{TL}(\Pi_2^{GT}\Pi_2^{LT}$$
$$-\Pi_1^{GT}\Pi_1^{LT})\}]G_1^{TG}-\frac{n^2}{\tilde{\zeta}^2}\frac{\Pi_2^{TL}G_1^{LG}}{\Pi_3^{T'T}G_1^{TG}}G_3^{(\lambda)T'T}\mathcal{D}+\frac{\tilde{P}^2}{\tilde{\zeta}^4}\frac{\Pi_1^{TG}}{\Pi_3^{T'T}}G_2^{GT}\mathcal{D},$$

$$\mathcal{D}G_3^{(\lambda)T'T} = \tilde{P}^2[-\tilde{P}^2\tilde{\zeta}^2 n^2\Pi_1^{GT}\{(\tilde{P}^2-\Pi_1^{LL})\Pi_3^{T'T}+n^2\Pi_1^{LT}\Pi_2^{TL}\}-\Pi_2^{GT}\{(\tilde{P}^2-\Pi_1'^{TT})(\tilde{P}^2-\Pi_1^{LL})-\tilde{P}^2\tilde{\zeta}^2 n^4\Pi_2^{TL}\Pi_2^{LT}\}$$
$$-n^2\Pi_1'^{GL}\{(\tilde{P}^2-\Pi_1'^{TT})\Pi_1^{LT}+\tilde{P}^2\tilde{\zeta}^2 n^2\Pi_2^{LT}\Pi_3^{T'T}\}]G_1^{TG},$$

$$\mathcal{D}G_1^{(\lambda)LL} = \vec{p}^2[(\tilde{P}^2-\Pi_1'^{TT}-\tilde{\zeta}^2\Pi_2^{TT})\{(\tilde{P}^2-\Pi_1'^{TT})\Pi_1'^{GL}+\tilde{P}^2\tilde{\zeta}^2 n^2\Pi_1^{GT}\Pi_2^{TL}\}$$
$$+\tilde{\zeta}^2\Pi_2^{GT}\{(\tilde{P}^2-\Pi_1'^{TT})\Pi_1^{TL}+\tilde{P}^2\tilde{\zeta}^2 n^2\Pi_2^{TL}\Pi_3^{TT'}\}-\tilde{P}^2\tilde{\zeta}^4 n^2\Pi_3^{T'T}\{\Pi_1'^{GL}\Pi_3^{TT'}-\Pi_1^{GT}\Pi_1^{TL}\}]G_1^{LG},$$

$$\mathcal{D}G_2^{(\lambda)TL} = \tilde{P}^2[-\Pi_1'^{GL}(\tilde{P}^2-\Pi_1'^{TT})^2+\tilde{P}^2\tilde{\zeta}^4 n^2\Pi_1^{GT}\{\Pi_2^{TT}\Pi_2^{TL}-\Pi_1^{TL}\Pi_3^{T'T}\}$$
$$+\tilde{\zeta}^2(\tilde{P}^2-\Pi_1'^{TT})\{-\tilde{P}^2 n^2\Pi_1^{GT}\Pi_2^{TL}+\Pi_2^{TT}\Pi_1'^{GL}-\Pi_2^{GT}\Pi_1^{TL}\}-\tilde{P}^2\tilde{\zeta}^4 n^2\Pi_3^{TT'}\{\Pi_2^{GT}\Pi_2^{TL}-\Pi_1'^{GL}\Pi_3^{T'T}\}]G_1^{TG},$$

$$G_1^{(\lambda)LT} = G_3^{(\lambda)T'T}\frac{G_1^{LG}}{G_1^{TG}}. \tag{3.54}$$

Here G_1^{TG} $(\equiv G_{R1}^{TG})$ and G_1^{LG} $(\equiv G_{R1}^{LG})$ are as in Eqs. (3.49) and (3.51), respectively. G_2^{GT} $(\equiv G_{R2}^{GT})$ is obtained from G_{A2}^{TG}, Eq. (3.50), with the help of Eq. (3.32).

$(\mathbf{G}_A)_{UV}$ is obtained from the above formulas with the substitutions Π_j^{UV} $(\equiv \Pi_{Rj}^{UV})\to\Pi_{Aj}^{UV}$. $G_{R3}^{TT'}$, G_{R1}^{TL}, and G_{R2}^{LT} are obtained, in respective order, from $G_{A3}^{T'T}$, G_{A1}^{LT}, and G_{A2}^{TL} with the help of Eq. (3.32) or Eq. (3.34).

2. Form for $(\tilde{H}_l)_{UV}$ in Eq. (3.42)

Straightforward computation using the formulas in Appendix C yields, for the SF elements of $\tilde{H}_l$,

$$(\tilde{H}_l)_1^{TT} = 2i\tilde{\zeta}^2 E_\perp^2 \mathrm{Im}(C_3^{TT'}\Pi_{R3}^{T'T}),$$

$$(\tilde{H}_l)_2^{TT} = 2i\,\mathrm{Im}\{-C_2^{TT}(\Pi_{R1}^{TT}+\tilde{\zeta}^2\Pi_{R2}^{TT})$$
$$+E_\perp^2 C_3^{T'T}(\Pi_{R3}^{TT'}+\Pi_{A3}^{TT'})\},$$

$$(\tilde{H}_l)_3^{TT'} = \tilde{\zeta}^2 C_2^{TT}\Pi_{A3}^{TT'}-C_3^{TT'}(2i\,\mathrm{Im}\Pi_{R1}^{TT}+\tilde{\zeta}^2\Pi_{R2}^{TT}),$$

$$(\tilde{H}_l)_1^{TL} = \tilde{\zeta}^2 C_2^{TT}\Pi_{A1}^{TL}-E_\perp^2 C_3^{TT'}\Pi_{A2}^{TL},$$

$$(\tilde{H}_l)_2^{TL} = \tilde{\zeta}^2 C_3^{T'T}\Pi_{A1}^{TL}. \tag{3.55}$$

$(\tilde{H}_l)_3^{T'T}$, $(\tilde{H}_l)_1^{LT}$, and $(\tilde{H}_l)_2^{LT}$ are obtained using Eqs. (3.32) and (3.35). Other SF-elements than the above ones vanish.

IV. GRADIENT PARTS OF THE PROPAGATORS AND THE GENERALIZED BOLTZMANN EQUATIONS

Here, we deduce the gradient terms of the quark and gluon propagators, and derive generalized Boltzmann equations and their relatives. Procedure goes parallel to those in [8–10], and then we describe briefly.

A. Quark sector

A configuration-space counterpart of $F(P,X)$ is denoted by $\underline{F}(x,y)$:

$$\underline{F}(x,y) = \int \frac{d^4P}{(2\pi)^4}e^{-iP\cdot(x-y)}F(P,X) \quad \left(X=\frac{x+y}{2}\right),$$
$$\equiv (F(P,X))_{\mathrm{IWT}}(x,y).$$

If $F(P,X)$ is independent of X, $\underline{F}(x,y)=\underline{F}(x-y)$ $\equiv (F(P))_{\mathrm{IFT}}(x-y)$. Here "IWT" ("IFT") stands for an inverse Wigner (Fourier) transform.

1. Preliminary

Configuration-space counterparts of Eqs. (2.4) and (2.5) are, with obvious notation,

$$\underline{A}(x,y)=\sum_{\rho,\sigma=\pm}[\underline{\mathcal{P}}_\rho\cdot\underline{A}^{\rho\sigma}\cdot\underline{\mathcal{P}}_\sigma](x,y), \tag{4.1}$$

$$\underline{A}^{\rho\rho}=\underline{A}_1^{\rho\rho}+\tfrac{1}{2}[\underline{A}_2^{\rho\rho}\cdot(i\partial\!\!\!/)+(i\partial\!\!\!/)\cdot\underline{A}_2^{\rho\rho}]+\tfrac{1}{2}[\underline{A}_3^{\rho\rho}\cdot\underline{N\!\!\!/}+\underline{N\!\!\!/}\cdot\underline{A}_3^{\rho\rho}]+\tfrac{1}{2}[\underline{A}_4^{\rho\rho}\cdot(i\partial\!\!\!/)\cdot\underline{N\!\!\!/}+(i\partial\!\!\!/)\cdot\underline{N\!\!\!/}\cdot\underline{A}_4^{\rho\rho}],$$

$$\underline{A}^{\rho-\rho}=\gamma_5[\underline{A}_1^{\rho-\rho}+\tfrac{1}{2}[\underline{A}_2^{\rho-\rho}\cdot(i\partial\!\!\!/)+(i\partial\!\!\!/)\cdot\underline{A}_2^{\rho-\rho}]+\tfrac{1}{2}(\underline{A}_3^{\rho-\rho}\cdot\underline{N\!\!\!/}+\underline{N\!\!\!/}\cdot\underline{A}_3^{\rho-\rho})+\tfrac{1}{2}[\underline{A}_4^{\rho-\rho}\cdot(i\partial\!\!\!/)\cdot\underline{N\!\!\!/}+(i\partial\!\!\!/)\cdot\underline{N\!\!\!/}\cdot\underline{A}_4^{\rho-\rho}]].$$

Here we have used the shorthand notation $\underline{F}\cdot\underline{G}$, which is a function whose "(x,y) component" is

$$[\underline{F}\cdot\underline{G}](x,y)=\int d^4z\,\underline{F}(x,z)\underline{G}(z,y). \tag{4.2}$$

For later use, we display the Wigner transform of $\underline{F}_G=\underline{F}\cdot\underline{G}$:

$$F_G(P,X)=F(P,X)G(P,X)-\frac{i}{2}\{F(P,X),\ \ G(P,X)\}, \tag{4.3}$$

which is valid to the gradient approximation. The "Poisson bracket" in Eq. (4.3) is defined as

$$\{F,G\}\equiv\frac{\partial F}{\partial X^\mu}\frac{\partial G}{\partial P_\mu}-\frac{\partial F}{\partial P_\mu}\frac{\partial G}{\partial X^\mu}. \tag{4.4}$$

2. Bare propagator and counter-Lagrangian

We proceed as in [9]. We start from the expression for the free propagator $\underline{\hat{S}}(x,y)$ [cf. Eqs. (2.8)–(2.10)],

$$\underline{\hat{S}}(x,y)=\sum_{\rho=\pm}\underline{\mathcal{P}}_\rho\cdot\underline{\hat{S}}_\rho\cdot\underline{\mathcal{P}}_\rho+\underline{S}_K\hat{M}_+,$$

$$\underline{\hat{S}}_\rho=\begin{pmatrix}\underline{S}_R & 0\\ \underline{S}_R-\underline{S}_A & -\underline{S}_A\end{pmatrix}-(\underline{S}_R\cdot\underline{f}_\rho-\underline{f}_\rho\cdot\underline{S}_A)\hat{M}_+,$$

$$\underline{S}_K=\sum_{\rho=\pm}\underline{\mathcal{P}}_\rho\cdot(\underline{S}_R\cdot\gamma_5\underline{N\!\!\!/}\cdot\underline{C}_{\rho-\rho}-\underline{C}_{\rho-\rho}\cdot\gamma_5\underline{N\!\!\!/}\cdot\underline{S}_A)\cdot\underline{\mathcal{P}}_{-\rho}.$$

For the time being, $\underline{f}_\rho(x,y)$ and $\underline{C}_{\rho-\rho}(x,y)$ in the above equations are left to be arbitrary. Specification of them will be made in Sec. IV 5. $\hat{S}$ in Eq. (2.8) is the leading part of the derivative expansion (DEX) of $\hat{S}(P,X)$ $[=(\underline{\hat{S}}(x,y))_{WT}]$. Here "WT" indicates to take Wigner transformation. Straightforward calculation within the gradient approximation yields

$$\underline{\hat{S}}^{-1}\cdot\underline{\hat{S}}=\underline{\hat{S}}\cdot\underline{\hat{S}}^{-1}=1, \tag{4.5}$$

$$\underline{\hat{S}}^{-1}(x,y)=(i\partial\!\!\!/_x-m)\delta^4(x-y)\hat{\tau}_3-\underline{L}_c(x,y)\hat{M}_-,$$

$$\underline{L}_c=\underline{L}_{c1}+\underline{L}_{c2},$$

$$L_{c1}=i\sum_{\rho=\pm}(\partial\!\!\!/_X f_\rho)\mathcal{P}_\rho=i\sum_{\rho=\pm}\mathcal{P}_\rho\left[\frac{P\cdot\partial_X}{P^2}P\!\!\!/+\frac{N\cdot\partial_X}{N^2}N\!\!\!/+\frac{\rho\epsilon(p_0)}{e_\perp^2}(e_\perp\cdot\partial_X)P\!\!\!/N\!\!\!/\right]f_\rho\mathcal{P}_\rho, \tag{4.6}$$

$$\begin{aligned}L_{c2}&=i\sum_{\rho=\pm}\mathcal{P}_\rho\gamma_5\left[\frac{\partial N\!\!\!/}{\partial P_\alpha}\frac{\partial C_{\rho-\rho}}{\partial X^\alpha}(P\!\!\!/-m)+(\partial\!\!\!/_X C_{\rho-\rho})N\!\!\!/\right]\mathcal{P}_{-\rho}\\&=i\sum_{\rho=\pm}\mathcal{P}_\rho\gamma_5\left[\left\{-\frac{\rho\epsilon(p_0)}{N^2}N\!\!\!/e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}-\frac{P\!\!\!/N\!\!\!/}{2N^2}\frac{\partial N^2}{\partial P_\alpha}-m\left(-\frac{P\!\!\!/}{P^2}N^\alpha+\frac{N\!\!\!/}{2N^2}\frac{\partial N^2}{\partial P_\alpha}-\frac{\rho\epsilon(p_0)}{e_\perp^2}P\!\!\!/N\!\!\!/e_\perp^\mu\frac{\partial N^\mu}{\partial P_\alpha}\right)\right\}\right.\\&\quad\left.\times\frac{\partial C_{\rho-\rho}}{\partial X^\alpha}+\left\{\frac{\rho\epsilon(p_0)}{P^2}P\!\!\!/(e_\perp\cdot\partial_X)+\frac{P\!\!\!/N\!\!\!/}{P^2}(P\cdot\partial_X)\right\}\times C_{\rho-\rho}\right]\mathcal{P}_{-\rho}.\end{aligned} \tag{4.7}$$

Here $1/P^2\equiv\mathbf{P}/P^2$, $1/N^2\equiv\mathbf{P}/N^2$, and $1/e_\perp^2=\mathbf{P}/e_\perp^2$, with $\mathbf{P}$ denoting to take the principal part.

Equation (4.5) tells us that the free action of the theory [6,9] is

$$\mathcal{A}_0=\int d^4x\,d^4y\,\bar{\hat{\psi}}(x)\underline{\hat{S}}^{-1}(x,y)\hat{\psi}(y), \tag{4.8}$$

$$\bar{\hat{\psi}}=(\bar{\psi}_1,\ \bar{\psi}_2),\quad \hat{\psi}=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

Since the term with $\underline{L}_c(x,y)$ ($\in\underline{\hat{S}}^{-1}$) in $\mathcal{A}_0$ is absent in the original action, we should introduce a counteraction to compensate it:

$$\mathcal{A}_c=\int d^4x\,d^4y\,\bar{\hat{\psi}}(x)\underline{L}_c(x,y)\hat{M}_-\hat{\psi}(y), \tag{4.9}$$

which yields a (two-point) vertex factor

$$i(\underline{L}_c(x,y))_{WT}\hat{M}_-=i[L_{c1}(P,X)+L_{c2}(P,X)]\hat{M}_-. \tag{4.10}$$

3. Dyson equation

Let us start with considering a "product" of $\underline{A}$ and $\underline{B}$ of the type (4.1) [cf. Eq. (4.2)],

$$\underline{C}(x,y)=[\underline{A}\cdot\underline{B}](x,y)=\left[\sum_{\rho,\xi,\sigma=\pm}\underline{\mathcal{P}}_\rho\cdot\underline{A}^{\rho\xi}\cdot\underline{\mathcal{P}}_\xi\cdot\underline{B}^{\xi\sigma}\cdot\underline{\mathcal{P}}_\sigma\right](x,y). \tag{4.11}$$

Using Eq. (4.3), we obtain, for the Wigner transform of $\underline{C}$ to the gradient approximation,

$$C(P,X)=\sum_{\rho,\xi,\sigma=\pm}\left[\mathcal{P}_\rho A^{\rho\xi}(P,X)B^{\xi\sigma}(P,X)\mathcal{P}_\sigma + \frac{i}{2}\mathcal{P}_\rho\left\{A^{\rho\xi}\frac{\partial\mathcal{P}_\xi}{\partial P_\mu}\frac{\partial B^{\xi\sigma}}{\partial X^\mu}-\frac{\partial A^{\rho\xi}}{\partial X^\mu}\frac{\partial\mathcal{P}_\xi}{\partial P_\mu}B^{\xi\sigma}\right\}\mathcal{P}_\sigma + \cdots\right], \tag{4.12}$$

where "$\cdots$" stands for other pieces of the gradient terms than the second term. Thanks to the relation

$$\mathcal{P}_{\pm\rho}\frac{\partial\mathcal{P}_\rho}{\partial P_\mu}=\frac{\partial\mathcal{P}_\rho}{\partial P_\mu}\mathcal{P}_{\mp\rho},$$

the second term vanishes, $\mathcal{P}_\rho\{\cdots\}\mathcal{P}_\sigma=0$. Then, to the gradient approximation, $\underline{C}(x,y)$ in Eq. (4.11) may be written as

$$\underline{C}(x,y)\simeq\left[\sum_{\rho,\xi,\sigma=\pm}\underline{\mathcal{P}}_\rho\cdot\underline{A}^{\rho\xi}\cdot\underline{B}^{\xi\sigma}\cdot\underline{\mathcal{P}}_\sigma\right](x,y).$$

Thus, as in Eq. (2.15), we can use the (2×2)-matrix notation in a polarization space, $(\underline{\mathbf{A}})^{\rho\sigma}=\underline{A}^{\rho\sigma}$ $(\rho,\sigma=\pm)$.

The self-energy-part $[\hat{\underline{\Sigma}}(x,y)]$ resummed propagator $\hat{\underline{\mathbf{G}}}$ obeys

$$\hat{\underline{\mathbf{G}}}=\hat{\underline{\mathbf{S}}}+\hat{\underline{\mathbf{S}}}\cdot\hat{\underline{\boldsymbol{\Sigma}}}\cdot\hat{\underline{\mathbf{G}}}=\hat{\underline{\mathbf{S}}}+\hat{\underline{\mathbf{G}}}\cdot\hat{\underline{\boldsymbol{\Sigma}}}\cdot\hat{\underline{\mathbf{S}}}, \tag{4.13}$$

$$\hat{\underline{\mathbf{S}}}=\begin{pmatrix}\underline{S}_R & 0\\ \underline{S}_R-\underline{S}_A & -\underline{S}_A\end{pmatrix}\mathbf{1}-(\underline{S}_R\cdot\underline{\mathbf{f}}-\underline{\mathbf{f}}\cdot\underline{S}_A-\underline{\mathbf{S}}_K)\hat{M}_+,$$

$$\underline{\mathbf{S}}_K=\underline{S}_R\cdot\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{C}}-\underline{\mathbf{C}}\cdot\gamma_5\underline{\not{N}}\cdot\underline{S}_A,$$

$$\underline{\mathbf{C}}=\begin{pmatrix}0 & \underline{C}_{+-}\\ \underline{C}_{-+} & 0\end{pmatrix}.$$

For $\hat{\underline{\mathbf{G}}}$ and $\hat{\underline{\boldsymbol{\Sigma}}}$, we have [cf. Eqs. (2.26)–(2.30), (2.20)–(2.24)]

$$\hat{\underline{\mathbf{G}}}=\begin{pmatrix}\underline{\mathbf{G}}_R & 0\\ \underline{\mathbf{G}}_R-\underline{\mathbf{G}}_A & -\underline{\mathbf{G}}_A\end{pmatrix}-[\underline{\mathbf{G}}_R\cdot\underline{\mathbf{f}}-\underline{\mathbf{f}}\cdot\underline{\mathbf{G}}_A-\underline{\mathbf{G}}_K]\hat{M}_+, \tag{4.14}$$

$$\hat{\underline{\boldsymbol{\Sigma}}}=\begin{pmatrix}\underline{\boldsymbol{\Sigma}}_R & 0\\ -\underline{\boldsymbol{\Sigma}}_R+\underline{\boldsymbol{\Sigma}}_A & -\underline{\boldsymbol{\Sigma}}_A\end{pmatrix}-[\underline{\boldsymbol{\Sigma}}_R\cdot\underline{\mathbf{f}}-\underline{\mathbf{f}}\cdot\underline{\boldsymbol{\Sigma}}_A+\underline{\boldsymbol{\Sigma}}_K]\hat{M}_-,$$

$$\underline{\mathbf{G}}_K=\underline{\mathbf{G}}_R\cdot\underline{\mathbf{f}}-\underline{\mathbf{f}}\cdot\underline{\mathbf{G}}_A+\underline{\mathbf{G}}_{12},$$

$$\underline{\boldsymbol{\Sigma}}_K=-\underline{\boldsymbol{\Sigma}}_R\cdot\underline{\mathbf{f}}+\underline{\mathbf{f}}\cdot\underline{\boldsymbol{\Sigma}}_A+\hat{\underline{\boldsymbol{\Sigma}}}_{12}. \tag{4.15}$$

Equation (4.13) may be solved to give

$$\underline{\mathbf{G}}_{R(A)}(x,y)=[\mathbf{1}(i\not{\partial}_x-m)^{-1}\delta^4(x-y)-\underline{\boldsymbol{\Sigma}}_{R(A)}(x,y)]^{-1}, \tag{4.16}$$

$$\underline{\mathbf{G}}_K=\underline{\mathbf{G}}_K^{[1]}+\underline{\mathbf{G}}_K^{[2]}+\underline{\mathbf{G}}_K^{[3]}, \tag{4.17}$$

$$\underline{\mathbf{G}}_K^{[1]}=-\underline{\mathbf{G}}_R\cdot\underline{\boldsymbol{\Sigma}}_K\cdot\underline{\mathbf{G}}_A, \tag{4.18}$$

$$\underline{\mathbf{G}}_K^{[2]}=\underline{\mathbf{G}}_R\cdot[\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{C}}\cdot\underline{\boldsymbol{\Sigma}}_A-\underline{\boldsymbol{\Sigma}}_R\cdot\underline{\mathbf{C}}\cdot\gamma_5\underline{\not{N}}]\cdot\underline{\mathbf{G}}_A \equiv\underline{\mathbf{G}}_R\cdot\underline{\mathbf{H}}\cdot\underline{\mathbf{G}}_A, \tag{4.19}$$

$$\underline{\mathbf{G}}_K^{[3]}=\underline{\mathbf{G}}_R\cdot\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{C}}-\underline{\mathbf{C}}\cdot\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{G}}_A. \tag{4.20}$$

The form for the leading part of the DEX of $\hat{G}(X,P)$ $[=(\underline{G}(x,y))_{WT}]$ is the $\hat{G}$ that is deduced in Sec. II.

4. Gradient piece of the self-energy-part resummed propagator

Form for $\mathbf{G}_{R(A)}$

From Eq. (4.16), we obtain, for the component $\underline{G}_R^{\rho\sigma}$,

$$\underline{G}_R^{\rho\rho}(x,y)=[(i\not{\partial}-m)\delta^4(x-y)-\underline{\Sigma}_R^{\rho\rho}-\underline{\Sigma}_R^{\rho-\rho}\cdot\underline{G}_R^{(\mathrm{pre})-\rho-\rho}\cdot\underline{\Sigma}_R^{-\rho\rho}]^{-1}, \tag{4.21}$$

$$\underline{G}_R^{\rho-\rho}(x,y)=\underline{G}_R^{(\mathrm{pre})\rho\rho}\cdot\underline{\Sigma}_R^{\rho-\rho}\cdot\underline{G}_R^{-\rho-\rho}=\underline{G}_R^{\rho\rho}\cdot\underline{\Sigma}_R^{\rho-\rho}\cdot\underline{G}_R^{(\mathrm{pre})-\rho-\rho}, \tag{4.22}$$

$$\underline{G}_R^{(\mathrm{pre})\rho\rho}(x,y)=[(i\not{\partial}-m)\delta^4(x-y)-\underline{\Sigma}_R^{\rho\rho}(x,y)]^{-1}. \tag{4.23}$$

Solving Eqs. (4.21) and (4.22), we write the solutions as $G_R^{\rho\sigma}=G_R^{(0)\rho\sigma}+G_R^{(1)\rho\sigma}$. Here, $G_R^{(0)\rho\sigma}$ is the leading part of the DEX of $G_R^{\rho\sigma}$, whose form has been obtained in Sec. II. The gradient part $G_R^{(1)\rho\sigma}$ is obtained as

$$\begin{aligned}G_R^{(1)\rho\rho}&=\frac{i}{2}G_R^{(0)\rho\rho}[\{(G_R^{(0)\rho\rho})^{-1},G_R^{(0)\rho\rho}\}\\&\quad-\{\Sigma_R^{\rho-\rho}|G_R^{(\mathrm{pre})-\rho-\rho}|\Sigma_R^{-\rho\rho}\}G_R^{(0)\rho\rho}\\&\quad-\Sigma_R^{\rho-\rho}\{G_R^{(\mathrm{pre})-\rho-\rho},\Sigma_R^{-\rho\rho}\}G_R^{(0)\rho\rho}\\&\quad-\{\Sigma_R^{\rho-\rho},G_R^{(\mathrm{pre})-\rho-\rho}\}\Sigma_R^{-\rho\rho}G_R^{(0)\rho\rho}]\\&=\frac{i}{2}[\{G_R^{(0)\rho\rho},(G_R^{(0)\rho\rho})^{-1}\}\\&\quad-G_R^{(0)\rho\rho}\{\Sigma_R^{\rho-\rho}|G_R^{(\mathrm{pre})-\rho-\rho}|\Sigma_R^{-\rho\rho}\}\\&\quad-G_R^{(0)\rho\rho}\{\Sigma_R^{\rho-\rho},G_R^{(\mathrm{pre})-\rho-\rho}\}\Sigma_R^{-\rho\rho}\\&\quad-G_R^{(0)\rho\rho}\Sigma_R^{\rho-\rho}\{G_R^{(\mathrm{pre})-\rho-\rho},\Sigma_R^{-\rho\rho}\}]G_R^{(0)\rho\rho},\end{aligned}$$

$$G_R^{(1)\rho-\rho} = -\frac{i}{2}\Big[\{G_R^{(\mathrm{pre})\rho\rho},\Sigma_R^{\rho-\rho}\}G_R^{(0)-\rho-\rho} + \{G_R^{(\mathrm{pre})\rho\rho}|\Sigma_R^{\rho-\rho}|G_R^{(0)-\rho-\rho}\} + G_R^{(\mathrm{pre})\rho\rho}\{\Sigma_R^{\rho-\rho},G_R^{(0)-\rho-\rho}\}\Big]$$
$$= -\frac{i}{2}\Big[\{G_R^{(0)\rho\rho},\Sigma_R^{\rho-\rho}\}G_R^{(\mathrm{pre})-\rho-\rho} + \{G_R^{(0)\rho\rho}|\Sigma_R^{\rho-\rho}|G_R^{(\mathrm{pre})-\rho-\rho}\} + G_R^{(0)\rho\rho}\{\Sigma_R^{\rho-\rho},G_R^{(\mathrm{pre})-\rho-\rho}\}\Big], \tag{4.24}$$

where

$$\{A|B|C\} \equiv \frac{\partial A}{\partial X^\mu}B\frac{\partial C}{\partial P_\mu} - \frac{\partial A}{\partial P_\mu}B\frac{\partial C}{\partial X^\mu}. \tag{4.25}$$

Then, $\underline{G}_R(x,y)$ is written as [cf. Eq. (4.1)]

$$\underline{G}_R(x,y) = \sum_{\rho,\sigma=\pm}[\underline{\mathcal{P}}_\rho\cdot(G_R^{\rho\sigma}(P,X))_{\mathrm{IWT}}\cdot\underline{\mathcal{P}}_\sigma](x,y).$$

We write the solution to Eq. (4.23) as $G_R^{(\mathrm{pre})\rho\rho} = G_R^{(\mathrm{pre})(0)\rho\rho} + G_R^{(\mathrm{pre})(1)\rho\rho}$. The form for $G_R^{(\mathrm{pre})(0)\rho\rho}$ is given in Sec. II, while the form for $G_R^{(\mathrm{pre})(1)\rho\rho}$ is given by Eq. (4.24) with the following replacements:

$$G_R^{(0)\rho\sigma}\to G_R^{(\mathrm{pre})(0)\rho\sigma}, \quad G_R^{(1)\rho\sigma}\to G_R^{(\mathrm{pre})(1)\rho\sigma}, \quad G_R^{(\mathrm{pre})\mp\rho\mp\rho}\to 0.$$

$G_A(P,X)$ is obtained from $G_R(P,X)$ with Σ_A's for Σ_R's.

Form for Σ_K, which is involved in $G_K^{[1]}$ in Eq. (4.18)

Computation of Eq. (4.15) to the gradient approximation yields

$$\underline{\Sigma}_K = \underline{L}_{c1} + \underline{L}_{c2} - \sum_{\rho,\sigma=\pm}\underline{\mathcal{P}}_\rho\cdot[(\Sigma_R^{\rho\sigma}f_\sigma)_{\mathrm{IWT}} - (f_\rho\Sigma_A^{\rho\sigma})_{\mathrm{IWT}} - \underline{\Sigma}_{12}^{\rho\sigma}]\cdot\underline{\mathcal{P}}_\sigma + \Sigma_K^{[1]}, \tag{4.26}$$

$$\Sigma_K^{[1]} = -\frac{i}{2}\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho[\{f_\sigma,\Sigma_R^{\rho\sigma}\} + \{f_\rho,\Sigma_A^{\rho\sigma}\}]\mathcal{P}_\sigma. \tag{4.27}$$

$\underline{L}_{c1}$ and $\underline{L}_{c2}$ in Eq. (4.26) come from $\mathcal{A}_c$ in Eq. (4.9) [see Eq. (4.10)]. The standard forms for $\Sigma_K^{[1]}$, H [Eq. (4.19)], and $\gamma_5\underline{\not N}\cdot\underline{\mathbf{C}}$ and $\underline{\mathbf{C}}\cdot\gamma_5\underline{\not N}$ in Eq. (4.20) are given in Appendix E.

5. Perturbation theories—generalized Boltzmann equations and their relatives

The aim of this section is to construct perturbation theories. We are employing the interaction picture in the sense of [11]. Then, the quark-gluon system of our concern is characterized by a density matrix ρ at an initial time $X^0 = X_i^0$, from which $f_\rho(P,X_i^0,\vec{X})$ and $C_{\rho-\rho}(P,X_i^0,\vec{X})$ [cf. Eqs. (2.8)–(2.10)] are determined. It should be emphasized that there is no information at this stage on how $f_\rho(P,X)$ and $C_{\rho-\rho}(P,X)$ evolve in spacetime. Then, in the course of construction of a perturbative framework, certain evolution equations that describe the spacetime evolution for f_ρ and $C_{\rho-\rho}$ should be settled. As a matter of fact, one can choose any forms for the evolution equations, on the basis of which a perturbative framework is constructed. Different frameworks are physically equivalent in the sense that they lead to the same result for the physical quantities (see below for more details). In the sequel, we construct two kinds of perturbative frameworks by employing two different forms for the evolution equations.

As seen from Eq. (4.14), the propagator $\hat{G}$ is written in terms of G_R, G_A, and G_K. $G_R(P,X)$ $[G_A(P,X)]$ is analytic in an upper [a lower] half complex p_0 plane. Then, in calculating some quantity, the parts of $\hat{G}$ that are proportional to G_R or to G_A yield well-defined contributions. Now, we observe that $G_K^{[1]}$ and $G_K^{[2]}$, Eqs. (4.18) and (4.19), contain G_RG_A. Since G_A is essentially the complex conjugate of G_R, G_RG_A is disastrously large on the energy shells,[2] $p_0 = \pm\omega_\pm(\pm\vec{p},X)$, on which

$$\mathrm{Re}[G_R^{\rho\rho}(P,X)]^{-1}|_{p_0=\pm\omega_\pm(\pm\vec{p},X)} = 0. \tag{4.28}$$

As a matter of fact, in the narrow-width approximation, $\mathrm{Im}(G_R^{\rho\rho})^{-1}\to\epsilon(p_0)0^+$, $G_R^{\rho\rho}G_A^{\rho\rho}$ develops pinch singularities at the energy shells in a p_0 plane.[3] Then, $G_K^{[1]}$ and $G_K^{[2]}$ yield diverging contribution. In practice, $\mathrm{Im}(G_R^{\rho\rho})^{-1}$ $(\propto g^2)$ is a small quantity, so that the contributions, although not divergent, are large, which invalidates the perturbative scheme. These large contributions come from the vicinities of the energy shells, on which $\mathrm{Re}(G_R^{\rho\rho})^{-1}\sim 0$.

Appropriate use of the first and second equalities of Eq. (2.36) together with Eq. (2.31) shows that $G_R^{\rho\rho}G_A^{\pm\rho\mp\rho}$ and $G_R^{\rho-\rho}G_A^{\pm\rho\mp\rho}$ do not yield large contributions. This is because, in general, the energy shells of $G_R^{\rho\rho}$ and $G_R^{-\rho-\rho}$, and of $G_R^{(\mathrm{pre})\rho\rho}$ and $G_R^{(\mathrm{pre})-\rho-\rho}$, do not coincide. For the case of $f_+ = f_-$, however, this is not the case. For $G_R^{\rho\rho}G_A^{\pm\rho\mp\rho}$, a large contribution emerges from the region $\mathrm{Re}(G_R^{\rho\rho})^{-1}\sim 0$, and, for $G_R^{\rho-\rho}G_A^{\pm\rho\mp\rho}$, large contributions emerge from the regions $\mathrm{Re}(G_R^{\rho\rho})^{-1}\sim 0$ and from the region $\mathrm{Re}(G_R^{(\mathrm{pre})\rho\rho})^{-1}\sim 0$.

From Eqs. (4.18) and (4.19) with Eq. (4.26), we have the following for the $(\rho\rho)$ and $(\rho-\rho)$ components $(\rho=\pm)$ of $\underline{H}-\underline{\Sigma}_K$ $[=\underline{G}_R^{-1}\cdot(\underline{G}_K^{[1]}+\underline{G}_K^{[2]})\cdot\underline{G}_A^{-1}]$:

$$(\underline{H}^{\rho\rho}-\underline{\Sigma}_K^{\rho\rho})_{WT} = -i\left[\frac{P\cdot\partial_X}{P^2}\not P + \frac{N\cdot\partial_X}{N^2}\not N + \frac{\rho\epsilon(p_0)}{e_\perp^2}(e_\perp\cdot\partial_X)\not P\not N\right]f_\rho + i\tilde{\Gamma}_p^{\rho\rho} - \Sigma_B^{\rho\rho}, \tag{4.29}$$

[2]How to find the solution to Eq. (4.28) is given in Appendix F.
[3]This is a characteristic feature of nonequilibrium dynamics [12].

$$(\underline{H}^{\rho-\rho}-\underline{\Sigma}_K^{\rho-\rho})_{WT} = -i\gamma_5\Bigg[\Bigg\{\frac{m}{P^2}(N\cdot\partial_X)+\frac{\rho\epsilon(p_0)}{P^2}(e_\perp\cdot\partial_X)\Bigg\}\not{P} - \Bigg\{\frac{\rho\epsilon(p_0)}{N^2}e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}+\frac{m}{2N^2}\frac{\partial N^2}{\partial P_\alpha}\Bigg\}\not{N}\frac{\partial}{\partial X^\alpha} - \Bigg\{\frac{1}{2N^2}\frac{\partial N^2}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}-m\frac{\rho\epsilon(p_0)}{e_\perp^2}e_\perp^\mu\frac{\partial N^\mu}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha} - \frac{P\cdot\partial_X}{P^2}\Bigg\}\not{P}\not{N}\Bigg]C_{\rho-\rho}+i\tilde{\Gamma}_p^{\rho-\rho}-\Sigma_B^{\rho-\rho}, \tag{4.30}$$

$$\tilde{\Gamma}_p^{\rho\sigma}=i[(1-f_\sigma)\Sigma_{12}^{\rho\sigma}+f_\rho\Sigma_{21}^{\rho\sigma}+(f_\rho-f_\sigma)\Sigma_{11}^{\rho\sigma}], \tag{4.31}$$

$$\Sigma_B^{\rho\sigma}=\Sigma_K^{[1]\rho\sigma}-[(\gamma_5\underline{\not{N}}\cdot\underline{C}\cdot\underline{\mathbf{\Sigma}}_A-\underline{\mathbf{\Sigma}}_R\cdot\underline{C}\cdot\gamma_5\underline{\not{N}})^{\rho\sigma}]_{\mathrm{WT}}. \tag{4.32}$$

The first term on the RHS of Eq. (4.29) [Eq. (4.30)] comes from the counter-Lagrangian $\underline{L}_{c1}$ ($\underline{L}_{c2}$), Eq. (4.6) [Eq. (4.7)], in Eq. (4.26).

For later reference, we note that the physical number densities $N_\pm^{(\mathrm{ph})}(P,X)$ and $\overline{N}_\pm^{(\mathrm{ph})}(P,X)$ are obtained through computing current density,

$$\langle j^\mu(x)\rangle \equiv \mathrm{Tr}[\overline{\psi}(x)\gamma^\mu\psi(x)\rho] = -\frac{i}{2}\mathrm{Tr}\{\gamma^\mu[\underline{G}_{21}(x,x)+\underline{G}_{12}(x,x)]\rho\}. \tag{4.33}$$

Similarly, the physical $C_{\pm\mp}(P,X)$, $C_{\pm\mp}^{(\mathrm{ph})}(P,X)$, is obtained from

$$\langle j_5^\mu(x)\rangle \equiv \mathrm{Tr}[\overline{\psi}(x)\gamma_5\gamma^\mu\psi(x)\rho] = -\frac{i}{2}\mathrm{Tr}\{\gamma_5\gamma^\mu[\underline{G}_{21}(x,x)+\underline{G}_{12}(x,x)]\rho\}. \tag{4.34}$$

Bare-N scheme

As has been emphasized at the beginning of this section, $f_\rho(P,X)$ in Eq. (4.29) and $C_{\rho-\rho}(P,X)$ in Eq. (4.30) ($X_i^0 < X^0$) have not been defined so far. For the purpose of determining them, we impose here the condition that the counter-Lagrangian L_c is absent, $L_c=0$:

$$P\cdot\partial_X f_\rho^{(\mathrm{B})}=N\cdot\partial_X f_\rho^{(\mathrm{B})}=e_\perp\cdot\partial_X f_\rho^{(\mathrm{B})}=0, \tag{4.35}$$

$$[m(N\cdot\partial_X)+\rho\epsilon(p_0)(e_\perp\cdot\partial_X)]C_{\rho-\rho}^{(\mathrm{B})} = \left[\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}+\frac{m}{2}\frac{\partial N^2}{\partial P_\alpha}\right]\frac{\partial C_{\rho-\rho}^{(\mathrm{B})}}{\partial X^\alpha} = \left[\frac{P^2}{2}\frac{\partial N^2}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}+m\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N^\mu}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha} - N^2(P\cdot\partial_X)\right]C_{\rho-\rho}^{(\mathrm{B})}=0, \tag{4.36}$$

where we have written $f_\rho^{(\mathrm{B})}$ ($C_{\rho-\rho}^{(\mathrm{B})}$) for f_ρ ($C_{\rho-\rho}$). Then, Eq. (4.29) [Eq. (4.30)], of which the first term on the RHS is absent, is to be solved under the given initial data $f_\rho(P,X_i^0,\vec{X})$ [$C_{\rho-\rho}(P,X_i^0,\vec{X})$]. Equation (4.35) is a "free Boltzmann equation" and its relatives for the bare number densities, $N_\rho^{(\mathrm{B})}(p_0,\vec{p},X)=\theta(p_0)f_\rho^{(\mathrm{B})}(P,X)$ and $\overline{N}_\rho^{(\mathrm{B})}(|p_0|,\vec{p},X)=1-\theta(-p_0)f_\rho^{(\mathrm{B})}(p_0,-\vec{p},X)$ [cf. Eq. (2.12)].

The physical number densities, which are obtained from $\langle j^\mu(x)\rangle$ [Eq. (4.33)], and the physical $C_{\rho-\rho}^{(\mathrm{ph})}$, which is obtained from $\langle j_5^\mu(x)\rangle$ [Eq. (4.34)], are functionals of $f_\sigma^{(\mathrm{B})}$ and $C_{\sigma-\sigma}^{(\mathrm{B})}$:

$$f_\rho^{(\mathrm{ph})}(P,X)=\theta(p_0)N_\rho^{(\mathrm{ph})}(p_0,\vec{p},X)+\theta(-p_0)\times[1-\overline{N}_\rho^{(\mathrm{ph})}(|p_0|,-\vec{p},X)] = \mathcal{F}_\rho(P,X;[f_\sigma^{(\mathrm{B})}],[C_{\sigma-\sigma}^{(\mathrm{B})}]),$$

$$C_{\rho-\rho}^{(\mathrm{ph})}(P,X)=\mathcal{G}_\rho(P,X;[f_\sigma^{(\mathrm{B})}],[C_{\sigma-\sigma}^{(\mathrm{B})}]).$$

$\mathcal{F}_\rho$ and $\mathcal{G}_\rho$ here contain large contributions mentioned above. Solving these equations for $f_\rho^{(\mathrm{B})}$ and $C_{\rho-\rho}^{(\mathrm{B})}$, one obtains

$$f_\rho^{(\mathrm{B})}=f_\rho^{(\mathrm{B})}(P,X;[f_\sigma^{(\mathrm{ph})}],[C_{\sigma-\sigma}^{(\mathrm{ph})}]), \tag{4.37}$$

$$C_{\rho-\rho}^{(\mathrm{B})}=C_{\rho-\rho}^{(\mathrm{B})}(P,X;[f_\sigma^{(\mathrm{ph})}],[C_{\sigma-\sigma}^{(\mathrm{ph})}]). \tag{4.38}$$

In the case of scalar theory [8], the physical number density is shown to obey the generalized Boltzmann equation.

Computation of some physical quantity yields the expression $F([f_\rho^{(\mathrm{B})}],[C_{\rho-\rho}^{(\mathrm{B})}])$, which includes large contribution. Substituting the RHS's of Eq. (4.37) and of Eq. (4.38) for, in respective order, $f_\rho^{(\mathrm{B})}$ and $C_{\rho-\rho}^{(\mathrm{B})}$ in F, one obtains the expression $F'([f_\rho^{(\mathrm{ph})}],[C_{\rho-\rho}^{(\mathrm{ph})}])$, which does not include large contributions.

The perturbation theory thus constructed is called the "bare-N scheme" in [8].

Physical-N scheme

Here we aim at constructing a perturbation theory, on the basis of which no large contributions appear. Then, in such a scheme, there are no large terms in the relations between $(f_\rho^{(\mathrm{ph})},C_{\rho-\rho}^{(\mathrm{ph})})$ and $(f_\rho,C_{\rho-\rho})$. This is achieved if the condition Eq. (4.29) = Eq. (4.30) = 0 could be imposed. This is, however, not possible. Nevertheless, it is possible to construct the scheme that is free from the large contributions.

For determining so far arbitrary $f_\rho(P,X)$ and $C_{\rho-\rho}(P,X)$ $(X_i^0<X^0)$, we impose the conditions

$$\mathrm{Tr}(\not{P}+\Omega_f(P,X))[\mathrm{Eq.\ (4.29)}]=\mathrm{Tr}\not{N}[\mathrm{Eq.\ (4.29)}] = \mathrm{Tr}\not{P}\not{N}[\mathrm{Eq.\ (4.29)}]=0, \tag{4.39}$$

$$\mathrm{Tr}\gamma_5(\not{P}-m)\not{N}[\text{Eq. (4.30)}]=\mathrm{Tr}\gamma_5[\not{P}-\Omega_C(P,X)]\times[\text{Eq. (4.30)}]=\mathrm{Tr}\gamma_5\not{N}[\text{Eq. (4.30)}]=0. \tag{4.40}$$

Here, $\Omega_f(P,X)$ and $\Omega_C(P,X)$ are arbitrary functions with the property

$$\Omega_f(p_0=\pm\omega_\pm(\pm\vec{p},X),\vec{p},X)=\Omega_C(p_0=\pm\omega_\pm(\pm\vec{p},X),\vec{p},X)=\{[\omega_\pm(\pm\vec{p},X)]^2-\vec{p}^{\,2}\}^{1/2}. \tag{4.41}$$

As has been discussed in [8], this arbitrariness does not matter (see also Sec. IV A 6 below). Computation of Eqs. (4.39) and (4.40) yields, in respective order,

$$P\cdot\partial_X f_\rho=\tfrac{1}{4}\mathrm{Tr}(\not{P}+\Omega_f(P,X))[\tilde{\Gamma}_p^{\rho\rho}+i\Sigma_B^{\rho\rho}], \tag{4.42}$$

$$N\cdot\partial_X f_\rho=\tfrac{1}{4}\mathrm{Tr}\not{N}[\tilde{\Gamma}_p^{\rho\rho}+i\Sigma_B^{\rho\rho}], \tag{4.43}$$

$$e_\perp\cdot\partial_X f_\rho=\tfrac{1}{4}\rho\epsilon(p_0)\mathrm{Tr}\not{P}\not{N}[\tilde{\Gamma}_p^{\rho\rho}+i\Sigma_B^{\rho\rho}], \tag{4.44}$$

and

$$\left[N^2P\cdot\partial_X-\frac{1}{2}(P^2-m^2)\frac{\partial N^2}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}\right]C_{\rho-\rho}=-\frac{1}{4}\mathrm{Tr}\gamma_5(\not{P}-m)\not{N}[\tilde{\Gamma}_p^{\rho-\rho}+i\Sigma_B^{\rho-\rho}], \tag{4.45}$$

$$[mN\cdot\partial_X+\rho\epsilon(p_0)e_\perp\cdot\partial_X]C_{\rho-\rho}=-\tfrac{1}{4}\mathrm{Tr}\gamma_5[\not{P}-\Omega_C(P,X)][\tilde{\Gamma}_p^{\rho-\rho}+i\Sigma_B^{\rho-\rho}], \tag{4.46}$$

$$\left[\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}+\frac{m}{2}\frac{\partial N^2}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}\right]C_{\rho-\rho}=\frac{1}{4}\mathrm{Tr}\gamma_5\not{N}[\tilde{\Gamma}_p^{\rho-\rho}+i\Sigma_B^{\rho-\rho}]. \tag{4.47}$$

These equations are the determining equations for f_ρ and $C_{\rho-\rho}$, which are to be solved under the given initial data $f_\rho(P,X_i^0,\vec{X})$ and $C_{\rho-\rho}(P,X_i^0,\vec{X})$, respectively.

After imposition of Eqs. (4.42)–(4.47), $H^{\rho\pm\rho}-\Sigma_K^{\rho\pm\rho}$, Eqs. (4.29) and (4.30), turns out to be

$$H^{\rho\rho}-\Sigma_K^{\rho\rho}=\frac{i}{4P^2}(P^2-\Omega_f\not{P})\mathrm{Tr}(\tilde{\Gamma}_p^{\rho\rho}+i\Sigma_B^{\rho\rho}), \tag{4.48}$$

$$H^{\rho-\rho}-\Sigma_K^{\rho-\rho}=\frac{i}{4P^2}\gamma_5(P^2-\Omega_C\not{P})\mathrm{Tr}\gamma_5(\tilde{\Gamma}_p^{\rho-\rho}+i\Sigma_B^{\rho-\rho}). \tag{4.49}$$

On the energy shells $p_0=\pm\omega_\pm$, these quantities vanish, since $(P^2-\Omega_{f(C)}\not{P})(P^2+\Omega_{f(C)}\not{P})=P^2(P^2-\Omega_{f(C)}^2)$. Then, the above mentioned large contributions, which turn out to be diverging contributions in the narrow-width approximation, do not appear. Thus, $G_K^{[1]}+G_K^{[2]}$ turns out to be a well-behaved function. As a matter of fact, in the narrow-width approximation,

$$G_K^{[1]}+G_K^{[2]}\propto\frac{p_0\mp\omega_\pm}{(p_0\mp\omega_\pm)^2+(0^+)^2}=\frac{\mathbf{P}}{p_0\mp\omega_\pm}\quad(p_0\simeq\pm\omega_\pm),$$

which is a well-defined distribution. In particular, the relations between the physical $(f_\rho^{(\mathrm{ph})},C_{\rho-\rho}^{(\mathrm{ph})})$ and $(f_\rho,C_{\rho-\rho})$ contain no large term:

$$f_\rho^{(\mathrm{ph})}=f_\rho+\Delta f_\rho,\quad C_{\rho-\rho}^{(\mathrm{ph})}=C_{\rho-\rho}+\Delta C_{\rho-\rho} \tag{4.50}$$

with Δf_ρ and $\Delta C_{\rho-\rho}$ the perturbative corrections.

Proceeding as in [9], from Eq. (4.42) on the energy shells, one obtains a generalized Boltzmann equation. In fact, the term with $\tilde{\Gamma}_p^{\rho\rho}$ on the RHS of Eq. (4.42) is proportional to the net production rate. To avoid complete repetition, we do not reproduce it here.

6. Discussion

Here we like to mention a similarity between the two schemes presented here, the bare-N scheme and the physical-N scheme, and those in the ultraviolet (UV) renormalization schemes in quantum-field theory. For simplicity of presentation, taking a complex-scalar theory, we focus on the mass renormalization and do not mention the coupling constant and wave function renormalizations.

Summary of the UV-renormalization theory

"Bare" UV renormalization scheme. The free Lagrangian density reads $\mathcal{L}_0=-\phi^\dagger(x)(\partial^2+m_B^2)\phi(x)$ with m_B the bare mass. Computation of the physical mass M_{ph} yields $M_{\mathrm{ph}}=M_{\mathrm{ph}}(m_B)$, which includes diverging terms. Solving this equation for m_B, we have $m_B=m_B(M_{\mathrm{ph}})$. Perturbative computation of some physical quantity F yields the expression $F=F(m_B)$, which contains, in general, UV divergences. Substituting the equation $m_B=m_B(M_{\mathrm{ph}})$ for m_B in $F(m_B)$, one gets $F_R(M_{\mathrm{ph}})\equiv F(m_B(M_{\mathrm{ph}}))$, which is free from UV divergence.

Physical UV-renormalization scheme. One introduces new free Lagrangian $\mathcal{L}_0'=-\phi^\dagger(x)(\partial^2+m^2)\phi(x)$ with m the renormalized mass. Then, the counter-Lagrangian should be introduced, $\mathcal{L}_c=\mathcal{L}_0-\mathcal{L}_0'=\phi^\dagger(x)[m^2-m_B^2]\phi(x)$. $m^2-m_B^2$ is determined so that the perturbatively computed physical mass M_{ph} is free from the UV divergence. Thus, no diverging term is involved in the relation $M_{\mathrm{ph}}=M_{\mathrm{ph}}(m)$. However, there is arbitrariness in the definition of the finite part of m, which is determined by imposing some condition. This arbitrariness is called a "renormalization scheme dependence" (see, e.g., [13]). It is well known that, when one computes some physical quantity F up to, say, nth order of perturbation theory, the above arbitrariness affects F at the next to the nth order. The renormalization scheme, in which $m=M^{(\mathrm{ph})}$, is convenient for many cases.

Summary of the two schemes presented above

Bare-N scheme. No counter-Lagrangian is introduced. Computation of the physical number densities (that are related to $f_\rho^{(\mathrm{ph})}$) and $C_{\rho-\rho}^{(\mathrm{ph})}$, which are the functionals of $f_\rho^{(\mathrm{B})}$ and $C_{\rho-\rho}^{(\mathrm{B})}$, include large contributions. Perturbative computation of some quantity yields the expression, which is written in terms of $f_\rho^{(\mathrm{B})}$ and $C_{\rho-\rho}^{(\mathrm{B})}$ and includes large contributions. Rewriting it in terms of the physical quantities, $f_\rho^{(\mathrm{ph})}$ and $C_{\rho-\rho}^{(\mathrm{ph})}$, one obtains the large-contribution free form.

Physical-N scheme. We introduce a counter-Lagrangian $\underline{L}_c=\underline{L}_{c1}+\underline{L}_{c2}$, which is determined so that the perturbatively computed physical number densities and $C_{\rho-\rho}^{(\mathrm{ph})}$ do not contain large contributions. There is arbitrariness in the definition of the "finite parts" of f_ρ and $C_{\rho-\rho}$. The arbitrariness in the choice of the functions Ω_f [Eqs. (4.42) and (4.48)] and Ω_C [Eqs. (4.46) and (4.49)] is this arbitrariness. It is worth mentioning that, if we could choose Ω_f and Ω_C so that $f_\rho(P,X)=f_\rho^{(\mathrm{ph})}(P,X)$, Eq. (4.42) on the energy shell turns out to be a genuine (generalized) Boltzmann equation. In the opposite case, the function that obeys the generalized Boltzmann equation is f_ρ and the physical $f_\rho^{(\mathrm{ph})}$ is written as in Eq. (4.50).

Similar comment to the above one at the end of the *Physical UV renormalization scheme* may be made here.

Correspondence

Above observation discloses the correspondence between the two schemes presented here and those in the UV-renormalization scheme.

Bare scheme:

$$\int d^4x\, \mathcal{L}_0(x)\leftrightarrow\mathcal{A}_0 \quad \text{in Eq. (4.8) with } \underline{L}_c=0,$$

$$m_B\leftrightarrow f_\rho^{(\mathrm{B})} \quad \text{and} \quad C_{\rho-\rho}^{(\mathrm{B})},$$

$$M_{\mathrm{ph}}\leftrightarrow f_\rho^{(\mathrm{ph})} \quad \text{and} \quad C_{\rho-\rho}^{(\mathrm{ph})}.$$

Physical scheme:

$$\int d^4x\, \mathcal{L}_0'(x)\leftrightarrow\mathcal{A}_0,$$

$$\int d^4x\, \mathcal{L}_c(x)\leftrightarrow\mathcal{A}_c \quad \text{in Eq. (4.9)},$$

$$m\leftrightarrow f_\rho \quad \text{and} \quad C_{\rho-\rho},$$

$$M_{\mathrm{ph}}\leftrightarrow f_\rho^{(\mathrm{ph})} \quad \text{and} \quad C_{\rho-\rho}^{(\mathrm{ph})},$$

absence of

divergence$\leftrightarrow$large contribution,

arbitrariness in $m\leftrightarrow$arbitrariness in Ω_f and Ω_C,

scheme with

$$m=M^{(\mathrm{ph})}\leftrightarrow f_\rho=f_\rho^{(\mathrm{ph})} \quad \text{and} \quad C_{\rho-\rho}=C_{\rho-\rho}^{(\mathrm{ph})}.$$

B. Gluon sector

1. Preliminary

Configuration-space counterparts of Eqs. (3.6) and (3.7) are, with obvious notation,

$$\underline{A}^{\mu\nu}(x,y)=\sum_{U,V=T,L,G}\sum_{j=1}^{J_{UV}}[\underline{\mathcal{R}}_{Lj}^{UV}\cdot\underline{\widetilde{A}}_j^{UV}\cdot\underline{\mathcal{R}}_{Rj}^{UV}]^{\mu\nu}(x,y), \tag{4.51}$$

where $J_{TT}=4$, $J_{LL}=J_{GG}=J_{LG}=J_{GL}=1$, and $J_{TL}=J_{LT}=J_{TG}=J_{GT}=2$, and

$$\sum_{j=1}^{4}[\underline{\mathcal{R}}_j^{TT}\cdot\underline{\widetilde{A}}_j^{TT}\cdot\underline{\mathcal{R}}_j^{TT}]^{\mu\nu}\equiv\underline{\mathcal{P}}_T^{\mu\rho}\cdot\underline{A}_1^{TT}\cdot(\underline{\mathcal{P}}_T)_\rho{}^\nu+\underline{\tilde{\zeta}}^\mu\cdot\underline{A}_2^{TT}\cdot\underline{\tilde{\zeta}}^\nu$$
$$-\underline{\tilde{\zeta}}^\mu\cdot\underline{A}_3^{TT'}\cdot\underline{E}_\perp^\nu+\underline{E}_\perp^\mu\cdot\underline{A}_3^{T'T}\cdot\underline{\tilde{\zeta}}^\nu,$$

$$[\underline{\mathcal{R}}_j^{LL}\cdot\underline{\widetilde{A}}_j^{LL}\cdot\underline{\mathcal{R}}_j^{LL}]^{\mu\nu}\equiv n^\mu\underline{A}_1^{LL}n^\nu,$$

$$[\underline{\mathcal{R}}_j^{GG}\cdot\underline{\widetilde{A}}_j^{GG}\cdot\underline{\mathcal{R}}_j^{GG}]^{\mu\nu}\equiv(i\tilde{\partial}^\mu)\underline{A}_1^{GG}(i\tilde{\partial}^\nu),$$

$$\sum_{j=1}^{2}[\underline{\mathcal{R}}_j^{TL}\cdot\underline{\widetilde{A}}_j^{TL}\cdot\underline{\mathcal{R}}_j^{TL}]^{\mu\nu}\equiv\underline{\tilde{\zeta}}^\mu\cdot\underline{A}_1^{TL}n^\nu+\underline{E}_\perp^\mu\cdot\underline{A}_2^{TL}n^\nu,$$

$$\sum_{j=1}^{2}[\underline{\mathcal{R}}_j^{LT}\cdot\underline{\widetilde{A}}_j^{LT}\cdot\underline{\mathcal{R}}_j^{LT}]^{\mu\nu}\equiv n^\mu\underline{A}_1^{LT}\cdot\underline{\tilde{\zeta}}^\nu-n^\mu\underline{A}_2^{LT}\cdot\underline{E}_\perp^\nu,$$

$$\sum_{j=1}^{2}[\underline{\mathcal{R}}_j^{TG}\cdot\underline{\widetilde{A}}_j^{TG}\cdot\underline{\mathcal{R}}_j^{TG}]^{\mu\nu}\equiv\underline{E}_\perp^\mu\cdot\underline{A}_1^{TG}(i\tilde{\partial}^\nu)+\underline{\tilde{\zeta}}^\mu\cdot\underline{A}_2^{TG}(i\tilde{\partial}^\nu),$$

$$\sum_{j=1}^{2}[\underline{\mathcal{R}}_j^{GT}\cdot\underline{\widetilde{A}}_j^{GT}\cdot\underline{\mathcal{R}}_j^{GT}]^{\mu\nu}\equiv(i\tilde{\partial}^\mu)\underline{A}_1^{GT}\cdot\underline{E}_\perp^\nu-(i\tilde{\partial}^\mu)\underline{A}_2^{GT}\cdot\underline{\tilde{\zeta}}^\nu,$$

$$[\underline{\mathcal{R}}_j^{LG}\cdot\underline{\widetilde{A}}_j^{LG}\cdot\underline{\mathcal{R}}_j^{LG}]^{\mu\nu}\equiv n^\mu\underline{A}_1^{LG}(i\tilde{\partial}^\nu),$$

$$[\underline{\mathcal{R}}_j^{GL}\cdot\underline{\widetilde{A}}_j^{GL}\cdot\underline{\mathcal{R}}_j^{GL}]^{\mu\nu}\equiv-(i\tilde{\partial}^\mu)\underline{A}_1^{GL}n^\nu,$$

with $\tilde{\partial}^\mu=\partial^\mu-n^\mu\partial_0$ and $\partial_0=\partial/\partial X_0$.

2. Bare propagator and counter-Lagrangian

We proceed as in [10]. We start from the expression for the bare propagator $\underline{\hat{D}}(x,y)$ [cf. Eqs. (3.10)–(3.13)],

$$\underline{\hat{D}}^{\mu\nu}=(\underline{\mathcal{P}_T}\cdot\underline{\hat{D}}_T\cdot\underline{\mathcal{P}_T})^{\mu\nu}+(\underline{\mathcal{P}_L}\cdot\underline{\hat{D}}_L\cdot\underline{\mathcal{P}_L})^{\mu\nu}+i\tilde{\partial}^\mu\underline{\hat{D}}_G(i\tilde{\partial}^\nu)$$
$$+n^\mu\underline{\hat{D}}_{LT}(i\tilde{\partial}^\nu)+i\tilde{\partial}^\mu\underline{\hat{D}}_{TL}n^\nu+\underline{D}_K^{\mu\nu}\hat{M}_+ ,$$

$$\underline{\hat{D}}_T=\begin{pmatrix}-\underline{\Delta}_R & 0\\ -\underline{\Delta}_R+\underline{\Delta}_A & \underline{\Delta}_A\end{pmatrix}-[\underline{\Delta}_R\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\underline{\Delta}_A]\hat{M}_+ ,$$

$$\underline{D}_L=-\left[\frac{1}{\tilde{P}^2}\left(1+\lambda\frac{p_0^2}{\tilde{P}^2}\right)\right]_{\rm IFT}\hat{\tau}_3$$
$$-\left[\left\{\frac{1}{\tilde{P}^2}\left(1+\lambda\frac{p_0^2}{\tilde{P}^2}\right)\right\}_{\rm IFT}\cdot\underline{\tilde{f}}\right.$$
$$\left.-\underline{\tilde{f}}\cdot\left\{\frac{1}{\tilde{P}^2}\left(1+\lambda\frac{p_0^2}{\tilde{P}^2}\right)\right\}_{\rm IFT}\right]\hat{M}_+ ,$$

$$\underline{\mathcal{D}}_G=-\lambda\left[\frac{1}{\tilde{P}^4}\right]_{\rm IFT}\hat{\tau}_3-\lambda\left[\left\{\frac{1}{\tilde{P}^4}\right\}_{\rm IFT}\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\left\{\frac{1}{\tilde{P}^4}\right\}_{\rm IFT}\right]\hat{M}_+ ,$$

$$\underline{D}_{LT}=\underline{\mathcal{D}}_{TL}$$
$$=-\lambda\left[\frac{p_0}{\tilde{P}^4}\right]_{\rm IFT}\hat{\tau}_3-\lambda\left[\left\{\frac{p_0}{\tilde{P}^4}\right\}_{\rm IFT}\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\left\{\frac{p_0}{\tilde{P}^4}\right\}_{\rm IFT}\right]\hat{M}_+ ,$$

$$\underline{D}_K^{\mu\nu}=-\underline{\tilde{\zeta}}^\mu\cdot[\underline{\Delta}_R\cdot\underline{C}_2^{TT}-\underline{C}_2^{TT}\cdot\underline{\Delta}_A]\cdot\underline{\tilde{\zeta}}^\nu$$
$$+\underline{\tilde{\zeta}}^\mu\cdot[\underline{\Delta}_R\cdot\underline{C}_3^{TT'}-\underline{C}_3^{TT'}\cdot\underline{\Delta}_A]\cdot\underline{E}_\perp^\nu$$
$$-\underline{E}_\perp^\mu\cdot[\underline{\Delta}_R\cdot\underline{C}_3^{T'T}-\underline{C}_3^{T'T}\cdot\underline{\Delta}_A]\cdot\underline{\tilde{\zeta}}^\nu .$$

$\hat{D}$ in Eq. (3.10) is the leading part of the DEX of $\hat{D}(P,X)$ $[(\underline{\hat{D}}(x,y))_{WT}]$. Calculation within the gradient approximation yields

$$(\underline{\hat{D}}^{-1})^{\mu\nu}\cdot\underline{\hat{D}}_\nu{}^\rho=g^{\mu\rho}, \tag{4.52}$$

$$(\underline{\hat{D}}^{-1})^{\mu\nu}(x,y)=\left(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu+\frac{1}{\lambda}\tilde{\partial}^\mu\tilde{\partial}^\nu\right)_{\rm IFT}\hat{\tau}_3-\underline{\hat{L}}_c^{\mu\nu}(x,y)$$
$$=\hat{\tau}_3\left[\mathcal{P}_T^{\mu\nu}(i\partial)\partial^2+\mathcal{P}_L^{\mu\nu}(i\partial)\tilde{\partial}^2\right.$$
$$-\partial_0(\tilde{\partial}^\mu n^\nu+n^\mu\tilde{\partial}^\nu)$$
$$\left.+\mathcal{P}_G^{\mu\nu}(i\partial)\left(\frac{\tilde{\partial}^2}{\lambda}+\partial_0^2\right)\right]_{\rm IFT}-\underline{\hat{L}}_c^{\mu\nu}(x,y), \tag{4.53}$$

$$\hat{L}_c^{\mu\nu}=L_c^{\mu\nu}\hat{M}_-=2i\hat{M}_-\left[(P\cdot\partial\tilde{f})\mathcal{P}_T^{\mu\nu}+\tilde{P}\cdot\tilde{\partial}\tilde{f}n^\mu n^\nu\right.$$
$$+\left\{\frac{1}{\tilde{P}^2}\left(\frac{p_0^2}{\tilde{P}^2}+\frac{2}{\lambda}\right)\tilde{P}\cdot\tilde{\partial}\tilde{f}+\frac{p_0}{\tilde{P}^2}\partial_0\tilde{f}\right\}\tilde{P}^\mu\tilde{P}^\nu$$
$$-\left\{\frac{p_0}{\tilde{P}^2}\tilde{P}\cdot\tilde{\partial}\tilde{f}+\frac{1}{2}\partial_0\tilde{f}\right\}(n^\mu\tilde{P}^\nu+\tilde{P}^\mu n^\nu)$$
$$+P\cdot\partial C_2^{TT}(P,X)\tilde{\zeta}^\mu\tilde{\zeta}^\nu-P\cdot\partial C_3^{TT'}(P,X)\tilde{\zeta}^\mu E_\perp^\nu$$
$$\left.+P\cdot\partial C_3^{T'T}(P,X)E_\perp^\mu\tilde{\zeta}^\nu\right]. \tag{4.54}$$

Here $P\cdot\partial\tilde{f}=P^\mu\partial\tilde{f}(P,X)/\partial X^\mu$, etc.

From Eq. (4.52), we see that the free action of the theory is

$$\mathcal{A}_0=\frac{1}{2}\int d^4x\, d^4y\, {}^t\hat{A}^\mu(x)(\hat{D}^{-1}(x,y))_{\mu\nu}\hat{A}^\nu(y),$$
$${}^t\hat{A}^\mu=(\hat{A}_1^\mu,\hat{A}_2^\mu), \tag{4.55}$$

where the color index is suppressed. Equation (4.55) with Eq. (4.53) tells us that there emerges a counteraction

$$\mathcal{A}_c=\frac{1}{2}\int d^4x\, d^4y\, {}^t\hat{A}^\mu(x)\underline{\hat{L}}_c^{\mu\nu}(x,y)\hat{A}^\nu,$$

which yields a (two-point) vertex factor

$$i(\underline{\hat{L}}_c^{\mu\nu}(x,y))_{WT}=iL_c^{\mu\nu}(P,X)\hat{M}_- .$$

3. Dyson equation

As in Sec. III, we use the (4×4)-matrix notation in Minkowski space. The self-energy-part $[\underline{\hat{\mathbf{\Pi}}}(x,y)]$ resummed propagator $\underline{\hat{\mathbf{G}}}(x,y)$ obeys

$$\underline{\hat{\mathbf{G}}}=\underline{\hat{\mathbf{D}}}-\underline{\hat{\mathbf{D}}}\cdot\underline{\hat{\mathbf{\Pi}}}\cdot\underline{\hat{\mathbf{G}}},$$
$$\underline{\hat{\mathbf{D}}}=\begin{pmatrix}\underline{\mathbf{D}}_R & 0\\ \underline{\mathbf{D}}_R-\underline{\mathbf{D}}_A & -\underline{\mathbf{D}}_A\end{pmatrix}+[\underline{\mathbf{D}}_R\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\underline{\mathbf{D}}_A+\underline{\mathbf{D}}_K]\hat{M}_+ . \tag{4.56}$$

For $\underline{\hat{\mathbf{G}}}$ and $\underline{\hat{\mathbf{\Pi}}}$, we have [cf. Eqs. (3.27)–(3.31) and (3.20)–(3.24)]

$$\underline{\hat{\mathbf{G}}}=\begin{pmatrix}\underline{\mathbf{G}}_R & 0\\ \underline{\mathbf{G}}_R-\underline{\mathbf{G}}_A & -\underline{\mathbf{G}}_A\end{pmatrix}+[\underline{\mathbf{G}}_R\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\underline{\mathbf{G}}_A+\underline{\mathbf{G}}_K]\hat{M}_+ ,$$
$$\underline{\hat{\mathbf{\Pi}}}=\begin{pmatrix}\underline{\mathbf{\Pi}}_R & 0\\ -\underline{\mathbf{\Pi}}_R+\underline{\mathbf{\Pi}}_A & -\underline{\mathbf{\Pi}}_A\end{pmatrix}+[\underline{\mathbf{\Pi}}_R\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\underline{\mathbf{\Pi}}_A-\underline{\mathbf{\Pi}}_K]\hat{M}_- ,$$
$$\underline{\mathbf{G}}_K=-\underline{\mathbf{G}}_R\cdot\underline{\tilde{f}}+\underline{\tilde{f}}\cdot\underline{\mathbf{G}}_A+\underline{\mathbf{G}}_{12},$$
$$\underline{\mathbf{\Pi}}_K=\underline{\mathbf{\Pi}}_R\cdot\underline{\tilde{f}}-\underline{\tilde{f}}\cdot\underline{\mathbf{\Pi}}_A+\underline{\mathbf{\Pi}}_{12}. \tag{4.57}$$

Equation (4.56) may be solved to give

$$\underline{\mathbf{G}}_{R(A)}=[\underline{\mathbf{D}}_{R(A)}^{-1}+\underline{\mathbf{\Pi}}_{R(A)}]^{-1}, \tag{4.58}$$

$$\underline{\mathbf{G}}_{K}=\underline{\mathbf{G}}_{K}^{[1]}+\underline{\mathbf{G}}_{K}^{[2]}+\underline{\mathbf{G}}_{K}^{[3]},$$

$$\underline{\mathbf{G}}_{K}^{[1]}=\underline{\mathbf{G}}_{R}\cdot\underline{\mathbf{\Pi}}_{K}\cdot\underline{\mathbf{G}}_{A}, \tag{4.59}$$

$$\underline{\mathbf{G}}_{K}^{[2]}=\underline{\mathbf{G}}_{R}\cdot[\underline{\mathbf{\Pi}}_{R}\cdot\underline{\mathbf{C}}-\underline{\mathbf{C}}\cdot\underline{\mathbf{\Pi}}_{A}]\cdot\underline{\mathbf{G}}_{A}\equiv-\underline{\mathbf{G}}_{R}\cdot\underline{\tilde{\mathbf{H}}}\cdot\underline{\mathbf{G}}_{A}, \tag{4.60}$$

$$\underline{\mathbf{G}}_{K}^{[3]}=\underline{\mathbf{G}}_{R}\cdot\underline{\mathbf{C}}-\underline{\mathbf{C}}\cdot\underline{\mathbf{G}}_{A}, \tag{4.61}$$

$$\underline{C}^{\mu\nu}=\underline{\tilde{\zeta}}^{\mu}\cdot\underline{C}_{2}^{TT}\cdot\underline{\tilde{\zeta}}^{\nu}-\underline{\tilde{\zeta}}^{\mu}\cdot\underline{C}_{3}^{TT'}\cdot\underline{E}_{\perp}^{\nu}+\underline{E}_{\perp}^{\mu}\cdot\underline{C}_{3}^{T'T}\cdot\underline{\tilde{\zeta}}^{\nu}.$$

The form for the leading part of the DEX of $\hat{G}(X,P)$, a Wigner transform of $\underline{G}(x,y)$, is the $\hat{G}$ that is deduced in Sec. III.

4. Gradient piece of the self-energy-part resummed propagator

Form for $G_{R(A)}$

We divide the Wigner transform of $\underline{\mathbf{\Pi}}_{R(A)}\cdot\underline{\mathbf{G}}_{R(A)}$ [cf. Eq. (4.58)] into two pieces [cf. Eq. (4.51)],

$$(\underline{\mathbf{\Pi}}_{R(A)}\cdot\underline{\mathbf{G}}_{R(A)})_{WT}=\mathbf{\Pi}_{R(A)}(P,X)\mathbf{G}_{R(A)}(P,X)+(\mathbf{\Pi}_{R(A)}\mathbf{G}_{R(A)})^{(1)}, \tag{4.62}$$

$$\begin{aligned}(\mathbf{\Pi}_{R(A)}\mathbf{G}_{R(A)})^{(1)\mu\nu}=\frac{i}{2}\sum_{U,V,V'=T,L,G}\sum_{j=1}^{J_{UV}}\sum_{j'=1}^{J_{VV'}}\Bigg[&\frac{\partial\mathcal{R}_{Lj}^{UV}}{\partial P_{\mu}}\frac{\partial\tilde{\Pi}_{R(A)j}^{UV}}{\partial X^{\mu}}\mathcal{R}_{Rj}^{UV}\mathcal{R}_{Lj'}^{VV'}\tilde{G}_{R(A)j'}^{VV'}\mathcal{R}_{Rj'}^{VV'}\\&-\mathcal{R}_{Lj}^{UV}\frac{\partial\tilde{\Pi}_{R(A)j}^{UV}}{\partial X^{\mu}}\frac{\partial\mathcal{R}_{Rj}^{UV}\mathcal{R}_{Lj'}^{VV'}\tilde{G}_{R(A)j'}^{VV'}\mathcal{R}_{Rj'}^{VV'}}{\partial P_{\mu}}+\frac{\partial\mathcal{R}_{Lj}^{UV}\tilde{\Pi}_{R(A)j}^{UV}\mathcal{R}_{Rj}^{UV}\mathcal{R}_{Lj'}^{VV'}}{\partial P_{\mu}}\frac{\partial\tilde{G}_{R(A)j'}^{VV'}}{\partial X^{\mu}}\mathcal{R}_{Rj'}^{VV'}\\&-\mathcal{R}_{Lj}^{UV}\tilde{\Pi}_{R(A)j}^{UV}\mathcal{R}_{Rj}^{UV}\mathcal{R}_{Lj'}^{VV'}\frac{\partial\tilde{G}_{R(A)j'}^{VV'}}{\partial X^{\mu}}\frac{\partial\mathcal{R}_{Rj'}^{VV'}}{\partial P_{\mu}}\Bigg]^{\mu\nu}.\end{aligned} \tag{4.63}$$

Here $\mathbf{\Pi}_{R(A)}(P,X)$ and $\mathbf{G}_{R(A)}(P,X)$ are, in respective order, $\mathbf{\Pi}_{R(A)}(P)$ and $\mathbf{G}_{R(A)}(P)$ in Sec. III. Using Eq. (4.62) in Eq. (4.58), we obtain the solution for $\mathbf{G}_{R(A)}$ $(=\mathbf{G}_{R(A)}^{(0)}+\mathbf{G}_{R(A)}^{(1)})$. The form for the leading part $\mathbf{G}_{R(A)}^{(0)}$ is given in Sec. III. The gradient part is

$$\mathbf{G}_{R(A)}^{(1)}=\mathbf{G}_{R(A)}^{(0)}\Bigg[\Bigg\{ig^{\mu\nu}P\cdot\partial-\frac{i}{2}(P^{\mu}\partial^{\nu}+\partial^{\mu}P^{\nu})+\frac{i}{2}\lambda(\tilde{P}^{\mu}\tilde{\partial}^{\nu}+\tilde{\partial}^{\mu}\tilde{P}^{\nu})\Bigg\}\mathbf{G}_{R(A)}^{(0)}-(\mathbf{\Pi}_{R(A)}\mathbf{G}_{R(A)}^{(0)})^{(1)}\Bigg].$$

Form for Π_K, which is involved in $G_K^{[1]}$ in Eq. (4.59)

In the following, we restrict ourselves to the strict Coulomb gauge $\lambda=0$, which is a physical gauge. Computation of Eq. (4.57) to the gradient approximation yields

$$\begin{aligned}\underline{\Pi}_{K}^{\mu\nu}=-\underline{L}_{c}^{\mu\nu}+\sum_{j}\sum_{UV=T,L}[\underline{\mathcal{R}}_{Lj}^{UV}\cdot\{(\tilde{f}[(\tilde{\Pi}_{12})_{j}^{UV}\\-(\tilde{\Pi}_{21})_{j}^{UV}])_{\mathrm{IWT}}+(\underline{\tilde{\Pi}}_{12})_{j}^{UV}\}\cdot\underline{\mathcal{R}}_{Rj}^{UV}]^{\mu\nu}\\+\underline{\Pi}_{K}^{[1]\mu\nu}+\underline{\Pi}_{K}^{[2]\mu\nu}.\end{aligned} \tag{4.64}$$

In $\underline{G}_{K}^{[1]\mu\nu}$, Eq. (4.59), $\underline{\Pi}_{K}^{\mu\nu}$, and then $\underline{L}_{c}^{\mu\nu}$ in Eq. (4.64), are sandwiched between $\underline{\mathbf{G}}_{R}$ and $\underline{\mathbf{G}}_{A}$. Then, in the case of $\lambda=0$, $\tilde{P}^{\mu}\tilde{P}^{\nu}$ and $(n^{\mu}\tilde{P}^{\nu}+\tilde{P}^{\mu}n^{\nu})$ terms in $\Pi_{c}^{\mu\nu}$, Eq. (4.54), do not contribute to $\mathbf{G}_{K}^{[1]}$ (cf. Sec. III).

The standard forms for the gradient terms $\Pi_{K}^{[1]\mu\nu}$ and $\Pi_{K}^{[2]\mu\nu}$, and $\tilde{H}$ [Eq. (4.60)] are given in Appendix G.

5. Generalized Boltzmann equation and its relatives

Structure of the theory is fully discussed in Sec. IV A 5, so that we restrict ourselves to giving a brief description of the physical-N scheme only.

Same reasoning as in Sec. IV A applies here: $\mathbf{G}_{K}^{[1]}$ and $\mathbf{G}_{K}^{[2]}$, Eqs. (4.59) and (4.60), bring about disaster. This disaster would be overcome if the condition

$$\mathbf{\Pi}_{K}-\tilde{\mathbf{H}}=0 \tag{4.65}$$

could be imposed. This is, however, not possible. Equation (4.65) may be imposed for the $\mathcal{P}_{T}^{\mu\nu}$, $n^{\mu}n^{\nu}$, $\tilde{\zeta}^{\mu}\tilde{\zeta}^{\nu}$, and $\tilde{\zeta}^{\mu}E_{\perp}^{\nu}$ components, which read, in respective order,

$$\begin{aligned}2P\cdot\partial\tilde{f}=-i[(1+\tilde{f})(\Pi_{12})_{1}^{TT}-\tilde{f}(\Pi_{21})_{1}^{TT}]-i\Pi_{K1}^{TT}\\-2\,\mathrm{Im}(\tilde{\zeta}^{2}E_{\perp}^{2}C_{3}^{TT'}\Pi_{R3}^{T'T})+iH_{1}^{(1)TT},\end{aligned} \tag{4.66}$$

$$2\tilde{P}\cdot\tilde{\partial}\tilde{f}=-i[(1+\tilde{f})(\Pi_{12})_{1}^{LL}-\tilde{f}(\Pi_{21})_{1}^{LL}]-i\Pi_{K1}^{LL}, \tag{4.67}$$

$$\begin{aligned}2P\cdot\partial C_{2}^{TT}=-i[(1+\tilde{f})(\Pi_{12})_{2}^{TT}-\tilde{f}(\Pi_{21})_{2}^{TT}]-i\Pi_{K2}^{(1)TT}\\-2\,\mathrm{Im}[-C_{2}^{TT}(\Pi_{R1}^{TT}+\tilde{\zeta}^{2}\Pi_{R2}^{TT})\\+E_{\perp}^{2}C_{3}^{T'T}(\Pi_{R3}^{TT'}+\Pi_{A2}^{TT'})]+iH_{2}^{(1)TT},\end{aligned} \tag{4.68}$$

$$2P\cdot\partial C_3^{TT'} = -i[(1+\tilde{f})(\Pi_{12})_3^{TT'} - \tilde{f}(\Pi_{21})_3^{TT'}] - i\Pi_{K3}^{(1)TT'} + i[\tilde{\zeta}^2 C_2^{TT}\Pi_{A3}^{TT'} + C_3^{TT'}(\Pi_{A1}^{TT} - \Pi_{R1}^{TT} - \tilde{\zeta}^2\Pi_{R2}^{TT})] + iH_3^{(1)TT'}. \quad (4.69)$$

Proceeding as in [10], from Eqs. (4.66) and (4.67) on the energy shells, we obtain the generalized Boltzmann equations for the transverse and the longitudinal modes, respectively. (As a matter of fact, on the energy shells, the first term on the RHS of Eq. (4.66) [Eq. (4.67)] is proportional to the net production rates of the transverse (longitudinal) mode.) We do not reproduce them here. It should be remarked that, in the case of L mode, Eq. (4.67), the "time-derivative term" $\partial_0\tilde{f}$ in the Boltzmann equation comes from Π_{K1}^{LL}. More precisely, the $\partial_0\tilde{f}$ term comes from $\Pi_K^{[1]\mu\nu}$ (with $UV=LL$), Eq. (G1) in Appendix G, which is in Π_{K1}^{LL} in Eq. (4.67). Equation (4.68) [Eq. (4.69)] determines spacetime evolution of C_2^{TT} [$C_3^{TT'}$] along P. An evolution equation for $C_3^{T'T}$ is obtained from Eqs. (4.69) and (3.16).

One cannot impose Eq. (4.65) for the remaining $\tilde{\zeta}^\mu n^\nu$, $n^\mu\tilde{\zeta}^\nu$, $E_\perp^\mu n^\nu$, and $n^\mu E_\perp^\nu$ components. This is because, for these components, there are no counterparts of $\tilde{f}$, C_2^{TT}, $C_3^{TT'}$, and $C_3^{T'T}$. For equilibrium systems, these modes are absent. Then, one can expect that, for the quasiuniform systems near equilibrium, these modes do not yield disastrously large contributions.

C. Ghost sector

The self-energy-part [$\underline{\hat{\hat{\mathbf{\Pi}}}}(x,y)$] resummed propagator $\underline{\hat{\hat{\mathbf{G}}}}(x,y)$ obeys

$$\underline{\hat{\hat{\mathbf{G}}}} = \underline{\hat{\hat{\mathbf{D}}}} + \underline{\hat{\hat{\mathbf{D}}}}\cdot\underline{\hat{\hat{\mathbf{\Pi}}}}\cdot\underline{\hat{\hat{\mathbf{G}}}}. \quad (4.70)$$

$\underline{\hat{\hat{\mathbf{D}}}}$ is an inverse Fourier transform of $\hat{\hat{\mathbf{D}}}$ in Eq. (3.17). As in Eq. (3.44), $\underline{\hat{\hat{\mathbf{D}}}}$, $\underline{\hat{\hat{\mathbf{\Pi}}}}$, and $\underline{\hat{\hat{\mathbf{G}}}}$ are diagonal (2×2)-matrix functions $\underline{\hat{\hat{\mathbf{D}}}} = \underline{\tilde{\mathbf{D}}}\hat{\tau}_3 =$, etc. Solving Eq. (4.70), we see that the gradient part of $\tilde{G}(P,X)$ vanishes and

$$\hat{\tilde{G}}(P,X) = (\hat{\tilde{G}}(P,X))^* = \frac{1}{\tilde{P}^2 - \tilde{\Pi}(P,X)}.$$

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APPENDIX A: RESUMMATION OF THE QUARK PROPAGATOR

Here we derive Eqs. (2.39)–(2.42). Formally solving Eq. (2.33), we obtain

$$\mathbf{G}_K = -\mathbf{G}_R\mathbf{\Sigma}_K\mathbf{G}_A + \mathbf{G}_K',$$

$$\mathbf{G}_K' = \mathbf{G}_R S_R^{-1}\mathbf{S}_K(1+\mathbf{\Sigma}_A\mathbf{G}_A), \quad (A1)$$

where use has been made of Eq. (2.32). Since $S_R^{-1}\mathbf{S}_K \propto (P^2 - m^2)\delta(P^2-m^2) = 0$, we have $\mathbf{G}_K' = 0$. This means that the piece $\mathbf{S}_K$ of the bare propagator disappears through resummation, which is unnatural.

A correct $\mathbf{G}_K'$ is obtained by substituting Eq. (2.18) for $\mathbf{S}_K$ in Eq. (A1) as follows:

$$\begin{aligned}\mathbf{G}_K' &= \mathbf{G}_R S_R^{-1}(S_R - S_A)\gamma_5 \not{N}\mathbf{C}(P)(1+\mathbf{\Sigma}_A\mathbf{G}_A)\\ &= [\mathbf{G}_R S_R^{-1}S_R\gamma_5\not{N} - \mathbf{G}_R S_R^{-1}\gamma_5\not{N}S_A]\mathbf{C}(P)(1+\mathbf{\Sigma}_A\mathbf{G}_A)\\ &= \mathbf{G}_R\gamma_5\not{N}\mathbf{C}(P)(1+\mathbf{\Sigma}_A\mathbf{G}_A) - (1+\mathbf{G}_R\mathbf{\Sigma}_R)\gamma_5\not{N}\mathbf{C}(P)\mathbf{G}_A,\end{aligned}$$

where use has been made of $\mathbf{G}_R S_R^{-1} = (\mathbf{1} + \mathbf{G}_R\mathbf{\Sigma}_R)$, which follows from Eq. (2.32). This is natural in the sense that

$$\mathbf{G}_K \xrightarrow{\hat{\mathbf{\Sigma}}\to 0} \mathbf{S}_K.$$

APPENDIX B: "MULTIPLICATIONS" OF THE TWO STANDARD FORMS FOR THE QUARK PART

We define "multiplications" of the functions of the type (2.5) as the products

$$(A\otimes B)^{\rho\sigma} \equiv A^{\rho\rho}B^{\rho\sigma}, \quad [A\otimes B]^{\rho\sigma} \equiv A^{\rho-\rho}B^{-\rho\sigma}. \quad (B1)$$

Straightforward manipulation yields the SF's [cf. Eq. (2.5)] of $(A\otimes B)^{\rho\sigma}$,

$$\begin{aligned}(A\otimes B)_1^{\rho\pm\rho} &= A_1^{\rho\rho}B_1^{\rho\pm\rho} \pm P^2 A_2^{\rho\rho}B_2^{\rho\pm\rho} \pm N^2 A_3^{\rho\rho}B_3^{\rho\pm\rho} - P^2N^2A_4^{\rho\rho}B_4^{\rho\pm\rho},\\ (A\otimes B)_2^{\rho\pm\rho} &= A_1^{\rho\rho}B_2^{\rho\pm\rho} \pm A_2^{\rho\rho}B_1^{\rho\pm\rho} \mp N^2A_3^{\rho\rho}B_4^{\rho\pm\rho} + N^2A_4^{\rho\rho}B_3^{\rho\pm\rho},\\ (A\otimes B)_3^{\rho\pm\rho} &= A_1^{\rho\rho}B_3^{\rho\pm\rho} \pm P^2A_2^{\rho\rho}B_4^{\rho\pm\rho} \pm A_3^{\rho\rho}B_1^{\rho\pm\rho} - P^2A_4^{\rho\rho}B_2^{\rho\pm\rho},\\ (A\otimes B)_4^{\rho\pm\rho} &= A_1^{\rho\rho}B_4^{\rho\pm\rho} \pm A_2^{\rho\rho}B_3^{\rho\pm\rho} \mp A_3^{\rho\rho}B_2^{\rho\pm\rho} + A_4^{\rho\rho}B_1^{\rho\pm\rho}. \end{aligned} \quad (B2)$$

$[A\otimes B]_j^{\rho-\rho}$ $(j=1\text{–}4)$ is given by $(A\otimes B)_j^{\rho\rho}$ in Eq. (B2) with $B_j^{-\rho-\rho}$ for $B_j^{\rho\rho}$, and $[A\otimes B]_j^{\rho\rho}$ is given by $(A\otimes B)_j^{\rho-\rho}$ in Eq. (B2) with $B_j^{-\rho\rho}$ for $B_j^{\rho-\rho}$.

APPENDIX C: MULTIPLICATION OF THE TWO STANDARD FORMS FOR THE GLUON PART

We define a multiplication of the functions $A^{\mu\nu}$ and $B^{\mu\nu}$ of the type (3.6) with Eq. (3.7), by $C^{\mu\nu} = A^{\mu\rho}B_\rho{}^\nu$. Straightforward computation yields the SF for $C^{\mu\nu}$:

$$C_1^{TT}=A_1^{TT}B_1^{TT}+\tilde{P}^2\tilde{\zeta}^4n^2A_3^{T'T}B_3^{TT'}+\tilde{P}^2\tilde{\zeta}^2n^4A_2^{TL}B_2^{LT}-\tilde{P}^4\tilde{\zeta}^2n^2A_1^{TG}B_1^{GT},$$

$$C_2^{TT}=A_1^{TT}B_2^{TT}+A_2^{TT}B_1^{TT}+\tilde{\zeta}^2A_2^{TT}B_2^{TT}+\tilde{P}^2\tilde{\zeta}^2n^2A_3^{TT'}B_3^{T'T}-\tilde{P}^2\tilde{\zeta}^2n^2A_3^{T'T}B_3^{TT'}+n^2A_1^{TL}B_1^{LT}-\tilde{P}^2n^4A_2^{TL}B_2^{LT}+\tilde{P}^4n^2A_1^{TG}B_1^{GT}-\tilde{P}^2A_2^{TG}B_2^{GT},$$

$$C_3^{TT'}=A_1^{TT}B_3^{TT'}+\tilde{\zeta}^2A_2^{TT}B_3^{TT'}+A_3^{TT'}B_1^{TT}+n^2A_1^{TL}B_2^{LT}-\tilde{P}^2A_2^{TG}B_1^{GT},$$

$$C_3^{T'T}=A_1^{TT}B_3^{T'T}+A_3^{T'T}B_1^{TT}+\tilde{\zeta}^2A_3^{T'T}B_2^{TT}+n^2A_2^{TL}B_1^{LT}-\tilde{P}^2A_1^{TG}B_2^{GT},$$

$$C_1^{LL}=\tilde{\zeta}^2n^2A_1^{LT}B_1^{TL}+\tilde{P}^2\tilde{\zeta}^2n^4A_2^{LT}B_2^{TL}+A_1^{LL}B_1^{LL}-\tilde{P}^2n^2A_1^{LG}B_1^{GL},$$

$$C_1^{GG}=-\tilde{P}^4\tilde{\zeta}^2n^2A_1^{GT}B_1^{TG}-\tilde{P}^2\tilde{\zeta}^2A_2^{GT}B_2^{TG}-\tilde{P}^2n^2A_1^{GL}B_1^{LG}+A_1^{GG}B_1^{GG},$$

$$C_1^{TL}=A_1^{TT}B_1^{TL}+\tilde{\zeta}^2A_2^{TT}B_1^{TL}+\tilde{P}^2\tilde{\zeta}^2n^2A_3^{TT'}B_2^{TL}+A_1^{TL}B_1^{LL}-P^2A_2^{TG}B_1^{GL},$$

$$C_2^{TL}=A_1^{TT}B_2^{TL}+\tilde{\zeta}^2A_3^{T'T}B_1^{TL}+A_2^{TL}B_1^{LL}-\tilde{P}^2A_1^{TG}B_1^{GL},$$

$$C_1^{LT}=A_1^{LT}B_1^{TT}+\tilde{\zeta}^2A_1^{LT}B_2^{TT}+\tilde{P}^2\tilde{\zeta}^2n^2A_2^{LT}B_3^{T'T}+A_1^{LL}B_1^{LT}-\tilde{P}^2A_1^{LG}B_2^{GT},$$

$$C_2^{LT}=\tilde{\zeta}^2A_1^{LT}B_3^{TT'}+A_2^{LT}B_1^{TT}+A_1^{LL}B_2^{LT}-\tilde{P}^2A_1^{LG}B_1^{GT},$$

$$C_1^{TG}=A_1^{TT}B_1^{TG}+\tilde{\zeta}^2A_3^{T'T}B_2^{TG}+n^2A_2^{TL}B_1^{LG}+A_1^{TG}B_1^{GG},$$

$$C_2^{TG}=A_1^{TT}B_2^{TG}+\tilde{\zeta}^2A_2^{TT}B_2^{TG}+\tilde{P}^2\tilde{\zeta}^2n^2A_3^{TT'}B_1^{TG}+n^2A_1^{TL}B_1^{LG}+A_2^{TG}B_1^{GG},$$

$$C_1^{GT}=A_1^{GT}B_1^{TT}+\tilde{\zeta}^2A_2^{GT}B_3^{TT'}+n^2A_1^{GL}B_2^{LT}+A_1^{GG}B_1^{GT},$$

$$C_2^{GT}=\tilde{P}^2\tilde{\zeta}^2n^2A_1^{GT}B_3^{TT'}+A_2^{GT}B_1^{TT}+\tilde{\zeta}^2A_2^{GT}B_2^{TT}+\tilde{n}^2A_1^{GL}B_1^{LT},$$

$$C_1^{LG}=\tilde{\zeta}^2A_1^{LT}B_2^{TG}+\tilde{P}^2\tilde{\zeta}^2n^2A_2^{LT}B_1^{TG}+A_1^{LL}B_1^{LG}+A_1^{LG}B_1^{GG},$$

$$C_1^{GL}=\tilde{P}^2\tilde{\zeta}^2n^2A_1^{GT}B_2^{TL}+\tilde{\zeta}^2A_2^{GT}B_1^{TL}+A_1^{GL}B_1^{LL}+A_1^{GG}B_1^{GL}.$$

APPENDIX D: GLUON PROPAGATOR IN A COVARIANT GAUGE

Here we present a ‘‘translation table’’ to get the expressions for the gluon propagator in a covariant-gauge from the Coulomb gauge counterparts given in Sec. III.

An orthogonal basis in Minkowski space is given by Eq. (3.1) with the replacements[4]

$$(\tilde{P}^\mu,\tilde{\zeta}^\mu,n^\mu,E_\perp^\mu)\Rightarrow(P^\mu,\tilde{\zeta}^\mu,\tilde{n}^\mu,E_\perp^\mu), \tag{D1}$$

$$\tilde{n}^\mu\equiv n^\mu-\frac{n\cdot P}{P^2}P^\mu=n^\mu-\frac{p_0}{P^2}P^\mu$$

$$\left(\tilde{n}^2=-\frac{\vec{p}^{\,2}}{P^2}\right).$$

Then, among the projection operators, Eqs. (3.2)–(3.4), $\mathcal{P}_L^{\mu\nu}$ and $\mathcal{P}_G^{\mu\nu}$ are replaced as

$$\mathcal{P}_L^{\mu\nu}(P)=\frac{n^\mu n^\nu}{n^2}\Rightarrow\mathcal{P}_L^{\mu\nu}(P)=\frac{\tilde{n}^\mu\tilde{n}^\nu}{\tilde{n}^2},$$

$$\mathcal{P}_G^{\mu\nu}(P)=\frac{\tilde{P}^\mu\tilde{P}^\nu}{\tilde{P}^2}\Rightarrow\mathcal{P}_G^{\mu\nu}(P)=\frac{P^\mu P^\nu}{P^2}.$$

$\mathcal{P}_T$ is the same as in Eq. (3.2).

Equation (3.9) is replaced with

$$(\hat{D}^{-1}(P))^{\mu\nu}=-P^2\left[\mathcal{P}_T^{\mu\nu}+\mathcal{P}_L^{\mu\nu}+\frac{1}{\lambda}\mathcal{P}_G^{\mu\nu}\right]\hat{\tau}_3,$$

which is already in SF.

The SF elements of $\mathbf{D}_K$ in Eq. (3.13) are replaced by

$$D_{K2}^{TT}(P)=2\pi iC_2^{TT}(P)\epsilon(p_0)\delta(P^2),$$

$$D_{K3}^{TT'}(P)=2\pi iC_3^{TT'}(P)\epsilon(p_0)\delta(P^2),$$

$$D_{K1}^{TL}(P)=2\pi iC_1^{TL}(P)\epsilon(p_0)\delta(P^2),$$

$$D_{K2}^{TL}(P)=2\pi iC_2^{TL}(P)\epsilon(p_0)\delta(P^2),$$

$$D_{Kj}^{UV}(P)=0\quad(\text{otherwise}).$$

In obtaining these, we have used the fact that $(\hat{D}_{UG})^{\mu\nu}=(\hat{D}_{GU})^{\mu\nu}=0$ $(U=T,L)$, which is verified from the ‘‘bare counterparts’’ of Eq. (D2), below. $D_R^{\mu\nu}$ in Eq. (3.15) is replaced with

$$D_R^{\mu\nu}=(D_A^{\mu\nu})^*=-\Delta_R\mathcal{P}_T^{\mu\nu}+\frac{d\Delta_R}{dP^2}P^2(\mathcal{P}_L^{\mu\nu}+\lambda\mathcal{P}_G^{\mu\nu}).$$

Equation (3.16) is replaced with

[4]It should be noted that $E_\perp^\mu=\epsilon^{\mu\nu\rho\sigma}P_\nu\tilde{\zeta}_\rho\tilde{n}_\sigma=\epsilon^{\mu\nu\rho\sigma}\tilde{P}_\nu\tilde{\zeta}_\rho n_\sigma$.

$$(C_2^{TT})^* = C_2^{TT}, \quad (C_3^{TT'})^* = -C_3^{T'T},$$

$$(C_1^{TL})^* = C_1^{LT}, \quad (C_2^{TL})^* = -C_2^{LT}.$$

Equation (3.17) is replaced by

$$\hat{\tilde{D}}(P) = \begin{pmatrix} \Delta_R & 0 \\ \Delta_R - \Delta_A & -\Delta_A \end{pmatrix} + \tilde{f}(\Delta_R - \Delta_A)\hat{M}_+ .$$

Introduction of $\mathbf{\Pi}'_R$, Eq. (3.39), is not necessary, $\mathbf{D}_0^{-1} = \mathbf{D}^{-1}$ and $\mathbf{\Pi}'_R = \mathbf{\Pi}_R$, and, Eqs. (3.39)–(3.41) are deleted.

Description after Eq. (3.43) is replaced with the following one: Solving Eq. (3.43), we obtain

$$\hat{\tilde{G}} = \begin{pmatrix} \tilde{G}_R & 0 \\ \tilde{G}_R - \tilde{G}_A & -\tilde{G}_A \end{pmatrix} + [\tilde{f}(\tilde{G}_R - \tilde{G}_A) + \tilde{G}_K]\hat{M}_+ ,$$

where

$$\tilde{G}_R(P) = \tilde{G}_A^*(P) = \frac{1}{P^2 - \tilde{\Pi}_R(P)},$$

$$\tilde{G}_K(P) = -\tilde{G}_R(P)\tilde{\Pi}_K(P)\tilde{G}_A(P),$$

$$\tilde{\Pi}_R = \tilde{\Pi}_A^* = \tilde{\Pi}_{11} + \tilde{\Pi}_{12} = -\tilde{\Pi}_{22} - \tilde{\Pi}_{21},$$

$$\tilde{\Pi}_K = (1+\tilde{f})\tilde{\Pi}_{11} - \tilde{f}\tilde{\Pi}_{21}.$$

Equation (3.46) is replaced with

$$\hat{G}_{\mu\nu}P^\nu = \lambda[\hat{\tau}_3 \hat{\tilde{\Pi}}_\mu - P_\mu]\hat{\tilde{G}}. \tag{D2}$$

Equation (3.47) is deleted.

Equations (3.48)–(3.52) are replaced by

$$G_{R1}^{GG}(P) = \lambda \frac{1}{P^2 + ip_0 0^+}[\tilde{\Pi}(P) - P^2]\tilde{G}_R(P)$$
$$= -\lambda \frac{1}{P^2 + ip_0 0^+},$$

$$G_{R1}^{TG}(P) = \lambda \frac{1}{(P^2 + ip_0 0^+)E_\perp^2}[E_\perp^\mu \tilde{\Pi}_{R\mu}(P)]\tilde{G}_R(P),$$

$$G_{R2}^{TG}(P) = \lambda \frac{1}{(P^2 + ip_0 0^+)\tilde{\zeta}^2}[\tilde{\zeta}^\mu \tilde{\Pi}_{R\mu}(P)]\tilde{G}_R(P),$$

$$G_{R1}^{LG}(P) = \lambda \frac{1}{(P^2 + ip_0 0^+)\tilde{n}^2}[\tilde{n}^\mu \tilde{\Pi}_{R\mu}(P)]\tilde{G}_R(P),$$

$$G_{K1}^{GG}(P) = -\lambda \left[\frac{P^2(P^2 - \tilde{\Pi}_R)\tilde{G}_K}{(P^2 + i0^+)(P^2 - i0^+)} + \frac{\tilde{\Pi}_K \tilde{G}_A}{P^2 - ip_0 0^+} \right]$$
$$= -i\pi\lambda\epsilon(p_0)\delta(P^2)\tilde{\Pi}_K\tilde{G}_A,$$

$$G_{K1}^{TG}(P) = \lambda \frac{1}{E_\perp^2}\left[\frac{E_\perp^\mu \tilde{\Pi}_{R\mu} P^2 \tilde{G}_K}{(P^2 + i0^+)(P^2 - i0^+)} - \frac{E_\perp^\mu \tilde{\Pi}_{K\mu}\tilde{G}_A}{P^2 - ip_0 0^+} \right],$$

$$G_{K2}^{TG}(P) = \lambda \frac{1}{\tilde{\zeta}^2}\left[\frac{\tilde{\zeta}^\mu \tilde{\Pi}_{R\mu} P^2 \tilde{G}_K}{(P^2 + i0^+)(P^2 - i0^+)} - \frac{\tilde{\zeta}^\mu \tilde{\Pi}_{K\mu}\tilde{G}_A}{P^2 - ip_0 0^+} \right],$$

$$G_{K1}^{LG}(P) = \lambda \frac{1}{\tilde{n}^2}\left[\frac{\tilde{n}^\mu \tilde{\Pi}_{R\mu} P^2 \tilde{G}_K}{(P^2 + i0^+)(P^2 - i0^+)} - \frac{\tilde{n}^\mu \tilde{\Pi}_{K\mu}\tilde{G}_A}{P^2 - ip_0 0^+} \right].$$

In Eqs. (3.53) and (3.54), the replacements (D1) and $(\Pi'_j)_{UV}$'s $\to$ $(\Pi_j)_{UV}$'s are made, and, in the formulas in Appendix C, the replacement (D1) is made.

APPENDIX E: STANDARD FORMS FOR THE QUANTITIES IN SEC. IV A

1. Standard form for $\Sigma_K^{[1]}$

From Eq. (4.27), we obtain, after some algebra,

$$\Sigma_K^{[1]} = -\frac{i}{2}\sum_{\rho=\pm}\mathcal{P}_\rho\Bigg[2\{f_\rho, \mathrm{Re}\Sigma_R^{\rho\rho}\}'\cdot\mathcal{P}_\rho + \{f_{-\rho}, \Sigma_R^{\rho-\rho}\}'\cdot\mathcal{P}_{-\rho} + \{f_\rho, \Sigma_A^{\rho-\rho}\}'\mathcal{P}_{-\rho}$$
$$+ 2\frac{\partial f_\rho}{\partial X^\alpha}\mathrm{Re}\Bigg\{\frac{\not{P}}{P^2}[P^\alpha\Sigma_{R2}^{\rho\rho} - N^\alpha\Sigma_{R3}^{\rho\rho} - \rho\epsilon(p_0)e_\perp^\alpha\Sigma_{R4}^{\rho\rho}] + \frac{\not{N}}{N^2}\left(N^\alpha\Sigma_{R2}^{\rho\rho} + \frac{1}{2}\frac{\partial N^2}{\partial P_\alpha}\Sigma_{R3}^{\rho\rho} - \rho\epsilon(p_0)e_\perp^\mu\frac{\partial N^\mu}{\partial P_\alpha}\Sigma_{R4}^{\rho\rho}\right)$$
$$+ \frac{\not{P}\not{N}}{P^2N^2}\rho\epsilon(p_0)\left(-e_\perp^\alpha\Sigma_{R2}^{\rho\rho} - e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\Sigma_{R3}^{\rho\rho} + \rho\epsilon(p_0)N^2\hat{P}^\alpha\Sigma_{R4}^{\rho\rho}\right)\Bigg\} + \gamma_5\Bigg\{\frac{\not{P}}{P^2}[P^\alpha(\Lambda_2^{\rho-\rho})_\alpha - N^\alpha(\Lambda_3^{\rho-\rho})_\alpha$$
$$+ \rho\epsilon(p_0)e_\perp^\alpha(\Lambda_4^{\rho-\rho})_\alpha] + \frac{\not{N}}{N^2}\left(N^\alpha(\Lambda_2^{\rho-\rho})_\alpha + \frac{1}{2}\frac{\partial N^2}{\partial P_\alpha}(\Lambda_3^{\rho-\rho})_\alpha + \rho\epsilon(p_0)e_\perp^\mu\frac{\partial N^\mu}{\partial P_\alpha}(\Lambda_4^{\rho-\rho})_\alpha\right)$$
$$+ \frac{\not{P}\not{N}}{P^2N^2}\rho\epsilon(p_0)\left(e_\perp^\alpha(\Lambda_2^{\rho-\rho})_\alpha + e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}(\Lambda_3^{\rho-\rho})_\alpha + \rho\epsilon(p_0)N^2\hat{P}^\alpha(\Lambda_4^{\rho-\rho})_\alpha\right)\Bigg\}\Bigg],$$

where

$$\hat{P}^{\alpha} \equiv P^{\alpha} + \frac{P^2}{2N^2}\frac{\partial N^2}{\partial P_\alpha},$$

$$(\Lambda_j^{\rho-\rho})_\alpha \equiv \Sigma_{Rj}^{\rho-\rho}\frac{\partial f^{-\rho}}{\partial X^\alpha} + \Sigma_{Aj}^{\rho-\rho}\frac{\partial f^{\rho}}{\partial X^\alpha} \quad (j=2,3,4),$$

$$\{f_{-\rho},\Sigma_R^{\rho-\rho}\}' \equiv \gamma_5[\{f_{-\rho},\Sigma_{R1}^{\rho-\rho}\} + \{f_{-\rho},\Sigma_{R2}^{\rho-\rho}\}\not{P} + \{f_{-\rho},\Sigma_{R3}^{\rho-\rho}\}\not{N} + \{f_{-\rho},\Sigma_{R4}^{\rho-\rho}\}\not{P}\not{N}],$$

etc. Here $\{\ldots,\ \ldots\}$ is as in Eq. (4.4).

2. Standard form for $\underline{H}$ in $G_K^{[2]}$ in Eq. (4.19)

Straightforward manipulation of Eq. (4.19) yields

$$\underline{H} = \sum_{\rho,\,\sigma=\pm} \underline{\mathcal{P}}_\rho \cdot [H_l^{\rho\sigma}]_{\mathrm{IWT}} \cdot \underline{\mathcal{P}}_\sigma + \underline{H}^{(1)},$$

$$H^{(1)} = \frac{i}{2}\sum_{\rho,\,\sigma=\pm} \mathcal{P}_\rho \gamma_5 \left(\frac{\partial \not{N}}{\partial P_\alpha}\frac{\partial C_{\rho-\rho}\Sigma_A^{-\rho\sigma}}{\partial X^\alpha} - \not{N}\{C_{\rho-\rho},\Sigma_A^{-\rho\sigma}\}\right)\mathcal{P}_\sigma + \frac{i}{2}\sum_{\rho,\sigma=\pm}\mathcal{P}_\rho\left(-\frac{\partial \Sigma_R^{\rho\sigma}C_{\sigma-\sigma}}{\partial X^\alpha}\frac{\partial \not{N}}{\partial P_\alpha} + \{C_{\sigma-\sigma},\Sigma_R^{\rho\sigma}\}\not{N}\right)\gamma_5 \mathcal{P}_{-\sigma}, \tag{E1}$$

where $H_l^{\rho\sigma}$ is as in Eq. (2.51). The SF for each term on the RHS of Eq. (E1) reads

$$\mathcal{P}_\rho\gamma_5\frac{\partial\not{N}}{\partial P_\alpha}\frac{\partial C_{\rho-\rho}\Sigma_A^{-\rho\rho}}{\partial X^\alpha}\mathcal{P}_\rho = \mathcal{P}_\rho\frac{\partial N_\mu}{\partial P_\alpha}\frac{\partial}{\partial X^\alpha}\Bigg[C_{\rho-\rho}\Bigg\{-P^\mu\Sigma_{A2}^{-\rho\rho} - N^\mu\Sigma_{A3}^{-\rho\rho} - \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A4}^{-\rho\rho} + \frac{\not{P}}{P^2}[-P^\mu\Sigma_{A1}^{-\rho\rho} + P^2N^\mu\Sigma_{A4}^{-\rho\rho} + \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A3}^{-\rho\rho}] + \frac{\not{N}}{N^2}[-N^2P^\mu\Sigma_{A4}^{-\rho\rho} - N^\mu\Sigma_{A1}^{-\rho\rho} - \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A2}^{-\rho\rho}] + \frac{\not{P}\not{N}}{P^2N^2}[-N^2P^\mu\Sigma_{A3}^{-\rho\rho} + P^2N^\mu\Sigma_{A2}^{-\rho\rho} + \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A1}^{-\rho\rho}]\Bigg\}\Bigg],$$

$$\mathcal{P}_\rho\gamma_5\frac{\partial\not{N}}{\partial P_\alpha}\frac{\partial C_{\rho-\rho}\Sigma_A^{-\rho-\rho}}{\partial X^\alpha}\mathcal{P}_{-\rho} = \mathcal{P}_\rho\frac{\partial N_\mu}{\partial P_\alpha}\gamma_5\frac{\partial}{\partial X^\alpha}\Bigg[C_{\rho-\rho}\Bigg\{P^\mu\Sigma_{A2}^{-\rho-\rho} + N^\mu\Sigma_{A3}^{-\rho-\rho} - \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A4}^{-\rho-\rho} + \frac{\not{P}}{P^2}(P^\mu\Sigma_{A1}^{-\rho-\rho} - P^2N^\mu\Sigma_{A4}^{-\rho-\rho} + \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A3}^{-\rho-\rho}) + \frac{\not{N}}{N^2}(N^2P^\mu\Sigma_{A4}^{-\rho-\rho} + N^\mu\Sigma_{A1}^{-\rho-\rho} - \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A2}^{-\rho-\rho}) + \frac{\not{P}\not{N}}{P^2N^2}[N^2P^\mu\Sigma_{A3}^{-\rho-\rho} - P^2N^\mu\Sigma_{A2}^{-\rho-\rho} + \rho\epsilon(p_0)e_\perp^\mu\Sigma_{A1}^{-\rho-\rho}]\Bigg\}\Bigg],$$

$$\mathcal{P}_{\rho}\frac{\partial\Sigma_R^{\rho\rho}C_{\rho-\rho}}{\partial X^{\alpha}}\frac{\partial\not{N}}{\partial P_{\alpha}}\gamma_5\mathcal{P}_{-\rho}=-\mathcal{P}_{\rho}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\gamma_5\frac{\partial}{\partial X^{\alpha}}\Bigg[\Bigg\{-P^{\mu}\Sigma_{R2}^{\rho\rho}-N^{\mu}\Sigma_{R3}^{\rho\rho}-\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R4}^{\rho\rho}$$
$$+\frac{\not{P}}{P^2}(P^{\mu}\Sigma_{R1}^{\rho\rho}+\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R3}^{\rho\rho}+P^2N^{\mu}\Sigma_{R4}^{\rho\rho})$$
$$+\frac{\not{N}}{N^2}(N^{\mu}\Sigma_{R1}^{\rho\rho}-\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R2}^{\rho\rho}-N^2P^{\mu}\Sigma_{R4}^{\rho\rho})$$
$$+\frac{\not{P}\not{N}}{P^2N^2}(\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R1}^{\rho\rho}-P^2N^{\mu}\Sigma_{R2}^{\rho\rho}+N^2P^{\mu}\Sigma_{R3}^{\rho\rho})\Bigg\}C_{\rho-\rho}\Bigg],$$

$$\mathcal{P}_{\rho}\frac{\partial\Sigma_R^{\rho-\rho}C_{-\rho\rho}}{\partial X^{\alpha}}\frac{\partial\not{N}}{\partial P_{\alpha}}\gamma_5\mathcal{P}_{\rho}=-\mathcal{P}_{\rho}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\frac{\partial}{\partial X^{\alpha}}\Bigg[\Bigg\{-P^{\mu}\Sigma_{R2}^{\rho-\rho}-N^{\mu}\Sigma_{R3}^{\rho-\rho}+\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R4}^{\rho-\rho}$$
$$+\frac{\not{P}}{P^2}(P^{\mu}\Sigma_{R1}^{\rho-\rho}-\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R3}^{\rho-\rho}+P^2N^{\mu}\Sigma_{R4}^{\rho-\rho})$$
$$+\frac{\not{N}}{N^2}[N^{\mu}\Sigma_{R1}^{\rho-\rho}+\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R2}^{\rho-\rho}-N^2P^{\mu}\Sigma_{R4}^{\rho-\rho}]$$
$$+\frac{\not{P}\not{N}}{P^2N^2}[-\rho\epsilon(p_0)e_{\perp}^{\mu}\Sigma_{R1}^{\rho-\rho}-P^2N^{\mu}\Sigma_{R2}^{\rho-\rho}+N^2P^{\mu}\Sigma_{R3}^{\rho-\rho}]\Bigg\}C_{-\rho\rho}\Bigg],$$

$$\mathcal{P}_{\rho}\gamma_5\not{N}\{C_{\rho-\rho},\Sigma_A^{-\rho\rho}\}\mathcal{P}_{\rho}=-\mathcal{P}_{\rho}[N^2\{C_{\rho-\rho},\Sigma_{A3}^{-\rho\rho}\}-N^2\not{P}\{C_{\rho-\rho},\ \Sigma_{A4}^{-\rho\rho}\}+\not{N}\{C_{\rho-\rho},\Sigma_{A1}^{-\rho\rho}\}-\not{P}\not{N}\{C_{\rho-\rho},\Sigma_{A2}^{-\rho\rho}\}]\mathcal{P}_{\rho}$$
$$-\frac{\partial C_{\rho-\rho}}{\partial X^{\alpha}}\mathcal{P}_{\rho}\Bigg[N^{\alpha}\Sigma_{A2}^{-\rho\rho}+\frac{1}{2}\frac{\partial N^2}{\partial P_{\alpha}}\Sigma_{A3}^{-\rho\rho}-\rho\epsilon(p_0)e_{\perp}^{\mu}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\Sigma_{A4}^{-\rho\rho}$$
$$+\frac{\not{P}}{P^2}\left(\rho\epsilon(p_0)e_{\perp}^{\alpha}\Sigma_{A2}^{-\rho\rho}+\rho\epsilon(p_0)e_{\perp}^{\mu}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\Sigma_{A3}^{-\rho\rho}-N^2\hat{P}^{\alpha}\Sigma_{A4}^{-\rho\rho}\right)$$
$$+\frac{\not{P}\not{N}}{P^2}[-P^{\alpha}\Sigma_{A2}^{-\rho\rho}+N^{\alpha}\Sigma_{A3}^{-\rho\rho}+\rho\epsilon(p_0)e_{\perp}^{\alpha}\Sigma_{A4}^{-\rho\rho}]\Bigg]\mathcal{P}_{\rho},$$

$$\mathcal{P}_{\rho}\gamma_5\not{N}\{C_{\rho-\rho},\Sigma_A^{-\rho-\rho}\}\mathcal{P}_{-\rho}=\mathcal{P}_{\rho}\gamma_5[N^2\{C_{\rho-\rho},\Sigma_{A3}^{-\rho-\rho}\}-N^2\not{P}\{C_{\rho-\rho},\Sigma_{A4}^{-\rho-\rho}\}$$
$$+\not{N}\{C_{\rho-\rho},\Sigma_{A1}^{-\rho-\rho}\}-\not{P}\not{N}\{C_{\rho-\rho},\Sigma_{A2}^{-\rho-\rho}\}]\mathcal{P}_{-\rho}$$
$$+\frac{\partial C_{\rho-\rho}}{\partial X^{\alpha}}\mathcal{P}_{\rho}\gamma_5\Bigg[N^{\alpha}\Sigma_{A2}^{-\rho-\rho}+\frac{1}{2}\frac{\partial N^2}{\partial P_{\alpha}}\Sigma_{A3}^{-\rho-\rho}+\rho\epsilon(p_0)e_{\perp}^{\mu}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\Sigma_{A4}^{-\rho-\rho}$$
$$+\frac{\not{P}}{P^2}\left(-\rho\epsilon(p_0)e_{\perp}^{\alpha}\Sigma_{A2}^{-\rho-\rho}-\rho\epsilon(p_0)e_{\perp}^{\mu}\frac{\partial N_{\mu}}{\partial P_{\alpha}}\Sigma_{A3}^{-\rho-\rho}-N^2\hat{P}^{\alpha}\Sigma_{A4}^{-\rho-\rho}\right)$$
$$+\frac{\not{P}\not{N}}{P^2}[-P^{\alpha}\Sigma_{A2}^{-\rho-\rho}+N^{\alpha}\Sigma_{A3}^{-\rho-\rho}-\rho\epsilon(p_0)e_{\perp}^{\alpha}\Sigma_{A4}^{-\rho-\rho}]\Bigg]\mathcal{P}_{-\rho},$$

$$\mathcal{P}_\rho\{C_{\rho-\rho},\Sigma_R^{\rho\rho}\}\not{N}\gamma_5\mathcal{P}_{-\rho}=\mathcal{P}_\rho\gamma_5[N^2\{C_{\rho-\rho},\Sigma_{R3}^{\rho\rho}\}-N^2\not{P}\{C_{\rho-\rho},\Sigma_{R4}^{\rho\rho}\}-\not{N}\{C_{\rho-\rho},\Sigma_{R1}^{\rho\rho}\}+\not{P}\not{N}\{C_{\rho-\rho},\Sigma_{R2}^{\rho\rho}\}]\mathcal{P}_{-\rho}$$
$$+\frac{\partial C_{\rho-\rho}}{\partial X^\alpha}\mathcal{P}_\rho\gamma_5\Bigg[N^\alpha\Sigma_{R2}^{\rho\rho}+\frac{1}{2}\frac{\partial N^2}{\partial P_\alpha}\Sigma_{R3}^{\rho\rho}-\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\Sigma_{R4}^{\rho\rho}$$
$$+\frac{\not{P}}{P^2}\left(\rho\epsilon(p_0)e_\perp^\alpha\Sigma_{R2}^{\rho\rho}+\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\Sigma_{R3}^{\rho\rho}-N^2\hat{P}^\alpha\Sigma_{R4}^{\rho\rho}\right)$$
$$+\frac{\not{P}\not{N}}{P^2}[P^\alpha\Sigma_{R2}^{\rho\rho}-N^\alpha\Sigma_{R3}^{\rho\rho}-\rho\epsilon(p_0)e_\perp^\alpha\Sigma_{R4}^{\rho\rho}]\Bigg]\mathcal{P}_{-\rho},$$

$$\mathcal{P}_\rho\{C_{-\rho\rho},\Sigma_R^{\rho-\rho}\}\not{N}\gamma_5\mathcal{P}_\rho=\mathcal{P}_\rho[N^2\{C_{-\rho\rho},\Sigma_{R3}^{\rho-\rho}\}-N^2\not{P}\{C_{\rho-\rho},\Sigma_{R4}^{\rho-\rho}\}-\not{N}\{C_{\rho-\rho},\Sigma_{R1}^{\rho-\rho}\}+\not{P}\not{N}\{C_{\rho-\rho},\Sigma_{R2}^{\rho-\rho}\}]\mathcal{P}_\rho$$
$$+\frac{\partial C_{-\rho\rho}}{\partial X^\alpha}\mathcal{P}_\rho\Bigg[N^\alpha\Sigma_{R2}^{\rho-\rho}+\frac{1}{2}\frac{\partial N^2}{\partial P_\alpha}\Sigma_{R3}^{\rho-\rho}+\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\Sigma_{R4}^{\rho-\rho}$$
$$+\frac{\not{P}}{P^2}\left(-\rho\epsilon(p_0)e_\perp^\alpha\Sigma_{R2}^{\rho-\rho}-\rho\epsilon(p_0)e_\perp^\mu\frac{\partial N_\mu}{\partial P_\alpha}\Sigma_{R3}^{\rho-\rho}-N^2\hat{P}^\alpha\Sigma_{R4}^{\rho-\rho}\right)$$
$$+\frac{\not{P}\not{N}}{P^2}[P^\alpha\Sigma_{R2}^{\rho-\rho}-N^\alpha\Sigma_{R3}^{\rho-\rho}+\rho\epsilon(p_0)e_\perp^\alpha\Sigma_{R4}^{\rho-\rho}]\Bigg]\mathcal{P}_\rho.$$

3. Standard forms for $\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{C}}$ and $\underline{\mathbf{C}}\cdot\gamma_5\underline{\not{N}}$ in $G_K^{[3]}$ in Eq. (4.20)

Form for $\gamma_5\underline{\not{N}}\cdot\underline{\mathbf{C}}$ in $\mathbf{G}_K^{[3]}$ in Eq. (4.20) is given by Eq. (E1) with

$$\Sigma_{A1}^{\rho\sigma}\to\delta^{\rho\sigma},\quad \Sigma_{Aj}^{\rho\sigma}\to 0\quad (j=2\text{–}4),$$
$$\Sigma_{Rj}^{\rho\sigma}\to 0\quad (j=1\text{–}4).$$

$\underline{\mathbf{C}}\cdot\gamma_5\underline{\not{N}}$ in Eq. (4.20) is given by Eq. (E1) with

$$\Sigma_{R1}^{\rho\sigma}\to-\delta^{\rho\sigma},\quad \Sigma_{Rj}^{\rho\sigma}\to 0\quad (j=2\text{–}4),$$
$$\Sigma_{Aj}^{\rho\sigma}\to 0\quad (j=1\text{–}4).$$

APPENDIX F: ENERGY SHELLS OF $G_R^{\rho\rho}(P,X)$

To find the energy shells of $G_R^{\rho\rho}$, we need $[G_R^{\rho\rho}(P,X)]^{-1}$, the inverse of $G_R^{\rho\rho}(P,X)$ [cf. Eq. (4.28)]. To the gradient approximation, we have

$$(G_R^{\rho\rho})^{-1}=(G_R^{(0)\rho\rho}+G_R^{(1)\rho\rho})^{-1}$$
$$\simeq(G_R^{(0)\rho\rho})^{-1}-(G_R^{(0)\rho\rho})^{-1}G_R^{(1)\rho\rho}(G_R^{(0)\rho\rho})^{-1}. \tag{F1}$$

Here $(G_R^{(0)\rho\rho})^{-1}$ is the (11)-element of Eq. (2.48) and $G_R^{(1)\rho\rho}$ is as in Eq. (4.24). If we ignore the gradient term in Eq. (F1), the energy shells are obtained through

$$\mathrm{Re}[G_R^{(0)\rho\rho}(P,X)]^{-1}\big|_{p_0=\pm\omega_\pm^{(0)}(\pm\vec{p},X)}\propto\mathcal{D}^{\rho\rho}\big|_{p_0=\pm\omega_\pm^{(0)}(\pm\vec{p},X)}=0,$$

where $\mathcal{D}^{\rho\rho}$ is given by Eq. (2.46) with the substitutions (2.50) being made. Then, the true energy shells $p_0=\pm\omega_\pm(\pm\vec{p},X)$ are obtained from Eq. (F1),

$$\pm\frac{\partial\mathrm{Re}[G_R^{(0)\rho\rho}(P,X)]^{-1}}{\partial p_0}\bigg|_{p_0=\pm\omega_\pm^{(0)}(\pm\vec{p},X)}$$
$$\times[\omega_\pm(\pm\vec{p},X)-\omega_\pm^{(0)}(\pm\vec{p},X)]$$
$$=\mathrm{Re}\Big\{[G_R^{(0)\rho\rho}(P,X)]^{-1}G_R^{(1)\rho\rho}(P,X)$$
$$\times[G_R^{(0)\rho\rho}(P,X)]^{-1}\Big\}\bigg|_{p_0=\pm\omega_\pm^{(0)}(\pm\vec{p},X)}.$$

APPENDIX G: STANDARD FORMS FOR THE QUANTITIES IN SEC. IV B

1. Standard forms for $\Pi_K^{[1]\mu\nu}$ and $\Pi_K^{[2]\mu\nu}$ in Eq. (4.64)

From Eq. (4.57) with Eq. (4.64), we obtain

$$\Pi_K^{[1]\mu\nu}=\frac{i}{2}\sum_j\sum_{UV=T,L}[\mathcal{R}_{Lj}^{UV}\{\tilde{f},\tilde{\Pi}_{Rj}^{UV}+\tilde{\Pi}_{Aj}^{UV}\}\mathcal{R}_{Rj}^{UV}]^{\mu\nu}, \tag{G1}$$

$$\Pi_K^{[2]\mu\nu}=2i\mathcal{P}_T^{\mu\nu}\mathrm{Re}\left[\Pi_{R1}^{TG}-\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}\Pi_{R3}^{T'T}\right](E_\perp\cdot\partial)\tilde{f}$$
$$+2i\frac{\tilde{\zeta}^\mu\tilde{\zeta}^\nu}{\tilde{\zeta}^2}\mathrm{Re}\left[\left(-\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}\Pi_{R2}^{TT}+\Pi_{R2}^{TG}\right)(\tilde{\zeta}\cdot\partial)\tilde{f}+\left\{\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}(\Pi_{R3}^{T'T}-\Pi_{R3}^{TT'})-\Pi_{R1}^{TG}\right\}(E_\perp\cdot\partial)\tilde{f}\right]$$
$$+i\tilde{\zeta}^\mu E_\perp^\nu\left[-\frac{1}{\tilde{P}^2}\Pi_{R3}^{TT'}(\tilde{P}\cdot\tilde{\partial})\tilde{f}+\frac{1}{E_\perp^2}\left(\Pi_{R2}^{TG}-\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}\Pi_{R2}^{TT}\right)(E_\perp\cdot\partial)\tilde{f}+\frac{1}{\tilde{\zeta}^2}\left(\Pi_{A1}^{GT}+\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}(\Pi_{R3}^{TT'}+\Pi_{A3}^{TT'})\right)(\tilde{\zeta}\cdot\partial)\tilde{f}\right]$$
$$+2in^\mu n^\nu\mathrm{Re}\Pi_{R1}^{LG}\partial_0\tilde{f}+i\tilde{\zeta}^\mu n^\nu\left[\Pi_{R2}^{TG}\partial_0\tilde{f}-\frac{1}{\tilde{\zeta}^2}\left(\Pi_{A1}^{GL}+\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}\Pi_{A1}^{TL}\right)(\tilde{\zeta}\cdot\partial)\tilde{f}+\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2\tilde{\zeta}^2}\Pi_{A2}^{TL}(E_\perp\cdot\partial)\tilde{f}\right]$$
$$+iE_\perp^\mu n^\nu\left[\frac{1}{\tilde{P}^2}\Pi_{A2}^{TL}(\tilde{P}\cdot\tilde{\partial})\tilde{f}+\Pi_{R1}^{TG}\partial_0\tilde{f}-\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2\tilde{\zeta}^2}\Pi_{A2}^{TL}(\tilde{\zeta}\cdot\partial)\tilde{f}-\frac{1}{E_\perp^2}\left(\Pi_{A1}^{GL}+\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}\Pi_{A1}^{TL}\right)(E_\perp\cdot\partial)\tilde{f}\right]+iE_\perp^\mu\tilde{\zeta}^\nu[\cdots]$$
$$+in^\mu\tilde{\zeta}^\nu[\cdots]+in^\mu E_\perp^\nu[\cdots]. \tag{G2}$$

$[\cdots]$'s are obtained using Eq. (3.26).

2. Standard form for $\tilde{H}$ in Eq. (4.60)

We write $\underline{\tilde{H}}^{\mu\nu}=\underline{\tilde{H}}^{(0)\mu\nu}+\underline{\tilde{H}}^{(1)\mu\nu}$, with $\underline{\tilde{H}}^{(0)\mu\nu}$ the leading term and $\underline{\tilde{H}}^{(1)\mu\nu}$ the gradient term of the DEX of $\tilde{H}^{\mu\nu}(P,X)$. Straightforward manipulation of Eq. (4.60) yields

$$\underline{\tilde{H}}^{(0)\mu\nu}=\underline{\mathcal{P}}_T^{\mu\rho}\cdot[(\tilde{H}_l)_1^{TT}]_{\mathrm{IWT}}\cdot(\underline{\mathcal{P}}_T)_\rho{}^\nu+\underline{\tilde{\zeta}}^\mu\cdot[(\tilde{H}_l)_2^{TT}]_{\mathrm{IWT}}\cdot\underline{\tilde{\zeta}}^\nu-\underline{\tilde{\zeta}}^\mu\cdot[(\tilde{H}_l)_3^{TT'}]_{\mathrm{IWT}}\cdot\underline{E}_\perp^\nu+\underline{E}_\perp^\mu\cdot[(\tilde{H}_l)_3^{T'T}]_{\mathrm{IWT}}\cdot\tilde{\zeta}^\nu$$
$$+\underline{\tilde{\zeta}}^\mu\cdot[(\tilde{H}_l)_1^{TL}]_{\mathrm{IWT}}n^\nu+\underline{E}_\perp^\mu\cdot[(\tilde{H}_l)_2^{TL}]_{\mathrm{IWT}}n^\nu+n^\mu[(\tilde{H}_l)_1^{LT}]_{\mathrm{IWT}}\cdot\underline{\tilde{\zeta}}^\nu-n^\mu[(\tilde{H}_l)_2^{LT}]_{\mathrm{IWT}}\cdot\underline{E}_\perp^\nu,$$

with $(\tilde{H}_l)_j^{UV}$ as in Eq. (3.55), and

$$\tilde{H}^{(1)\mu\nu}=i\mathcal{P}_T^{\mu\nu}\mathrm{Re}\left[\tilde{\zeta}^2E_\perp^2\{C_3^{TT'},\Pi_{R3}^{T'T}\}-2(\zeta\cdot\tilde{P})\left\{\tilde{\zeta}^2C_3^{TT'}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{R3}^{T'T}-\frac{1}{\tilde{P}^2}C_3^{TT'}(E_\perp\cdot\partial)\Pi_{R1}^{TT}\right\}\right]$$
$$+i\tilde{\zeta}^\mu\tilde{\zeta}^\nu\mathrm{Re}\Bigg[-\{C_2^{TT},\Pi_{R1}^{TT}\}-\tilde{\zeta}^2\{C_2^{TT},\Pi_{R2}^{TT}\}+E_\perp^2\{C_3^{T'T},\Pi_{R3}^{TT'}-\Pi_{A3}^{TT'}\}-2\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}C_2^{TT}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{R2}^{TT}+2\tilde{\zeta}^2C_3^{T'T}(\tilde{P}\cdot\overleftrightarrow{\partial})\Pi_{R3}^{TT'}$$
$$+2(\zeta\cdot\tilde{P})[C_3^{TT'}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{R3}^{T'T}-C_3^{T'T}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{R3}^{TT'}]-2\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2\tilde{\zeta}^2}[C_2^{TT}(\tilde{\zeta}\cdot\partial)+2\mathrm{Re}C_3^{TT'}(E_\perp\cdot\partial)]\Pi_{R1}^{TT}\Bigg]$$
$$+\frac{i}{2}\frac{\tilde{\zeta}^\mu E_\perp^\nu}{\tilde{P}^2}\Bigg[\tilde{P}^2\tilde{\zeta}^2(\{C_2^{TT},\Pi_{A3}^{TT'}\}+\{C_3^{TT'},\Pi_{R2}^{TT}\})+2\tilde{P}^2\{C_3^{TT'},\mathrm{Re}\Pi_{R1}^{TT}\}-2C_3^{TT'}(\tilde{P}\cdot\tilde{\partial})\Pi_{A1}^{TT}+2(\zeta\cdot\tilde{P})$$
$$\times\left\{-C_2^{TT}\frac{E_\perp\cdot\partial}{E_\perp^2}\Pi_{A1}^{TT}+C_2^{TT}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{A3}^{TT'}+C_3^{TT'}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{R2}^{TT}+2i(E_\perp\cdot\partial)\mathrm{Im}(C_3^{TT'}\Pi_{R3}^{T'T})+\frac{2}{\tilde{\zeta}^2}C_3^{TT'}(\tilde{\zeta}\cdot\partial)\mathrm{Re}\Pi_{R1}^{TT}\right\}\Bigg]$$
$$+\frac{i}{2}\tilde{\zeta}^\mu n^\nu\left[-\tilde{\zeta}^2\{C_2^{TT},\ \Pi_{A1}^{TL}\}+E_\perp^2\{C_3^{TT'},\Pi_{A2}^{TL}\}-\frac{2\zeta\cdot\tilde{P}}{\tilde{P}^2}C_2^{TT}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{A1}^{TL}+2C_3^{TT'}[\tilde{\zeta}^2(\tilde{P}\cdot\overleftrightarrow{\partial})-(\zeta\cdot\tilde{P})(\tilde{\zeta}\cdot\overleftrightarrow{\partial})]\Pi_{A2}^{TL}\right]$$
$$-iE_\perp^\mu n^\nu\left[\frac{\tilde{\zeta}^2}{2}\{C_3^{T'T},\ \Pi_{A1}^{TL}\}+\frac{\zeta\cdot\tilde{P}}{\tilde{P}^2}C_3^{T'T}(\tilde{\zeta}\cdot\overleftrightarrow{\partial})\Pi_{A1}^{TL}\right]+iE_\perp^\mu\tilde{\zeta}^\nu[\cdots]+in^\mu\tilde{\zeta}^\nu[\cdots]-in^\mu E_\perp^\nu[\cdots],$$

where $A\overleftrightarrow{\partial}B\equiv A\partial B-A\overleftarrow{\partial}B$, and $[\cdots]$'s are obtained using Eqs. (3.32) and (3.35).

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