Nucleon properties at finite lattice spacing in chiral perturbation theory

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Properties of the proton and neutron are studied in partially quenched chiral perturbation theory at finite lattice spacing. Masses, magnetic moments, the matrix elements of isovector twist-2 operators and axial-vector currents are examined at the one-loop level in a double expansion in the light-quark masses and the lattice spacing. This work will be useful in extrapolating the results of simulations using Wilson valence and sea quarks, as well as simulations using Wilson sea quarks and Ginsparg-Wilson valence quarks, to the continuum.

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I. INTRODUCTION

Impressive progress is currently being made in understanding properties and interactions of the low-lying hadrons using lattice QCD. However, computational limitations necessitate the use of quark masses m_q that are significantly larger than those of nature, lattice spacings a that are not significantly smaller than the physical scale of interest, and lattice sizes L that are not significantly larger than the physical scale of interest. In order to make a connection between lattice QCD calculations of the foreseeable future and nature, extrapolations in the quark masses, lattice spacing and lattice volume are required. Assuming a hierarchy of mass scales,

$$L^{-1} \ll m_q \ll \Lambda_\chi \ll a^{-1}, \tag{1}$$

where Λ_χ is the scale of chiral symmetry breaking (a typical QCD scale), and working in the infinite volume limit, the appropriate tool for incorporating the light quark masses and the finite lattice spacing into hadronic observables is effective field theory (EFT).

Chiral perturbation theory (χPT) provides a systematic description of low-energy QCD near the chiral limit and is therefore an EFT which exploits the hierarchy $m_q \ll \Lambda_\chi$. This technology has been extended to describe both quenched QCD [1–5] with quenched chiral perturbation theory (QχPT) and partially quenched QCD (PQQCD) [6–10] with partially quenched chiral perturbation theory (PQχPT). It is hoped that future lattice simulations can be performed with sufficiently small quark masses where the chiral expansion is convergent, and can be used to extrapolate down to the quark masses of nature. Recently, meson and baryon properties have been studied extensively in both QχPT [4,5] and PQχPT [11–14]. The EFT describing the low-energy dynamics of two-nucleon systems in PQQCD has also been explored [15,16].

In order to construct the low-energy EFT at finite lattice spacing, written in terms of the hadronic fields, one must first construct the underlying lattice theory, written in terms of the quark and gluon fields. The lattice theory and the continuum theory coincide in the $a=0$ limit, but away from this limit the theories differ. The lattice breaks the Lorentz group [$O(4)$ in Euclidean space] down to the discrete symmetry group of the lattice, which we will take to be the symmetry group $H(4)$ of a hypercubic lattice. As first discussed by Symanzik [17], the strong-interaction Lagrange density at $a \neq 0$ will receive contributions from an infinite series of operators $\sim \Sigma a^k \mathcal{O}^{(4+k)}$. Therefore, the contribution from terms of $\mathcal{O}(a^n)$ to a given strong-interaction observable will, according to Eq. (1), be suppressed by factors of $\sim a^n \Lambda_\chi^n$. For Wilson fermions [18], where chiral symmetry is not a good symmetry, it is straightforward to show that at $\mathcal{O}(a)$ the Symanzik Lagrange density has the form, once appropriate redefinitions and renormalizations have been performed,

$$\mathcal{L} = \overline{\psi}(\not{D} + m_q)\psi + a c_{sw} \overline{\psi} \sigma^{\mu\nu} G_{\mu\nu} \psi + \cdots \tag{2}$$

where c_{sw} is the Sheikholeslami-Wohlert [19] coefficient that must be determined numerically. For lattice fermions that satisfy the Ginsparg-Wilson (GW) condition [20], such as Kaplan fermions [21] and overlap fermions [22], where chiral symmetry is a good symmetry, the coefficient of the Sheikholeslami-Wohlert [19] term vanishes, $c_{sw}=0$. The power counting we will use in this work treats both m_q and the lattice spacing a as small. The small dimensionless parameters that we will use in our low-energy EFT are

$$p^2 \sim \frac{m_q}{\Lambda_\chi} \sim a\Lambda_\chi \sim \frac{\partial^2}{\Lambda_\chi^2}, \tag{3}$$

where ∂ represents the derivative operator.

Following earlier work of Sharpe and Singleton [23] (see also Ref. [24]), Rupak and Shoresh [25] have extended χPT to $\mathcal{O}(p^4)$ including the effects of a finite lattice spacing at $\mathcal{O}(a)$ for Wilson fermions and computed the Goldstone-boson masses. Together with Bär [26] these same authors have generalized the results to "mixed" actions where different types of lattice fermions are used for the sea and valence quarks. Recently, this work was extended to $\mathcal{O}(a^2)$ for both Wilson and mixed actions [27,28]. The pion decay constant has also been computed to $\mathcal{O}(a^2)$ for Wilson fermions in QCD by Aoki [28]. When considering the matrix elements of operators coupled to external sources, such as the axial-vector current matrix elements or the matrix elements of twist-2 operators, there are contributions at $\mathcal{O}(a)$ from the operator itself, in addition to the Sheikholeslami-Wohlert term in the strong-interaction sector. In this work we consider the leading $\mathcal{O}(a)$ corrections to nucleon properties. We compute the contributions to the nucleon masses at $\mathcal{O}(p^3)$, to their magnetic moments at $\mathcal{O}(p)$, to their isovector axial-vector matrix elements at $\mathcal{O}(p^2)$ and to the matrix element of the $n=2$ isovector twist-2 operator at $\mathcal{O}(p^2)$.

II. PQQCD AT FINITE LATTICE SPACING

The Symanzik effective Lagrange density at $\mathcal{O}(p)$ which describes the quark sector of PQQCD for two light flavors is

$$\mathcal{L}=\overline{Q}[i\not{D}-m_Q]Q+\overline{Q}\sigma^{\mu\nu}G_{\mu\nu}\,A_Q Q, \tag{4}$$

where the valence, sea, and ghost quarks are combined into the column vector

$$Q=(u,d,j,l,\tilde{u},\tilde{d})^T. \tag{5}$$

The u and d are valence quarks, the $\tilde{u}$ and $\tilde{d}$ are ghost quarks, and the j and l are sea quarks. The mass matrix m_Q is $m_Q=\mathrm{diag}(m_u,m_d,m_j,m_l,m_u,m_d)$ and the Sheikholeslami-Wohlert coefficient matrix is $A_Q=a\,\mathrm{diag}(c_{sw}^{(V)},c_{sw}^{(V)},c_{sw}^{(S)},c_{sw}^{(S)},c_{sw}^{(V)},c_{sw}^{(V)})$ [25,26]. As mentioned previously, when both the valence and sea quarks are Wilson fermions $c_{sw}^{(V)}=c_{sw}^{(S)}$, but when the valence quarks are GW fermions while the sea quarks are Wilson fermions, $c_{sw}^{(V)}=0$.

The graded equal-time commutation relations for two fields are

$$Q_i^\alpha(\mathbf{x})Q_k^{\beta\dagger}(\mathbf{y})-(-)^{\eta_i\eta_k}Q_k^{\beta\dagger}(\mathbf{y})Q_i^\alpha(\mathbf{x})=\delta^{\alpha\beta}\delta_{ik}\delta^3(\mathbf{x}-\mathbf{y}), \tag{6}$$

where α,β are spin indices and i,k are flavor indices. The objects η_k correspond to the parity of the component of Q_k, with $\eta_k=+1$ for $k=1,2,3,4$ and $\eta_k=0$ for $k=5,6$, and the graded commutation relations for two Q's or two $Q^\dagger$'s are analogous. The left- and right-handed quark fields, $Q_{L,R}$ in Eq. (5), transform in the fundamental representation of $SU(4|2)_{L,R}$, respectively. The ground floor of Q_L transforms as a $(\mathbf{4},\mathbf{1})$ of $SU(4)_{qL}\otimes SU(2)_{\tilde{q}L}$ while the first floor transforms as $(\mathbf{1},\mathbf{2})$, and the right handed field Q_R transforms analogously. In the absence of the quark mass and Sheikholeslami-Wohlert terms, $m_Q=A_Q=0$, the Lagrange density in Eq. (4) has a graded symmetry $U(4|2)_L\otimes U(4|2)_R$, where the left- and right-handed quark fields transform as $Q_L\rightarrow U_L Q_L$ and $Q_R\rightarrow U_R Q_R$, respectively. The strong anomaly reduces the symmetry of the theory to $SU(4|2)_L\otimes SU(4|2)_R\otimes U(1)_V$ [10]. It is assumed that this symmetry is spontaneously broken according to the pattern $SU(4|2)_L\otimes SU(4|2)_R\otimes U(1)_V\rightarrow SU(4|2)_V\otimes U(1)_V$ so that an identification with QCD can be made.

A. The pseudo-Goldstone bosons

In order to construct the Lagrange density describing the dynamics of the pseudo-Goldstone bosons at $\mathcal{O}(p^2)$, we allow m_Q and A_Q to transform under the graded chiral group [23,25,26]. This leads to

$$\mathcal{L}=\frac{f^2}{8}\mathrm{str}[\partial^\mu\Sigma^\dagger\partial_\mu\Sigma]+\lambda_M\frac{f^2}{4}\mathrm{str}[m_Q\Sigma^\dagger+m_Q\Sigma]+\lambda_A\frac{f^2}{4}\mathrm{str}[A_Q\Sigma^\dagger+A_Q\Sigma]+\alpha_\Phi\partial^\mu\Phi_0\partial_\mu\Phi_0-m_0^2\Phi_0^2, \tag{7}$$

where str denotes a supertrace, and α_Φ and m_0 are quantities that do not vanish in the chiral limit. The meson field is incorporated in Σ via

$$\Sigma=\exp\left(\frac{2i\Phi}{f}\right)=\xi^2, \qquad \Phi=\begin{pmatrix} M & \chi^\dagger \\ \chi & \tilde{M}\end{pmatrix}, \tag{8}$$

where M and $\tilde{M}$ are matrices containing bosonic mesons while χ and $\chi^\dagger$ are matrices containing fermionic mesons, with

$$M=\begin{pmatrix} \eta_u & \pi^+ & J^0 & L^+ \\ \pi^- & \eta_d & J^- & L^0 \\ \overline{J^0} & J^+ & \eta_j & Y_{jl}^+ \\ L^- & \overline{L^0} & Y_{jl}^- & \eta_l \end{pmatrix}, \qquad \tilde{M}=\begin{pmatrix} \tilde{\eta}_u & \tilde{\pi}^+ \\ \tilde{\pi}^- & \tilde{\eta}_d \end{pmatrix},$$

$$\chi=\begin{pmatrix} \chi_{\eta_u} & \chi_{\pi^+} & \chi_{J^0} & \chi_{L^+} \\ \chi_{\pi^-} & \chi_{\eta_d} & \chi_{J^-} & \chi_{L^0} \end{pmatrix}. \tag{9}$$

The convention we use corresponds to $f\sim 132$ MeV, and the charge assignments have been made using an electromagnetic charge matrix of $\mathcal{Q}^{(PQ)}=\frac{1}{3}\mathrm{diag}(2,-1,2,-1,2,-1)$. The singlet field is defined to be $\Phi_0=\mathrm{str}(\Phi)/\sqrt{2}$, and its mass m_0 can be taken to be of the order of the scale of chiral symmetry breaking, $m_0\rightarrow\Lambda_\chi$ [10]. Hence the parameters α_Φ and m_0 decouple from the low-energy EFT in PQχPT [10].

It is straightforward to show that the meson masses resulting from this Lagrange density are

$$m_{ud}^2=\lambda_M(m_u+m_d)+2\lambda_A a c_{sw}^{(V)},$$

$$m_{uu}^2=2\lambda_M m_u+2\lambda_A a c_{sw}^{(V)},$$

$$m_{ju}^2=\lambda_M(m_j+m_u)+\lambda_A(a c_{sw}^{(V)}+a c_{sw}^{(S)}),$$

$$m_{jl}^2=\lambda_M(m_j+m_l)+2\lambda_A a c_{sw}^{(S)}, \tag{10}$$

and so forth, where m_{ab}^2 denotes the mass squared of a meson containing a (anti-) quark of flavor a and one of flavor b (either valence, sea or ghost). The meson masses have been computed out to $\mathcal{O}(m_q^2 a)$ in Refs. [25,26].

B. The nucleons and Δ resonances

The free Lagrange density for the 70-dimensional baryon supermultiplet $\mathcal{B}_{ijk}$ containing the nucleon and for the 44-dimensional baryon supermultiplet $\mathcal{T}^{\mu}_{ijk}$ containing the Δ resonances is [11,12], at leading order [$\mathcal{O}(p)$],

$$\begin{aligned}\mathcal{L}=&\,i(\overline{\mathcal{B}}v\cdot\mathcal{D}\mathcal{B})+2\alpha_M^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{B}\mathcal{M}_+)+2\beta_M^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{M}_+\mathcal{B})\\&+2\sigma_M^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{B})\mathrm{str}(\mathcal{M}_+)+2\alpha_A^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{B}\mathcal{A}_+)\\&+2\beta_A^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{A}_+\mathcal{B})+2\sigma_A^{(\mathrm{PQ})}(\overline{\mathcal{B}}\mathcal{B})\mathrm{str}(\mathcal{A}_+)\\&-i(\overline{\mathcal{T}}^{\mu}v\cdot\mathcal{D}\mathcal{T}_\mu)+\Delta(\overline{\mathcal{T}}^{\mu}\mathcal{T}_\mu)-2\gamma_M^{(\mathrm{PQ})}(\overline{\mathcal{T}}^{\mu}\mathcal{M}_+\mathcal{T}_\mu)\\&-2\overline{\sigma}_M^{(\mathrm{PQ})}(\overline{\mathcal{T}}^{\mu}\mathcal{T}_\mu)\mathrm{str}(\mathcal{M}_+)-2\gamma_A^{(\mathrm{PQ})}(\overline{\mathcal{T}}^{\mu}\mathcal{A}_+\mathcal{T}_\mu)\\&-2\overline{\sigma}_A^{(\mathrm{PQ})}(\overline{\mathcal{T}}^{\mu}\mathcal{T}_\mu)\mathrm{str}(\mathcal{A}_+),\end{aligned}\tag{11}$$

where Δ is the mass splitting between the 70 and 44, $\mathcal{M}_+=\frac{1}{2}(\xi^\dagger m_Q\xi^\dagger+\xi m_Q\xi)$, $\xi=\sqrt{\Sigma}$, and $\mathcal{A}_+=\frac{1}{2}(\xi^\dagger A_Q\xi^\dagger+\xi A_Q\xi)$. The terms that arise at $\mathcal{O}(p)$ from the Sheikholeslami-Wohlert [19] operator have coefficients $\alpha_A^{(\mathrm{PQ})}$, $\beta_A^{(\mathrm{PQ})}$, $\sigma_A^{(\mathrm{PQ})}$, $\gamma_A^{(\mathrm{PQ})}$, and $\overline{\sigma}_A^{(\mathrm{PQ})}$.

The Lagrange density describing the interactions of the 70 and 44 with the pseudo-Goldstone bosons at LO in the chiral expansion is [4]

$$\begin{aligned}\mathcal{L}=&\,2\alpha(\overline{\mathcal{B}}S^\mu\mathcal{B}A_\mu)+2\beta(\overline{\mathcal{B}}S^\mu A_\mu\mathcal{B})+2\mathcal{H}(\overline{\mathcal{T}}^\nu S^\mu A_\mu\mathcal{T}_\nu)\\&+\sqrt{\frac{3}{2}}\mathcal{C}\,[(\overline{\mathcal{T}}^\nu A_\nu\mathcal{B})+(\overline{\mathcal{B}}A_\nu\mathcal{T}^\nu)],\end{aligned}\tag{12}$$

where S^μ is the covariant spin vector [29–31]. Restricting ourselves to the valence sector, we can compare Eq. (12) with the LO interaction Lagrange density of QCD,

$$\begin{aligned}\mathcal{L}=&\,2g_A\overline{N}S^\mu A_\mu N+g_1\overline{N}S^\mu N\mathrm{tr}[A_\mu]\\&+g_{\Delta N}[\overline{T}^{abc,\nu}A^d_{a,\nu}N_b\epsilon_{cd}+\mathrm{H.c.}]\\&+2g_{\Delta\Delta}\overline{T}^\nu S^\mu A_\mu T_\nu+2g_X\overline{T}^\nu S^\mu T_\nu\mathrm{tr}[A_\mu],\end{aligned}\tag{13}$$

and find that at tree level

$$\alpha=\frac{4}{3}g_A+\frac{1}{3}g_1,\quad \beta=\frac{2}{3}g_1-\frac{1}{3}g_A,\quad \mathcal{H}=g_{\Delta\Delta},\qquad \mathcal{C}=-g_{\Delta N},\tag{14}$$

with $g_X=0$. Considering only the nucleons, and decomposing the Lagrange density in Eq. (13) into the mass eigenstates of the isospin-symmetric limit, $\pi^\pm,\pi^0$ and η, we have

$$\mathcal{L}=2g_A\overline{N}S^\mu\widetilde{A}_\mu N+\frac{\sqrt{2}}{f}(g_A+g_1)\overline{N}S^\mu N\partial_\mu\eta,\tag{15}$$

where $\widetilde{A}_\mu$ is the axial-vector field of pions only (excluding the isosinglet meson). In the isospin-symmetric limit, with the mass of η being of the order of $\sim\Lambda_\chi$, all expressions must be independent of the coupling g_1. At the order we work in this paper, higher-order interactions do not contribute.

III. NUCLEON MASSES

The mass of the ith baryon in the 70-dimensional baryon supermultiplet has an expansion in m_q and a of the form

$$M_i=M_0(\mu)-M_i^{(1)}(\mu)-M_i^{(3/2)}(\mu)+\cdots,\tag{16}$$

and we will be interested only in the proton and neutron masses, i.e., $i=p,n$. The superscript denotes the order in the expansion, i.e., $M_p^{(3/2)}(\mu)$ denotes a contribution of $\mathcal{O}(p^3)$. The term $M_0(\mu)$ is the same for all baryons in the supermultiplet, and is non-zero in the $m_Q,a\to0$ limits. The $a=0$ values of $M_i^{(1)}(\mu)$ and $M_i^{(3/2)}(\mu)$ can be found in Ref. [12] for arbitrary quark masses. For $a\neq0$ we find that $M_i^{(1)}(\mu)$ becomes

$$\begin{aligned}M_p^{(1)}=&\,\frac{1}{3}m_u(5\alpha_M^{(PQ)}+2\beta_M^{(PQ)})+\frac{1}{3}m_d(\alpha_M^{(PQ)}+4\beta_M^{(PQ)})\\&+2\sigma_M^{(PQ)}(m_j+m_l)+2ac_{sw}^{(V)}(\alpha_A^{(PQ)}+\beta_A^{(PQ)})\\&+4\sigma_A^{(PQ)}ac_{sw}^{(S)},\\M_n^{(1)}=&\,\frac{1}{3}m_u(\alpha_M^{(PQ)}+4\beta_M^{(PQ)})+\frac{1}{3}m_d(5\alpha_M^{(PQ)}+2\beta_M^{(PQ)})\\&+2\sigma_M^{(PQ)}(m_j+m_l)+2ac_{sw}^{(V)}(\alpha_A^{(PQ)}+\beta_A^{(PQ)})\\&+4\sigma_A^{(PQ)}ac_{sw}^{(S)}.\end{aligned}\tag{17}$$

At this order, the finite a contributions are the same for the proton and neutron. This has to be the case as the lattice spacing, being the same for the u and d quarks, transforms as an isoscalar. The $M_i^{(3/2)}(\mu)$ contributions, arising from one-loop diagrams in PQχPT, have exactly the same form as in Eqs. (38) and (40) of Ref. [12], however, it is understood that the meson masses are evaluated with the relations in Eq. (10), and hence have implicit dependence on the lattice spacing. We see that this introduces non-analytic dependence on the lattice spacing in the chiral limit.

As the explicit expressions for the loop contributions are quite long, we will simply quote the results in the isospin limit (i.e. the sea quarks are degenerate, but different in mass from the degenerate valence quarks and ghosts),

$$\begin{aligned}M_p^{(3/2)}=&\,\frac{1}{8\pi f^2}\left(\frac{g_A^2}{12}[9m_{SS}^2m_{VV}+16m_{SV}^3-7m_{VV}^3]\right.\\&+\frac{g_1^2}{12}[9m_{SS}^2m_{VV}+10m_{SV}^3-19m_{VV}^3]\\&+\frac{g_Ag_1}{6}[9m_{SS}^2m_{VV}+4m_{SV}^3-13m_{VV}^3]\\&\left.+\frac{2g_{\Delta N}^2}{3\pi}[F_{VV}+F_{SV}]\right),\end{aligned}\tag{18}$$

where S denotes a sea quark and V denotes a valence quark. The function $F_c=F(m_c,\Delta,\mu)$ is

$$F(m,\Delta,\mu)=(m^2-\Delta^2)\left[\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right)-\Delta\log\left(\frac{m^2}{\mu^2}\right)\right]-\frac{1}{2}\Delta m^2\log\left(\frac{m^2}{\mu^2}\right), \tag{19}$$

where μ is the renormalization scale, and the meson masses depend upon light-quark masses and the lattice spacing through Eq. (10). Note that we use dimensional regularization with $\overline{MS}$ to regulate divergent integrals. Of course, all regulators must give equivalent results.

In the QCD and isospin limit, $m_j,m_l,m_u,m_d\to\overline{m}$, while at finite a, the nucleon mass is

$$M_N=M_0-2\overline{m}(\alpha_M^{(PQ)}+\beta_M^{(PQ)}+2\sigma_M^{(PQ)})-\frac{1}{8\pi f^2}\left[\frac{3}{2}g_A^2m_\pi^3+\frac{4g_{\Delta N}^2}{3\pi}F_\pi\right]-2ac_{sw}^{(V)}(\alpha_A^{(PQ)}+\beta_A^{(PQ)})-4ac_{sw}^{(S)}\sigma_A^{(PQ)}. \tag{20}$$

IV. NUCLEON MAGNETIC MOMENTS

The most general analysis of the nucleon magnetic moments requires us to determine the vector-current operator out to the order in the lattice spacing that we are working. While the $\mathcal{O}(a)$ corrections are known [32], in order to compute the magnetic moments at $\mathcal{O}(p)$ only the continuum limit of the vector-current operator is required. Up to $\mathcal{O}(p)$, it is convenient to write the magnetic moment of the ith nucleon as

$$\mu_i=\alpha_i+\frac{M_N}{4\pi f^2}[\beta_i+\beta_i']+\cdots, \tag{21}$$

where the constants α_i, β_i and β_i' in PQQCD are given in Eqs. (49)–(51) in Ref. [12]. The only modification required at finite lattice spacing is to use Eq. (10) in evaluating the meson masses. Therefore, we see that the lattice spacing first appears through the meson masses at $\mathcal{O}(p)$, and hence the leading contribution of the finite lattice spacing is nonanalytic in the lattice spacing in the chiral limit.

In the isospin limit one finds

$$\beta_p=-\frac{g_A^2}{9}[8m_{SV}+m_{VV}]-\frac{4g_Ag_1}{9}[m_{SV}-m_{VV}]-\frac{g_1^2}{18}[m_{SV}-m_{VV}]+(q_j+q_l)[m_{SV}-m_{VV}]\times\frac{1}{3}\left(2g_A^2+g_Ag_1+\frac{5}{4}g_1^2\right),$$

$$\beta_p'=\frac{g_{\Delta N}^2}{9}\left[-2\mathcal{F}_{VV}+\frac{3}{2}(q_j+q_l)(\mathcal{F}_{VV}-\mathcal{F}_{SV})\right],$$

$$\beta_n=\frac{g_A^2}{9}[4m_{SV}+5m_{VV}]+\frac{2g_Ag_1}{9}[m_{SV}-m_{VV}]-\frac{2g_1^2}{9}[m_{SV}-m_{VV}]+(q_j+q_l)[m_{SV}-m_{VV}]\times\frac{1}{3}\left(2g_A^2+g_Ag_1+\frac{5}{4}g_1^2\right),$$

$$\beta_n'=\frac{g_{\Delta N}^2}{9}\left[\mathcal{F}_{VV}+\mathcal{F}_{SV}+\frac{3}{2}(q_j+q_l)(\mathcal{F}_{VV}-\mathcal{F}_{SV})\right], \tag{22}$$

where the function $\mathcal{F}_i=\mathcal{F}(m_i,\Delta,\mu)$ is

$$\pi\mathcal{F}(m,\Delta,\mu)=\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right)-\Delta\log\left(\frac{m^2}{\mu^2}\right). \tag{23}$$

We have used $\mathcal{Q}=\mathrm{diag}(+\frac{2}{3},-\frac{1}{3},q_j,q_l,q_j,q_l)$ for the electromagnetic charge matrix in PQQCD [11,12]. The charges of the sea quarks and ghosts, q_j and q_l, arise due to the fact that their charge assignments are not unique, only constrained by the requirement that electromagnetic observables computed in PQQCD reproduce those of QCD in the QCD limit [11,12,36]. It is clear from the expressions in Eq. (22) that this is indeed the case.

In the isospin-symmetric QCD limit at finite lattice spacing, the magnetic moments become [33,34]

$$\mu_p=\mu_0+\mu_1-\frac{M_N}{4\pi f^2}\left[g_A^2m_\pi+\frac{2}{9}g_{\Delta N}^2\mathcal{F}_\pi\right],$$

$$\mu_n=\mu_0-\mu_1+\frac{M_N}{4\pi f^2}\left[g_A^2m_\pi+\frac{2}{9}g_{\Delta N}^2\mathcal{F}_\pi\right]. \tag{24}$$

The isoscalar and isovector magnetic moment contributions from the LO dimension-5 operators are μ_0 and μ_1, respectively, and are independent of the lattice spacing.

V. NUCLEON AXIAL MATRIX ELEMENTS

The leading effects of a finite lattice spacing on the matrix elements of the axial-vector current enter at $\mathcal{O}(p^2)$ from both one-loop diagrams and from local counterterms. In addition to the contribution from the Sheikholeslami-Wohlert term, there is also a contribution from the $\mathcal{O}(a)$ corrections to the axial-current operator. As discussed in detail in Ref. [35], there are only two operator structures that contribute to the axial-current operator at $\mathcal{O}(a)$, using the notation of Ref. [35],

$$\mathcal{O}_{7,\mu}^a=\overline{q}\tau^a\gamma_5(i\overleftrightarrow{D}_\mu)q,\quad \mathcal{O}_{8,\mu}^a=\overline{q}\tau^a\gamma_\mu\gamma_5m_qq, \tag{25}$$

in QCD, where $\overleftrightarrow{D}_\mu=\vec{D}_\mu-\overleftarrow{D}_\mu$. The appearance of m_q in $\mathcal{O}_{8,\mu}^a$ renders it $\mathcal{O}(p^4)$ in the power counting, and so we neglect it, leaving the contribution from $\mathcal{O}_{7,\mu}^a$. The extension to PQQCD is straightforward. In analogy with the electromagnetic interaction, the isovector axial-charge matrix must be extended to PQQCD and the axial charges of the sea quarks and ghost must be defined as the extension is not unique [11,12,36]. Requiring the axial charge matrix to be supertraceless implies that the most general extension to the leading operator is, e.g.,

$$\tau^3 \to \overline{\tau}^3 = \mathrm{diag}(1,-1,y_j,y_l,y_j,y_l). \tag{26}$$

In order to construct the nucleon axial matrix element at leading order one uses the spurion construction in which $\overline{\tau}_L^a \to L\overline{\tau}_L^a L^\dagger$ and $\overline{\tau}_R^a \to R\overline{\tau}_L^a R^\dagger$, where the axial-current operator is decomposed into contributions from the left- and right-handed quark fields.

At subleading order $\mathcal{O}(p^2)$, the contribution to the axial current in PQQCD is

$$\delta A_\mu^a = \overline{Q}\overline{\tau}_{A,7}^a \gamma_5 (i\overleftrightarrow{D}_\mu) Q, \tag{27}$$

where, for example, the matrix $\overline{\tau}_{A,7}^3$ is

$$\overline{\tau}_{A,7}^3 = a\,\mathrm{diag}(c_{A7}^{(V)}, -c_{A7}^{(V)}, y_j c_{A7}^{(S)}, y_l c_{A7}^{(S)}, y_j c_{A7}^{(V)}, y_l c_{A7}^{(V)}), \tag{28}$$

for the charge matrix in Eq. (26), where $c_{A7}^{(S)}$ and $c_{A7}^{(V)}$ are the coefficients of the axial-current corrections for the sea quarks and valence quarks, respectively. Under chiral transformations the spurion field transforms as $\overline{\tau}_{A,7}^3 \to L\overline{\tau}_{A,7}^3 R^\dagger$, like the quark mass matrix.

The leading-order contribution to the matrix elements of the axial current arises from the operators [11,12]

$$\begin{aligned} {}^{(PQ)}j_{\mu,5}^a = {} & 2\alpha(\overline{\mathcal{B}}S_\mu \mathcal{B}\overline{\tau}_{\xi+}^a) + 2\beta(\overline{\mathcal{B}}S_\mu \overline{\tau}_{\xi+}^a \mathcal{B}) \\ & + 2\mathcal{H}(\overline{\mathcal{T}}^\nu S_\mu \overline{\tau}_{\xi+}^a \mathcal{T}_\nu) \\ & + \sqrt{\frac{3}{2}}\mathcal{C}\,[(\overline{\mathcal{T}}_\mu \overline{\tau}_{\xi+}^a \mathcal{B}) + (\overline{\mathcal{B}}\overline{\tau}_{\xi+}^a \mathcal{T}_\mu)], \end{aligned} \tag{29}$$

where $\overline{\tau}_{\xi+}^a = \frac{1}{2}(\xi\overline{\tau}^a\xi^\dagger + \xi^\dagger\overline{\tau}^a\xi)$.

At $\mathcal{O}(p^2)$ there are three different contributions, which we denote as ${}^{(PQ)}\delta j_{\mu,5}^{(1),a}$, ${}^{(PQ)}\delta j_{\mu,5}^{(2),a}$ and ${}^{(PQ)}\delta j_{\mu,5}^{(3),a}$. The contribution from a single insertion of the leading $\mathcal{O}(a)$ corrections to the axial-current operator is

$${}^{(PQ)}\delta j_{\mu,5}^{(1),a} = 2\gamma_{A1}(\overline{\mathcal{B}}S_\mu \mathcal{B}\overline{\tau}_{A,7,\xi+}^a) + 2\gamma_{A2}(\overline{\mathcal{B}}S_\mu \overline{\tau}_{A,7,\xi+}^a \mathcal{B}), \tag{30}$$

where $\overline{\tau}_{A,7,\xi+}^a = \frac{1}{2}(\xi\overline{\tau}_{A,7}^a\xi + \xi^\dagger\overline{\tau}_{A,7}^a\xi^\dagger)$. The remaining two contributions arise from a single insertion of the light-quark mass matrix and from the Sheikholeslami-Wohlert term, which are of the form

$$\begin{aligned} {}^{(PQ)}j_{\mu,5}^{(2,3),a} = {} & 2[b_{1,\Gamma}\overline{\mathcal{B}}^{kji}\{\overline{\tau}_{\xi+}^a,\Gamma_+\}_i^n S_\mu \mathcal{B}_{njk} + b_{2,\Gamma}(-)^{(\eta_i+\eta_j)(\eta_k+\eta_n)}\overline{\mathcal{B}}^{kji}\{\overline{\tau}_{\xi+}^a,\Gamma_+\}_k^n S_\mu \mathcal{B}_{ijn} \\ & + b_{3,\Gamma}(-)^{\eta_l(\eta_j+\eta_n)}\overline{\mathcal{B}}^{kji}(\overline{\tau}_{\xi+}^a)_i^l(\Gamma_+)_j^n S_\mu \mathcal{B}_{lnk} + b_{4,\Gamma}(-)^{\eta_l\eta_j+1}\overline{\mathcal{B}}^{kji}((\overline{\tau}_{\xi+}^a)_i^l(\Gamma_+)_j^n \\ & + (\Gamma_+)_i^l(\overline{\tau}_{\xi+}^a)_j^n)S_\mu \mathcal{B}_{nlk} + b_{5,\Gamma}(-)^{\eta_i(\eta_l+\eta_j)}\overline{\mathcal{B}}^{kji}(\overline{\tau}_{\xi+}^a)_j^l(\Gamma_+)_i^n S_\mu \mathcal{B}_{nlk} + b_{6,\Gamma}\overline{\mathcal{B}}^{kji}(\overline{\tau}_{\xi+}^a)_i^l S_\mu \mathcal{B}_{ljk}\mathrm{str}(\Gamma_+) \\ & + b_{7,\Gamma}(-)^{(\eta_i+\eta_j)(\eta_k+\eta_n)}\overline{\mathcal{B}}^{kji}(\overline{\tau}_{\xi+}^a)_k^n S_\mu \mathcal{B}_{ijn}\mathrm{str}(\Gamma_+) + b_{8,\Gamma}\overline{\mathcal{B}}^{kji}S_\mu \mathcal{B}_{ijk}\mathrm{str}(\overline{\tau}_{\xi+}^a\Gamma_+)], \end{aligned} \tag{31}$$

where $\Gamma = \mathcal{M}$ and $\mathcal{A}$.

We write the axial matrix elements as [12]

$$\begin{aligned} \langle N_b|{}^{(PQ)}j_{\mu,5}|N_a\rangle = {} & \Bigg[\rho_{ab} + \frac{1}{16\pi^2 f^2}\Bigg(\eta_{ab} - \rho_{ab}\frac{1}{2}[w_a + w_b] \\ & + y_j\eta_{ab}^{(j)} + y_l\eta_{ab}^{(l)}\Bigg) + c_{ab} + y_j c_{ab}^{(j)} + y_l c_{ab}^{(l)} \\ & + d_{ab} + y_j d_{ab}^{(j)} + y_l d_{ab}^{(l)}\Bigg] 2\overline{U}_b S_\mu U_a, \end{aligned} \tag{32}$$

where the constants ρ_{ab}, η_{ab}, $\eta_{ab}^{(k)}$, w_a, c_{ab}, $c_{ab}^{(k)}$ can be found in Ref. [12] with $b_j \to b_{j,M}$. In extending the τ^+ isovector operator from QCD to PQQCD, one can simply replace the τ^3 in the upper 2×2 block of $\overline{\tau}^3$ with τ^+, as described in Ref. [12]. The meson masses in ρ_{ab}, η_{ab}, $\eta_{ab}^{(k)}$, w_a are understood to be evaluated at finite lattice spacing via Eq. (10). The constants d_{ab} and $d_{ab}^{(k)}$ arise from ${}^{(PQ)}\delta j_{\mu,5}^{(1),a}$ and ${}^{(PQ)}\delta j_{\mu,5}^{(3),a}$, and are

$$\begin{aligned} d_{pp} = {} & \frac{ac_{sw}^{(V)}}{3}[-2b_{1,A} + 4b_{2,A} - b_{3,A} + b_{4,A} + 2b_{5,A}] \\ & + \frac{2ac_{sw}^{(S)}}{3}[2b_{7,A} - b_{6,A}] + \frac{ac_{A7}^{(V)}}{3}(2\gamma_{A,1} - \gamma_{A,2}), \\ d_{pp}^{(j)} = {} & d_{pp}^{(l)} = d_{nn}^{(j)} = d_{nn}^{(l)} = b_{8,A}(ac_{sw}^{(S)} - ac_{sw}^{(V)}), \\ d_{np} = {} & -d_{nn} = d_{pp}, \quad d_{np}^{(j)} = d_{np}^{(l)} = 0. \end{aligned} \tag{33}$$

In the isospin-symmetric QCD limit at finite lattice spacing, the proton isovector axial matrix element is

$$\langle p|j^3_{\mu,5}|p\rangle = g_A - \frac{1}{8\pi^2 f^2}\Bigg[g_A(1+2g_A^2)L_\pi + \left(2g_A + \frac{50}{81}g_{\Delta\Delta}\right)g^2_{\Delta N}J_\pi - \frac{16}{9}g_A g^2_{\Delta N}K_\pi\Bigg] + \frac{\overline{m}}{3}(-2b_{1,M}+4b_{2,M}-b_{3,M}+b_{4,M} + 2b_{5,M}-2b_{6,M}+4b_{7,M}) + \frac{ac_{sw}^{(V)}}{3}[-2b_{1,A} + 4b_{2,A}-b_{3,A}+b_{4,A}+2b_{5,A}] + \frac{2ac_{sw}^{(S)}}{3}[2b_{7,A} - b_{6,A}] + \frac{ac_{A7}^{(V)}}{3}(2\gamma_{A,1}-\gamma_{A,2}) + (y_j+y_l)b_{8,A}(ac_{sw}^{(S)}-ac_{sw}^{(V)}), \quad (34)$$

where $L_\pi = m_\pi^2\log(m_\pi^2/\mu^2)$, $J_\pi = J(m_\pi,\Delta,\mu)$ is

$$J(m,\Delta,\mu) = (m^2-2\Delta^2)\log\left(\frac{m^2}{\mu^2}\right) + 2\Delta\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right), \quad (35)$$

and $K_\pi = K(m_\pi,\Delta,\mu)$ is

$$K(m,\Delta,\mu) = \left(m^2-\frac{2}{3}\Delta^2\right)\log\left(\frac{m^2}{\mu^2}\right) + \frac{2}{3}\Delta\sqrt{\Delta^2-m^2}\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right) + \frac{2}{3}\frac{m^2}{\Delta}\left[\pi m - \sqrt{\Delta^2-m^2} \times\log\left(\frac{\Delta-\sqrt{\Delta^2-m^2+i\epsilon}}{\Delta+\sqrt{\Delta^2-m^2+i\epsilon}}\right)\right], \quad (36)$$

where μ is the renormalization scale. The last contribution in Eq. (34) is somewhat interesting as it depends upon the extension from QCD to PQQCD and also upon the choice of lattice fermions and does not vanish in the isospin-symmetric QCD limit.

VI. MATRIX ELEMENTS OF THE ISOVECTOR TWIST-2 OPERATORS

The forward matrix elements of twist-2 operators play an important role in hadronic structure as they are directly related to the moments of the parton distribution functions. In QCD the long-distance contributions to these matrix elements have been computed order by order in the chiral expansion using χPT [37–40] and have been applied to results from both quenched and unquenched lattice data [37], with interesting results. Further, the analogous contributions in QχPT and PQχPT have been computed in Refs. [11,12,41].

In QCD, in the continuum limit the twist-2 operators are

$$\mathcal{O}^{(n),b}_{\mu_1\mu_2\cdots\mu_n} = \overline{q}\tau^b\gamma_{\mu_1}(i\overleftrightarrow{D}_{\mu_2})\cdots(i\overleftrightarrow{D}_{\mu_n})q - \text{traces}, \quad (37)$$

where it is understood that the operator is symmetrized on its Lorentz indices. For the forward matrix elements in the proton and neutron, as are relevant for deep inelastic scattering, we only need to consider $b=3$. The extension to PQQCD is straightforward,

$$^{(PQ)}\mathcal{O}^{(n),b}_{\mu_1\mu_2\cdots\mu_n} = \overline{Q}\overline{\tau}^b\gamma_{\mu_1}(i\overleftrightarrow{D}_{\mu_2})\cdots(i\overleftrightarrow{D}_{\mu_n})Q - \text{traces}, \quad (38)$$

where $\overline{\tau}^b$ has the same form as that in Eq. (26), and we will use the same charges, but it should be remembered that they are unrelated to those of the axial-current operators.

The higher-dimensional operators that enter at finite lattice spacing are, in general, quite complicated. This is due to the fact that the operators must be classified not only by charge conjugation, parity and so forth, but also by the representation theory of the hypercubic group, $H(4)$. Such a classification has been performed up to $n=4$ [42], but as n increases the number and complexity of the operator basis increase significantly.

The $n=1$ operator $\mathcal{O}^{(1),3}_\mu$ is the isovector vector-current operator with $\mathcal{O}(a)$ corrections [32]

$$\overline{q}\tau^3\gamma^\mu m_q q, \quad \partial_\nu(\overline{q}\tau^3\sigma^{\mu\nu}q). \quad (39)$$

The forward matrix element of the second operator vanishes for obvious reasons and the first operator is $\mathcal{O}(p^4)$ in the expansion, and thus there are no operator corrections to $\mathcal{O}^{(1),3}_\mu$ to the order we are working. Therefore, there are no modifications to the matrix element of $\mathcal{O}^{(1),3}_\mu$.

By contrast, the forward matrix element of the $n=2$ operator,

$$\mathcal{O}^{(2),3}_{\mu\nu} = \overline{q}\tau^3\gamma_\mu(i\overleftrightarrow{D}_\nu)q, \quad (40)$$

does receive corrections at $\mathcal{O}(p^2)$ [32]. The correction to $\mathcal{O}^{(2),3}_{\mu\nu}$ at $\mathcal{O}(p^2)$ in QCD is

$$\delta\mathcal{O}^{(2),3}_{\mu\nu} = ac_1^{(2)}\overline{q}\tau^3\sigma_{\mu\lambda}[i\overleftrightarrow{D}_\nu, i\overleftrightarrow{D}_\lambda]q + ac_2^{(2)}\overline{q}\tau^3\{i\overleftrightarrow{D}_\mu, i\overleftrightarrow{D}_\nu\}q. \quad (41)$$

When extended to PQQCD, this correction becomes

$$^{(PQ)}\delta\mathcal{O}^{(2),3}_{\mu\nu} = \overline{Q}\overline{\tau}^3_{A,1}\sigma_{\mu\lambda}[i\overleftrightarrow{D}_\nu, i\overleftrightarrow{D}_\lambda]Q + \overline{Q}\overline{\tau}^3_{A,2}\{i\overleftrightarrow{D}_\mu, i\overleftrightarrow{D}_\nu\}Q, \quad (42)$$

where the charge matrices are

$$\overline{\tau}^3_{A,1} = a\,\text{diag}(c_1^{(2)(V)}, -c_1^{(2)(V)}, y_j c_1^{(2)(S)}, y_l c_1^{(2)(S)}, y_j c_1^{(2)(V)}, y_l c_1^{(2)(V)}),$$

$$\overline{\tau}^3_{A,2} = a\,\mathrm{diag}(c_2^{(2)(V)}, -c_2^{(2)(V)}, y_j c_2^{(2)(S)}, y_l c_2^{(2)(S)}, y_j c_2^{(2)(V)}, y_l c_2^{(2)(V)}) \tag{43}$$

for the charge matrix given in Eq. (26). The coefficients $c_{1,2}^{(2)(V)}$ and $c_{1,2}^{(2)(S)}$ are the valence- and sea-quark coefficients, respectively. In order to construct the nucleon matrix elements we define the fields

$$\overline{\tau}^{(2),3}_{A,1,\xi+} = \frac{1}{2}\,(\xi \overline{\tau}^3_{A,1}\xi + \xi^\dagger \overline{\tau}^3_{A,1}\xi^\dagger),$$

$$\overline{\tau}^{(2),3}_{A,2,\xi+} = \frac{1}{2}\,(\xi \overline{\tau}^3_{A,2}\xi + \xi^\dagger \overline{\tau}^3_{A,2}\xi^\dagger). \tag{44}$$

In QCD, the matrix elements of the continuum operator $\mathcal{O}^{(2),3}_{\mu\nu}$ are reproduced at leading order by

$$\mathcal{O}^{(2),3}_{\mu\nu} \to \rho^{(2)} v_\mu v_\nu \overline{N}\tau^3_{\xi+}N + \gamma^{(2)} v_\mu v_\nu \overline{T}^\alpha \tau^3_{\xi+} T_\alpha + \sigma^{(2)}\,\frac{1}{2}\,[\overline{T}_\mu \tau^3_{\xi+} T_\nu + \overline{T}_\nu \tau^3_{\xi+} T_\mu] - \text{traces}, \tag{45}$$

and in PQQCD this becomes

$$^{PQ}\mathcal{O}^{(2),3}_{\mu\nu} \to \alpha_0^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\mathcal{B}\overline{\tau}^3_{\xi+}) + \beta_0^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\overline{\tau}^3_{\xi+}\mathcal{B}) + \gamma_0^{(2)} v_\mu v_\nu (\overline{\mathcal{T}}^\alpha \overline{\tau}^3_{\xi+}\mathcal{T}_\alpha) + \sigma_0^{(2)}\frac{1}{2}\,[(\overline{\mathcal{T}}_\mu \overline{\tau}^3_{\xi+}\mathcal{T}_\nu) + (\overline{\mathcal{T}}_\nu \overline{\tau}^3_{\xi+}\mathcal{T}_\mu)] - \text{traces}. \tag{46}$$

The $\mathcal{O}(a)$ corrections to the operator give rise to $\mathcal{O}(p^2)$ corrections to the nucleon matrix elements of the form

$$^{PQ}\delta\mathcal{O}^{\mathcal{O},(2),3}_{\mu\nu} = \alpha_{11}^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\mathcal{B}\overline{\tau}^{(2),3}_{A,1,\xi+}) + \beta_{11}^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\overline{\tau}^{(2),3}_{A,1,\xi+}\mathcal{B}) + \alpha_{12}^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\mathcal{B}\overline{\tau}^{(2),3}_{A,2,\xi+}) + \beta_{12}^{(2)} v_\mu v_\nu (\overline{\mathcal{B}}\overline{\tau}^{(2),3}_{A,2,\xi+}\mathcal{B}) - \text{traces}, \tag{47}$$

while the contributions from a single insertion of the light-quark mass matrix and from the Sheikholeslami-Wohlert term at $\mathcal{O}(p^2)$ are

$$\begin{aligned}
^{PQ}\delta\mathcal{O}^{\Gamma,(2),3}_{\mu\nu} = {}& 2[b_{1,\Gamma}^{(2)}\overline{\mathcal{B}}^{kji}\{\overline{\tau}^a_{\xi+},\Gamma_+\}^n_i \mathcal{B}_{njk} + b_{2,\Gamma}^{(2)}(-)^{(\eta_i+\eta_j)(\eta_k+\eta_n)}\overline{\mathcal{B}}^{kji}\{\overline{\tau}^a_{\xi+},\Gamma_+\}^n_k \mathcal{B}_{ijn} \\
&+ b_{3,\Gamma}^{(2)}(-)^{\eta_l(\eta_j+\eta_n)}\overline{\mathcal{B}}^{kji}(\overline{\tau}^a_{\xi+})^l_i(\Gamma_+)^n_j\mathcal{B}_{lnk} + b_{4,\Gamma}^{(2)}(-)^{\eta_l\eta_j+1}\overline{\mathcal{B}}^{kji}((\overline{\tau}^a_{\xi+})^l_i(\Gamma_+)^n_j \\
&+ (\Gamma_+)^l_i(\overline{\tau}^a_{\xi+})^n_j)\mathcal{B}_{nlk} + b_{5,\Gamma}^{(2)}(-)^{\eta_i(\eta_l+\eta_j)}\overline{\mathcal{B}}^{kji}(\overline{\tau}^a_{\xi+})^l_j(\Gamma_+)^n_i\mathcal{B}_{nlk} + b_{6,\Gamma}^{(2)}\overline{\mathcal{B}}^{kji}(\overline{\tau}^a_{\xi+})^l_i\mathcal{B}_{ljk}\mathrm{str}(\Gamma_+) \\
&+ b_{7,\Gamma}^{(2)}(-)^{(\eta_i+\eta_j)(\eta_k+\eta_n)}\overline{\mathcal{B}}^{kji}(\overline{\tau}^a_{\xi+})^n_k\mathcal{B}_{ijn}\mathrm{str}(\Gamma_+) + b_{8,\Gamma}^{(2)}\overline{\mathcal{B}}^{kji}\mathcal{B}_{ijk}\mathrm{str}(\overline{\tau}^a_{\xi+}\Gamma_+)]v_\mu v_\nu - \text{traces},
\end{aligned} \tag{48}$$

where $\Gamma = \mathcal{M}$ and $\mathcal{A}$.

The forward matrix elements between nucleon states can be written as [12]

$$\langle {}^{PQ}\mathcal{O}^{(2),3}_{\mu\nu}\rangle_i = \left[\rho_i^{(2)} + \frac{1}{16\pi^2 f^2}(\eta_i^{(2),0} - \rho_i^{(2)} w_i + y_j\eta_i^{(2),j} + y_l\eta_i^{(2),l}) + c_i^{(2),0} + y_j c_i^{(2),j} + y_l c_i^{(2),l} + d_i^{(2),0} + y_j d_i^{(2),j} + y_l d_i^{(2),l}\right]\overline{U}_i v_\mu v_\nu U_i - \text{traces}, \tag{49}$$

where expressions for $\rho_i^{(2)}$, $\eta_i^{(2),0}$, w_i, $\eta_i^{(2),k}$, $c_i^{(2),0}$ and the $c_i^{(2),k}$ can be found in Ref. [12] with the understanding that the meson masses are evaluated at finite lattice spacing according to Eq. (10). The constants $d_i^{(2),0}$ and $d_i^{(2),k}$ arise from the operators $^{PQ}\delta\mathcal{O}^{\mathcal{O},(2),3}_{\mu\nu}$, $^{PQ}\delta\mathcal{O}^{\mathcal{M},(2),3}_{\mu\nu}$ and $^{PQ}\delta\mathcal{O}^{\mathcal{A},(2),3}_{\mu\nu}$, and are found to be

$$d_p^{(2),0} = \frac{ac_1^{(2)(V)}}{3}(2\alpha_{11}^{(2)} - \beta_{11}^{(2)}) + \frac{ac_2^{(2)(V)}}{3}(2\alpha_{12}^{(2)} - \beta_{12}^{(2)}) + \frac{1}{3}ac_{sw}^{(V)}(-2b_{1,A}^{(2)} + 4b_{2,A}^{(2)} - b_{3,A}^{(2)} + b_{4,A}^{(2)} + 2b_{5,A}^{(2)}) + \frac{2}{3}ac_{sw}^{(S)}(-b_{6,A}^{(2)} + 2b_{7,A}^{(2)}),$$

$$d_p^{(2),j} = d_p^{(2),l} = (ac_{sw}^{(S)} - ac_{sw}^{(V)})b_{8,A}^{(2)}, \tag{50}$$

and those for the neutron are related by $d_n^{(2),0} = -d_p^{(2),0}$ and $d_p^{(2),j} = d_n^{(2),j} = d_n^{(2),l}$.

In the isospin-symmetric QCD limit at finite lattice spacing these expressions reduce to

$$\langle {}^{PQ}\mathcal{O}^{(2),3}_{\mu\nu}\rangle_p = \Bigg[\rho_p^{(2)}\left(1-\frac{(3g_A^2+1)}{8\pi^2 f^2}L_\pi\right) - \frac{g_{\Delta N}^2}{4\pi^2 f^2}J_\pi\left[\rho_p^{(2)}+\frac{5}{9}\gamma^{(2)}-\frac{5}{27}\sigma^{(2)}\right] + \frac{\overline{m}}{3}(-2b_{1,M}^{(2)}+4b_{2,M}^{(2)}-b_{3,M}^{(2)}+b_{4,M}^{(2)}+2b_{5,M}^{(2)}-2b_{6,M}^{(2)}+4b_{7,M}^{(2)}) + \frac{ac_1^{(2)(V)}}{3}(2\alpha_{11}^{(2)}-\beta_{11}^{(2)}) + \frac{ac_2^{(2)(V)}}{3}(2\alpha_{12}^{(2)}-\beta_{12}^{(2)}) + \frac{1}{3}ac_{sw}^{(V)}(-2b_{1,A}^{(2)}+4b_{2,A}^{(2)}-b_{3,A}^{(2)}+b_{4,A}^{(2)}+2b_{5,A}^{(2)}) + \frac{2}{3}ac_{sw}^{(S)}(-b_{6,A}^{(2)}+2b_{7,A}^{(2)}) + (y_j+y_l)(ac_{sw}^{(S)}-ac_{sw}^{(V)})b_{8,A}^{(2)}\Bigg]\overline{U}_p v_\mu v_\nu U_p - \text{traces}. \tag{51}$$

Like the matrix elements of the axial current, the choice of charges in the sea quark and ghost sectors remain at finite lattice spacing through the last term in Eq. (51) with coefficient $b_{8,A}^{(2)}$.

VII. CONCLUSIONS

As lattice QCD moves closer to its ultimate goal of computing strong-interaction observables directly from first principles, effective field theory calculations must be developed in parallel in order to facilitate comparison with data and to make rigorous predictions. Significant attention has been paid to the chiral extrapolation of existing quenched and unquenched data with the goal of making a connection between lattice calculations performed at unphysically large quark masses and nature. With this program maturing pleasantly, the time is ripe to address other extrapolations that need to be performed in order to make a rigorous connection with data: the continuum extrapolation, $a\to 0$, and the infinite volume extrapolation.

In this work we have computed the leading effects of a finite lattice spacing, at $\mathcal{O}(a)$, on some nucleon properties. One source of this dependence is the leading $\mathcal{O}(a)$ corrections to the strong-interaction Lagrange density, i.e. the Sheikholeslami-Wohlert term. However, when considering matrix elements of operators, there are additional contributions from the $\mathcal{O}(a)$ corrections to the operators themselves. If the lattice calculations are performed with lattice fermions that respect chiral symmetry, then all the finite lattice spacing contributions we have computed in this work will vanish. This will also be the case for $\mathcal{O}(a)$-improved lattice simulations. The continuum extrapolation of unimproved simulations of nucleon properties with Wilson quarks or mixed quarks, e.g. Ginsparg-Wilson valence quarks and Wilson sea quarks, can be performed with the expressions we have determined in this work.

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