

Direct J/ψ and ψ' hadroproduction via fragmentation in the collinear parton model and k_T -factorization approach

V.A. Saleev*

Samara State University, Samara, Russia and Samara Municipal Nayanova University, Samara, Russia

D.V. Vasin†

Samara State University, Samara, Russia

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The p_T spectra for direct J/ψ and ψ' hadroproduction at Fermilab Tevatron energy are calculated within the framework of the NRQCD formalism and the fragmentation model in the collinear parton model as well as in the k_T -factorization approach. We describe the Collider Detector at Fermilab data and obtain a good agreement between the predictions of the parton model and the k_T -factorization approach. We perform the calculations using the relevant leading order in α_s hard amplitudes and taking equal values of the long-distance matrix elements for both models.

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I. INTRODUCTION

During the years after the measurement of the charmonium production cross sections and polarization effects for J/ψ and ψ' mesons at the Fermilab Tevatron collider [1] the phenomenology of quarkonium production had a phase of intensive developments. Nowadays, it is understood that heavy quarkonium production is a very complicated physical process and needs many new theoretical ideas. Starting from the color singlet model [2] the so-called nonrelativistic QCD (NRQCD) formalism [3] was developed to describe the non-perturbative transition of a $Q\bar{Q}$ pair into a final heavy quarkonium.

The perturbative fragmentation functions for partons which split into heavy quarkonia have been obtained [4] within the framework of the NRQCD formalism. It is supposed that the fragmentation model is more adequate for the description of quarkonium production at a large quarkonium transverse momentum $p_T \gg M_{Q\bar{Q}}$ than the fusion model.

In charmonium production at the energy range of the Tevatron collider we deal with the gluon distribution function from a proton which is taken at a very small x but the relevant virtuality $\mu^2 \sim M_{Q\bar{Q}}^2 + p_T^2$ is large. In the region under consideration the collinear parton model can be generalized within the framework of the k_T -factorization approach [5–8]. This fact leads to some interesting effects in the quarkonium production at the high energies, which were discussed ten years ago in Ref. [9] and recently in Refs. [10–13].

In this paper we calculate the p_T -spectra of the unpolarized direct J/ψ and ψ' mesons produced via the fragmentation mechanism. In the case of direct J/ψ and ψ' it is supposed that the production via the gluon fragmentation into the color-octet $Q\bar{Q}$ [$^3S_1, 8$] state is a dominant contribution [14,15]. We compare the predictions which are obtained as in

the collinear parton model as in the k_T -factorization approach. In both cases we performed calculations with the hard amplitudes in the leading order of the QCD running constant α_s . So, in the collinear parton model we take into consideration the partonic subprocess:

$$g + g \rightarrow g + g. \quad (1)$$

In the k_T -factorization approach we take into consideration the subprocess with off-shell or reggeized initial gluons:

$$g^* + g^* \rightarrow g. \quad (2)$$

We have described the Collider Detector at Fermilab (CDF) data [1] and obtained a good agreement between the parton model and the k_T -factorization calculations based on the hard amplitudes for subprocesses (1) and (2) with the equal values of the long-distance matrix elements $\langle O^{J/\psi} [^3S_1, 8] \rangle$ and $\langle O^{\psi'} [^3S_1, 8] \rangle$ in the fragmentation functions $D_{g \rightarrow J/\psi, \psi'}(z, \mu^2)$ for both models. The QCD evolution of the fragmentation function is described by the homogeneous equation with a boundary condition proportional to the delta function $\delta(1-z)$.

The results obtained in the paper within the k_T -factorization approach based on the fragmentation function model differ from results of Refs. [11–13], which were obtained using the gluon fusion model. The possible reasons for a disagreement will be discussed below.

II. NRQCD FORMALISM

Within the framework of the NRQCD, the cross section or the fragmentation function for quarkonium H production can be expressed as a sum of terms, which are factorized into a short-distance coefficient and a long-distance matrix element [3]:

$$d\sigma(H) = \sum_n d\hat{\sigma}(Q\bar{Q}[n]) \langle O^H[n] \rangle, \quad (3)$$

*Electronic address: saleev@ssu.samara.ru

†Electronic address: vasin@ssu.samara.ru

$$D(a \rightarrow H) = \sum_n D(a \rightarrow Q\bar{Q}[n]) \langle O^H[n] \rangle. \quad (4)$$

Here the n denotes the set of color and angular momentum numbers of the $Q\bar{Q}$ pair, in which the production cross section is $d\hat{\sigma}(Q\bar{Q}[n])$ or which the fragmentation function is $D(a \rightarrow Q\bar{Q}[n])$. The last ones can be calculated perturbatively in the strong coupling α_s . Of course, in the case of production in a hadron collision, the short-distance cross-section $d\hat{\sigma}$ has to be convoluted with the parton distribution function from the hadrons. The nonperturbative transition from the $Q\bar{Q}$ state n into the final quarkonium H is described by a long-distance matrix element $\langle O^H[n] \rangle$ which have to be calculated using nonperturbative methods or determined from experimental data. The fit of the Tevatron data for the p_T -spectra of J/ψ , ψ' , and χ_c charmonium states has been done recently by different authors (see a review in Ref. [16]). As it was shown in Refs. [14,15], at the large charmonium transverse momentum ($p_T > 7$ GeV) the gluon fragmentation into color-octet state

$$g^* \rightarrow Q\bar{Q} [{}^3S_{1,8}] \quad (5)$$

gives dominated contribution to the direct J/ψ and ψ' hadro-production. The values of the color-octet matrix elements under consideration are the following: $\langle O^{J/\psi} [{}^3S_{1,8}] \rangle = 4.4 \times 10^{-3} \text{ GeV}^3$, $\langle O^{\psi'} [{}^3S_{1,8}] \rangle = 4.2 \times 10^{-3} \text{ GeV}^3$ [15]. Note that the fit of CDF data for the p_T -spectrum of a direct J/ψ production within the framework of the fusion model in the collinear parton model gives another numerical value of the long-distance matrix element $\langle O^{J/\psi} [{}^3S_{1,8}] \rangle = 1.2 \times 10^{-2} \text{ GeV}^3$ [16]. It will be important to compare the results obtained in the k_T -factorization approach based on the fusion and fragmentation models.

III. FRAGMENTATION FUNCTION

The gluon fragmentation into the 3S_1 charmonium state is determined by the color-singlet [Fig. 1(a)] and the color-octet [Fig. 1(b)] contributions. The previous analysis has shown that the probability of a gluon fragmentation into the color-singlet state is only a small part of the probability of a fragmentation into the color-octet state [14]. The leading order in the α_s fragmentation functions for the transition (4) is known and can be written at the scale $\mu^2 = \mu_0^2 = 4m_c^2$ as follows [4]:

$$D_{g \rightarrow J/\psi, \psi'}^T(z, \mu^2) = 2d^T(z, \mu_0^2) \langle O^{J/\psi, \psi'} [{}^3S_{1,8}] \rangle, \quad (6)$$

$$D_{g \rightarrow J/\psi, \psi'}^L(z, \mu^2) = d^L(z, \mu_0^2) \langle O^{J/\psi, \psi'} [{}^3S_{1,8}] \rangle, \quad (7)$$

where

$$d^T(z, \mu_0^2) = \frac{\pi \alpha_s(\mu_0^2)}{48m_c^3} \delta(1-z), \quad (8)$$

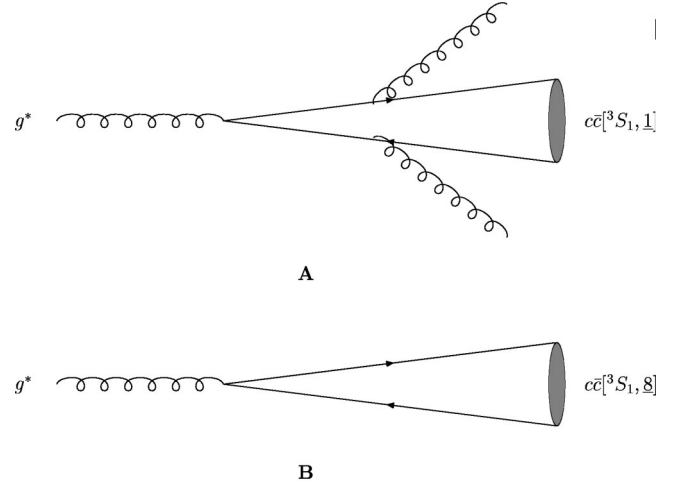


FIG. 1. Diagrams used for description LO in the α_s fragmentation $g^* \rightarrow J/\psi gg$ (color-singlet state, A) and $g^* \rightarrow J/\psi$ (color-octet state, B).

$$d^L(z, \mu_0^2) = \frac{\alpha_s^2(\mu_0^2)}{8m_c^3} \frac{(1-z)}{z}. \quad (9)$$

It is obvious that the probability of the gluon fragmentation into the longitudinally polarized charmonium is negligibly small and J/ψ or ψ' mesons have transverse polarization.

The fragmentation functions (6) and (7) are evolved in μ^2 using the standard homogeneous DGLAP evolution equation [17]:

$$\mu^2 \frac{\partial D_g}{\partial \mu^2}(z, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \int_z^1 \frac{dx}{x} P_{gg}\left(\frac{x}{z}\right) D_g(x, \mu^2), \quad (10)$$

where $P_{gg}(x)$ is the usual leading order gluon-gluon splitting function. To solve Eq. (10) we use the well known method based on Mellin transform. It is easy to obtain that the Mellin-momentum at the scale μ^2 can be written as follows:

$$D_g(n, \mu^2) = D_g(n, \mu_0^2) \exp \left[\frac{P_{gg}(n)}{2\pi} \int_{\mu_0^2}^{\mu^2} \frac{d\mu^2}{\mu^2} \alpha_s(\mu^2) \right], \quad (11)$$

where

$$P_{gg}(n) = 3 \left[-2S_1(n) + \frac{11}{6} + \frac{2}{n(n-1)} + \frac{2}{(n+1)(n+2)} \right] - 1, \quad (12)$$

$$S_1(n) = \sum_{j=1}^n \frac{1}{j}.$$

In the one-loop approximation for the running constant $\alpha_s(\mu^2)$ with the three active flavors ($b_0=9$) one has

$$\alpha_s(\mu^2) = \frac{4\pi}{b_0 \log(\mu^2/\Lambda^2)}, \quad (13)$$

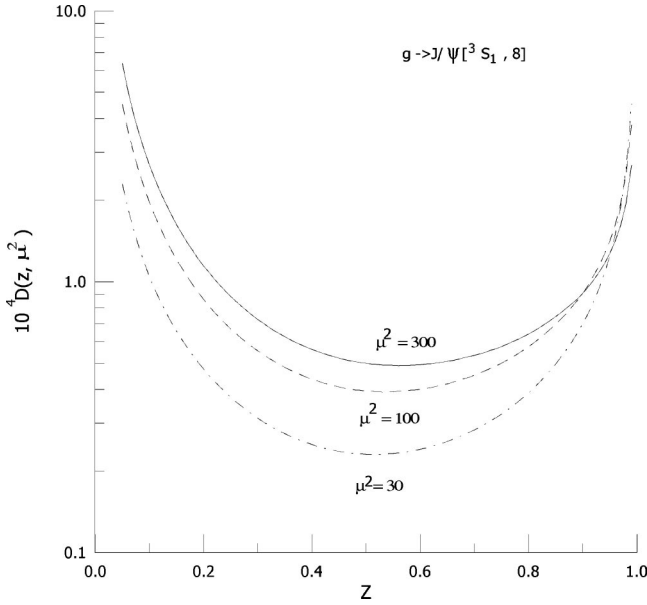


FIG. 2. $D_{g \rightarrow J/\psi}(z, \mu^2)$ at the $\mu^2 = 30, 100,$ and 300 GeV^2 as a function of z .

and Eq. (11) can be presented as follows:

$$D_g(n, \mu^2) = D_g(n, \mu_0^2) \exp \left[\frac{2}{b_0} P_{gg}(n) \log \left(\frac{\log(\mu^2/\Lambda^2)}{\log(\mu_0^2/\Lambda^2)} \right) \right]. \quad (14)$$

Taking into consideration that

$$D_g^T(n, \mu_0^2) = \frac{\pi \alpha_s(\mu_0^2)}{24m_c^3} \langle O^{J/\psi, \psi'} [^3S_1, 8] \rangle \quad (15)$$

we have performed an inverse Mellin transform numerically using the following rule:

$$D_g(z, \mu^2) \approx D_g^T(z, \mu^2) = \int_C dn z^{-n} D_g^T(n, \mu^2). \quad (16)$$

The integration contour C can be transformed such as

$$D_g(z, \mu^2) = \frac{1}{\pi} \int_0^\infty dt \text{Im} [e^{i\phi} z^{-n} D_g^T(n, \mu^2)], \quad (17)$$

where $n = c + te^{i\phi}$ and $c \approx 2$, $\phi = \pi/2$. In Fig. 2 the obtained fragmentation function $D_{g \rightarrow J/\psi}(z, \mu^2)$ multiplied by 10^4 is shown at the different $\mu^2 = 30, 100,$ and 300 GeV^2 . We see that our result at the $\mu^2 = 300 \text{ GeV}^2$ agrees well with the same one obtained in Ref. [18]. In a stage of convoluting of the fragmentation function $D_{g \rightarrow J/\psi}(z, \mu^2)$ with the partonic cross section for the subprocesses (1) or (2) we will use the following definition for the variable z :

$$z = \frac{E_\psi + |\vec{p}_\psi|}{2E_g}. \quad (18)$$

Thus we consider that the massless parton fragments into the massive meson. As it will be seen the definition (18) is more correct at a not so large p_T than the massless one between the parton four-momentum and the meson four-momentum

$$p_\psi^\mu = z p_g^\mu. \quad (19)$$

We suggest also that the meson has a small transverse momentum, respectively, initial gluon jet and we approximately can accept that in the laboratory frame

$$\eta_\psi \approx \eta_g, \quad (20)$$

where η_ψ, η_g are the meson and gluon pseudorapidities.

Within the framework of the fragmentation model, the meson production cross section and the relevant gluon production cross section are connected as follows:

$$\hat{\sigma}(gg \rightarrow J/\psi X) = \int dz D_{g \rightarrow J/\psi}(z, \mu^2) \hat{\sigma}(gg \rightarrow gg) \quad (21)$$

or

$$\hat{\sigma}(g^* g^* \rightarrow J/\psi X) = \int dz D_{g \rightarrow J/\psi}(z, \mu^2) \hat{\sigma}(g^* g^* \rightarrow g). \quad (22)$$

IV. LEADING ORDER HARD AMPLITUDES

The squared amplitude for the partonic process (1) is well known and it can be presented as follows:

$$\overline{|M(gg \rightarrow gg)|^2} = 18\pi^2 \alpha_s^2 \frac{(\hat{s}^4 + \hat{t}^4 + \hat{u}^4)(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)}{(\hat{s}\hat{t}\hat{u})^2}, \quad (23)$$

where \hat{s} , \hat{t} , and \hat{u} are the usual Mandelstam variables.

There are two approaches for calculation of a partonic amplitude for the subprocess (2) in the k_T -factorization approach [6,8]. The effective Feynman rules for processes with off-shell gluons were suggested in Ref. [6]. The special trick is a choice of the initial gluon polarization 4-vector as follows:

$$\varepsilon^\mu(q) = \frac{q_T^\mu}{|\vec{q}_T|}. \quad (24)$$

In Ref. [8] the initial gluons are considered as reggeons (reggeized gluons) and the effective reggeon-reggeon-gluon vertex function was obtained

$$C^\lambda(k_1, k_2) = -(k_1 - k_2)^\lambda + P_1^\lambda \left(\frac{k_1^2}{(kP_1)} + 2 \frac{(kP_2)}{(P_1 P_2)} \right) - P_2^\lambda \left(\frac{k_2^2}{(kP_2)} + 2 \frac{(kP_1)}{(P_1 P_2)} \right), \quad (25)$$

where $P_1 = (\sqrt{s}/2)(1, 0, 0, 1)$ and $P_2 = (\sqrt{s}/2)(1, 0, 0, -1)$ are the colliding protons four-momenta, $k_1 = x_1 P_1 + k_{1T}$ and $k_2 = x_2 P_2 + k_{2T}$ are the initial gluons four-momenta, k_T

$= (0, \vec{k}_T, 0)$, $k = k_1 + k_2$ is the final real gluon four-momentum. It is easy to show that the vertex function $C^\lambda(k_1, k_2)$ satisfies the gauge invariance condition $(k_1 + k_2)_\lambda C^\lambda(k_1, k_2) = 0$.

Omitting the color factor f^{abc} we can write the amplitude of the subprocess (2) according to Ref. [6] as follows:

$$\begin{aligned} \mathcal{M} = & -g\varepsilon^\lambda(k) \frac{k_{1T}^\mu k_{2T}^\nu}{|\vec{k}_{1T}| |\vec{k}_{2T}|} [(k+k_1)_\nu g_{\lambda\mu} + (-k_1+k_2)_\lambda g_{\mu\nu} \\ & + (-k_1-k)_\mu g_{\nu\lambda}]. \end{aligned} \quad (26)$$

We have obtained after simple transformations that

$$\mathcal{M} = -\frac{g\varepsilon^\lambda(k)}{2|\vec{k}_{1T}| |\vec{k}_{2T}|} x_1 x_2 s \tilde{C}_\lambda(k_1, k_2), \quad (27)$$

where

$$\begin{aligned} \tilde{C}^\lambda(k_1, k_2) = & -(k_1 - k_2)^\lambda + \frac{2P_1^\lambda}{x_2 s} (k_1^2 + x_1 x_2 s) \\ & - \frac{2P_2^\lambda}{x_1 s} (k_2^2 + x_1 x_2 s) \\ = & C^\lambda(k_1, k_2). \end{aligned} \quad (28)$$

In such a way, the approaches [6,8] are equivalent and give the equal answer for the squared vertex function and amplitude:

$$C^\lambda(k_1, k_2) C_\lambda(k_1, k_2) = -\frac{4k_1^2 k_2^2}{x_1 x_2 s}, \quad (29)$$

and

$$\overline{|M(g^* g^* \rightarrow g)|^2} = \frac{3}{2} \pi \alpha_s \vec{p}_T^2, \quad (30)$$

where $\vec{p}_T^2 = (\vec{k}_{1T} + \vec{k}_{2T})^2 = x_1 x_2 s$, \vec{p}_T is the transverse momentum of the final gluon.

V. CROSS SECTIONS FOR THE PROCESS $pp \rightarrow J/\psi(\psi')X$

In the conventional collinear parton model it is suggested that the hadronic cross section, in our case, $\sigma(pp \rightarrow J/\psi X, s)$, and the relevant partonic cross section $\hat{\sigma}(gg \rightarrow J/\psi X, \hat{s})$ are connected as follows:

$$\begin{aligned} \sigma^{PM}(pp \rightarrow J/\psi, s) = & \int dx_1 \int dx_2 G(x_1, \mu^2) G(x_2, \mu^2) \\ & \times \hat{\sigma}(gg \rightarrow J/\psi, \hat{s}), \end{aligned} \quad (31)$$

where $\hat{s} = x_1 x_2 s$, $G(x, \mu^2)$ is the collinear gluon distribution function in a proton, $x_{1,2}$ are the fractions of a proton momentum, and μ^2 is the typical scale of a hard process. The μ^2 evolution of the gluon distribution $G(x, \mu^2)$ is described by the DGLAP evolution equation [17].

In the k_T -factorization approach hadronic and partonic cross sections are related by the following condition [5–7]:

$$\begin{aligned} \sigma^{KT}(pp \rightarrow J/\psi X) \\ = & \int \frac{dx_1}{x_1} \int d\vec{k}_{1T}^2 \int \frac{d\phi_1}{2\pi} \Phi(x_1, \vec{k}_{1T}^2, \mu^2) \\ & \times \int \frac{dx_2}{x_2} \int d\vec{k}_{2T}^2 \int \frac{d\phi_2}{2\pi} \Phi(x_2, \vec{k}_{2T}^2, \mu^2) \\ & \times \hat{\sigma}(g^* g^* \rightarrow J/\psi X, \hat{s}), \end{aligned} \quad (32)$$

where $\hat{\sigma}(g^* g^* \rightarrow J/\psi X, \hat{s})$ is the J/ψ production cross section on off-shell gluons, $k_1^2 = k_1^2 = -\vec{k}_{1T}^2$, $k_2^2 = k_2^2 = -\vec{k}_{2T}^2$, $\hat{s} = x_1 x_2 s - (\vec{k}_{1T} + \vec{k}_{2T})^2$, and $\phi_{1,2}$ are the azimuthal angles in the transverse XOY plane between vectors $\vec{k}_{1T}(\vec{k}_{2T})$ and the fixed OX axis ($\vec{p}_\psi \in XOZ$). The unintegrated gluon distribution function $\Phi(x_1, \vec{k}_{1T}^2, \mu^2)$ satisfies the BFKL evolution equation [19].

Our calculation in the parton model is done using the GRV LO [20] and CTEQ5L [21] parametrizations for a collinear gluon distribution function $G(x, \mu^2)$. In the case of the k_T -factorization approach we use the following parametrizations for an unintegrated gluon distribution function $\Phi(x_1, \vec{k}_{1T}^2, \mu^2)$: JB by Bluemlein [22], JS by Jung and Salam [23], and KMR by Kimber, Martin, and Ryskin [24]. The direct comparison between different parametrizations as functions of x , \vec{k}_T^2 , and μ^2 was presented in paper [10].

The doubly differential cross sections for the process $pp \rightarrow J/\psi(\psi')X$ can be written as follows:

$$\begin{aligned} \frac{d\sigma^{PM}}{d\eta_\psi dp_{\psi T}} = & \int dx_1 \int dz G(x_1, \mu^2) G(x_2, \mu^2) \\ & \times D_{g \rightarrow \psi}(z, \mu^2) \frac{p_{gT} E_g}{E_\psi} \frac{\overline{|M(gg \rightarrow gg)|^2}}{8\pi x_1 x_2 s (u + x_1 s)}, \end{aligned} \quad (33)$$

where

$$u = -\sqrt{s}(E_g - p_{gz}), \quad t = -\sqrt{s}(E_g + p_{gz}),$$

$$x_2 = -\frac{x_1 t}{u + x_1 s}, \quad x_{1, \min} = -\frac{u}{s + t},$$

$$\hat{t} = x_1 t, \quad \hat{u} = x_2 u, \quad \hat{s} = x_1 x_2 s;$$

$$\begin{aligned} \frac{d\sigma^{KT}}{d\eta_\psi dp_{\psi T}} = & \int dz \int d\phi_1 \int d\vec{k}_{1T}^2 \Phi(x_1, \vec{k}_{1T}^2, \mu^2) \\ & \times \Phi(x_2, \vec{k}_{2T}^2, \mu^2) D_{g \rightarrow \psi}(z, \mu^2) \\ & \times \frac{E_g}{p_{gT} E_\psi} \frac{\overline{|M(g^* g^* \rightarrow g)|^2}}{x_1 x_2 s}, \end{aligned} \quad (34)$$

where

$$\vec{k}_{2T} = \vec{p}_{gT} - \vec{k}_{1T}, \quad x_1 = \frac{E_g + p_{gz}}{\sqrt{s}}, \quad x_2 = \frac{E_g - p_{gz}}{\sqrt{s}}.$$

The energy (E_g) of a fragmenting gluon and the energy of a final meson (E_ψ) are related as follows:

$$E_g = \frac{E_\psi + |\vec{p}_\psi|}{2z}, \quad |\vec{p}_\psi| = \frac{p_{\psi T}}{\sin(\theta_\psi)}, \quad p_{gz} = E_g \cos(\theta_\psi),$$

$$\theta_\psi = 2 \arctg[\exp(-\eta_\psi)].$$

VI. THE RESULTS

We compare our predictions with the CDF data [1] for unpolarized direct J/ψ and ψ' production. The direct J/ψ cross section does not include contributions from B meson decays into J/ψ as well as from χ_c radiative decays. For direct ψ' production, indirect contribution from B meson decays are removed. Our results obtained in the collinear parton model do not depend on a choice of the parametrization for the gluon distribution function and coincide approximately (± 10 – 20%) to the results obtained in Ref. [15] using a similar approach. We can see in Figs. 3 and 4 that the curves denoted as ‘‘GRV,’’ which were obtained in the collinear parton model, are below the data especially at the small p_T , where the gluon fusion into 1S_0 and 3P_J color octet states is dominant [15,16].

The curves which were obtained in the k_T -factorization approach strongly depend on a choice of the unintegrated gluon distribution function. In the region of a large p_T , where the fragmentation approach is more adequate, the results obtained with JB [22] and KMR [24] parametrizations

coincide well. However, JS parametrization [23] predicts the values smaller by a factor of 2, which are near the values obtained in the collinear parton model with the GRV gluon distribution function. In such a way, the p_T spectra of direct J/ψ and ψ' mesons obtained in the collinear parton model and in the k_T -factorization approach approximately coincide. Note that our conclusions disagree with the previous results obtained in Refs. [11–13] using the fusion model. Opposite our result, the fit of the CDF data accordingly [11–13] needs strong suppression (in 10–30 times) for the long-distance matrix elements $\langle O^{J/\psi, \psi'} [^3S_1, 8] \rangle$ to compare the values obtained in the collinear parton model.

There are several reasons for such a disagreement. First, we used the fragmentation functions, which take into account effectively high order corrections via the DGLAP evolution equation. Second, in Refs. [11–13] the argument μ^2 of the strong coupling constant $\alpha_s(\mu^2)$ is equal to \vec{q}_{1T}^2 or \vec{q}_{2T}^2 , our choice is $\mu^2 = \vec{p}_T^2 + 4m_c^2$. This fact gives the additional factor of 3 in a cross section [13]. Third, in Refs. [11,12] the KMS [25] parametrization for an unintegrated gluon distribution function was used. We have shown (Figs. 2 and 3) that the difference between predictions based on the different parametrizations may be about a factor of 2 to 3.

The obtained results for the direct unpolarized J/ψ and ψ' hadroproduction as well as our previous results for the J/ψ photoproduction at HERA energies [10] show that the predictions for spectra on p_T with the LO in α_s hard amplitudes in the k_T -factorization approach coincide well with the predictions with the NLO in α_s hard amplitudes in the collinear parton model. The NLO in α_s calculation for the J/ψ photoproduction cross section was performed in Ref. [26].

It is obvious that the NLO hard subprocess for a gluon production in the k_T -factorization approach with off-shell initial gluons is the LO subprocess used in the collinear par-

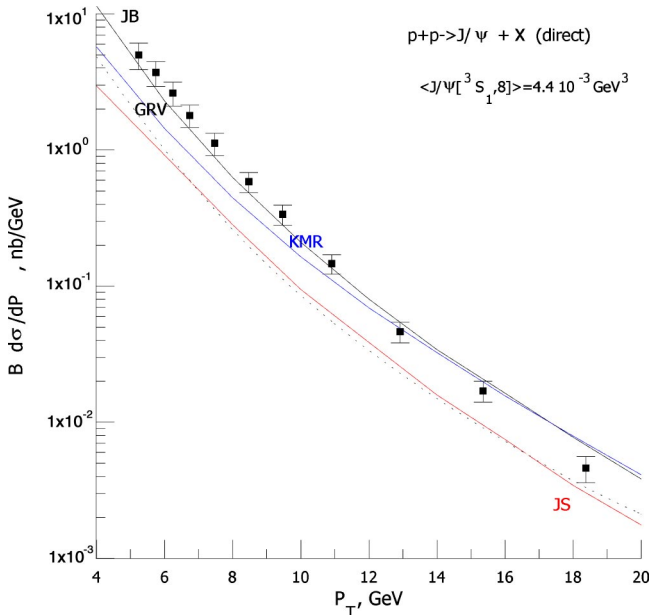


FIG. 3. The spectrum of direct J/ψ on p_T at the $\sqrt{S} = 1800$ GeV and $|\eta| < 0.6$. The data points are from [1]. The B is the J/ψ lepton branching.

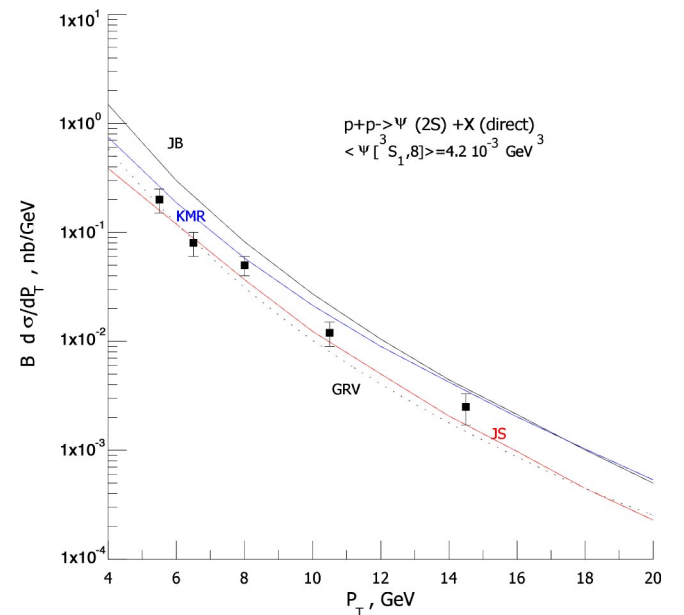


FIG. 4. The spectrum of direct ψ' on p_T at the $\sqrt{S} = 1800$ GeV and $|\eta| < 0.6$. The data points are from [1]. The B is the ψ' lepton branching.

ton model with on-shell initial gluons:

$$g^* + g^* \rightarrow g + g. \quad (35)$$

An amplitude of the subprocess (35) has infrared singularities even at the large value of p_T for the final gluon which fragments into a meson. Oppositely, in the collinear parton model both gluons are hard in a similar case. The procedure of a calculation of an amplitude for the subprocess (35) in the k_T -factorization approach has been suggested in Ref. [8], where initial gluons are considered as reggeons and the infrared divergencies are removed. It should be interesting to

calculate $J/\psi(\psi')$ production cross section using the results of Ref. [8]. This study is in progress.

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