QCD status of factorization ansatz for double parton distributions

A. M. Snigirev*
M.V. Lomonosov Moscow State University, D.V. Skobeltsyn Institute of Nuclear Physics 119992, Vorobievy Gory, Moscow, Russia
(Received 19 September 2003; revised 8 October 2003; published 29 December 2003)

This article brings attention to what is knowable from perturbative QCD theory on two-parton distribution functions in the light of recent CDF measurements [Phys. Rev. D **56**, 3811 (1997)] of the inclusive cross section for double parton scattering. It is shown that the solution of generalized Lipatov-Altarelli-Parisi-Dokshitzer equations for the two-parton distributions is not at all the product of two single-parton distributions, which is usually applied to the current analysis as an ansatz.

DOI: 10.1103/PhysRevD.68.114012 PACS number(s): 12.38.−t

The Collider Detector at Fermilab (CDF) Collaboration has recently measured a large number of double parton scatterings [1]. Thus new and complementary information on the structure of the proton can be obtained by identifying and analyzing events in which two parton-parton hard scatterings take place within one $p\bar{p}$ collision. Double parton scattering provides information on both the spatial distribution of partons within the proton, and possible parton-parton correlations. Both the absolute rate for the double parton process and any dynamics that correlations may introduce are therefore of interest. The theoretical estimations of the effect under consideration have been done in a number of works [2–9].

For instance, the differential cross section for the four-jet process (due to the simultaneous interaction of two parton pairs) is given by [4,5]

$$d\sigma=\sum_{q/g}\frac{d\sigma_{12}d\sigma_{34}}{\sigma_{\rm eff}}D_p(x_1,x_3)D_{\bar{p}}(x_2,x_4), \qquad (1)$$

where $d\sigma_{ij}$ stands for the two-jet cross section. The dimensional factor $\sigma_{\rm eff}$ in the denominator represents the total inelastic cross section which is an estimate of the size of the hadron, $\sigma_{\rm eff}\simeq 2\pi r_p^2$ (the factor 2 is introduced due to the identity of the two parton processes). With the effective cross section measured by CDF, $(\sigma_{\rm eff})_{\rm CDF}=(14.5\pm 1.7^{+1.7}_{-2.3})$ mb [1], one can estimate the transverse size $r_p\simeq 0.5$ fm, which is too small in comparison with the proton radius extracted from *ep* elastic scattering experiments. The relatively small value of $(\sigma_{\rm eff})_{\rm CDF}$ with respect to the naive expectation $2\pi r_p^2$ was, in fact, considered [7,8] as evidence of nontrivial correlation effects in transverse space. But, apart from these correlations, the longitudinal momentum correlations can also exist and they are the subject of the present investigation. Usually the two-parton distributions are supposed to be the product of two single-parton distributions times a momentum conserving phase space factor

$$D_p(x_i,x_j)=D_p(x_i,Q^2)D_p(x_j,Q^2)(1-x_i-x_j), \qquad (2)$$

where $D_p(x_i,Q^2)$ are the single quark/gluon momentum distributions at the scale Q^2 (determined by a hard process).

The main purpose of this paper is to analyze the status of the factorization ansatz (2) for many parton distributions in the perturbative QCD theory. Here one should note that the generalized Lipatov-Altarelli-Parisi-Dokshitzer equations for many parton distribution functions have been derived for the first time in Refs. [10,11] within the leading logarithm approximation of QCD using a method by Lipatov [12]. Under certain initial conditions these equations lead to solutions which are identical with the jet calculus rules proposed for multiparton fragmentation functions by Konishi-Ukawa-Veneziano [13]. Because of a very old affair it is necessary to recall some features of that investigation to be clear.

In Ref. [14] the structure functions of *ep* scattering and e^+e^- annihilation were calculated in the leading logarithmic approximation for vector and pseudoscalar theories. Similar calculations in QCD were made in Ref. [15]. Lipatov shown [12] that the results of these calculations admit a simple interpretation in the framework of the parton model with a variable cutoff parameter $\Lambda\sim Q^2$ with respect to the transverse momenta, and derived an equation for the scaling violation of the parton distribution $D_i^j(x,\Lambda)$ inside a dressed quark or gluon, which is in fact equivalent to the one proposed by Altarelli and Parisi [16], within the difference that it was not applied by Lipatov to QCD.

After introducing the natural variable

$$t=\frac{1}{2\pi b}\ln\left[1+\frac{g^2(\mu^2)}{4\pi}b\ln\left(\frac{\Lambda}{\mu^2}\right)\right],\quad b=\frac{33-2n_f}{12\pi}\text{ in QCD,}$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2, n_f is the number of flavors, this equation reads [12,15,16]

$$\frac{dD_i^j(x,t)}{dt}=\sum_{j'}\int_x^1\frac{dx'}{x'}D_i^{j'}(x',t)P_{j'\to j}\left(\frac{x}{x'}\right). \qquad (3)$$

It is interesting that expression for the kernels P in Lipatov method already includes a regularization at $x\to x'$, which was introduced in Ref. [16] afterwards.

This method allows one to obtain also the generalized Lipatov-Altarelli-Parisi-Dokshitzer equation for two-parton distributions $D_i^{j_1j_2}(x_1,x_2,t)$, representing the probability that in a dressed constituent i one finds two bare partons of

*Email: snigirev@lav01.sinp.msu.ru

types j_1 and j_2 with the given momentum fractions x_1 and x_2, namely (see Refs. [10,11,17] for details),

$$\frac{dD_i^{j_1j_2}(x_1,x_2,t)}{dt} = \sum_{j_1'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} D_i^{j_1'j_2}(x_1',x_2,t) P_{j_1'\to j_1}\left(\frac{x_1}{x_1'}\right) + \sum_{j_2'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} D_i^{j_1j_2'}(x_1,x_2',t) P_{j_2'\to j_2}\left(\frac{x_2}{x_2'}\right) + \sum_{j'} D_i^{j'}(x_1+x_2,t)\frac{1}{x_1+x_2} P_{j'\to j_1j_2}\left(\frac{x_1}{x_1+x_2}\right), \tag{4}$$

where the kernel $[1/(x_1+x_2)]P_{j'\to j_1j_2}[x_1/(x_1+x_2)]$ is defined without δ-function regularization. The result for the m-parton functions can be found in Ref. [10].

It is readily verified by direct substitution that the solution of Eq. (4) can be written via the convolution of single distributions [10,11]

$$D_i^{j_1j_2}(x_1,x_2,t) = \sum_{j'j_1'j_2'} \int_0^t dt' \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} D_i^{j'}(z_1+z_2,t') \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right) \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1},t-t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2},t-t'\right). \tag{5}$$

This coincides with the jet calculus rules [13] proposed originally for the fragmentation functions and is the generalization of the well-known Gribov-Lipatov relation installed for single functions [14,15] (the distribution of bare partons inside a dressed constituent is identical to the distribution of dressed constituents in the fragmentation of a bare parton in the leading logarithm approximation). The equations for the multiparton fragmentation functions are obtained by Lipatov's method in a similar way [11] and beyond the given investigation.

The solution (5) shows that the distribution of partons is *correlated* in the leading logarithm approximation

$$D_i^{j_1j_2}(x_1,x_2,t) \neq D_i^{j_1}(x_1,t) D_i^{j_2}(x_2,t). \tag{6}$$

Of course, it is interesting to find out the phenomenological issue of the equations under consideration. This can be done within the well-known factorization of soft and hard stages (physics of short and long distances) [18]. As a result, Eqs. (3) and (4) describe the evolution of parton distributions in a hadron with $t(Q^2)$, if one replaces the index i by index h only. However, the initial conditions for new equations at $t=0$ $(Q^2=\mu^2)$ are unknown *a priori* and must be introduced phenomenologically or must be extracted from experiments or some models dealing with physics of long distances [at the parton level: $D_i^j(x,t=0)=\delta_{ij}\delta(x-1)$; $D_i^{j_1j_2}(x_1,x_2,t=0)=0$]. Nevertheless the solution of Eq. (4) with the given initial condition may be written as before via the convolution of single distributions [11]

$$D_h^{j_1j_2}(x_1,x_2,t) = \sum_{j_1'j_2'} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} D_h^{j_1'j_2'}(z_1,z_2,0) \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1},t\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2},t\right) + \sum_{j'j_1'j_2'} \int_0^t dt' \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} D_h^{j'}(z_1+z_2,t') \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right) \times D_{j_1'}^{j_1}\left(\frac{x_1}{z_1},t-t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2},t-t'\right). \tag{7}$$

The reckoning for the unsolved confinement problem (physics of long distances) is the unknown two-parton correlation function $D_h^{j_1'j_2'}(z_1,z_2,0)$ at some scale μ^2. One can suppose that this function is the product of two single-parton distributions times a momentum conserving factor at this scale μ^2:

$$D_h^{j_1j_2}(z_1,z_2,0) = D_h^{j_1}(z_1,0) D_h^{j_2}(z_2,0)\, \theta(1-z_1-z_2). \tag{8}$$

Then

$$D_h^{j_1j_2}(x_1,x_2,t) = \Bigg\{ D_h^{j_1}(x_1,t) D_h^{j_2}(x_2,t) + \sum_{j_1'j_2'} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} D_h^{j_1'}(z_1,0) D_h^{j_2'}(z_2,0) D_{j_1'}^{j_1}\left(\frac{x_1}{z_1},t\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2},t\right) \times [\theta(1-z_1-z_2)-1] \Bigg\} \theta(1-x_1-x_2) + \sum_{j'j_1'j_2'} \int_0^t dt' \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} D_h^{j'}(z_1+z_2,t') \frac{1}{z_1+z_2} P_{j'\to j_1'j_2'}\left(\frac{z_1}{z_1+z_2}\right) D_{j_1'}^{j_1}\left(\frac{x_1}{z_1},t-t'\right) D_{j_2'}^{j_2}\left(\frac{x_2}{z_2},t-t'\right), \tag{9}$$

where

$$D_h^j(x,t)=\sum_{j'}\int_x^1\frac{dz}{z}D_h^{j'}(z,0)D_{j'}^j\left(\frac{x}{z},t\right) \tag{10}$$

is the solution of Eq. (3) with the given initial condition $D_h^j(x,0)$ for parton distributions inside a hadron expressed via distributions at the parton level.

This result (9) is, as a matter of fact, the answer to the question set. If the two-parton distributions are factorized at some scale μ^2 then the evolution violates this factorization *inevitably* at any different scale ($Q^2\neq\mu^2$), apart from the violation due to the kinematic correlations induced by the momentum conservation (given by θ functions).[1]

For a practical employment it is interesting to know the degree of this violation. It can be done numerically using, for instance, the CTEQ parametrization [19] for single distributions as an input in Eq. (9) and considering the kinematics of some specific process. Partly this problem was investigated theoretically in Refs. [11,20] and for the two-particle correlations of fragmentation functions in Ref. [21]. That technique is based on the Mellin transformation of distribution functions as

$$M_h^j(n,t)=\int_0^1 dx\ x^n D_h^j(x,t). \tag{11}$$

After that, the integrodifferential equations (3) and (4) become systems of ordinary linear-differential equations of first order with constant coefficients and can be solved explicitly [11,20]. In order to obtain the distributions in x representation an inverse Mellin transformation must be performed

$$D_h^j(x,t)=\int\frac{dn}{2\pi i}x^{-n}M_h^j(n,t), \tag{12}$$

where the integration runs along the imaginary axis to the right from all n singularities. This can be done numerically again. However the asymptotic behavior can be estimated. Namely, with the growth of $t(Q^2)$ the second term in Eq. (7) becomes *dominant* for finite x_1 and x_2 [20]. Thus the two-parton distribution functions ''forget'' the initial conditions unknown *a priori* and the correlations perturbatively calculated appear.

The CDF Collaboration found no evidence for the kinematic correlation between the two scatterings in double parton events. This can mean only that the factorization ansatz (2) is the acceptable approximation at the scale $Q_{\text{CDF}}\sim 5$ GeV accessible to CDF measurements [$E_T^{\text{jet}}(\text{min})\simeq 5$ GeV [1]]. The explanation is simple: in the kinematical region of finite x_1,x_2 and large Q^2, where correlations are important, double parton collisions give a negligible contribution to the cross section. Thus, the typical hard scale and longitudinal momentum fractions of CDF measurements are relatively small to observe the correlations under consideration. There are no arguments to assert that the ansatz (2) is acceptable at larger scales of hard processes accessible to LHC measurements. For instance, one can believe that the evolution term will be relevant if one observes double parton scatterings in equal sign heavy W boson production at the LHC. This relatively rare process was studied in Ref. [22], has a good signature and large enough Q^2 to probe longitudinal momentum correlations.

To summarize, the analysis shows that within the leading logarithm approximation of the perturbative QCD theory and the factorization of physics of short and long distances, the two-parton distribution functions being the product of two single distributions at some reference scale become to be dynamically correlated at any different scale of a hard process. These correlations are perturbatively calculable using Eq. (9).

It is pleasure to thank V.A. Ilyin drawing the attention to the problem of a double parton scattering. Discussions with E.E. Boos, I.P. Lokhtin, S.V. Molodtsov, A.S. Proskuryakov, V.I. Savrin, and G.M. Zinovjev are gratefully acknowledged.

[1] This is the analogue of the momentum conserving phase space factor in Eq. (2).

[1] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **56**, 3811 (1997).
[2] C. Goebel, F. Halzen, and D.M. Scott, Phys. Rev. D **22**, 2789 (1980).
[3] N. Paver and D. Treleani, Nuovo Cimento A **70**, 215 (1982).
[4] B. Humpert, Phys. Lett. **131B**, 461 (1983).
[5] B. Humpert and R. Odorico, Phys. Lett. **154B**, 211 (1985).
[6] T. Sjostrand and M. van Zijl, Phys. Rev. D **36**, 2019 (1987).
[7] G. Calucci and D. Treleani, Nucl. Phys. B (Proc. Suppl.) **71**, 392 (1999).
[8] G. Calucci and D. Treleani, Phys. Rev. D **60**, 054023 (1999).
[9] A. Del Fabbro and D. Treleani, Phys. Rev. D **61**, 077502 (2000); Nucl. Phys. B (Proc. Suppl.) **92**, 130 (2001).
[10] R. Kirschner, Phys. Lett. **84B**, 266 (1979).
[11] V.P. Shelest, A.M. Snigirev, and G.M. Zinovjev, Phys. Lett. **113B**, 325 (1982); Sov. Theor. Math. Phys. **51**, 523 (1982).
[12] L.N. Lipatov, Sov. J. Nucl. Phys. **20**, 94 (1974).
[13] K. Konishi, A. Ukawa, and G. Veneziano, Phys. Lett. **78B**, 243 (1978); Nucl. Phys. **B157**, 45 (1979).
[14] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972); **15**, 675 (1972).
[15] Yu.L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
[16] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[17] A.M. Snigirev, hep-ph/0304172.
[18] J.C. Collins, D.E. Soper, and G. Sterman, in *Perturbative QCD*, edited by A. Mueller (World Scientific, Singapore, 1989).
[19] CTEQ Collaboration, H.L. Lai *et al.*, Eur. Phys. J. C **12**, 375 (2000).
[20] V.P. Shelest, A.M. Snigirev, and G.M. Zinovjev, JINR Communication, No. E2-82-194, Dubna, 1982; Report No. ITP-83-46E, Kiev, 1983 (unpublished).
[21] M.J. Puhala, Phys. Rev. D **22**, 1087 (1980).
[22] A. Kulesza and W.J. Stirling, Phys. Lett. B **475**, 168 (2000).