Understanding the $D_{sJ}^{+}(2317)$ and $D_{sJ}^{+}(2460)$ with sum rules in heavy quark effective theory

Yuan-Ben Dai,[1] Chao-Shang Huang,[1] Chun Liu,[1] and Shi-Lin Zhu[2]
[1]*Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, China*
[2]*Department of Physics, Peking University, Beijing 100871, China*
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In the framework of heavy quark effective theory we use QCD sum rules to calculate the masses of the $\bar{c}s(0^+,1^+)$ and $(1^+,2^+)$ excited states. The results are consistent with the states $D_{sJ}(2317)$ and $D_{sJ}(2460)$ observed by BABAR and CLEO being the 0^+ and 1^+ states in the $j_l=\frac{1}{2}^+$ doublet.

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I. INTRODUCTION

Recently the BaBar Collaboration announced a positive-parity narrow state with a rather low mass 2317 ± 3 MeV in the $D_s^+(1969)\pi$ channel [1], which was confirmed by CLEO [2] and BELLE [3] later. Because of its low mass and decay angular distribution, its J^P is believed to be 0^+. In the same experiment CLEO [2] observed a state at 2460 MeV with a possible spin-parity $J^P=1^+$ in the $D_s^*\pi$ channel. BaBar [1] also found a signal near there. Since these two states lie below the DK and D^*K thresholds, respectively, the potentially dominant s-wave decay modes $D_{sJ}(2317)\to D_sK$, etc., are kinematically forbidden. Thus the radiative decays and isospin-violating strong decays become favorable decay modes. The later decay goes in two steps with the help of virtual $D_s\eta$ intermediate states $D_{sJ}(2317)\to D_s\eta\to D_s\pi^0$ where the second step arises from the tiny η-π^0 isospin-violating mixing due to $m_u\neq m_d$.

The experimental discovery of these two states has recently triggered a heated debate on their nature in the literature. The key point is to understand their low masses. The $D_{sJ}(2317)$ mass is significantly lower than the values of a 0^+ mass in the range of 2.4–2.6 GeV calculated in quark models of [4]. The model using the heavy quark mass expansion of the relativistic Bethe-Salpeter equation in [5] predicted a lower value 2.369 GeV of 0^+ mass which is still 50 MeV higher than the experimental data. Bardeen, Eichten, and Hill interpreted them as the $\bar{c}s(0^+,1^+)$ spin doublet and as the parity conjugate states of the $(0^-,1^-)$ doublet in the framework of chiral symmetry[1] [6] (see also Ref. [7]). A quark-antiquark picture was also advocated by Colangelo and De Fazio [9], Cahn and Jackson [10], and Godfrey [11]. Based on such a "conventional" picture, the various decay modes were discussed in Refs. [6,9,11].

Apart from the quark-antiquark interpretation, $D_{sJ}(2317)$ was suggested to be a four-quark state by Cheng and Hou [12] and Barnes, Close, and Lipkin [13]. Szczepaniak even argued that it could be a strong $D\pi$ atom [14]. But we think it would be very exceptional for a molecule or atom to have a binding energy as large as 40 MeV.

van Beveren and Rupp [15] argued from the experience with $a_0/f_0(980)$ that the low mass of $D_{sJ}(2317)$ could arise from the mixing between the DK continuum and lowest scalar nonet. In this way the 0^+ $\bar{c}s$ state is artificially pushed much lower than that expected from quark models.

A recent lattice calculation suggests a value around 2.57 GeV for the 0^+ state mass [16], much larger than the experimentally observed $D_{sJ}(2317)$ and compatible with quark model predictions. The conclusion in Ref. [16] is that $D_{sJ}(2317)$ might receive a large component of DK and the physics might resemble $a_0/f_0(980)$. Such a large DK component makes lattice simulations very difficult.

In this paper we shall use QCD sum rules [20] in the framework of the heavy quark effective theory (HQET) [21] to extract the masses since HQET provides a systematic method to compute the properties of heavy hadrons containing a single heavy quark via the $1/m_Q$ expansion, where m_Q is the heavy quark mass. The masses of ground state heavy mesons have been studied with QCD sum rules in HQET in [22–24]. In [17–19] masses of the lowest excited nonstrange heavy meson doublets $(0^+,1^+)$ and $(1^+,2^+)$ were studied with the sum rules in HQET up to the order of $O(1/m_Q)$. These masses were also analyzed in the earlier works [25] with sum rules in full QCD. In this paper we extend the same formalism in [17,18] to include the light quark mass in calculating the $\bar{c}s$ mesons. We shall also use more stringent criteria for the stability windows of the sum rules in the numerical analysis.

II. m_M-m_Q AT THE LEADING ORDER OF HQET

The proper interpolating current $J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}$ for a state with quantum numbers j, P, j_ℓ in HQET was given in [17]. These currents proved to satisfy the conditions

$$\langle 0|J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(0)|j',P',j'_\ell\rangle=f_{Pj_l}\delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}\eta^{\alpha_1\cdots\alpha_j}, \tag{1}$$

$$\begin{aligned} i\langle 0|T(J^{\alpha_1\cdots\alpha_j}_{j,P,j_\ell}(x)J^{\dagger\beta_1\cdots\beta_{j'}}_{j',P',j'_\ell}(0))|0\rangle \\ =\delta_{jj'}\delta_{PP'}\delta_{j_\ell j'_\ell}(-1)^j\mathcal{S}g_t^{\alpha_1\beta_1}\cdots g_t^{\alpha_j\beta_j} \\ \times\int dt\,\delta(x-vt)\Pi_{P,j_\ell}(x), \end{aligned} \tag{2}$$

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[1]The existence of parity doublets has also been shown by combining chiral symmetry and heavy quark symmetry in the Bethe-Salpeter approach in [8].

in the $m_Q \to \infty$ limit, where $\eta^{\alpha_1 \cdots \alpha_j}$ is the polarization tensor for the spin j state, v is the velocity of the heavy quark, $g_t^{\alpha\beta} = g^{\alpha\beta} - v^\alpha v^\beta$ is the transverse metric tensor, $\mathcal{S}$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$, and f_{P,j_ℓ} and Π_{P,j_ℓ} are a constant and a function of x, respectively, which depend only on P and j_ℓ.

We consider the correlator

$$\Pi(\omega) = i \int d^4x e^{ikx} \langle 0 | T(J_{j,P,j_\ell}^{\alpha_1 \cdots \alpha_j}(x) J_{j',P',j'_\ell}^{\dagger \beta_1 \cdots \beta_{j'}}(0)) | 0 \rangle, \tag{3}$$

where $\omega = 2v \cdot k$. It can be written as

$$\Pi(\omega) = \frac{f_{P,j_\ell}^2}{2\overline{\Lambda}_{j,P,j_\ell} - \omega} + \text{higher states}, \tag{4}$$

where $\overline{\Lambda}_{j,P,j_\ell} = \lim_{m_Q \to \infty} (m_{M_{j,P,j_\ell}} - m_Q)$. On the other hand, it will be calculated in terms of quarks and gluons. Invoking Borel transformation to Eq. (3) we get

$$f_{P,j_\ell}^2 e^{-2\overline{\Lambda}_{j,P,j_\ell}/T} = \frac{1}{\pi} \int_{2m_q}^{\omega_c} \rho(\omega) e^{-\omega/T} + \text{condensates}, \tag{5}$$

where m_q is the light quark mass, $\rho(\omega)$ is the perturbative spectral density, and ω_c is the threshold parameter used to subtract the higher-state contribution with the help of the quark-hadron duality assumption.

We shall study the low-lying $(0^+,1^+),(1^+,2^+)(\bar{c}s)$ states. The values of various QCD condensates are

$$\langle \bar{s}s \rangle = -(0.8 \pm 0.1) * (0.24 \ \text{GeV})^3,$$
$$\langle \alpha_s GG \rangle = 0.038 \ \text{GeV}^4,$$
$$m_0^2 = 0.8 \ \text{GeV}^2. \tag{6}$$

We use $m_s(1 \text{ GeV}) = 0.15$ GeV for the strange quark mass in the modified minimal subtraction (MS) scheme. We use $\Lambda_{QCD} = 375$ MeV for three active flavors and $\Lambda_{QCD} = 220$ MeV for four active flavors. The sum rules with massless light quarks have been obtained in [18,17].

In the numerical analysis of the QCD sum rules we require that the high-order power corrections be less than 30% of the perturbation term. This condition yields the minimum value T_{min} of the allowed Borel parameter. We also require that the pole term, which is equal to the sum of the cutoff perturbative term and the condensation terms, be larger than 60% of the perturbative term, which leads to the maximum value T_{max} of the allowed T. Thus we have the working interval $T_{min} < T < T_{max}$ for a fixed ω_c. If $T_{min} \geqslant T_{max}$, we are unable to extract useful information from such a sum rule. In the ideal case, the difference between the meson mass m_M and the heavy quark mass m_Q (or other observables) is almost independent of T for certain values of ω_c. Namely, the dependence on both the Borel parameter and continuum threshold is minimum. In realistic cases, the variation of a sum rule with both T and ω_c will contribute to the errors of the extracted value, together with the truncation of the operator product expansion and the uncertainty of vacuum condensate values.

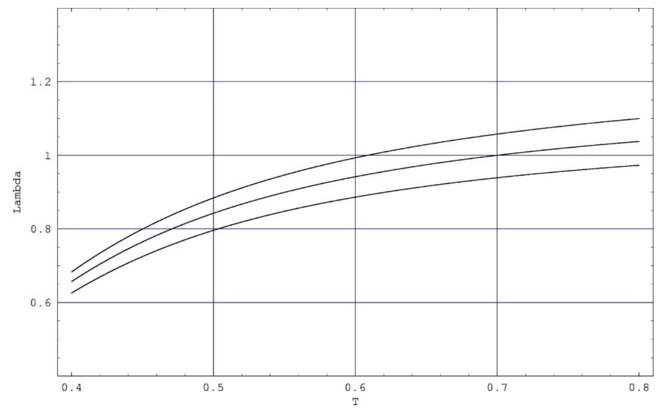

FIG. 1. The variation of $\overline{\Lambda}(\frac{1}{2}^+)$ (in units of GeV) of the $(0^+,1^+)$ doublet with T and ω_c for the derivative currents. The vertical and horizontal axes correspond to $\overline{\Lambda}$ and T. From top to bottom, the curves correspond to ω_c being 3.1, 2.9, 2.7 GeV, respectively.

In Secs. II A and II B we give sum rules for $(0^+,1^+)$ and $(1^+,2^+)$ doublets using interpolating currents with derivatives, respectively. For comparison we discuss sum rules for $(0^+,1^+)$ using interpolating currents without derivatives in Sec. II C.

A. $(0^+,1^+)$ doublet with derivative currents

We employ the interpolating currents [17]

$$J'^{\dagger}_{0,+,2} = \frac{1}{\sqrt{2}} \bar{h}_v (-i) \not{\mathcal{D}}_t s,$$

$$J'^{\alpha\dagger}_{1,+,2} = \frac{1}{\sqrt{2}} \bar{h}_v \gamma^5 \gamma_t^\alpha (-i) \not{\mathcal{D}}_t s, \tag{7}$$

where $D_t^\mu = D^\mu - (v \cdot D) v^\mu$, with $D^\mu = \partial^\mu - igA^\mu$ being the gauge-covariant derivative. We obtain the sum rule relevant to $\overline{\Lambda}$:

$$f'^2_{+,1/2} e^{-2\overline{\Lambda}/T} = \frac{3}{64\pi^2} \int_{2m_s}^{\omega_c} [\omega^4 + 2m_s\omega^3 - 6m_s^2\omega^2 - 12m_s^3\omega] e^{-\omega/T} d\omega - \frac{1}{16} m_0^2 \langle \bar{s}s \rangle + \frac{3}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{16\pi} \langle \alpha_s G^2 \rangle. \tag{8}$$

Taking the derivative of the logarithm of the above equation with respect to $(1/T)$ one obtains the sum rule for $\overline{\Lambda}$. Substituting the obtained value of $\overline{\Lambda}$ in Eq. (8) one obtains the sum rule for $f'_{+,1/2}$. In Fig. 1, the variation of $\overline{\Lambda}(1/2^+)$

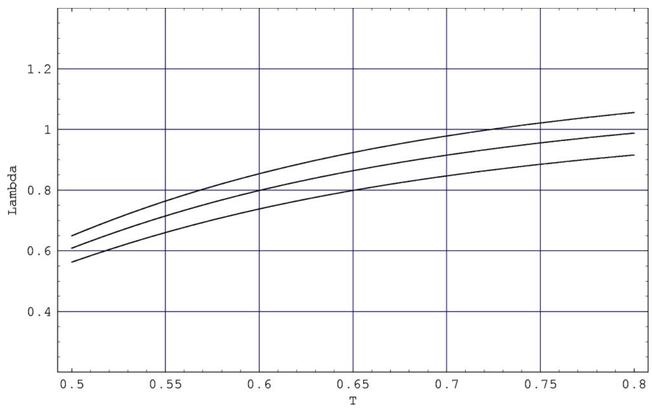

FIG. 2. The variation of $\overline{\Lambda}(\frac{3}{2}^{+})$ (in units of GeV) of the $(1^{+},2^{+})$ doublet with T and ω_c. From top to bottom, $\omega_c=3.2$, 3.0, 2.8 GeV, respectively.

with T and ω_c is shown. According to the criteria stated above the working range is $0.38<T<0.58$ GeV. We have

$$\overline{\Lambda}(1/2^{+})=(0.86\pm0.10)\ \text{GeV},$$

$$f'_{+,1/2}=(0.31\pm0.05)\ \ \text{GeV}^{5/2}, \tag{9}$$

where the central value corresponds to $T=0.52$ GeV and $\omega_c=2.9$ GeV.

B. $(1^{+},2^{+})$ doublet

For the $(1^{+},2^{+})$ doublet, by using the interpolating currents [17]

$$J^{\dagger\alpha}_{1,+,1}=\sqrt{\frac{3}{4}}\bar{h}_v\gamma^5(-i)\left(\mathcal{D}_t^{\alpha}-\frac{1}{3}\gamma_t^{\alpha}\not{\mathcal{D}}_t\right)q,$$

$$J^{\dagger\alpha_1,\alpha_2}_{2,+,1}=\sqrt{\frac{1}{2}}\bar{h}_v\frac{(-i)}{2}\left(\gamma_t^{\alpha_1}\mathcal{D}_t^{\alpha_2}+\gamma_t^{\alpha_2}\mathcal{D}_t^{\alpha_1}-\frac{2}{3}g_t^{\alpha_1\alpha_2}\not{\mathcal{D}}_t\right)q, \tag{10}$$

we obtain the sum rule

$$f^2_{+,3/2}e^{-2\overline{\Lambda}/T}=\frac{1}{64\pi^2}\int_{2m_s}^{\omega_c}[\omega^4+2m_s\omega_c^3-6m_s^2\omega^2-12m_s^3\omega]e^{-\omega/T}d\omega-\frac{1}{12}m_0^2\langle\bar{s}s\rangle-\frac{1}{32}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle T+\frac{1}{8}m_s^2\langle\bar{s}s\rangle-\frac{m_s}{48\pi}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle. \tag{11}$$

From the sum rule, the variation of $\overline{\Lambda}(3/2^{+})$ with T and ω_c is plotted in Fig. 2. The working range is $0.55<T<0.65$ GeV. We have

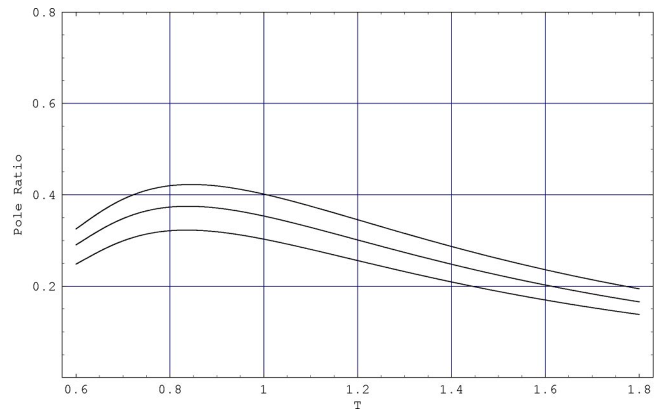

FIG. 3. The variation of the ratio of the pole contribution and the perturbative piece of the $(0^{+},1^{+})$ doublet with T and ω_c for the nonderivative currents. From top to bottom, $\omega_c=3.1$, 2.9, 2.7 GeV, respectively.

$$\overline{\Lambda}(3/2^{+})=(0.83\pm0.10)\ \text{GeV},$$

$$f_{+,3/2}=(0.19\pm0.03)\ \ \text{GeV}^{5/2}, \tag{12}$$

where the central value corresponds to $T=0.62$ GeV and $\omega_c=3.0$ GeV.

C. $(0^{+},1^{+})$ doublet with currents without derivative

Possible different currents for $(0^{+},1^{+})$ doublet are the nonderivative currents

$$J^{\dagger}_{0,+,2}=\frac{1}{\sqrt{2}}\bar{h}_v s,$$

$$J^{\dagger\alpha}_{1,+,2}=\frac{1}{\sqrt{2}}\bar{h}_v\gamma^5\gamma_t^{\alpha}s. \tag{13}$$

With the nonderivative currents the sum rule reads

$$f^2_{+,1/2}e^{-2\overline{\Lambda}/T}=\frac{3}{16\pi^2}\int_{2m_s}^{\omega_c}[\omega^2-2m_s\omega-2m_s^2]e^{-\omega/T}d\omega+\frac{1}{2}\langle\bar{s}s\rangle+\frac{m_s}{4T}\langle\bar{s}s\rangle-\frac{1}{8T^2}m_0^2\langle\bar{s}s\rangle. \tag{14}$$

Requiring that the condensate contribution be less than 30% of the perturbative term, we get $T_{\rm min}=0.75$ GeV. In Fig. 3, we show the ratio of the pole term—i.e, the sum of the cutoff perturbative term and condensation terms—for the sum rule (14). In the whole range of $T>T_{\rm min}$, this pole contribution is less than 40%. Hence, there is no stability window for this sum rule satisfying our criteria stated before.

If we arbitrarily loosen the analysis criteria and require the pole contribution to be greater than 30% only, we get the working range $0.75<T<1.2$ GeV. In Fig. 4, the variation of $\overline{\Lambda}(1/2^{+})$ with T and ω_c is shown. Numerically,

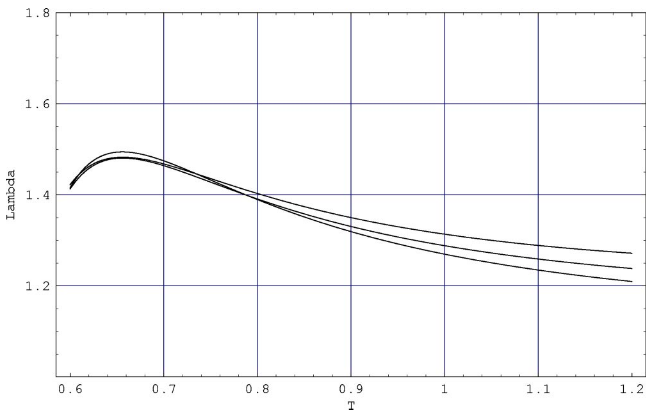

FIG. 4. The variation of $\overline{\Lambda}$ (in units of GeV) of the $(0^+,1^+)$ doublet with T and ω_c for the nonderivative currents. From top to bottom, $\omega_c = 3.1,\ 2.9,\ 2.7$ GeV, respectively.

$$\overline{\Lambda}(1/2^+) = (1.30 \pm 0.15) \ \ \text{GeV},$$

$$f_{+,1/2} = (0.39 \pm 0.05) \ \ \text{GeV}^{3/2}, \tag{15}$$

where the central value corresponds to $T = 1.0$ GeV and $\omega_c = 2.9$ GeV. Because of the weaker criteria used, these results are less reliable and will not be used in the final numerical results.

Here we would like to make some remarks. Usually, the currents with the least number of derivatives are used in QCD sum rule approaches. The sum rules then are less sensitive to the threshold energy ω_c. However, it is pointed out in [17] that in the nonrelativistic limit the coupling constant of these currents to the P-wave states vanishes. If this coupling constant is suppressed due to this reason, the relative importance of the contribution of the DK and other states in continuum in the sum rules which are usually neglected would be enhanced. Besides, it is shown in [26,27] by using the soft pion theorem that the contribution of the $D\pi$ continuum is large in the sum rule with the nonderivative current for the 0^+ state of the nonstrange D system and significantly decreases the value of $\overline{\Lambda}$. A similar method of calculation is not good in the present case, but it indicates that the contribution of the DK continuum with the nonderivative current may be large too.

III. SUM RULES AT THE $1/m_Q$ ORDER

To the order of $1/m_Q$, the Lagrangian of HQET is

$$\mathcal{L}_{\text{eff}} = \overline{h}_v iv \cdot D h_v + \frac{1}{2m_Q}\mathcal{K} + \frac{1}{2m_Q}\mathcal{S} + \mathcal{O}(1/m_Q^2), \tag{16}$$

where $h_v(x)$ is the velocity-dependent effective field related to the original heavy quark field $Q(x)$ by

$$h_v(x) = e^{im_Q v \cdot x}\frac{1+\not{v}}{2}Q(x). \tag{17}$$

Here $\mathcal{K}$ is the kinetic operator defined as

$$\mathcal{K} = \overline{h}_v (iD_t)^2 h_v, \tag{18}$$

and $\mathcal{S}$ is the chromomagnetic operator

$$\mathcal{S} = \frac{g}{2}C_{mag}(m_Q/\mu)\overline{h}_v \sigma_{\mu\nu}G^{\mu\nu}h_v, \tag{19}$$

where $C_{mag} = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$, $\beta_0 = 11 - 2n_f/3$.

Considering $O(1/m_Q)$ corrections the pole term of the correlator on the hadron side becomes

$$\Pi(\omega)_{pole} = \frac{(f+\delta f)^2}{2(\overline{\Lambda}+\delta m) - \omega}$$
$$= \frac{f^2}{2\overline{\Lambda}-\omega} - \frac{2\,\delta m f^2}{(2\overline{\Lambda}-\omega)^2} + \frac{2 f \delta f}{2\overline{\Lambda}-\omega}, \tag{20}$$

where δm and δf are of order $O(1/m_Q)$.

To extract δm in Eq. (20) we follow the approach of [23] to consider the three-point correlation functions

$$\delta_O \Pi^{\alpha_1 \cdots \alpha_j, \beta_1 \cdots \beta_j}_{j,P,i}(\omega,\omega')$$
$$= i^2 \int d^4x d^4y e^{ik\cdot x - ik'\cdot y}$$
$$\times \langle 0 | T(J^{\alpha_1 \cdots \alpha_j}_{j,P,i}(x) O(0) J^{\dagger \beta_1 \cdots \beta_j}_{j,P,i}(y)) | 0 \rangle, \tag{21}$$

where $O = \mathcal{K}$ or $\mathcal{S}$. The scalar function corresponding to Eq. (21) can be represented as the double dispersion integral

$$\delta_O \Pi(\omega,\omega') = \frac{1}{\pi^2}\int \frac{\rho_o(s,s')ds ds'}{(s-\omega)(s'-\omega')}.$$

The pole parts of $\delta_O \Pi(\omega,\omega'), O = \mathcal{K},\mathcal{S}$, are

$$\delta_{\mathcal{K}}\Pi(\omega,\omega')_{pole} = \frac{f^2 K}{(2\overline{\Lambda}-\omega)(2\overline{\Lambda}-\omega')}$$
$$+ \frac{f^2 G_{\mathcal{K}}(\omega')}{2\overline{\Lambda}-\omega} + \frac{f^2 G_{\mathcal{K}}(\omega)}{2\overline{\Lambda}-\omega'}, \tag{22}$$

$$\delta_{\mathcal{S}}\Pi(\omega,\omega')_{pole} = \frac{d_M f^2 \Sigma}{(2\overline{\Lambda}-\omega)(2\overline{\Lambda}-\omega')}$$
$$+ d_M f^2\left[\frac{G_{\mathcal{S}}(\omega')}{2\overline{\Lambda}-\omega} + \frac{G_{\mathcal{S}}(\omega)}{2\overline{\Lambda}-\omega'}\right], \tag{23}$$

where

$$K_{j,P,j_l}=\langle j,P,j_l|\overline{h}_v(iD_\perp)^2h_v|j,P,j_l\rangle,$$

$$2d_M\Sigma_{j,P,j_l}=\langle j,P,j_l|\overline{h}_v g\sigma_{\mu\nu}G^{\mu\nu}h_v|j,P,j_l\rangle,$$

$$d_M=d_{j,j_l},\quad d_{j_l-1/2,j_l}=2j_l+2,$$

$$d_{j_l+1/2,j_l}=-2j_l\,.$$

Letting $\omega=\omega'$ in Eqs. (22) and (23) and comparing with Eq. (20), one obtains [23]

$$\delta m=-\frac{1}{4m_Q}(K+d_MC_{mag}\Sigma). \tag{24}$$

The single pole terms in Eqs. (22) and (23) come from the region in which $s(s')=2\overline{\Lambda}$ and $s'(s)$ is at the pole for a radial excited state or in the continuum. They are suppressed by making a double Borel transformation for both ω and ω'. The Borel parameters corresponding to ω and ω' are taken to be equal. One obtains thus the sum rules for K and Σ as

$$f^2Ke^{-2\overline{\Lambda}/T}=\int_{2m_s}^{\omega_c}\int_{2m_s}^{\omega_c}d\omega d\omega' e^{-(\omega+\omega')/2T}\rho_{\mathcal{K}}(\omega,\omega'), \tag{25}$$

$$f^2\Sigma e^{-2\overline{\Lambda}/T}=\int_{2m_s}^{\omega_c}\int_{2m_s}^{\omega_c}d\omega d\omega' e^{-(\omega+\omega')/2T}\rho_{\mathcal{S}}(\omega,\omega'). \tag{26}$$

In this section we shall neglect the m_s corrections to K and Σ. We obtain for the $j_l^P=\frac{1}{2}^+$ doublet with derivative currents

$$f'^2_{+,1/2}Ke^{-2\overline{\Lambda}/T}=-\frac{3}{2^7\pi^2}\int_{2m_s}^{\omega_c}\omega^6e^{-\omega/T}d\omega+\frac{3}{2^4\pi}\langle\alpha_sGG\rangle T^3,$$

$$f'^2_{+,1/2}\Sigma e^{-2\overline{\Lambda}/T}=\int_{2m_s}^{\omega_c}\int_{2m_s}^{\omega_c}\rho(s,s')e^{-(s+s')/2T}dsds'+\frac{1}{48\pi}\langle\alpha_sGG\rangle T^3, \tag{27}$$

with

$$\rho(s,s')=\frac{1}{96}\frac{\alpha_s(2T)}{\pi^3}C_{mag}ss'\times\{s'^2(3s-s')\theta(s-s')+(s\leftrightarrow s')\}.$$

We can eliminate the dependence of the K and Σ on $f_{+,1/2}$ and $\overline{\Lambda}$ through dividing the above sum rules (27) by the sum rule (8). We use the same working windows as those of two-point sum rules for $\overline{\Lambda}$ in evaluating $O(1/m_Q)$ corrections. Numerically we have

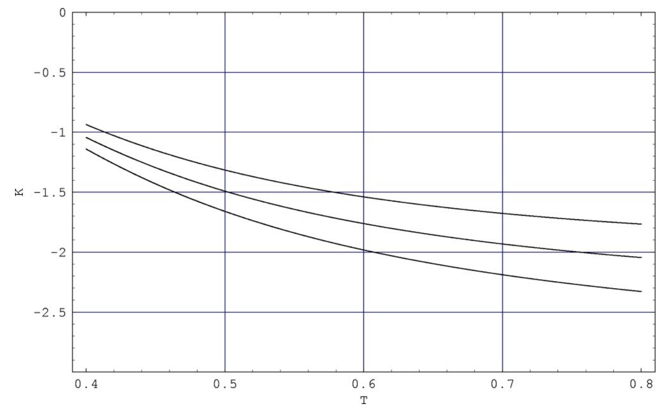

FIG. 5. The variation of $K_{1/2}$ (in units of GeV^2) of the $(0^+,1^+)$ doublet with T and ω_c for the derivative currents. From top to bottom, $\omega_c=3.1,\ 2.9,\ 2.7$ GeV, respectively.

$$K_{1/2}=(-1.60\pm0.30)\ \ \mathrm{GeV}^2,$$

$$\Sigma_{1/2}=(0.28\pm0.05)\ \ \mathrm{GeV}^2. \tag{28}$$

The variations of K and Σ with T and ω_c for the $j_l^P=\frac{1}{2}^+$ doublet are shown in Figs. 5 and 6, respectively.

For $j_l^P=\frac{3}{2}^+$ doublet, we have

$$f^2_{+,3/2}Ke^{-2\overline{\Lambda}/T}=-\frac{1}{2^7\pi^2}\int_{2m_s}^{\omega_c}\omega^6e^{-\omega/T}d\omega+\frac{7}{3\times2^5\pi}\langle\alpha_sGG\rangle T^3,$$

$$f^2_{+,3/2}\Sigma e^{-2\overline{\Lambda}/T}=\int_{2m_s}^{\omega_c}\int_{2m_s}^{\omega_c}\rho(s,s')e^{-(s+s')/2T}dsds'+\frac{1}{72\pi}\langle\alpha_sGG\rangle T^3, \tag{29}$$

with

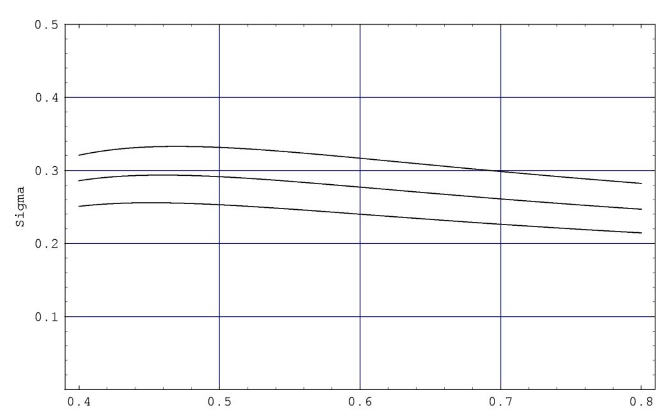

FIG. 6. The variation of $\Sigma_{1/2}$ (in units of GeV^2) of the $(0^+,1^+)$ doublet with T and ω_c for the derivative currents. From top to bottom, $\omega_c=3.1,\ 2.9,\ 2.7$ GeV, respectively.

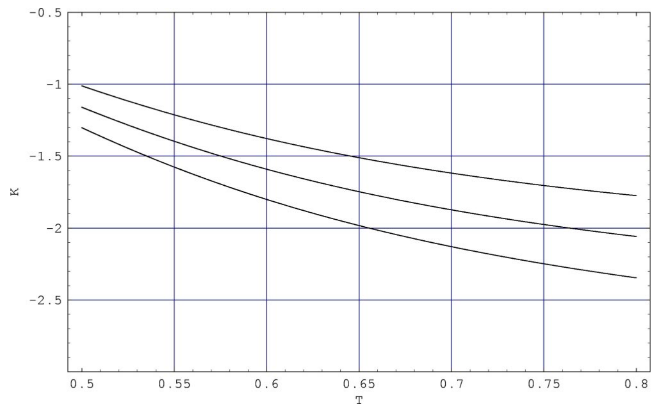

FIG. 7. The variation of $K_{3/2}$ (in units of GeV^2) of the $(1^+,2^+)$ doublet with T and ω_c. From top to bottom, $\omega_c=3.2$, 3.0, 2.8 GeV, respectively.

$$\rho(s,s')=\frac{1}{288}\frac{\alpha_s(2T)}{\pi^3}C_{mag}\times\left\{s'^4\left(s-\frac{3}{5}s'\right)\theta(s-s')+(s\leftrightarrow s')\right\}.$$

The variations of K and Σ with T and ω_c for the $j_l^P=\frac{3}{2}^+$ doublet are plotted in Figs. 7 and 8, respectively. Numerically we have

$$K_{3/2}=(-1.64\pm 0.40)\ \ \mathrm{GeV}^2,$$

$$\Sigma_{3/2}=(0.058\pm 0.01)\ \ \mathrm{GeV}^2. \tag{30}$$

The spin-symmetry-violating term $\mathcal{S}$ not only causes a splitting of the masses within the same doublet, but also causes a mixing of states with the same j, P but different j_l. In Ref. [18] corrections from the mixing are found to be negligible. We omit this effect here.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We present our results for the $c\bar{s}$ system assuming the HQET is good enough for excited D mesons. For the doublet $(0^+,1^+)$ with the derivative current, the weighted average mass is

$$\frac{1}{4}(m_{D_{s0}^*}+3m_{D_{s1}^*})=m_c+(0.86\pm 0.10)+\frac{1}{m_c}[(0.40\pm 0.08)\ \ \mathrm{GeV}^2].$$

The mass splitting is

$$m_{D_{s1}^*}-m_{D_{s0}^*}=\frac{1}{m_c}[(0.28\pm 0.05)\ \ \mathrm{GeV}^2].$$

For the $(1^+,2^+)$ doublet we have

$$\frac{1}{8}(3m_{D_{s1}}+5m_{D_{s2}^*})=m_c+(0.83\pm 0.10)+\frac{1}{m_c}\times[(0.41\pm 0.10)\ \ \mathrm{GeV}^2]. \tag{31}$$

The $1^+,2^+$ mass splitting is

$$m_{D_{s2}^*}-m_{D_{s1}}=\frac{1}{m_c}[(0.116\pm 0.06)\ \ \mathrm{GeV}^2].$$

The results for $\bar{b}s$ system are obtained by replacing m_c by m_b and multiplying Σ by 0.8 (since $C_{mag}\approx 0.8$ for the B system by using the values of Λ_{QCD} given in Sec. II) in the above equations.

Choosing m_c to fit the experimental value

$$\frac{1}{8}(3m_{D_{s1}}+5m_{D_{s2}^*})=2.56\ \ \mathrm{GeV},$$

where we again neglect the mixing between two 1^+ states, we obtain $m_c=1.44$ GeV. Using this m_c value we obtain the following numerical results. The $1^+,2^+$ mass splitting is

$$m_{D_{s2}^*}-m_{D_{s1}}=(0.080\pm 0.042)\ \ \mathrm{GeV},$$

which is consistent with the experimental value 37 MeV within a large theoretical uncertainty. Experimentally, the mass splitting in the $(1^+,2^+)$ doublet in the D_s system is almost equal to that in the D system. This justifies neglecting the m_s correction to the Σ term in our calculation. For the $(0^+,1^+)$ doublet, the weighted average mass is

$$\frac{1}{4}(m_{D_{s0}^*}+3m_{D_{s1}^*})=(2.57\pm 0.12)\ \ \mathrm{GeV}.$$

The mass splitting is

$$m_{D_{s1}^*}-m_{D_{s0}^*}=(0.19\pm 0.04)\ \ \mathrm{GeV}. \tag{32}$$

Therefore, the 0^+ mass is predicted to be $m_{0^+}=(2.42\pm 0.13)$ GeV. This is consistent with the experimental value 2.317 GeV, though the central value is 100 MeV larger than

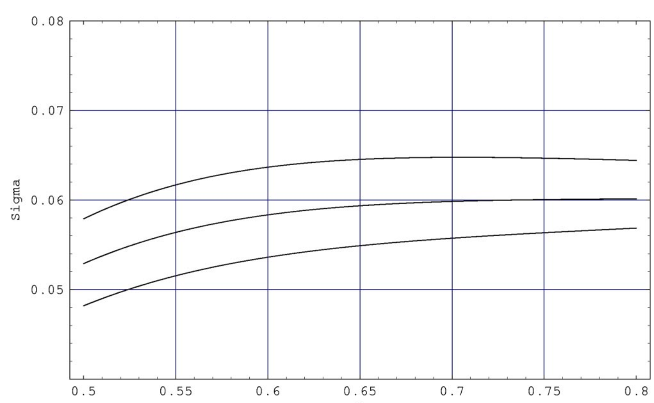

FIG. 8. The variation of $\Sigma_{3/2}$ (in units of GeV^2) of the $(1^+,2^+)$ doublet with T and ω_c. From top to bottom, $\omega_c=3.2$, 3.0, 2.8 GeV, respectively.

data. If the 1^+ D_s state of mass 2.460 GeV found by CLEO [2,1] is assigned as the other member of the $(0^+,1^+)$ doublet, then the observed mass splitting 0.143 GeV is consistent with Eq. (32). Notice that the $\Sigma_{1/2}$ value in Fig. 6 changes very slowly between $T=0.35$ and 0.8 GeV. Therefore the result (32) is insensitive to the stability window used.

We would like to note that the stability of the sum rules (in particular, those for the K and Σ) obtained by us is not as good as those for the ground states, as can be seen from the figures. And for the charm flavor hadrons, the $1/m_c$ corrections are significant. So the predictions on masses have a relatively large uncertainties, which are estimated by given errors.

If we replace the strange quark condensate by up and down quark condensates and m_s by the zero up and down quark masses, we can extract the nonstrange excited D meson masses. For the $(1^+,2^+)$ doublet, the sum rule window is $0.57<T<0.67$ GeV for $\omega_c=2.8$–3.2 GeV. The results are

$$\Lambda_{3/2}=(0.73\pm0.08)\ \text{GeV},$$

$$K_{3/2}=-(1.6\pm0.4)\ \text{GeV}^2,$$

$$\Sigma_{3/2}=(0.057\pm0.01)\ \text{GeV}^2, \tag{33}$$

with the central value at $T=0.62$ GeV. For the $(0^+,1^+)$ doublet, the sum rule window is $0.46<T<0.6$ GeV for $\omega_c=2.7$–3.1 GeV. The results are

$$\Lambda_{1/2}=(0.79\pm0.08)\ \text{GeV},$$

$$K_{1/2}=-(1.57\pm0.4)\ \text{GeV}^2,$$

$$\Sigma_{1/2}=(0.286\pm0.05)\ \text{GeV}^2, \tag{34}$$

with the central value at $T=0.53$ GeV. The effect of strange quarks is to make the mass of D_s mesons a little bit larger, as expected. Experimentally, the Belle Collaboration found $m_{D_0}=(2290\pm22\pm20)$ MeV and $m_{D_1^*}=(2400\pm30\pm20)$ MeV recently [28] while the CLEO Collaboration found $m_{D_1^*}=2400^{+48}_{-42}$ MeV [29]. The $j_{3/2}$ nonstrange doublet mass is known precisely [30], $m_{D_2}=2460$ MeV and $m_{D_1}=2420$ MeV. Our results (33), (34) are consistent with the experimental data within the theoretical uncertainty.

In summary, we have calculated the masses of the excited $(0^+,1^+)$ and $(1^+,2^+)$ doublets for the $\overline{c}s$ system to the $1/m_Q$ order in the HQET sum rules. The numerical results imply that the $D_{sJ}(2317)$ and $D_{sJ}(2460)$ observed by BABAR and CLEO can be consistently identified as the $(0^+,1^+)$ doublet with $j_l=\frac{1}{2}^+$ within the theoretical uncertainty. Especially, the mass splitting in the $(0^+,1^+)$ doublet is reproduced quite well. The repulsion between these states and the DK and DK^* continuum may help to lower their masses. In the framework of the sum rule this effect should come from the contribution from the DK and DK^* continuum to the dispersion integral.

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