Strong phases and factorization for color suppressed decays

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We prove a factorization theorem in QCD for the color suppressed decays $\bar{B}^0 \to D^0 M^0$ and $\bar{B}^0 \to D^{*0} M^0$ where M is a light meson. Both the color-suppressed and W-exchange or annihilation amplitudes contribute at lowest order in $\Lambda_{\rm QCD}/Q$ where $Q=\{m_b, m_c, E_\pi\}$, so no power suppression of annihilation contributions is found. A new mechanism is given for generating nonperturbative strong phases in the factorization framework. Model-independent predictions that follow from our results include the equality of the $\bar{B}^0 \to D^0 M^0$ and $\bar{B}^0 \to D^{*0} M^0$ rates and the equality of nonperturbative strong phases between isospin amplitudes, $\delta^{(DM)} = \delta^{(D^*M)}$. Relations between amplitudes and phases for $M=\pi,\rho$ are also derived. These results do not follow from large N_c factorization with heavy quark symmetry.

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I. INTRODUCTION

Many of the most frequent hadronic decay channels of B mesons are mediated by the quark level transition $b \to c d \bar{u}$. The same hadronic dynamics also governs the Cabibbo suppressed $b \to c s \bar{u}$ decays. Typical decays of this kind are $\bar{B} \to D\pi$, $\bar{B} \to D^*\pi$, $\bar{B} \to D\rho$, $\bar{B} \to D^*\rho$, $\bar{B} \to DK$, $\bar{B} \to D^*K$, $\bar{B} \to DK^*$, $\bar{B} \to D^*K^*$, $\bar{B} \to D_s K^-$, $\bar{B} \to D_s K^{*-}$, etc., and will be generically referred to as $\bar{B} \to D\pi$ decays. Since these decays are the simplest of a complicated array of hadronic channels, a great deal of theoretical work has been devoted to their understanding [1–15].

After integrating out the W boson the weak Hamiltonian for $\bar{B} \to D\pi$ decays is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu)(\bar{c}b)_{V-A}(\bar{d}u)_{V-A} + C_2(\mu)(\bar{c}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}], \tag{1}$$

where i,j are color indices, and for $\mu_b = 5$ GeV, $C_1(\mu_b) = 1.072$ and $C_2(\mu_b) = -0.169$ at next-to-leading-logarithm order in the naive dimensional regularization l scheme [16]. For the Cabibbo suppressed $\mathcal{H}_W$ we replace $\bar{d} \to \bar{s}$ and $V_{ud}^* \to V_{us}^*$. It is convenient to categorize the decays into three classes [1], depending on the role played by the spectator in the B meson (where "spectator" is a generic term for the flavor structure carried by the light degrees of freedom in B). Class-I decays receive contributions from graphs where the pion is emitted at the weak vertex (Fig. 1T), while in class-II decays the spectator quark ends up in the pion (Figs. 1C,1E). Finally, class-III decays receive both types of contributions. Many of these channels have been well studied experimentally [17–22]; see Table I. Another method to categorize these decays makes use of amplitudes corresponding to the different Wick contractions of flavor topologies. These can be read off from Fig. 1 and are denoted as T (tree), C (color suppressed), and E (W exchange or weak annihilation).

Long ago, it was observed that approximating the matrix elements by the factorized product $\langle D|(\bar{c}b)_{V-A}|B\rangle \times \langle \pi|(\bar{d}u)_{V-A}|0\rangle$ gives an accurate prediction for the branching fractions of type-I decays, and a fair prediction for type-III decays. For all class-I and -II amplitudes a similar procedure was proposed [1]. In terms of two phenomenological parameters $a_{1,2}$,

$$iA(\bar{B}^0 \to D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1(D\pi) \langle D^+|(\bar{c}b)_{V-A}|\bar{B}^0\rangle \times \langle \pi^-|(\bar{d}u)_{V-A}|0\rangle,$$

$$iA(\bar{B}^0 \to D^0\pi^0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2(D\pi) \langle \pi^0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle \times \langle D^0|(\bar{c}u)_{V-A}|0\rangle. \tag{2}$$

Type-III amplitudes are related by isospin to linear combinations of type-I and -II decays. Naive factorization[1] predicts the universal values $a_1 = C_1 + C_2/N_c$ and $a_2 = C_2 + C_1/N_c$.

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[1]In this paper we will use the phrase naive factorization to refer to factoring matrix elements of four quark operators even though this may not be a justified procedure, and will use the phrase factorization for results which follow from a well-defined limit of QCD.

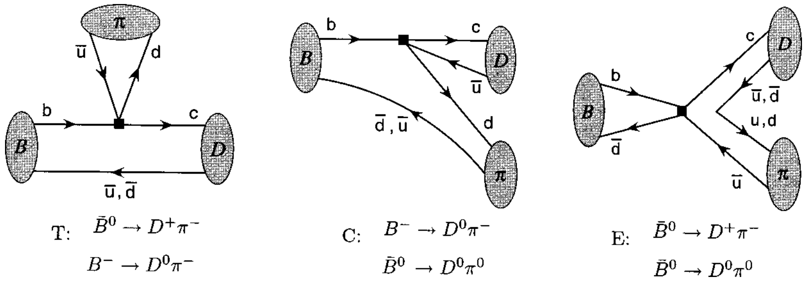

FIG. 1. Decay topologies referred to as tree (T), color suppressed (C), and W exchange (E) and the corresponding hadronic channels to which they contribute.

Phenomenological analyses testing the validity of the factorization hypothesis have been presented in [3], where typically contributions from E are not included. These contributions can be modeled using the vacuum insertion approximation which gives the $D\to\pi$ form factor at a large timelike momentum transfer $q^2=m_B^2$. For this reason, they are often estimated to be suppressed relative to the T amplitudes by $\Lambda_{\rm QCD}^2/m_b^2$ [7].

One rigorous method for investigating factorization in these decays is based on the large N_c limit of QCD. In this limit the amplitudes for type-I decays start at $O(N_c^{1/2})$ while type-II decays are suppressed by $1/N_c$ (hence the name color suppressed). The type-I amplitudes have a form similar to Eq. (2) since nonfactorizable diagrams are suppressed, while type-II decays simultaneously receive contributions from factorized and nonfactorizable diagrams. For a typical class-II decay, a Fierz transformation puts the amplitude into the form

$$iA(\overline{B}{}^0\to D^0\pi^0)=\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*\bigg\{\bigg(C_2+\frac{C_1}{N_c}\bigg)\times\langle D^0\pi^0|(\overline{d}b)(\overline{c}u)|\overline{B}{}^0\rangle+2C_1\times\langle D^0\pi^0|(\overline{d}T^a b)(\overline{c}T^a u)|\overline{B}{}^0\rangle\bigg\}, \quad (3)$$

where the $(V-A)\otimes(V-A)$ structure is implicit. The two matrix elements have expansions in $1/N_c$ which start with terms of the order of $N_c^{1/2}$ and $N_c^{-1/2}$, respectively,

$$\frac{1}{N_c^{1/2}}\langle D^{(*)0}\pi^0|(\overline{d}b)(\overline{c}u)|\overline{B}{}^0\rangle=F_0^{(*)}+\frac{1}{N_c^2}F_2^{(*)}+\cdots,$$
$$\frac{1}{N_c^{1/2}}\langle D^{(*)0}\pi^0|(\overline{d}T^a b)(\overline{c}T^a u)|\overline{B}{}^0\rangle=\frac{1}{N_c}G_1^{(*)}+\frac{1}{N_c^3}G_3^{(*)}+\cdots, \quad (4)$$

where $F_i^{(*)}\sim N_c^0$, $G_i^{(*)}\sim N_c^0$. The Wilson coefficients in Eq. (1) can be assigned scalings with N_c following from their perturbative expansions $C_1\sim O(1)$, $C_2\sim N_c^{-1}$, which roughly corresponds to the hierarchy in their numerical values at μ_b. The leading terms are the matrix elements $F_0^{(*)}$, which factor in terms of large N_c form factors and decay constants

$$N_c^{1/2}F_0^{(*)}\sim\langle D^{(*)0}|\overline{c}u|0\rangle\langle\pi^0|\overline{d}b|\overline{B}{}^0\rangle+\langle D^{(*)0}\pi^0|\overline{c}u|0\rangle\times\langle 0|\overline{d}b|\overline{B}{}^0\rangle \quad (5)$$

plus the matrix elements $G_1^{(*)}$ which are nonfactorizable. The naive factorization assumption would keep only $F_0^{(*)}$ and neglect $G_1^{(*)}$. This approximation is not justified in the $1/N_c$ expansion since $G_1^{(*)}$ is enhanced by the large Wilson coefficient C_1. In either case, no prediction is obtained for the ratio of the $\overline{B}\to D\pi$ and $\overline{B}\to D^*\pi$ amplitudes,

TABLE I. Data on $B\to D^{(*)}\pi$ and $B\to D^{(*)}\rho$ decays from various references. If not otherwise indicated data are from Ref. [17].

Type	Decay	BR (10^{-3})	$\lvert A\rvert$ $(10^{-7}$ GeV)	Decay	BR (10^{-3})	$\lvert A\rvert$ $(10^{-7}$ GeV)
I	$\bar{B}^0\to D^+\pi^-$	2.68 ± 0.29 [a]	5.89 ± 0.32	$\bar{B}^0\to D^{*+}\pi^-$	2.76 ± 0.21	6.05 ± 0.23
III	$B^-\to D^0\pi^-$	4.97 ± 0.38 [a]	7.70 ± 0.29	$B^-\to D^{*0}\pi^-$	4.6 ± 0.4	7.49 ± 0.33
II	$\bar{B}^0\to D^0\pi^0$	0.292 ± 0.045 [b]	1.94 ± 0.15	$\bar{B}^0\to D^{*0}\pi^0$ [b]	0.25 ± 0.07	1.82 ± 0.25
I	$\bar{B}^0\to D^+\rho^-$	7.8 ± 1.4	10.2 ± 0.9	$\bar{B}^0\to D^{*+}\rho^-$	6.8 ± 1.0 [c]	9.08 ± 0.68 [e]
III	$B^-\to D^0\rho^-$	13.4 ± 1.8	12.8 ± 0.9	$B^-\to D^{*0}\rho^-$	9.8 ± 1.8 [c]	10.5 ± 0.97 [e]
II	$\bar{B}^0\to D^0\rho^0$	0.29 ± 0.11 [d]	1.97 ± 0.37	$\bar{B}^0\to D^{*0}\rho^0$	<0.56	<2.77

[a] Reference [18].
[b] References [19,20].
[c] Reference [21].
[d] Reference [22].
[e] For $\bar{B}\to D^*\rho$ the amplitudes for longitudinally polarized ρ's are displayed.

$$R_0^\pi \equiv \frac{A(\overline{B}^0 \to D^{*0}\pi^0)}{A(\overline{B}^0 \to D^0\pi^0)} = \frac{(C_2 + C_1/N_c)F_0^* + (2C_1/N_c)G_1^*}{(C_2 + C_1/N_c)F_0 + (2C_1/N_c)G_1}. \tag{6}$$

Heavy quark symmetry does not operate with large N_c factorization because for C and E it is broken by the allowed exchange of energetic hard gluons between the heavy quarks and the quarks in the pion. In contrast, we will show in this paper that expanding about the limit $E_\pi \gg \Lambda$ this ratio is predicted to be 1 at leading order in Λ/Q. Here $\Lambda \sim \Lambda_{\rm QCD}$ is a typical hadronic scale.

Another rigorous approach to factorization becomes possible in the limit $E_\pi \gg \Lambda_{\rm QCD}$, which corresponds to having an energetic light hadron in the final state. In this paper we analyze type-II decays using QCD and an expansion in $\Lambda_{\rm QCD}/m_b$, $\Lambda_{\rm QCD}/m_c$, and $\Lambda_{\rm QCD}/E_\pi$ (or generically $\Lambda_{\rm QCD}/Q$ where $Q = \{m_b, m_c, m_b - m_c\}$). We derive a factorization theorem and show that E and C appear at the same order in the power counting and are suppressed by $\Lambda_{\rm QCD}/Q$ relative to T. Arguments for the suppression of C by $(\Lambda_{\rm QCD}/Q)^1$ and E by $(\Lambda_{\rm QCD}/Q)^{1,2}$ appear in the literature [7], but we are unaware of a derivation that is model independent. Our leading order result disagrees with the a_2-factorization result. Instead the amplitudes for $\overline{B}^0 \to D^{(*)0}\pi^0$ and $\overline{B}^0 \to D^{(*)0}\rho^0$ are determined by the leading light-cone wave functions $\phi_{\pi,\rho}$ and two new universal $\overline{B} \to D^{(*)}$ distribution functions. Long-distance contributions also occur at this order in $\Lambda_{\rm QCD}/Q$, but are shown to be suppressed relative to the short-distance contributions by an additional $\alpha_s(Q)/\pi$.

For type-I decays a color transparency [23] argument given by Bjorken suggested $A(\overline{B}^0 \to D^+\pi^-) \simeq (C_1 + C_2/N_c) f_\pi F_0^{BD}(m_\pi^2) + O(\alpha_s(Q))$. In Ref. [2] it was argued that this factorization is the leading order prediction in the large energy limit $E_\pi \gg \Lambda_{\rm QCD}$, and in Refs. [6,7] that α_s corrections can be rigorously included. This factorization was extended to all orders in α_s with the proof of a factorization theorem using the soft-collinear effective theory [9]

$$A(B \to D^{(*)}\pi) = N^{(*)} \xi(w_0,\mu) \int_0^1 dx\, T^{(*)}(x, m_c/m_b, \mu) \times \phi_\pi(x,\mu) + \dots, \tag{7}$$

where the ellipses denote power suppressed terms. This result is similar to predictions obtained from the hard exclusive scattering formalism of Brodsky-Lepage [24], except for the presence of the Isgur-Wise function $\xi(\omega_0,\mu)$. The normalization factor is given by[2]

$$N^{(*)} = \frac{G_F V_{cb} V_{ud}^*}{\sqrt{2}} E_\pi f_\pi \sqrt{m_{D^{(*)}} m_B} \left(1 + \frac{m_B}{m_{D^{(*)}}}\right). \tag{8}$$

The proof of Eq. (7) uses the heavy quark limit, so $m_D = m_{D^*}$ and $N = N^*$. In Eq. (7), $\phi_\pi(x,\mu)$ is the nonperturbative pion light-cone wave function and $\xi(w_0,\mu)$ is evaluated at maximum recoil $v \cdot v' \to w_0 = (m_B^2 + m_{D^{(*)}}^2)/(2 m_B m_{D^{(*)}})$. The hard coefficient $T^{(*)}(x,\mu) = C_{L\pm R}^{(0)}[(4x-2)E_\pi, \mu, m_b]$, where $\pm$ correspond to the D and D^* respectively, and $C_{L\pm R}^{(0)} = C_L^{(0)} \pm C_R^{(0)}$ is the calculable Wilson coefficient of the operators defined in Eq. (20) below. The renormalization scale dependence of the hard scattering function $T(x,\mu)$ cancels the μ dependence in the Isgur-Wise function and pion wave function. In this framework [7] there is no longer a need to identify by hand a factorization scale.[3] In the language of soft-collinear effective theory(SCET) [9], the scale dependence is understood from the matching and running procedure.

Equation (7) implies equal rates for $\overline{B}^0 \to D^+\pi^-$ and $\overline{B}^0 \to D^{*+}\pi^-$ up to the $\alpha_s(m_b)$ corrections in $T^{(*)}$ and power corrections. This prediction is in good agreement with the observed data for type-I and -III decays to π, ρ, K, and K^* as shown in Tables I and II. For two-body type-I decays both the large N_c and large energy mechanisms make similar phenomenological predictions. However, these mechanisms can be distinguished with $B \to DX$ decays where X is a multihadron state [12].

So far, no results of comparable theoretical rigor exist for the color suppressed type-II decays. In fact existing results in $B \to D\pi$ and $B \to \psi K^{(*)}$ do not support naive factorization with a universal coefficient a_2 [11]. Furthermore, it has been argued that in general factorization will not hold for type-II decays [7].

Using the SCET [25,26], we prove in this paper a factorization theorem for color suppressed (type-II) $\overline{B} \to DM$ decays, $M = \{\pi^0, \rho^0, \dots\}$. These decays are power suppressed relative to the type-I decays, and our results are valid at leading nonvanishing order in Λ/Q. The main results of our paper are as follows.

(1) The color suppressed (C) and exchange (E) contributions to $B^0 \to D^{(*)0}\pi^0$ are both suppressed by Λ/Q relative to the amplitude (T). The C and E amplitudes are found to be of comparable size since the factorization theorem relates them to the same perturbative and nonperturbative quantities. Our result is incompatible with the naive a_2-type factorization.

(2) When our result is combined with heavy quark symmetry it predicts the equality of the amplitudes for $\overline{B}^0 \to D^0\pi^0$ and $\overline{B}^0 \to D^{*0}\pi^0$ (in fact for any DM and D^*M).

[2]Note for longitudinal D^*, $n \cdot \varepsilon_{D^*} = n \cdot v'$. Production of transverse ρ's is suppressed by Λ/Q.

[3]In naive factorization the hadronic matrix elements in Eq. (2) are independent of the scale that separates hard and soft physics. The scale dependence in a_1 and a_2 then causes the physical amplitudes to become scale dependent. The parameters a_1 and a_2 were therefore assumed to be evaluated at a specific scale called the "factorization scale." In other words, the nonfactorizable effects were accounted for by allowing a_1 and a_2 to be free parameters that are fit to data. The factorization scale can then be extracted from the scale dependence of a_1 and a_2 [3].

This prediction is in good agreement with existing data and will be tested by future measurements.

(3) Our result gives a new mechanism for generating nonperturbative strong phases for exclusive decays within the framework of factorization. For DM and D^*M it implies the equality of the strong phases δ between isospin amplitudes. Furthermore, certain cases with different light mesons M are predicted to also have a universal nonperturbative strong phase ϕ in their isospin triangle.

(4) The power suppressed amplitudes for all color suppressed $\bar{B}\to D^{(*)}M$ decays are factorizable into two types of terms, which we refer to as short-distance ($\mu^2\sim E_M\Lambda$) and long-distance ($\mu^2\sim\Lambda^2$) contributions. The short-distance contributions depend on complex soft $B^0\to D^{(*)0}$ distribution functions, $S_{L,R}^{(0,8)}(k_+,\ell_+)$, which depend only on the direction of M (the superscripts indicate that two color structures contribute). For $M=\pi,\rho$ the long-distance contributions vanish at lowest order in $\alpha_s(Q)/\pi$.

Combined with Eq. (7) the results here give a complete leading order description of the $B\to D\pi$ isospin triangles.

In Sec. II we review the current data for $B\to D\pi$ decays. The derivation of a factorization theorem for the color suppressed channels $\bar{B}^0\to D^{(*)0}\pi^0$ and $\bar{B}^0\to D^{(*)0}\rho^0$ is carried out in Sec. III using SCET. Then in Sec. IV the formalism is applied to decays with kaons, $\bar{B}^0\to D^{(*)0}K^0$, $\bar{B}^0\to D^{(*)0}K^{*0}$, $\bar{B}^0\to D_s^{(*)}K^-$, and $\bar{B}^0\to D_s^{(*)}K^{*-}$. In Sec. V we contrast our results with the large N_c limit of QCD and prior theoretical expectations. Readers only interested in final results can safely skip Secs. III, IV, and V. In Sec. VI we discuss the phenomenological predictions that follow from our new formalism for color-suppressed channels. Conclusions are given in Sec. VII. In Appendix A we prove that for π^0 and ρ^0 the long-distance contributions are suppressed. Finally in Appendices B and C we elaborate on the properties of the jet functions and our new soft $B\to D^{(*)}$ distribution functions, respectively.

II. DATA

We start by reviewing existing data on the $\bar{B}\to D^{(*)}\pi$ decays. The branching ratios for most of these modes have been measured and the existing results are collected in Table I. Taking into account that the $D^{(*)}\pi$ final state can have isospin $I=1/2,3/2$, these decays can be parametrized by two isospin amplitudes $A_{1/2}$, $A_{3/2}$:

$$
\begin{aligned}
A_{+-}&=A(\bar{B}^0\to D^+\pi^-)=\frac{1}{\sqrt{3}}A_{3/2}+\sqrt{\frac{2}{3}}A_{1/2}=T+E,\\
A_{0-}&=A(B^-\to D^0\pi^-)=\sqrt{3}A_{3/2}=T+C,\\
A_{00}&=A(\bar{B}^0\to D^0\pi^0)=\sqrt{\frac{2}{3}}A_{3/2}-\frac{1}{\sqrt{3}}A_{1/2}\\
&=\frac{1}{\sqrt{2}}(C-E). \qquad (9)
\end{aligned}
$$

Similar expressions can be written for the decay amplitudes of $B\to D^*\pi$, $B\to D\rho$, $B\to D^*\rho$ with well-defined helicity of the final state vector mesons. Equation (9) also gives the alternative parametrization of these amplitudes in terms of the amplitudes T,C,E discussed in Sec. I.

Using the data in Table I, the individual isospin amplitudes A_I and their relative phase $\delta=\arg(A_{1/2}A_{3/2}^*)$ can be extracted using

$$
\begin{aligned}
\mathrm{Br}(\bar{B}\to D^{(*)}M)&=\tau_B\Gamma(\bar{B}\to D^{(*)}M)\\
&=\frac{\tau_B|\mathbf{p}|}{8\pi m_B^2}\sum_{\mathrm{pol}}|A(\bar{B}\to D^{(*)}M)|^2. \qquad (10)
\end{aligned}
$$

with $\tau_{\bar{B}^0}=2.343\times10^{12}\ \mathrm{GeV}^{-1}$ and $\tau_{B^-}=2.543\times10^{12}\ \mathrm{GeV}^{-1}$. We find

$$
\begin{aligned}
&|A_{1/2}^D|=(4.33\pm0.47)\times10^{-7}\ \mathrm{GeV},\quad \delta^{D\pi}=30.5^{\circ}{}^{+7.8}_{-13.8},\\
&|A_{3/2}^D|=(4.45\pm0.17)\times10^{-7}\ \mathrm{GeV},\\
&|A_{1/2}^{D^*}|=(4.60\pm0.36)\times10^{-7}\ \mathrm{GeV},\quad \delta^{D^*\pi}=30.2\pm6.6^{\circ},\\
&|A_{3/2}^{D^*}|=(4.33\pm0.19)\times10^{-7}\ \mathrm{GeV}. \qquad (11)
\end{aligned}
$$

The ranges for δ correspond to 1σ uncertainties for the experimental branching ratios. A graphical representation of these results is given in Fig. 5, where we show contour plots for the ratios of isospin amplitudes $R_I=A_{1/2}/(\sqrt{2}A_{3/2})$ for both $D\pi$ and $D^*\pi$ final states. For $\bar{B}\to D\pi$ an isospin analysis was performed recently by CLEO [18] including error correlations among the decay modes; we used their analysis in quoting errors on $\delta^{D\pi}$.

For later convenience we define the amplitude ratios

$$
\begin{aligned}
&R_0^M\equiv\frac{A(\bar{B}^0\to D^{*0}M^0)}{A(\bar{B}^0\to D^0M^0)},\quad R_0^{M/M'}\equiv\frac{A(\bar{B}^0\to D^{(*)0}M^0)}{A(\bar{B}^0\to D^{(*)0}M'^0)},\\
&R_I\equiv\frac{A_{1/2}}{\sqrt{2}A_{3/2}}=1-\frac{3}{2}\,\frac{C-E}{T+C},\\
&R_c\equiv\frac{A(\bar{B}^0\to D^{(*)+}M^-)}{A(B^-\to D^{(*)0}M^-)}=1-\frac{C-E}{T+C}, \qquad (12)
\end{aligned}
$$

where the ratios R_I and R_c are defined for each $D^{(*)}M$ mode. Predictions are obtained for the ratios in Eq. (12), including the leading power corrections to R_I and R_c. The relation $R_I=1+O(\Lambda/Q)$ can be represented graphically by a triangle with base normalized to 1 (see Fig. 5 in Sec. VI). The two angles adjacent to the base are the strong isospin phase δ, and another strong phase ϕ. The usual prediction is that $\delta\sim1/Q^k$ [7,11], and that there is no constraint on the strong phase ϕ which can be large. In Sec. VI we show that at lowest order the angle ϕ is predicted to be the same for all channels in Table I, and that δ can be dominated by a constrained nonperturbative strong phase. From R_I in Eq. (12)

TABLE II. Data on Cabibbo suppressed $\bar{B}\to DK^{(*)}$ decays. Unless otherwise indicated, the data are taken from Ref. [17].

Type	Decay	BR (10^{-5})	$\lvert A\rvert$ (10^{-7} GeV)	Decay	BR (10^{-5})	$\lvert A\rvert$ (10^{-7} GeV)
I	$\bar{B}^0\to D^+K^-$	20.0 ± 6.0	1.62 ± 0.24	$\bar{B}^0\to D^{*+}K^-$	20 ± 5	1.64 ± 0.20
III	$B^-\to D^0K^-$	37.0 ± 6.0	2.11 ± 0.17	$B^-\to D^{*0}K^-$	36 ± 10	2.11 ± 0.29
II	$\bar{B}^0\to D^0\bar{K}^0$	$5.0^{+1.1}_{-1.2}\pm 0.6$ [27]	0.81 ± 0.11	$\bar{B}^0\to D^{*0}\bar{K}^0$		
I	$\bar{B}^0\to D^+K^{*-}$	37.0 ± 18.0	2.24 ± 0.54	$\bar{B}^0\to D^{*+}K^{*-}$	38 ± 15	2.30 ± 0.45 [a]
III	$B^-\to D^0K^{*-}$	61.0 ± 23.0	2.76 ± 0.52	$B^-\to D^{*0}K^{*-}$	72 ± 34	3.03 ± 0.72 [a]
II	$\bar{B}^0\to D^0\bar{K}^{*0}$	$4.8^{+1.1}_{-1.0}\pm 0.5$ [27]	0.81 ± 0.11	$\bar{B}^0\to D^{*0}\bar{K}^{*0}$		

[a]Since no helicity measurements for D^*K^* are available we show effective amplitudes which include contributions from all three helicities.

we note that for a leading order prediction of δ it is not necessary to know the power corrections to the T amplitude.

A similar analysis can be given for the Cabibbo suppressed $\bar{B}\to DK^{(*)}$ decays. Although several of these modes had been seen for some time, it is only recently that some of the corresponding class-II decays have been seen by the Belle Collaboration [27] (see Table II). For this case the final $D^{(*)}K^{(*)}$ states can have isospins $I=0,1$, so these decays are parametrized in terms of two isospin amplitudes $A_{I=0,1}$ (for given spins of the final particles),

$$A_{+-}=A(\bar{B}^0\to D^+K^-)=\tfrac{1}{2}A_0+\tfrac{1}{2}A_1=T,$$
$$A_{0-}=A(B^-\to D^0\,K^-)=A_1=T+C, \qquad (13)$$
$$A_{00}=A(\bar{B}^0\to D^0\,K^0)=\tfrac{1}{2}A_1-\tfrac{1}{2}A_0=C.$$

Isospin symmetry implies the amplitude relation among these modes $A_{+-}+A_{00}=A_{0-}$, which can be used to extract the isospin amplitudes $A_{0,1}$ and their relative phase $\delta=\arg(A_0A_1^*)$. Using Gaussian error propagation we obtain

$$|A_0^{DK}|=(1.45\pm 0.62)\times 10^{-7}\ \text{GeV},\quad \delta^{DK}=49.9\pm 9.5°$$
$$|A_1^{DK}|=(2.10\pm 0.17)\times 10^{-7}\ \text{GeV},$$
$$|A_0^{DK*}|=(1.93\pm 1.49)\times 10^{-7}\ \text{GeV},\quad \delta^{DK*}=34.9\pm 19.4° \qquad (14)$$
$$|A_1^{DK*}|=(2.76\pm 0.52)\times 10^{-7}\ \text{GeV}. \qquad (15)$$

However, note that scanning the amplitudes A_{+-}, and A_{00}, A_{0-} in their 1σ allowed regions still allows a flat isospin triangle [13].

III. SOFT-COLLINEAR EFFECTIVE THEORY ANALYSIS

The key idea of the soft-collinear effective theory [25,26] is to separate perturbative and nonperturbative scales directly at the level of operators. The relevant scales have virtualities $p^2\sim m_b^2,m_c^2,E_M^2$ (hard), $p^2\sim E_M\Lambda$ (intermediate), and $p^2\simeq\Lambda^2$ (soft). The $p^2\sim\Lambda^2$ scales are described by soft $(p^+,p^-,p^\perp)\sim(\Lambda,\Lambda,\Lambda)$ and collinear $(p^+,p^-,p^\perp)\sim(\Lambda^2/E_M,E_M,\Lambda)$ degrees of freedom. We follow the notation in Refs. [9,28].

The weak Hamiltonian $\mathcal{H}_W$ in Eq. (1) is matched onto effective operators containing soft and collinear fields. In exclusive processes this matching can be simplified by a two-stage procedure [29]. We first match QCD onto a theory SCET_I with ultrasoft fields with $p^2\sim\Lambda^2$, but intermediate-collinear fields with $p^2\sim E_M\Lambda$. This theory gives a simplified description of the $p^2\sim E_M\Lambda$ exchanges that necessarily mediate interactions between soft and collinear particles. Then at a scale $\mu^2\sim E_M\Lambda$ we match SCET_I onto the final theory SCET_II which has only the propagating long-distance soft and collinear particles. This procedure determines which factors of $\alpha_s(\mu)$ belong[4] at the hard scale $\mu^2=Q^2$, which belong at the intermediate scale $\mu^2=E_M\Lambda$, and what nonperturbative matrix elements appear.

Since the collinear fields do not interact with soft fields at lowest order, if one can rearrange the fields in the SCET_II operator to express it as a product of collinear fields and soft fields, the factorization of matrix elements is achieved. This is precisely what happens in type-I decays, and as we will see also type-II decays, with operators of the form

$$\text{Type I:}\quad [\bar{h}_{v'}S\Gamma S^\dagger h_v][(\bar{\xi}_nW)\Gamma'(W^\dagger\xi_n)]$$
$$\text{Type II:}\quad [(\bar{h}_{v'}S)\Gamma(S^\dagger h_v)(\bar{q}S)\Gamma''(S^\dagger q)]$$
$$\times[(\bar{\xi}_nW)\Gamma'(W^\dagger\xi_n)],$$
$$\int d^4x\,T[(\bar{h}_{v'}S)\Gamma(S^\dagger h_v)(\bar{\xi}_nW)\Gamma'(W^\dagger\xi_n)](0)$$
$$\times[(\bar{q}S)\Gamma''(S^\dagger q)(\bar{\xi}_nW)\Gamma'(W^\dagger\xi_n)](x). \qquad (16)$$

In type-II decays the first and second SCET_II operators give short- and long-distance contributions, respectively. We use here the notation in Ref. [26] so that h_v are HQET fields, ξ_n are collinear quark fields, q are soft quark fields, and S,W are soft and collinear Wilson lines. Since collinear particles do not connect with the heavy meson states and soft particles do not connect with the collinear light meson state, the matrix elements of these operators factor into the product of a soft $B\to D^{(*)}$ matrix element and a collinear matrix element involving M.

[4]A more accurate statement is that the scale dependence is determined by anomalous dimensions of operators in SCET_I and SCET_II.

We start by reviewing type-I decays. Using SCET, the factorization of the leading amplitude for type-I decays has been proven in Ref. [9] at leading order in $1/Q$ (and nonperturbatively to all orders in α_s). The operators in Eq. (1) are matched onto effective operators at a scale $\mu_Q \simeq Q$

$$\sum_{1,2} C_i O_i \to 4 \sum_{j=L,R} \int d\tau_1 d\tau_2 [C_j^{(0)}(\tau_1,\tau_2) \mathcal{Q}_j^{(0)}(\tau_1,\tau_2) + C_j^{(8)}(\tau_1,\tau_2) \mathcal{Q}_j^{(8)}(\tau_1,\tau_2)]. \tag{17}$$

At leading order in $\mathrm{SCET_I}$ there are four operators [$j = L,R$]

$$\mathcal{Q}_j^{(0)}(\tau_1,\tau_2) = [\bar{h}_{v'}^{(c)} \Gamma_j^h h_v^{(b)}][(\bar{\xi}_n^{(d)} W)_{\tau_1} \Gamma_n (W^\dagger \xi_n^{(u)})_{\tau_2}],$$

$$\mathcal{Q}_j^{(8)}(\tau_1,\tau_2) = [\bar{h}_{v'}^{(c)} Y \Gamma_j^h T^a Y^\dagger h_v^{(b)}] \times [(\bar{\xi}_n^{(d)} W)_{\tau_1} \Gamma_n T^a (W^\dagger \xi_n^{(u)})_{\tau_2}]. \tag{18}$$

The superscript (0,8) denotes the $1\otimes 1$ and $T^a \otimes T^a$ color structures. The Dirac structures on the heavy side are $\Gamma_{L,R}^h = \not{n} P_{L,R}$ with $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$, while on the collinear side we have $\Gamma_n = \not{\bar{n}} P_L/2$. The momenta labels are defined by $(W^\dagger \xi_n)_{\omega_2} = [\delta(\omega_2 - \bar{\mathcal{P}})\, W^\dagger \xi_n]$.

The matching conditions for the Wilson coefficients at tree level at $\mu = E_\pi$ are

$$C_L^{(0)}(\tau_i) = C_1 + \frac{C_2}{N_c}, \quad C_L^{(8)}(\tau_i) = 2C_2, \quad C_R^{(0,8)}(\tau_i) = 0. \tag{19}$$

Matching corrections of order $O(\alpha_s)$ can be found in Ref. [7].

The operators in Eq. (18) are written in terms of collinear fields which do not couple to usoft particles at leading order. This was achieved by a decoupling field redefinition [26] on the collinear fields $\xi_n \to Y \xi_n$, etc. The operators in Eq. (18) are then matched onto $\mathrm{SCET_{II}}$ to give [$\omega_i = \tau_i$]

$$\mathcal{Q}_j^{(0)}(\omega_1,\omega_2) = [\bar{h}_{v'}^{(c)} \Gamma_j^h h_v^{(b)}][(\bar{\xi}_n^{(d)} W)_{\omega_1} \Gamma_n (W^\dagger \xi_n^{(u)})_{\omega_2}],$$

$$\mathcal{Q}_j^{(8)}(\omega_1,\omega_2) = [\bar{h}_{v'}^{(c)} S \Gamma_j^h T^a S^\dagger h_v^{(b)}] \times [(\bar{\xi}_n^{(d)} W)_{\omega_1} \Gamma_n T^a (W^\dagger \xi_n^{(u)})_{\omega_2}], \tag{20}$$

where the collinear and soft Wilson lines W and S are defined in Eq. (C2) of Appendix C. At leading order in $1/Q$ only the operators $\mathcal{Q}_{L,R}^{(0)}$ and the leading order collinear and soft Lagrangians ($\mathcal{L}_c^{(0)}$, $\mathcal{L}_s^{(0)}$) contribute to the $B^- \to D^{(*)0}\pi^-$ and $\bar{B}^0 \to D^{(*)+}\pi^-$ matrix elements. The matrix elements of $\mathcal{Q}_{L,R}^{(8)}$ vanish because they factorize into a product of bilinear matrix elements and the octet currents give vanishing contribution between color singlet states [9].

Note that we take the pion state or interpolating field to be purely collinear and the B and $D^{(*)}$ states to be purely soft. Power corrections to these states are included as time-ordered products. This includes asymmetric configurations containing one soft and one collinear quark which involve T products with subleading Lagrangians [29].

Next we consider type-II decays. The matrix elements of the leading order operators vanish, $\langle D^0 \pi^0 | Q_j^{(0,8)} | \bar{B}^0 \rangle = 0$. This occurs due to a mismatch between the type of quarks produced by $\mathcal{Q}_j^{(0,8)}$ and those required for the light meson state, where we need two collinear quarks of the same flavor. The operator $\mathcal{Q}_j^{(0,8)}$ produces collinear quarks with (du) flavor. Therefore it cannot produce a π^0 since the leading order SCET Lagrangian only produces or annihilates collinear quark pairs of the same flavor. For this reason the leading contributions to $\bar{B}^0 \to D^{(*)0}\pi^0$ are power suppressed.

In $\mathrm{SCET_I}$ there are several sources of power suppressed contributions obtained by including higher order four-quark operators, higher order contributions from the Lagrangians, or both. However, there is only a *single type of $SCET_I$ operator* which contributes to $\bar{B}^0 \to D^{(*)0} M^0$ decays at leading order. They are given by T-ordered products of the leading operators in Eq. (18) with two insertions of the usoft-collinear Lagrangian $\mathcal{L}_{\xi q}^{(1)}$:

$$T_j^{(0,8)} = \frac{1}{2} \int d^4x\, d^4y\, T\{\mathcal{Q}_j^{(0,8)}(0), i\mathcal{L}_{\xi q}^{(1)}(x), i\mathcal{L}_{\xi q}^{(1)}(y)\}. \tag{21}$$

Here the subleading Lagrangian is [30,29]

$$\mathcal{L}_{\xi q}^{(1)} = (\bar{\xi}_n W)\left(\frac{1}{\bar{\mathcal{P}}} W^\dagger ig \mathcal{B}_\perp^c W\right) q_{us} - \bar{q}_{us}\left(W^\dagger ig\, \mathcal{B}_\perp^c W \frac{1}{\bar{\mathcal{P}}^\dagger}\right) \times (W^\dagger \xi_n), \tag{22}$$

where $ig\mathcal{B}_\perp^c = [i\bar{n}\cdot D^c, i\mathcal{D}_\perp^c]$. The two factors of $i\mathcal{L}_{\xi q}^{(1)}$ in Eq. (21) are necessary to swap one u quark and one d quark from ultrasoft to collinear. In contrast to the tree amplitude, for this case both the $\mathcal{Q}_j^{(0)}$ and $\mathcal{Q}_j^{(8)}$ operators can contribute. By power counting, the $T_j^{(0,8)}$'s are suppressed by $\lambda^2 = \Lambda/Q$ relative to the leading operators. They will give order Λ/Q contributions in $\mathrm{SCET_{II}}$, in agreement with our earlier statements.

In Fig. 2 we show graphs contributing to the matching of $\mathrm{SCET_I}$ operators (a,b) onto operators in $\mathrm{SCET_{II}}$ (c,d,e). In Figs. 2(a,b) the gluon always has off shellness $p^2 \sim E_M \Lambda$ due to momentum conservation, and is shrunk to a point in $\mathrm{SCET_{II}}$. However, the collinear quark propagator in (a,b) can either have $p^2 \sim E_M \Lambda$, giving rise to the short-distance $\mathrm{SCET_{II}}$ contribution in Fig. 2(e), or it can have $p^2 \sim \Lambda^2$, which gives the long-distance $\mathrm{SCET_{II}}$ contribution in Figs. 2(c,d). To match onto the short-distance contribution in Fig. 2(e) we subtract the $\mathrm{SCET_{II}}$ diagrams (c,d):

$$(a) + (b) - (c) - (d) = (e). \tag{23}$$

The operators in Figs. 2(a,b) are from the T products $T_j^{(0,8)}$ in Eq. (21), while Figs. 2(c,d) involve the $\mathrm{SCET_{II}}$ T-products $\overline{O}_j^{(i)}$ in Eq. (27), and Fig. 2(e) involves $O_j^{(i)}$ in Eq. (28).

To generate connected $\mathrm{SCET_I}$ diagrams from the time-ordered product in Eq. (21) requires at least two contractions,

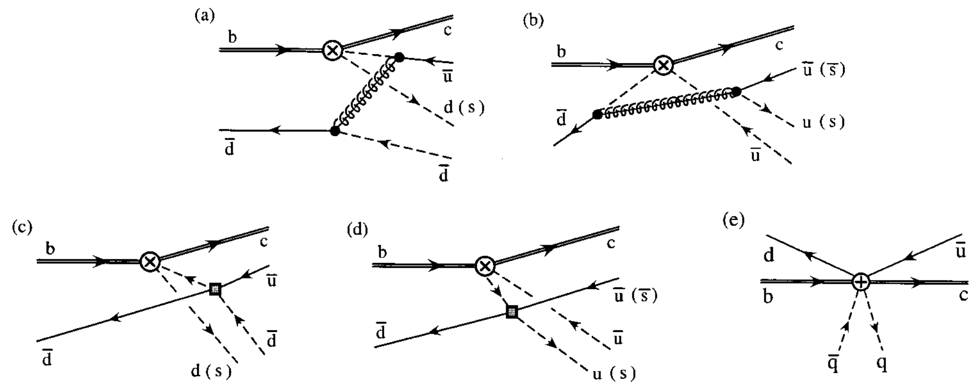

FIG. 2. Graphs for the tree level matching calculation from $\mathrm{SCET_I}$ (a,b) onto $\mathrm{SCET_{II}}$ (c,d,e). The dashed lines are collinear quark propagators and the spring with a line is a collinear gluon. Solid lines in (a,b) are ultrasoft and those in (c,d,e) are soft. $\otimes$ denotes an insertion of the weak operator, given in Eq. (18) for (a,b) and in Eq. (20) in (c,d). $\oplus$ in (e) is a six-quark operator from Eq. (28). The two solid dots in (a,b) denote insertions of the mixed usoft-collinear quark action $\mathcal{L}_{\xi q}^{(1)}$. The boxes denote the $\mathrm{SCET_{II}}$ operator $\mathcal{L}_{\xi\xi qq}^{(1)}$ in Eq. (25).

of which the minimum basic possibilities can be grouped as follows. (1) Contraction of $\xi_n^{(u)}\bar{\xi}_n^{(u)}$ and the $\perp$ gluon in $B_\perp^\mu B_\perp^\nu$ [C topology, Fig. 2(a)], (2) contraction of $\xi_n^{(d)}\bar{\xi}_n^{(d)}$ and the $\perp$ gluon in $B_\perp^\mu B_\perp^\nu$ [E topology, Fig. 2(b)], and (3) contraction of $\xi_n^{(u)}\bar{\xi}_n^{(u)}$ and $\xi_n^{(d)}\bar{\xi}_n^{(d)}$ (topology with two external collinear gluons and no external collinear quarks, not shown).

All more complicated contractions have one of these three as a root. Case (3) only contributes for light mesons with an isosinglet component (η, η', ω, ϕ), which we will not consider here.

Each of the $\mathrm{SCET_I}$ T products is matched onto $\mathrm{SCET_{II}}$ operators at scale $\mu=\mu_0$, and

$$\int d\tau_1\, d\tau_2\, C_j^{(0,8)} T_j^{(0,8)} \to [T_j^{(0,8)}]_{\rm short} + [T_j^{(0,8)}]_{\rm long},$$

$$[T_{L,R}^{(0,8)}]_{\rm short} = \int d\tau_i\, dk_\ell^+\, d\omega_k C_{L,R}^{(0,8)}(\tau_i,\mu_0) \times J^{(0,8)}(\tau_i,k_\ell^+,\omega_k,\mu_0,\mu) O_{L,R}^{(0,8)}(k_\ell^+,\omega_k,\mu),$$

$$[T_{L,R}^{(0,8)}]_{\rm long} = \int dk^+ d\omega_1\, d\omega_2\, d\omega\, C_{L,R}^{(0,8)}(\omega_i,\mu_0) \times \overline{J}^{(0,8)}(k^+\omega,\mu_0,\mu)\overline{O}_{L,R}^{(0,8)}(\omega_i,k^+,\omega,\mu), \tag{24}$$

where the subscripts i,ℓ,k run over values 1,2. Here J, $\overline{J}$ are jet functions containing effects at the $p^2\sim E_M\Lambda$ scale and are Wilson coefficients for the $\mathrm{SCET_{II}}$ operators O and $\overline{O}$. The $[T_{L,R}^{(0,8)}]_{\rm short}$ and $[T_{L,R}^{(0,8)}]_{\rm long}$ terms are, respectively, Fig. 2(e) and Figs. 2(c,d) (after they are dressed with all possible gluons). The μ_0 and μ dependence in Eq. (24) signifies the scale dependence in $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$, respectively. The jet functions are generated by the contraction of intermediate collinear fields with couplings $\alpha_s(\mu_0)$ (where $\mu_0^2\sim E_\pi\Lambda$). In general the jet functions depend on the large light-cone momenta τ_i coming out of the hard vertex, the large light-cone momenta ω_k of the external collinear $\mathrm{SCET_{II}}$ fields, and the k_j^+ momenta of the external soft $\mathrm{SCET_{II}}$ fields. No other soft momentum dependence is possible since the leading $\mathrm{SCET_I}$ collinear Lagrangian depends on only $n\cdot\partial_{us}$.

The difference between the time-ordered products $T_{L,R}^{(0,8)}$ and the time-ordered products $\overline{O}_{L,R}^{(0,8)}$ gives the six-quark $\mathrm{SCET_{II}}$ operator $O_{L,R}^{(0,8)}$, whose coefficients are the jet functions $J^{(0,8)}$. In this $\mathrm{SCET_I}$ -$\mathrm{SCET_{II}}$ matching calculation the $\overline{O}_{L,R}^{(0,8)}$ graphs subtract long-distance contributions from the $T_{L,R}^{(0,8)}$ graphs so that $J^{(0,8)}$ are free from infrared singularities. In general the matrix elements for color suppressed decays then include both short- and long-distance contributions as displayed in Eq. (24). However, for the isotriplet π and ρ a dramatic simplification occurs at leading order in $C_J^{(i)}$. In this case it can be proven that the long-distance contributions $[T_j^{(i)}]_{\rm long}$ *vanish* to all orders in the α_s couplings in $\mathrm{SCET_I}$, and with the α_s couplings in $\mathrm{SCET_{II}}$ treated nonperturbatively. The proof of this fact uses the G-parity invariance of QCD and is carried out in Appendix A. At leading order in the coefficients $C_{L,R}^{(0,8)}$ the $M=\pi,\rho$ factorization theorem is therefore more predictive since possible long-distance contribution from $\overline{O}_j^{(i)}$ are absent. Most of the following discussion will focus on $O_j^{(i)}$, but $\overline{O}_j^{(i)}$ is fully included in the final factorization theorem.

In the $\mathrm{SCET_{II}}$ diagrams in Figs. 2(c,d) a power suppressed four-quark Lagrangian appears. It is similar to an operator introduced in Ref. [31], and can be obtained from $T\{i\mathcal{L}_{\xi q}^{(1)},i\mathcal{L}_{\xi q}^{(1)}\}$ in $\mathrm{SCET_I}$ by a simple matching calculation [32]. Summing over flavors q,q' we find

$$\mathcal{L}_{\xi\xi qq}^{(1)} = \sum_{j=L,R}\sum_{\omega}\sum_{k^+}[\overline{J}^{(0)}(\omega k^+)L_j^{(0)}(\omega,k^+,x) + \overline{J}^{(8)}(\omega k^+)L_j^{(8)}(\omega,k^+,x)],$$

$$L_j^{(0)}(\omega,k^+,x) = \sum_{q,q'}[(\bar{\xi}_n^{(q)}W)_\omega \bar{n}\!\!\!/ P_j (W^\dagger\xi_n^{(q')})_\omega] \times[(\overline{q'}S)_{k^+} n\!\!\!/ P_j (S^\dagger q)_{k^+}](x). \tag{25}$$

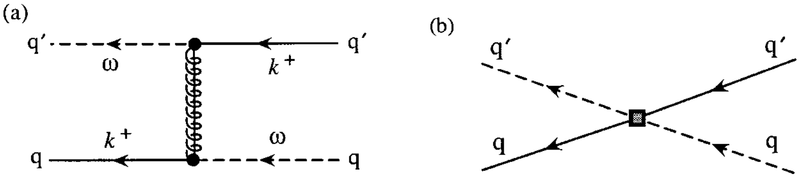

FIG. 3. Tree level matching calculation for the $L_{L,R}^{(0,8)}$ operators, with (a) the T product in $\mathrm{SCET_I}$ and (b) the operator in $\mathrm{SCET_{II}}$. Here q,q' are flavor indices and $\omega\sim\lambda^0$ are minus-momenta.

In Eq. (25) the soft momenta labels are defined by $(S^\dagger q)_{k^+}=[\delta(k^+-n\cdot P)S^\dagger q]$, and the positions $(x^+,x^-,x_\perp)\sim(1/\Lambda,Q/\Lambda^2,1/\Lambda)$. For the soft fields the x^- coordinates encode small residual plus-momenta, and for the collinear fields the x^+ coordinates encode small residual minus-momenta. Thus, we used the summation/integration notation for label/residual momenta from Ref. [33]. The operator $L_j^{(8)}(\omega,k^+,x)$ has the same form as Eq. (25) except with color structure $T^a\otimes T^a$. At tree level the coefficient functions are given by the calculation in Fig. 3,

$$\bar{J}^{(0)}(\omega k^+)=-\frac{C_F}{2N_c}\frac{4\pi\alpha_s(\mu)}{\omega k^+},$$

$$\bar{J}^{(8)}(\omega k^+)=\frac{1}{2N_c}\frac{4\pi\alpha_s(\mu)}{\omega k^+}. \qquad (26)$$

Beyond tree level they obtain contributions from loop diagrams with additional $\mathcal{L}_{\xi\xi}^{(0)}$ vertices. In terms of the operator in Eq. (25) the $\mathrm{SCET_{II}}$ operators that contribute to $[T_j^{(i)}]_{\rm long}$ in the factorization theorem are

$$\bar{O}_j^{(0,8)}(\omega_i,k^+,\omega,\mu)=\int d^4x\, T\, \mathcal{Q}_j^{(0,8)}(\omega_i,x=0)\, iL^{(0,8)}(\omega,k^+,x). \qquad (27)$$

The operators $\bar{O}$ generate the diagrams (c) and (d) in Fig. 2.

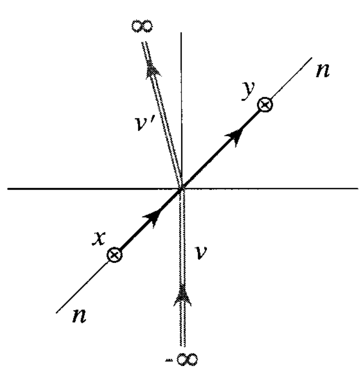

FIG. 4. Nonperturbative structure of the soft operators in Eq. (29) which arise from $O_j^{(0,8)}$. Wilson lines are shown for the paths $S_n(x,0)$, $S_n(0,y)$, $S_{v'}(-\infty,0)$ and $S_{v'}(0,\infty)$, plus two interacting QCD quark fields inserted at the locations x and y. The S_v and $S_{v'}$ Wilson lines are from interactions with the fields h_v and $h_{v'}$ fields, respectively. The nonperturbative structure of soft fields in $\bar{O}_j^{(0,8)}$ is similar except that we separate the single and double Wilson lines by an amount $x_\perp$.

At any order in perturbation theory the jet functions J from the C topology and E topology generate one spin structure and two color structures for the $\mathrm{SCET_{II}}$ operators. For the six-quark operators we find

$$O_j^{(0)}(k_i^+,\omega_k)=[\bar{h}_{v'}^{(c)}\Gamma_j^h h_v^{(b)}(\bar{d}S)_{k_1^+}\bar{n}\!\!\!/P_L(S^\dagger u)_{k_2^+}]\times[(\bar{\xi}_nW)_{\omega_1}\Gamma_c(W^\dagger\xi_n)_{\omega_2}],$$

$$O_j^{(8)}(k_i^+,\omega_k)=[(\bar{h}_{v'}^{(c)}S)\Gamma_j^h\, T^a\,(S^\dagger h_v^{(b)})\times(\bar{d}S)_{k_1^+}\bar{n}\!\!\!/P_LT^a(S^\dagger u)_{k_2^+}]\times[(\bar{\xi}_nW)_{\omega_1}\Gamma_c(W^\dagger\xi_n)_{\omega_2}], \qquad (28)$$

where here the d, u, $h_{v'}^{(c)}$, and $h_v^{(b)}$ fields are soft, and the ξ_n fields are collinear isospin doublets $(\xi_n^{(u)},\xi_n^{(d)})$. In Eq. (28) $\Gamma_{L,R}^h=\bar{n}\!\!\!/P_{L,R}$ as in Eq. (18), while for the collinear isospin triplet $\Gamma_c=\tau^3\bar{n}\!\!\!/P_L/2$.[5] We do not list operators with a T^a next to Γ_c since they will give vanishing contribution in the collinear matrix element. For light vector mesons the spin structure Γ_c only produces the longitudinal polarization. This result follows from the quark helicity symmetry of $\mathcal{L}_{\xi\xi}^{(0)}$ and is discussed in further detail in Appendix B.

In position space the $O_j^{(i)}$ are bilocal operators, with the two soft light quarks aligned on the n_μ light-cone direction $(x^-=\frac{1}{2}n_\mu\bar{n}\cdot x, y^-=\frac{1}{2}n_\mu\bar{n}\cdot y)$ passing through the point $x=0$,

$$(\bar{h}_{v'}^{(c)}S)\Gamma_h(S^\dagger h_v^{(b)})(\bar{d}S)_{r^+}\Gamma_q(S^\dagger u)_{\ell^+}=\int\frac{dx^-dy^-}{(4\pi)^2}e^{i/2(r^+x^--\ell^+y^-)}[\bar{h}_{v'}^{(c)}\Gamma_hh_v^{(b)}](0)\times[\bar{d}(x^-)S_n(x^-,0)\Gamma_qS_n(0,y^-)u(y^-)]. \qquad (29)$$

The gluon interactions contained in matrix elements of $O_j^{(0,8)}$ include attachments to the light quarks q, to the heavy quarks $h_{v,v'}$, and to the Wilson lines S_n as shown in Fig. 4. The interactions with $h_{v,v'}$ have been drawn as Wilson lines $S_{v,v'}$ along v,v' [34].

Even though we have factored the collinear and soft degrees of freedom in the two final state hadrons, the presence of the soft Wilson lines bring in information about the vector

[5]There are also isosinglet contributions with $\Gamma_c=\bar{n}\!\!\!/P_L/2$.

n^μ. This allows the soft operators $O_j^{(i)}$ to be nontrivial functions of $n\cdot k_j$, $n\cdot v$, and $n\cdot v'$, and this information gives rise to a *complex phase* in the soft functions $S_{L,R}^{(0,8)}$ as shown in Appendix C. Thus, the S_n Wilson lines are directly responsible for producing final state interactions, and the soft fields in $O_j^{(0,8)}$ encode nonperturbative rescattering information.[6] This makes good sense, given that the soft gluons in the S_n's were originally generated by integrating out attachments to the collinear quarks and gluons making up the light energetic hadron.

The above procedure provides a *new* mechanism for generating nonperturbative strong phases for exclusive decays within factorization. In the soft $B\to D^{(*)}$ matrix elements the information about the light energetic meson is limited to its direction of motion n^μ. Since these matrix elements know nothing further about the nature of the light meson, these strong phases are universal. In particular the same strong phase ϕ is generated for the decays $\bar{B}\to D^{(*)}\pi$ and $\bar{B}\to D^{(*)}\rho$. (We caution that this is not the isospin strong phase, but rather a different angle in the triangle.) The same mechanism produces another universal strong phase for color suppressed decays to $D\bar{K}^{(*)0}$, and a third for decays to $D_sK^{(*)-}$. The different phases in the three classes arise in part due to the appearance of different moments of the matrix elements of the soft operators. However, for the kaons there are additional long-distance contributions to the strong phases from $[T]_{\rm long}$, which make the universality of the phase ϕ from $[T]_{\rm short}$ hard to test. A more complete set of phenomenological predictions is given in Sec. VI, including a comparison with existing data. Further details on the properties of the soft functions $S^{(0,8)}$ are given in Appendix C.

The matrix elements of the short-distance operators $O_j^{(i)}$ in Eq. (28) factor into products of soft and collinear parts, respectively. The collinear part of the matrix elements are simply given in terms of the light-cone wave function of the light meson. For π and ρ the definitions are [we suppress prefactors of $\int_0^1 dx\, \delta(\omega_1 - x\,\bar{n}\cdot p_M)\,\delta(\omega_2+(1-x)\bar{n}\cdot p_M)$ on the right-hand side (RHS)][7]

$$\langle \pi_n^0|(\bar{\xi}_n W)_{\omega_1}\bar{n}\!\!\!/\gamma_5\tau_3(W^\dagger\xi_n)_{\omega_2}|0\rangle = -i\sqrt{2}f_\pi\bar{n}\cdot p_\pi\,\phi_\pi(\mu,x),$$

$$\begin{aligned}\langle \rho_n^0(\varepsilon)|(\bar{\xi}_n W)_{\omega_1}\bar{n}\!\!\!/\tau_3(W^\dagger\xi_n)_{\omega_2}|0\rangle &= i\sqrt{2}f_\rho m_\rho\bar{n}\cdot\varepsilon^*\,\phi_\rho(\mu,x)\\ &= i\sqrt{2}f_\rho\bar{n}\cdot p_\rho\,\phi_\rho(\mu,x).\end{aligned} \quad (30)$$

In the last equality we have used the fact that at this order the collinear operator only produces longitudinal ρ's, for which $m_\rho\bar{n}\cdot\varepsilon_L^* = \bar{n}\cdot p_\rho$.

Since it no longer contains couplings to energetic gluons, the soft part of the matrix elements of $O_j^{(0,8)}$ can be constrained using heavy quark symmetry. In other words, heavy quark symmetry relations can be derived for matrix elements of soft fields. The constraints can be implemented most compactly using the trace formalism of the HQET [36]. First, consider the matrix element of the soft fields in $O_j^{(0,8)}$. For $O_j^{(0)}$ we have

$$\begin{aligned}&\frac{\langle D^{(*)0}(v')|(\bar{h}_{v'}^{(c)}S)\Gamma\,(S^\dagger h_v^{(b)})(\bar{d}S)_{k_1^+}n\!\!\!/P_L\,(S^\dagger u)_{k_2^+}|\bar{B}^0(v)\rangle}{\sqrt{m_Bm_D}}\\ &\quad = \mathrm{Tr}[\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}X^{(0)}],\end{aligned} \quad (31)$$

where $X^{(0)} = X^{(0)}(k_j^+,n,v,v')$ and we use the standard relativistic normalization for the states [and note that the left-hand side (LHS) is independent of $m_{b,c}$ in the heavy quark limit]. An identical equation holds for $O_j^{(8)}$ with an $X^{(8)}$. In writing the trace formula in Eq. (31) we have used the fact that the d and u quarks must end up in the $\bar{B}$ and $D^{(*)}$ states.[8] The heavy mesons (D,D^*) and (B,B^*) are grouped together into superfields [36], defined as

$$H_v = \frac{1+v\!\!\!/}{2}(P_v^{*\mu}\gamma_\mu + P_v\gamma_5). \quad (32)$$

Now $X^{(0,8)}$ are the most general structures compatible with the symmetries of QCD. They involve four functions $a_{1-4}^{(0,8)}(k_1^+,k_2^+,v\cdot v',n\cdot v,n\cdot v')$,

$$X^{(0,8)} = a_1^{(0,8)}n\!\!\!/P_L + a_2^{(0,8)}n\!\!\!/P_R + a_3^{(0,8)}P_L + a_4^{(0,8)}P_R. \quad (33)$$

Structures proportional to $v\!\!\!/$ and $v\!\!\!/'$ can be eliminated by using $H_v v\!\!\!/ = -H_v$, etc.

The presence of four functions in Eq. (33) would appear to restrict the predictive power of heavy quark symmetry. However, using the properties of H_v and $\bar{H}_{v'}$ and the fact that the two-body kinematics relates n to v and v' via $m_Bv = m_Dv' + E_Mn$, it is easy to see that the four functions a_i appear only in two distinct combinations. (Note that we are taking $m_M/m_B \sim \Lambda/m_B \ll 1$.) For $\Gamma_{L,R}^h$ they give soft functions $S_{L,R}$ defined as $S_L = (n\cdot v')(a_1 - a_3/2) - a_4/2$, $S_R = (n\cdot v')(a_2 - a_4/2) - a_3/2$ and

[6]Note that in semi-inclusive processes a different mechanism is responsible for the phases in single-spin asymmetries which has to do with the boundary conditions on Wilson lines [35].

[7]Our vector meson states are defined with an extra minus sign relative to the standard convention.

[8]The matrix element of the analogous soft operators with $(\bar{u}u)+(\bar{d}d)$ would contain a second term in Eq. (31) of the form $\mathrm{Tr}\,[\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}X]\mathrm{Tr}[Y]$, which arises from contracting the light quarks in the operator. These types of traces also show up for power corrections to $\bar{B}^0\to D^{(*)+}M^-$ and $B^-\to D^{(*)0}M^-$.

$$\frac{\langle D^0(v')|(\overline{h}_{v'}^{(c)}S)\not{n}P_{L,R}(S^\dagger h_v^{(b)})(\overline{d}S)_{k_1^+}\not{n}P_L(S^\dagger u)_{k_2^+}|\overline{B}^0(v)\rangle}{\sqrt{m_B m_D}}=S_{L,R}^{(0)}(k_j^+),$$

$$\frac{\langle D^{*0}(v',\varepsilon)|(\overline{h}_{v'}^{(c)}S)\not{n}P_{L,R}(S^\dagger h_v^{(b)})(\overline{d}S)_{k_1^+}\not{n}P_L(S^\dagger u)_{k_2^+}|\overline{B}^0(v)\rangle}{\sqrt{m_B m_{D^*}}}=\pm\frac{n\cdot\varepsilon^*}{n\cdot v'}S_{L,R}^{(0)}(k_j^+), \tag{34}$$

where $\pm$ for D^* refers to the choice of P_L or P_R. Identical definitions hold for the matrix elements of the color-octet operators which give $S_{L,R}^{(8)}(k_j^+)$. We will see in Sec. VI that the result in Eq. (34) relates decay amplitudes and strong phases for $\overline{B}^0\to D^0M^0$ and $\overline{B}^0\to D^{*0}M^0$ at leading order in the power expansion, and up to terms suppressed by $\alpha_s(Q)/\pi$. If one takes $n\cdot v=1$, then $n\cdot v'=m_B/m_D^{(*)}$, $v\cdot v'=(m_B^2+m_{D^{(*)}}^2)/(2m_B m_{D^{(*)}})$. The D,D^* variables are equal in the heavy quark limit.

For the long-distance operators $\overline{O}_j^{(i)}$ the same set of arguments in Eqs. (31)–(34) can be applied except that now we must add terms $a_5^{(0,8)}\not{x}_\perp P_L+a_6^{(0,8)}\not{x}_\perp P_R$ to $X^{(0,8)}$, and the a_i's can also depend on $x_\perp^2$. The functions analogous to $S_{L,R}^{(0,8)}$ are defined as $\Phi_{L,R}^{(0,8)}(k^+,x_\perp,\varepsilon_{D^*}^*)$. In this case the D and D^* decompositions are no longer related since the matrix element involves both $n\cdot\varepsilon^*$ and $x_\perp\cdot\varepsilon^*$ terms for D^*. Thus, due to the long-distance contributions for light vector meson we must restrict ourselves to the longitudinal polarization in order to have equality for the D and D^* amplitudes. In the case of ρ this restriction is not important since the long-distance contributions vanish (see Appendix A). However this observation does have phenomenological implications for decays to K^*'s.

We are now in a position to write down the most general factorized result for the amplitude for the decays $\overline{B}^0\to D^{(*)0}M^0$. Combining all the factors, this formula contains the soft functions $S^{(0,8)}(k_1^+,k_2^+)$ from Eq. (34), the jet functions $J^{(i)}$ from Eq. (24), and the Wilson coefficients $C_{L,R}^{(0,8)}$ from Eq. (17). In $J^{(i)}(\tau_i,k_\ell^+,\omega_k)$ we can pull out a factor of $\delta(\tau_1-\tau_2-\omega_1+\omega_2)$ by momentum conservation. This leaves the variables $\tau_1+\tau_2=2E_M(2z-1)$ and $\omega_1+\omega_2=2E_M(2x-1)$ unconstrained, which give convolutions with the momentum fractions z and x, respectively. In defining $J^{(i)}(z,x,k_\ell^+)$ we multiply $J^{(i)}(\tau_i,k_\ell^+,\omega_k)$ by $\omega_1-\omega_2=\overline{n}\cdot p_M$. Altogether the result for the $\overline{B}^0\to D^{(*)0}M^0$ amplitude is

$$\begin{aligned}A_{00}^{D^{(*)}}=N_0^M\int_0^1 dx\,dz\int dk_1^+dk_2^+[C_L^{(i)}(z)J^{(i)}(z,x,k_1^+,k_2^+)\\ \times S_L^{(i)}(k_1^+,k_2^+)\phi_M(x)\pm C_R^{(i)}(z)J^{(i)}(z,x,k_1^+,k_2^+)\\ \times S_R^{(i)}(k_1^+,k_2^+)\phi_M(x)]+A_{\rm long}^{D^{(*)}M},\end{aligned} \tag{35}$$

where we sum over $i=0,8$ and the μ_0,μ dependence is as in Eq. (24). The $A_{\rm long}^{D^{(*)}M}$ in Eq. (35) denotes the contributions from the matrix elements of the SCET$_{\rm II}$ time-ordered products $[T]_{\rm long}$. Also, $\pm$ refer to D/D^*, $C_{L,R}^{(i)}(z)=C_{L,R}^{(i)}(\tau_1+\tau_2,E_M,m_b,m_c,\mu)$, and

$$N_0^M=\frac{G_F V_{cb}V_{ud}^*}{2}f_M\sqrt{m_B m_{D^{(*)}}}. \tag{36}$$

The normalization factor is common since $m_D=m_D^*$ and $n\cdot\varepsilon^{(D^*)}=n\cdot v'$. This follows since the M's produced by $O_j^{(0,8)}$ are longitudinally polarized.

The long-distance amplitudes also obey a factorization theorem which can be derived by examining the matrix elements of the $\overline{O}_{L,R}^{(0,8)}$ operators in Eq. (27). First factorize the collinear fields into the matrix element with M and the soft fields into the matrix element with $B,D^{(*)}$. The independence of the collinear propagators on the residual soft minus-momenta leads to a $\delta(x^+)$ and the independence of the soft propagators on the residual collinear plus-momenta leads to a $\delta(x^-)$ (somewhat similar to the calculation for $B\to X_s\gamma$ as described in Ref. [26]). The result is

$$\begin{aligned}A_{\rm long}^{D^{(*)}M}=N_0^M\int_0^1 dz\int dk^+d\omega\int d^2x_\perp[C_L^{(i)}(z)\overline{J}^{(i)}(\omega k^+)\\ \times\Phi_L^{(i)}(k^+,x_\perp,\varepsilon_{D^*}^*)\Psi_M^{(i)}(z,\omega,x_\perp,\varepsilon_M^*)\\ \pm C_R^{(i)}(z)\overline{J}^{(i)}(\omega k^+)\Phi_R^{(i)}(k^+,x_\perp,\varepsilon_{D^*}^*)\\ \times\Psi_M^{(i)}(z,\omega,x_\perp,\varepsilon_M^*)],\end{aligned} \tag{37}$$

where $\pm$ is for D and D^* and we defined the nonperturbative functions in a way which gives the same prefactor as in Eq. (35). Here $C_{L,R}^{(i)}$ are the Wilson coefficients of the weak operators in Eq. (20), and the jet functions $\overline{J}^{(0,8)}$ are the coefficients of the SCET$_{\rm II}$ Lagrangian in Eq. (25). $\Phi_{L,R}^{(i)}$ and $\Psi_M^{(i)}$ are soft and collinear matrix elements from the operators $\overline{O}$ and are given by [with prefactor $\int_0^1 dz\,\delta(\omega_1-z\overline{n}\cdot p_M)\delta(\omega_2+(1-z)\overline{n}\cdot p_M)$ for $\Psi_M^{(0)}$]

$$\langle M^0(p_M,\epsilon_M)|[(\overline{\xi}_n^{(d)}W)_{\omega_1}\bar{n}\!\!\!/ P_L(W^\dagger\xi_n^{(u)})_{\omega_2}](0_\perp)\times[(\overline{\xi}_n^{(u)}W)_\omega \bar{n}\!\!\!/ P_L(W^\dagger\xi_n^{(d)})_\omega](x_\perp)|0\rangle = if_M/\sqrt{2}\Psi_M^{(0)}(z,\omega,x_\perp,\varepsilon_M^*),$$

$$\langle D^{(*)0}(v',\epsilon_{D*})|[(\overline{h}_{v'}^{(c)}S)\Gamma_{L,R}^h(S^\dagger h_v^{(b)})](0_\perp)\times[(\overline{d}S)_{k^+}n\!\!\!/ P_L(S^\dagger u)_{k^+}](x_\perp)|\overline{B}^0\rangle = \pm\sqrt{m_B m_{D^{(*)}}}\Phi_{L,R}^{(0)}(k^+,x_\perp,\varepsilon_{D*}^*), \tag{38}$$

and $\Psi_M^{(8)}$ and $\Phi_{L,R}^{(8)}$ are defined by analogous equations with color structure $T^a\otimes T^a$. $\pm$ are for P_L and P_R, respectively. In a more traditional language the $A_{\rm long}^{D^{(*)}M}$ contributions might be referred to as "nonfactorizable" since they involve a direct $x_\perp$ convolution between nonperturbative functions. Equations (35) and (37) are the main results of our paper. Additional details about the derivation of Eq. (37) will be presented in Ref. [37].

Using the $\mathrm{SCET_{II}}$ power counting in $\eta=\Lambda/Q$ we can verify that the short- and long-distance contributions to the factorization theorem are indeed of the same order. The coefficients $C_{L,R}^{(i)}\sim\eta^0$. The results in Eqs. (26) and (42) for the jet functions imply $J^{(i)}\sim 1/\Lambda^2$ and $\overline{J}^{(i)}\sim 1/(Q\Lambda)$. Furthermore, $\phi_M\sim\eta^0$ from the definitions in Eq. (30). For the soft function in Eq. (34) we get $(\eta^{3/2})^4$ from the fields, η^{-3} from the states, times η^{-2} from the delta functions indicated by the momentum subscripts. This gives $S(k_1^+,k_2^+)\sim\eta$, i.e., $S(k_1^+,k_2^+)\sim\Lambda$. A similar calculation for the collinear and soft long-distance matrix elements in Eq. (38) gives $\Psi_M^{(0,8)}\sim\Lambda^2/Q$ and $\Phi_{L,R}^{(0,8)}\sim\Lambda$. In the factorization theorem the measures have scaling $(dk_1^+dk_2^+)\sim\Lambda^2$ and $(dk^+d^2x_\perp)\sim 1/\Lambda$. Combining all the factors for the short-distance amplitude gives $(\Lambda)(\Lambda^2)(1/\Lambda^2)(\Lambda)(\Lambda^0)=\Lambda^2$, while for the long-distance amplitude we find $(\Lambda)(1/\Lambda)(1/\Lambda)(\Lambda)(\Lambda^2)=\Lambda^2$ also. Therefore, both terms in $A_{00}^{D^{(*)}}$ are of the same order in the power counting as expected. They also give the complete set of contributions at this order.

For numerical results with $M=\pi,\rho$ the $A_{\rm long}^{D^{(*)}M}$ contributions are very small since taking $C_{L,R}^{(i)}(z)$ independent of z gives $A_{\rm long}^{D^{(*)}M}=0$, as shown in Appendix A. This implies that $A_{\rm long}^{D^{(*)}M}/A_{00}\sim\alpha_s(Q)/\pi$, and together with the helicity structure of the jet function discussed in Appendix B implies that the production of transverse ρ mesons is suppressed. In Sec. VI we explore further phenomenological implications.

Next, tree level results are presented for the jet functions $J^{(0,8)}$. The $\mathrm{SCET_I}$ graphs in Fig. 2 are computed with insertions of $\mathcal{Q}_j^{(0,8)}$ and taking momenta $-k_1$ and $-k_2$ for the initial and final light soft antiquarks, together with momenta p_1 and p_2 for the collinear quark and antiquark. The diagrams in Figs. 2(a,b) with insertions of $\{\mathcal{Q}_j^{(0)},\mathcal{Q}_j^{(8)}\}$ are

$$C:\quad g^2\frac{(\overline{u}_{v'}^{(c)}\gamma^\nu P_L\{1,T^B\}u_v^{(b)})(\overline{u}_n^{(d)}\gamma_\nu P_L n\!\!\!/ /2\{1,T^B\}T^A\gamma_\perp^\mu v_s^{(u)})(\overline{v}_s^{(d)}T^A\gamma_\mu^\perp v_n^{(d)})}{[n\cdot(k_1-k_2)+i\epsilon][\overline{n}\cdot p_2\, n\cdot k_1+i\epsilon]},$$

$$E:\quad -g^2\frac{(\overline{u}_{v'}^{(c)}\gamma^\nu P_L\{1,T^B\}u_v^{(b)})(\overline{u}_n^{(u)}T^A\gamma_\perp^\mu v_s^{(u)})(\overline{v}_s^{(d)}T^A\{1,T^B\}\gamma_\mu^\perp n\!\!\!/ /2\gamma_\nu P_L v_n^{(u)})}{[n\cdot(k_1-k_2)+i\epsilon][-\overline{n}\cdot p_1\, n\cdot k_2+i\epsilon]}. \tag{39}$$

Adding these contributions with factors of $C_L^{(0)}$ and $C_L^{(8)}$ to distinguish the two color structures, and then Fierzing gives

$$C_L^{(0)}[\overline{u}_{v'}^{(c)}n\!\!\!/ P_L u_v^{(b)}\overline{v}_s^{(d)}n\!\!\!/ P_L v_s^{(u)}]\frac{2\pi\alpha_s C_F}{N_c}\left(\frac{\overline{u}_n^{(d)}\bar{n}\!\!\!/ P_L v_n^{(d)}}{[n\cdot(k_1-k_2)+i\epsilon][\overline{n}\cdot p_2 n\cdot k_1+i\epsilon]}-\frac{\overline{u}_n^{(u)}\bar{n}\!\!\!/ P_L v_n^{(u)}}{[n\cdot(k_1-k_2)+i\epsilon][-\overline{n}\cdot p_1 n\cdot k_2+i\epsilon]}\right)$$
$$-C_L^{(8)}[\overline{u}_{v'}^{(c)}n\!\!\!/ P_L T^a u_v^{(b)}\overline{v}_s^{(d)}n\!\!\!/ P_L T^a v_s^{(u)}]\frac{\pi\alpha_s}{N_c^2}\left(\frac{\overline{u}_n^{(d)}\bar{n}\!\!\!/ P_L v_n^{(d)}}{[n\cdot(k_1-k_2)+i\epsilon][\overline{n}\cdot p_2 n\cdot k_1+i\epsilon]}-\frac{\overline{u}_n^{(u)}\bar{n}\!\!\!/ P_L v_n^{(u)}}{[n\cdot(k_1-k_2)+i\epsilon][-\overline{n}\cdot p_1 n\cdot k_2+i\epsilon]}\right), \tag{40}$$

where $C_F=(N_c^2-1)/(2N_c)$ and we set $C_R^{(0,8)}=0$. The first term in each round bracket originates from the C-type graph (Fig. 2a) and the second term from the E-type graph (Fig. 2b). It is convenient to group the result into isosinglet and isotriplet terms for the collinear spinors. Since π^0 and ρ^0 have definite charge conjugation, we can freely interchange the positive momenta $\overline{n}\cdot p_1\leftrightarrow\overline{n}\cdot p_2$, so a factor of $1/\overline{n}\cdot p_1$ can be pulled out front. For the terms in round brackets we find

$$\left(\frac{1}{2}\frac{[\overline{u_n^{(d)}}\bar{n}P_L v_n^{(d)}-\overline{u_n^{(u)}}\bar{n}P_L v_n^{(u)}]}{[n\cdot k_1+i\epsilon][-n\cdot k_2+i\epsilon]} -\frac{1}{2}\frac{[\overline{u_n^{(d)}}\bar{n}P_L v_n^{(d)}+\overline{u_n^{(u)}}\bar{n}P_L v_n^{(u)}]n\cdot(k_2+k_1)}{[n\cdot(k_1-k_2)+i\epsilon][n\cdot k_1+i\epsilon][-n\cdot k_2+i\epsilon]}\right). \tag{41}$$

For $\bar{B}^0\to D^{(*)0}\pi^0$ and $\bar{B}^0\to D^{(*)0}\rho^0$ where we have isotriplet M^0's the contributions from the SCET$_{\mathrm{II}}$ diagrams in Figs. 2(c,d) cancel. Thus, the denominator in Eq. (41) directly gives the tree level isotriplet jet functions

$$J^{(0)}(z,x,k_1^+,k_2^+)=-\frac{4\pi\alpha_s(\mu)C_F}{N_c}\times\frac{\delta(z-x)}{x[n\cdot k_1+i\epsilon][-n\cdot k_2+i\epsilon]},$$

$$J^{(8)}(z,x,k_1^+,k_2^+)=\frac{2\pi\alpha_s(\mu)}{N_c^2}\times\frac{\delta(z-x)}{x[n\cdot k_1+i\epsilon][-n\cdot k_2+i\epsilon]}, \tag{42}$$

where $\bar{n}\cdot p_1=x\,\bar{n}\cdot p_M$. These jet functions are nonsingular, given that the nonperturbative soft function $S(k_1^+,k_2^+)$ vanishes for $k_1^+=0$ or $k_2^+=0$, and that $\phi_{\pi,\rho}(x)$ vanishes at $x=0$ and $x=1$. On the other hand for isosinglet M^0's the result in Eq. (41) has a singular denominator $1/[n\cdot(k_1-k_2)+i\epsilon]$. The singularity occurs when the collinear quark propagators in Figs. 2(a,b) get too close to their mass shells, ie. when $n\cdot(k_1-k_2)\lesssim\Lambda^2/Q$. This singularity is exactly what is canceled by subtracting the SCET$_{\mathrm{II}}$ diagrams in Figs. 2(c,d), which then gives a nonsingular isosinglet jet function.

Next we consider the result for the factorization theorem for $M=\pi,\rho$ with these tree level jet functions. Taking the matrix elements of the $O_L^{(0,8)}$ operators, the collinear part factors from the soft operators as explained above. Their matrix elements are given in terms of the M^0 light-cone wave function and the $S^{(0,8)}(k_+,l_+)$ functions. This gives the explicit result for the $\bar{B}^0\to D^{(*)0}\pi^0$ and $\bar{B}^0\to D^{(*)0}\rho^0$ decay amplitudes, at lowest order in the matching for C and J,

$$A(\bar{B}^0\to D^{(*)0}\pi^0)=N_0^{\pi}\left\{-\frac{4\pi\alpha_s(\mu_0)C_F}{N_c}C_L^{(0)}s^{(0)}+\frac{2\pi\alpha_s(\mu_0)}{N_c^2}C_L^{(8)}s^{(8)}\right\}\langle x^{-1}\rangle_\pi,$$

$$A(\bar{B}^0\to D^{(*)0}\rho^0)=N_0^{\rho}\left\{-\frac{4\pi\alpha_s(\mu_0)C_F}{N_c}C_L^{(0)}s^{(0)}+\frac{2\pi\alpha_s(\mu_0)}{N_c^2}C_L^{(8)}s^{(8)}\right\}\langle x^{-1}\rangle_\rho. \tag{43}$$

We choose to evaluate $C_L^{(0,8)}$, $s^{(0,8)}$, and $\langle x^{-1}\rangle$ at the common scales $\mu=\mu_0\sim\sqrt{E_\pi\Lambda}$ since one of the hard scales m_c^2 is not much different from $E_\pi\Lambda$. In Eq. (43) the convolutions of the soft and collinear matrix elements are defined by

$$s^{(0,8)}=|s^{(0,8)}|e^{i\phi^{(0,8)}}=\int dk_1^+dk_2^+\frac{S_L^{(0,8)}(k_1^+,k_2^+,\mu)}{(k_1^++i\epsilon)(-k_2^++i\epsilon)},$$

$$\langle x^{-1}\rangle_M=\int_0^1 dx\frac{\phi_M(x,\mu)}{x}. \tag{44}$$

From Eq. (44) we can immediately verify the result of the power counting for operators described earlier. Since $\langle x^{-1}\rangle_M\sim\langle x^0\rangle_M\sim\lambda^0$, comparing Eqs. (7) and (8) and (43) we see that

$$\frac{A(\bar{B}^0\to D^0\pi^0)}{A(\bar{B}^0\to D^+\pi^-)}\sim 4\pi\alpha_s(\mu_0)\frac{N_0\,s^{(0)}}{N\,E_\pi}\sim 4\pi\alpha_s(\mu_0)\frac{s^{(0)}}{E_\pi}\sim 4\pi\alpha_s(\mu_0)\frac{\Lambda_{\mathrm{QCD}}}{E_\pi}, \tag{45}$$

where we have used the standard HQET power counting for the soft matrix elements to determine that $s^{(0,8)}\sim\Lambda_{\mathrm{QCD}}$. Thus, the ratio of type-II to type-I amplitudes scales as Λ/Q just as predicted. Due to the factor of 4π the suppression by α_s does not have much effect numerically. 4π arises because α_s is generated at tree level. It is expected that perturbative corrections to the matching for C and J will be suppressed by factors of $\alpha_s(Q)/\pi$ and $\alpha_s(\sqrt{E_\pi\Lambda})/\pi$, respectively. In Eq. (45) grouping $g^2N_c\sim1$ gives an extra factor of $1/N_c$, so with this counting the ratio is color suppressed as expected.

IV. ADDING STRANGE QUARKS

In this section we consider how the factorization theorem derived in Sec. III is modified in the case of color suppressed decays involving kaons, which include $\bar{B}^0\to D_s^{(*)}K^-$, $\bar{B}^0\to D_s^{(*)}K^{*-}$, as well as the Cabbibo suppressed decays $\bar{B}^0\to D^{(*)0}K^0$ and $\bar{B}^0\to D^{(*)0}K^{*0}$.

If strange quarks are included in the final state, then operators with different flavor structure appear. In the exchange topology we can have the production of an $s\bar{s}$ pair (as shown by the s quarks in brackets in Fig. 2b). This gives SCET$_{\mathrm{II}}$ six-quark operators

$$O_j^{(0)}(k_i^+,\omega_k)=[\overline{h}_{v'}^{(c)}\Gamma_j^h h_v^{(b)}(\overline{d}\,S)_{k_1^+}\not{n}P_L(S^\dagger s)_{k_2^+}]\times[(\overline{\xi}_n^{(s)}W)_{\omega_1}\Gamma_c(W^\dagger\xi_n^{(u)})_{\omega_2}],$$

$$O_j^{(8)}(k_i^+,\omega_k)=[(\overline{h}_{v'}^{(c)}S)\Gamma_j^h T^a(S^\dagger h_v^{(b)})\times(\overline{d}\,S)_{k_1^+}\not{n}P_L T^a(S^\dagger s)_{k_2^+}]\times[(\overline{\xi}_n^{(s)}W)_{\omega_1}\Gamma_c(W^\dagger\xi_n^{(u)})_{\omega_2}], \tag{46}$$

which mediate $\overline{B}^0\to D_s^{(*)}K^{(*)-}$. For the long-distance contribution we take flavors $q'=d$ and $q=s$ in the Lagrangian in Eq. (25), which leads to $s,\overline{s}$ quarks replacing $u,\overline{u}$ quarks in $\overline{O}_{L,R}^{(i)}$. The result for the factorization theorem is then identical to Eqs. (35) and (37), except that only the E topology contributes. For this case the long-distance contribution is not suppressed and serves to regulate the singularity when matching onto the E-topology jet functions $J^{(0,8)}=J_E^{(0,8)}$. Further discussion of the singularities is left to Ref. [37]. The hard coefficients $C_{L,R}^{(0,8)}$ are the same as in the previous section.

The remaining difference for $\overline{B}^0\to D_s^{(*)}K^{(*)-}$ are the nonperturbative functions. The light-cone wave functions for K^-, $\overline{K}^0$, K^{*-}, and $\overline{K}^{*0}$ are [with $q=u,d$, $\omega_1=\overline{n}\cdot px_s$, $\omega_2=-\overline{n}\cdot px_q$, and a prefactor as in Eq. (30)]

$$\langle K_n|(\overline{\xi}_n^{(s)}W)_{\omega_1}\bar{\not{n}}\gamma_5(W^\dagger\xi_n^{(q)})_{\omega_2}|0\rangle=-2if_K\overline{n}\cdot p_K\,\phi_K(\mu,x_s),$$

$$\langle K_n^*(\epsilon)|(\overline{\xi}_n^{(s)}W)_{\omega_1}\bar{\not{n}}(W^\dagger\xi_n^{(q)})_{\omega_2}|0\rangle=-2if_{K*}m_{K*}\overline{n}\cdot\epsilon^*\,\phi_{K*}(\mu,x_s)=-2if_{K*}\overline{n}\cdot p_{K*}\,\phi_{K*}(\mu,x_s). \tag{47}$$

The collinear functions $\Psi_M^{(0,8)}$ also depend on the light meson M. The nonperturbative soft functions involve strange quarks and are also different from Sec. III, $S\to\tilde{S}_{L,R}^{(i)}$ and $\Phi_{L,R}^{(0,8)}\to\tilde{\Phi}_{L,R}^{(0,8)}$. The nonperturbative functions are related to those in the previous section in the SU(3) flavor symmetry limit. However, the jet functions are not related in this limit, they differ since different topologies contribute. This leads to different convolutions over the nonperturbative functions.

Next consider the Cabibbo suppressed $b\to cs\overline{u}$ transition with the color suppressed topology (as shown by the brackets in Fig. 2a). For the six-quark operators we have[9]

$$O_j^{(0)}(k_i^+,\omega_k)=[\overline{h}_{v'}^{(c)}\Gamma_j^h h_v^{(b)}(\overline{d}\,S)_{k_1^+}\not{n}P_L(S^\dagger u)_{k_2^+}]\times[(\overline{\xi}_n^{(s)}W)_{\omega_1}\Gamma_c(W^\dagger\xi_n^{(d)})_{\omega_2}],$$

$$O_j^{(8)}(k_i^+,\omega_k)=[(\overline{h}_{v'}^{(c)}S)\Gamma_j^h T^a(S^\dagger h_v^{(b)})\times(\overline{d}\,S)_{k_1^+}\not{n}P_L T^a(S^\dagger u)_{k_2^+}]\times[(\overline{\xi}_n^{(s)}W)_{\omega_1}\Gamma_c(W^\dagger\xi_n^{(d)})_{\omega_2}], \tag{48}$$

which mediate the decays $\overline{B}^0\to D^{(*)0}\overline{K}^{(*)0}$. In this case the $\mathrm{SCET_{II}}$ Lagrangian in Eq. (25) has the same flavor structure as in Sec. III. Since only the C topology contributes, the long-distance contribution is not suppressed in the factorization theorem, and the jet function $J^{(0,8)}\to J_C^{(0,8)}$. For both the short- and long-distance nonperturbative functions the change of flavor appears only through the collinear quarks in the weak operator, so the collinear functions depend on the $K^{(*)0}$ but the soft functions $S_{L,R}^{(0,8)}$ and $\Phi_{L,R}^{(0,8)}$ are identical to those in Sec. III. (However, now $J_C^{(0,8)}$ appears, so the moments over the soft function $S_{L,R}^{(0,8)}$ will be different.) Finally note that if we allow a strange quark in the initial state (for B_s decays), then the E topology can also contribute and more operators are generated.

Due to the non-negligible long-distance contributions the number of model-independent phenomenological predictions for kaons are more limited. The main predictions are the equality of branching fractions and strong phase shifts for decays to D versus D^*. For $M=K^0,K^-$ an identical proof to the one for π^0 and ρ^0 can be used. For the vector mesons the proof can also be used if we restrict our attention to longitudinal polarizations, so the final states $D^{(*)}K_\parallel^{*0}$ are related, and so are $D^{(*)}K_\parallel^{*-}$. The factorization theorem allows for transversely polarized kaons at the same order in the power counting, but only through the long-distance contribution.

V. DISCUSSION AND COMPARISON WITH THE LARGE N_c LIMIT

It is instructive to compare the N_c scaling of the different terms in the SCET result, Eq. (35) [or Eq. (43)], with that expected from QCD before expanding in $1/Q$ given in Eq. (3). Combining the matrix elements in Eq. (3) written in a form similar to Eq. (43) gives the decay amplitude at leading order in $1/Q$ as

$$A(\overline{B}^0\to D^0M^0)=N_0^M\left(C_1+\frac{C_2}{N_c}\right)\left[\frac{1}{N_c}(F_0+2G_1)+\cdots\right]+N_0^M C_2\left[F_0+\frac{1}{N_c^2}(-F_0+F_2-2G_1)+\cdots\right]+\cdots. \tag{49}$$

[9] Note that the flavor structure was not distinguished in naming the operators in Eqs. (28),(46),(48). This should not cause confusion since they always contribute to different decays.

The ellipses denote power suppressed terms. This reproduces the $1/N_c$ expansion of the SCET amplitude in Eq. (43) with the identification

$$F_0=0, \quad G_1=-\pi\alpha_s C_F\, s^{(0)}|_{N_c\to\infty},$$
$$F_2-2G_1=2\pi\alpha_s\, s^{(8)}|_{N_c\to\infty}, \tag{50}$$

where $s^{(0)}\sim N_c^0$ and $s^{(8)}\sim N_c$. This implies that the factorizable term F_0 is power suppressed in the limit of an energetic pion relative to the leading order amplitude in Eq. (43).

The naive factorization approach in Eq. (2) keeps only the F_0 term, which is expressed in terms of the $\bar{B}\to\pi$ form factor in the large N_c limit. We comment here on the form of this contribution in the effective theory. They appear in the matching of the $(\bar{d}b)_{V\text{-}A}(\bar{c}u)_{V\text{-}A}$ operator onto SCET_I T products such as

$$T_1^{(4)}=T\{\mathcal{Q}_0^{(2)},i\mathcal{L}_{q\xi}^{(2)}\}, \quad T_2^{(4)}=T\{\mathcal{Q}_{1a,1b}^{(3)},i\mathcal{L}_{q\xi}^{(1)}\}, \tag{51}$$

where the operators $\mathcal{Q}^{(2,3)}$ contain one usoft light quark. From the leading order operators in Eq. (20) they can be constructed by switching $\xi_n\to q$ to give $\mathcal{Q}^{(2)}$, and adding a further $W^\dagger iD_\perp W$ to get $\mathcal{Q}^{(3)}$. Their precise form is different depending on whether they are introduced by matching from the color suppressed (C) or the W-exchange (E) graph. Schematically

$$C\text{-type}\quad \mathcal{Q}_0^{(2)}=[(\bar{\xi}_n^{(d)}W)_\omega\Gamma_c h_v^{(b)}][\bar{h}_{v'}^{(c)}\Gamma_h u],$$
$$\mathcal{Q}_{1a}^{(3)}=\left[\left(\bar{\xi}_n^{(d)}\frac{\bar{n}\!\!\!/}{2}i\mathcal{D}\!\!\!\!/^{\,\dagger}_{\perp c}W\right)_\omega\frac{1}{\bar{n}\cdot\mathcal{P}^\dagger}\Gamma_c h_v^{(b)}\right][\bar{h}_{v'}^{(c)}\Gamma_h u],$$
$$\mathcal{Q}_{1b}^{(3)}=\frac{1}{m_c}[(\bar{\xi}_n^{(d)}W)_{\omega_1}\Gamma_c h_v^{(b)}]\times[\bar{h}_{v'}^{(c)}[W^\dagger i\mathcal{D}\!\!\!\!/_{\perp c}W]_{\omega_2}\Gamma_h u], \tag{52}$$

$$E\text{-type}\quad \mathcal{Q}_0^{(2)}=[\bar{d}\Gamma_h h_v^{(b)}][\bar{h}_{v'}^{(c)}\Gamma_c(W^\dagger\xi_n^{(u)})_\omega],$$
$$\mathcal{Q}_{1a}^{(3)}=[\bar{d}\Gamma_h h_v^{(b)}]\left[\bar{h}_{v'}^{(c)}\Gamma_c\frac{1}{\bar{n}\cdot\mathcal{P}}\left(W^\dagger i\mathcal{D}\!\!\!\!/_{\perp c}\frac{\bar{n}\!\!\!/}{2}\xi_n^{(u)}\right)_\omega\right],$$
$$\mathcal{Q}_{1b}^{(3)}=\frac{1}{m_c}[\bar{d}\Gamma_h h_v^{(b)}][\bar{h}_{v'}^{(c)}[W^\dagger i\mathcal{D}\!\!\!\!/_{\perp c}W]_{\omega_1}\times\Gamma_c(W^\dagger\xi_n^{(u)})_{\omega_2}]. \tag{53}$$

The presence of the usoft quark field q in these operators introduces an additional suppression factor of λ^2, such that the T products $T_{1,2}^{(4)}$ are $O(\lambda^4)\sim\Lambda^2/Q^2$ down relative to the operators $\mathcal{Q}_{L,R}^{(0,8)}$ in Eq. (20). (Note that since the form factors enter as time-ordered products we do not expect a different α_s suppression for $T_{1,2}^{(4)}$ relative to those in Eq. (21) [29].) This explains the absence of the F_0 contributions at order Λ/Q, as noted in Eq. (50). Although F_0 is part of the leading order result in the large N_c limit, it is subleading in the $1/Q$ expansion.

After soft-collinear factorization, the T products (51) match onto factorizable operators in SCET_II. For example, the C type time-ordered product containing $Q_{1a}^{(3)}$ gives (schematically)

$$T_2^{(4)}\to\int d\omega_1 d\omega_2\,\mathcal{J}(\omega_i,k_1^+)[(\bar{d}S)_{k_1}\Gamma(S^\dagger h_v^{(b)})][\bar{h}_{v'}^{(c)}\Gamma_h u]\times[(\bar{\xi}_n^{(u)}W)_{\omega_1}\Gamma_c(W^\dagger\xi_n^{(u)})_{\omega_2}]. \tag{54}$$

Apart from the $(\bar{c}u)$ soft bilinear, this is similar to a factorizable operator contributing to the $B\to\pi$ form factor [29]. The presence of the D meson in the final state implies that the matrix element of the soft operator in Eq. (54) is different from that appearing in $\bar{B}\to\pi$. Therefore, naive factorization of type-II decay amplitudes, as written in Eq. (2), does not follow in general from the large energy limit. Still, in the large N_c limit, the matrix element of $T_1^{(4)}$ above can be indeed expressed in terms of the $B\to\pi$ form factor, as required by Eq. (5)

Recently an analysis of color-suppressed decays was performed using the "pQCD" approach working at leading order in an expansion in $m_{D^{(*)}}/m_B$ and $\Lambda_\text{QCD}/m_{D^{(*)}}$ [38]. This differs from the expansion used here, in that we do not expand in $m_{D^{(*)}}/m_B$. The nonperturbative functions in their proposed factorization formula include the light-cone wave functions $\phi_\pi^{(p)}(x_3)$, $\phi_D(x_2)$ and a B light-cone wave function that depends on a transverse coordinate $\phi_B(x_1,b_1)$. This differs from our result which involves a $B\to D$ function $S(k_1^+,k_2^+)$ and also has additional long-distance contributions, $A_\text{long}^{D^{(*)}M}$, at the same order in our power counting. Our long-distance contributions are "non-factorizable" in the sense that the nonperturbative functions $\Phi_{L,R}^{(i)}(k^+,x_\perp)$ and $\Psi_M^{(i)}(z,\omega,x_\perp)$ communicate directly through their $x_\perp$ dependence without going through a hard kernel. In Ref. [38] strong phases only occur from the perturbative $\mu_0^2\simeq E_M\Lambda$ scale, whereas we also find nonperturbative strong phases from the Λ^2 scale [in $S(k_1^+,k_2^+)$]. The nonperturbative phases are expected to dominate in our result. Finally, the results in Ref. [38] do not manifestly predict the equality of the D and D^* amplitudes since at the order they are working contributions from different $B\to M$ form factors show up. For example their pQCD prediction $\text{BR}(\bar{B}^0\to D^{*0}\rho^0)/\text{BR}(\bar{B}^0\to D^0\rho^0)=2.7$ is much different from the prediction of 1.0 that we obtain in the next section using heavy quark symmetry.

The time-ordered products presented in this paper in Eq. (21) are only Λ/Q down from the class-I T amplitudes. Therefore, they give the dominant contribution to the color suppressed and W-exchange amplitudes in the limit of an energetic pion ($\Lambda/Q\ll 1$). This is a new result, not noticed previously in the literature. The power counting of factorizable F_0-type contributions are indeed suppressed by Λ^2/Q^2

in our analysis in agreement with the literature. However, these terms do not give the dominant contribution.

VI. PHENOMENOLOGICAL PREDICTIONS

A factorization theorem for color suppressed $\bar{B}^0 \to D^0 M^0$ decays was proven in Sec. III and extended to decays to kaons in Sec. IV. The amplitudes at leading order in $\Lambda_{\rm QCD}/Q$ with $Q=\{m_b, m_c, E_\pi\}$ have the form

$$A_{00}=A(\bar{B}^0 \to D^{(*)0}M^0)$$
$$=N_0^M \int_0^1 dx\, dz \int dk_1^+ dk_2^+ \sum_{i=0,8} [C_L^{(i)}(z) S_L^{(i)}(k_j^+)$$
$$\pm C_R^{(i)}(z)\, S_R^{(i)}(k_j^+)] J^{(i)}(z,x,k_j^+)\, \phi_M(x) + A_{\rm long}^{D^{(*)}M}, \quad (55)$$

where the sign $\pm$ corresponds to a D^0 or D^{*0} meson in the final state, respectively. In this section the implications of Eq. (55) for the phenomenology of color suppressed decays are discussed. One class of predictions follow without any assumptions about the form of J:

(1) Heavy quark symmetry relates the nonperturbative soft matrix elements appearing in the $\bar{B}^0 \to D^0 M^0$ and $\bar{B}^0 \to D^{*0} M^0$ decays with the same light meson at leading order in $\alpha_s(Q)/\pi$. This implies relations among their branching fractions and equal strong phases in their isospin triangles. These relations are encoded in the ratios R_0^M in Eq. (12).

A second class of predictions depend on using a perturbative expansion of J in $\alpha_s(\mu_0)$ for $\mu_0^2 \sim E_M \Lambda$:

(2) Using a perturbative description of J the amplitudes and strong phases for decays to different light mesons M can be related at leading order in $\alpha_s(\mu_0)/\pi$.

These predictions are encoded in the ratios $R_0^{M/M'}$, R_c, and strong phase ϕ in R_I, as defined in Eq. (12). We consider the two classes of predictions in turn.

First, consider relations between color suppressed $\bar{B} \to DM$ and $\bar{B} \to D^* M$ decays with the same light meson. At tree level in the matching at the hard scale $\mu \simeq Q$, two of the Wilson coefficients vanish $C_R^{(0,8)}=0$. Therefore both amplitudes for D and D^* contain only the soft functions $S_L^{(0,8)}(k_j^+)$ appearing in the same linear combination. This implies model-independent predictions, which can be made even in the absence of any information about the jet functions $J^{(i)}$ and the nonperturbative functions $S_L^{(i)}$, ϕ_M, and without knowing $A_{\rm long}^{D^{(*)}M}$. For $M=\pi^0,\rho^0$, we have $A_{\rm long}^{D^{(*)}M}=0$ so Eq. (55) gives

$$R_0^\pi \equiv \frac{A(\bar{B}^0 \to D^{*0}\pi^0)}{A(\bar{B}^0 \to D^0\pi^0)}=1, \quad R_0^\rho \equiv \frac{A(\bar{B}^0 \to D^{*0}\rho^0)}{A(\bar{B}^0 \to D^0\rho^0)}=1. \quad (56)$$

For decays to $D_s^{(*)}K^-$, $D_s^{(*)}K_\parallel^{*-}$, $D^{(*)0}\bar{K}^0$, and $D^{(*)0}\bar{K}_\parallel^{*0}$ it was shown that $A_{\rm long}^{DM}=A_{\rm long}^{D^*M}$ and so

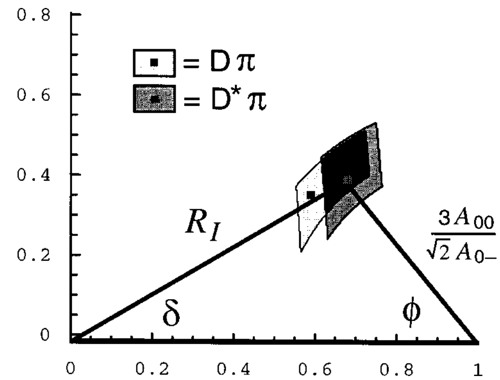

FIG. 5. The ratio of isospin amplitudes $R_I=A_{1/2}/(\sqrt{2}A_{3/2})$ and strong phases δ and ϕ in $\bar{B} \to D\pi$ and $\bar{B} \to D^*\pi$. The central values following from the D and D^* data in Table I are denoted by squares, and the shaded regions are the 1σ ranges computed from the branching ratios. The overlap of the D and D^* regions show that the two predictions embodied in Eq. (56) work well.

$$R_0^{K^-}=\frac{A(\bar{B}^0 \to D_s^* K^-)}{A(\bar{B}^0 \to D_s K^-)}=1, \quad R_0^{K_\parallel^{*-}}=\frac{A(\bar{B}^0 \to D_s^* K_\parallel^{*-})}{A(\bar{B}^0 \to D_s K_\parallel^{*-})}=1,$$

$$R_0^{K^0}=\frac{A(\bar{B}^0 \to D^* \bar{K}^0)}{A(\bar{B}^0 \to D\bar{K}^0)}=1, \quad R_0^{K_\parallel^{*0}}=\frac{A(\bar{B}^0 \to D^* \bar{K}_\parallel^{*0})}{A(\bar{B}^0 \to D\bar{K}_\parallel^{*0})}=1. \quad (57)$$

The ratios in Eqs. (56) and (57) have calculable corrections of the order of $\alpha_s(Q)/\pi$ and power corrections[10] of the order of Λ/Q, which can be expected to be $\sim$20%.

These amplitude relations imply the equality of the branching fractions. They also imply the equality of the nonperturbative strong phases between isospin amplitudes, namely, the phases $\delta^{D^{(*)}M}$ in the ratios $R_I^{D^{(*)}M}$ as shown in Fig. 5. Thus for each of $M=\pi^0,\rho^0,K^0,K_\parallel^{*0}$,

$$\mathrm{BR}(\bar{B}^0 \to D^{*0}M^0)=\mathrm{BR}(\bar{B}^0 \to D^0 M^0), \quad \delta^{D^{*0}M^0}=\delta^{D^0M^0}, \quad (58)$$

and for $M=K^-,K_\parallel^{*-}$

$$\mathrm{BR}(\bar{B}^0 \to D_s^* M)=\mathrm{BR}(\bar{B}^0 \to D_s M), \quad \delta^{D_s^*M}=\delta^{D_sM}. \quad (59)$$

The predictions in Eqs. (56) and (58) agree well with the data for $D^{(*)}\pi$ in Table I, which give

$$|R_0^\pi|^{\rm exp}=0.94 \pm 0.21, \quad \delta^{D\pi}=30.3°{}^{+7.8}_{-13.8},$$
$$\delta^{D^*\pi}=30.1° \pm 6.1°. \quad (60)$$

[10]Note that using the observed D and D^* masses $R_0^M=N_0^*/N_0 = 1.04$. This small difference corresponds to keeping an incomplete set of higher order corrections.

This agreement is represented graphically by the overlap of the 1σ regions in Fig. 5, with small squares indicating the central values. The dominant contribution to the phase δ is generated by the $(C-E)$ amplitudes which have complex phases from $J^{(i)} S_L^{(0,8)}$ in Eq. (55). Since the phases in $S_L^{(0,8)}$ are nonperturbative and can be large, it is expected that they will dominate. Note that with this choice of triangle the power suppressed side in Fig. 5 is enlarged by a isospin prefactor of $3/\sqrt{2}=2.1$.

For $\overline{B}^0$ decays to $D^{(*)0}\rho^0$, $D^{(*)0}\overline{K}^0$, $D^{(*)0}\overline{K}_{\parallel}^{*0}$, $D_s^{(*)}K^-$, and $D_s^{(*)}K_{\parallel}^{*-}$ only upper bounds on the branching ratios exist, so our relation between D and D^* triangles has not yet been tested. For each of these channels similar triangles to the one in Fig. 5 can be constructed once data become available.

The results in Eqs. (56) and (57) can be contrasted with the absence of a definite prediction in the large N_c limit as in Eq. (6). Even when only the F_0 term is included (naive factorization), R_π is given by a ratio of $B\to\pi$ form factors, which for generic $m_{b,c}$ are not related by heavy quark symmetry. Thus, one does not expect a relation between the branching fractions or strong phases unless the $1/Q$ expansion is used.

Next consider the second class of predictions, which follow from the perturbative expansion of the jet function in Eq. (55). We now assume that $\alpha_s(\mu_0)$ is perturbative and focus on $M=\pi,\rho$ since the kaons are contaminated by contributions from $A_{\rm long}^{D^{(*)}M}$. The tree level result for J is given in Eq. (42), and was used to define the nonperturbative parameters $s^{(0,8)}$ through convolutions with the soft distribution functions $S_L^{(0,8)}(k_i^+)$ as in Eq. (44). It is convenient to introduce an effective moment parameter

$$s_{\rm eff}=-s^{(0)}+\frac{1}{2N_cC_F}\frac{C_L^{(8)}}{C_L^{(0)}}s^{(8)}=|s_{\rm eff}|e^{-i\phi}. \qquad (61)$$

In terms of the effective moment the result in Eq. (55) at lowest order in $\alpha_s(Q)$ and $\alpha_s(\mu_0)$ becomes

$$A(\overline{B}^0\to D^{(*)0}M^0)=N_0^M\,C_L^{(0)}\frac{16\pi\alpha_s(\mu_0)}{9}s_{\rm eff}(\mu_0)\langle x^{-1}\rangle_M, \qquad (62)$$

where N_0^M is defined in Eq. (36). Since $s_{\rm eff}$ is independent of $M=\pi,\rho$ the same phase ϕ is predicted for these two light mesons.

At leading order in $1/Q$ the type-I amplitude $A_{0-}=A(B^-\to D^0\pi^-)$ factors as in Eq. (7) giving the product of a form factor and decay constant, both of which are real (with the usual phase conventions for the states, and neglecting tiny $\alpha_s(m_b)$ strong phases ($\sim 2°$) generated by the coefficients $C_{L,R}^{(0)}$ at one loop [7]). Therefore the amplitude A_{0-} is real at leading order in $1/Q$ up to calculable corrections of the order of $\alpha_s(Q)$. Choosing the orientation of the triangle so that A_{0-} lies on the real axis, the phase ϕ can be directly extracted as one of the angles in the isospin triangle

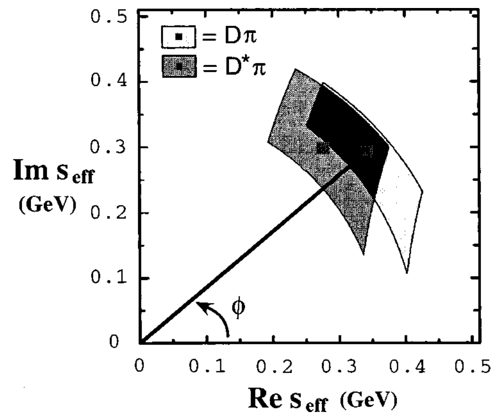

FIG. 6. Fit to the soft parameter $s_{\rm eff}$ defined in the text, represented in the complex plane with the convention that A_{0-} is real. The regions are derived by scanning the 1σ errors on the branching fractions (which may slightly overestimate the uncertainty). The light gray area gives the constraint from $\overline{B}\to D\pi$ and the dark gray area gives the constraint from $\overline{B}\to D^*\pi$.

$$\sqrt{2}A_{00}+A_{+-}=A_{0-}. \qquad (63)$$

This is shown in Fig. 6 where we divide by A_{0-} to normalize the base. The data on $\overline{B}^0\to D^0\rho^0$ are not yet sensitive enough to test the prediction that ϕ is the same for π^0 and ρ^0.

Using Eqs. (7) and (62) it is possible to make a prediction for the ratio R_c in Eq. (12) at next-to-leading order in the power expansion. Since $R_c=A_{+-}/A_{0-}$ contains only charged light mesons, it is easier to measure than neutral pion channels. Data are available for all four of the $D^{(*)}\pi$ and $D^{(*)}\rho$ channels. Using the triangle relation in Eq. (63) one finds for the ratio of any two such modes $[M=\pi,\rho]$

$$R_c^{D^{(*)}M}=1-\sqrt{2}\,\frac{A_{00}}{A_{0-}} =1-\frac{16\pi\alpha_s(\mu_0)m_{D^{(*)}}}{9\,E_M(m_B+m_{D^{(*)}})}\frac{s_{\rm eff}(\mu_0)}{\xi(w_0,\mu_0)}\langle x^{-1}\rangle_M. \qquad (64)$$

It is easy to see that the ratio of amplitudes on the right-hand side is common to final states containing a D or D^*, and has only a mild dependence on the light meson, introduced through the inverse moment $\langle x^{-1}\rangle_M$. In particular we note that there is no dependence on the decay constant f_M on the RHS of Eq. (64), since it cancels in the ratio A_{00}/A_{0-}. This implies that the ratios R_c are comparable for all four channels $D^{(*)}\pi$ and $D^{(*)}\rho$, up to corrections introduced by $\langle x^{-1}\rangle_\pi\neq\langle x^{-1}\rangle_\rho$. These corrections can be smaller than the correction one might expect from the ratio of decay constants $f_\rho/f_\pi\simeq 1.6$ (which appear in the naive a_2 factorization). The experimental values of these ratios can be extracted from Table I and are in good agreement with a quasiuniversal prediction,

$$|R_c^{(D\pi)}|=\frac{|A(\bar{B}^0\to D^+\pi^-)|}{|A(B^-\to D^0\pi^-)|}=0.77\pm0.05,$$

$$|R_c^{(D^*\pi)}|=\frac{|A(\bar{B}^0\to D^{*+}\pi^-)|}{|A(B^-\to D^{*0}\pi^-)|}=0.81\pm0.05,\tag{65}$$

$$|R_c^{(D\rho)}|=\frac{|A(\bar{B}^0\to D^+\rho^-)|}{|A(B^-\to D^0\rho^-)|}=0.80\pm0.09,$$

$$|R_c^{(D_L^*\rho_L)}|=\frac{|A(\bar{B}^0\to D^{*+}\rho^-)|}{|A(B^-\to D^{*0}\rho^-)|}=0.86\pm0.10.$$

This lends support to our prediction for the universality of the strong phase ϕ in $\bar{B}\to D^{(*)}\pi$ and $\bar{B}\to D^{(*)}\rho$ decays from the $s_{\rm eff}$ in Eq. (64). The central values of $R_c\simeq0.8$ are well described by $s_{\rm eff}$ of the expected size ($\sim\Lambda_{\rm QCD}$), as discussed in the fit to the isospin triangle below. Further data on these channels may expose other interesting questions, such as whether $R_c^{(D^*M)}$ is closer to $R_c^{(DM)}$ than $R_c^{(D^{(*)}\pi)}$ is to $R_c^{(D^{(*)}\rho)}$.

An alternative use of Eq. (64) and the R_c amplitude ratios is to give us a method for extracting the ratio of ρ and π moments. Using the $D\pi$ and $D\rho$ measurements which have smaller errors than for D^*, we find

$$\frac{\langle x^{-1}\rangle_\rho}{\langle x^{-1}\rangle_\pi}=\frac{|R_c^{(D\rho)}|-1}{|R_c^{(D\pi)}|-1}=0.87\pm0.42,\tag{66}$$

where only the experimental uncertainty is shown. The extraction in Eq. (66) is smaller, but still in agreement with the ratio extracted from light-cone QCD sum rules. The best fit from the $\gamma^*\gamma\to\pi^0$ data performed in Ref. [39] gives $\langle x^{-1}\rangle_\pi=3.2\pm0.4$ in agreement with sum-rule estimates of the moment. The QCD sum-rule result $\langle x^{-1}\rangle_\rho=3.48\pm0.27$ [40] then implies

$$\left.\frac{\langle x^{-1}\rangle_\rho}{\langle x^{-1}\rangle_\pi}\right|_{\rm SR}=1.10\pm0.16.\tag{67}$$

The result that this ratio is close to unity is consistent with the universality of the data in Eq. (65). These data can be contrasted with cases where the single light meson is replaced by a multibody state such as [17]

$$\frac{{\rm BR}(\bar{B}^0\to D^{*+}\pi^-\pi^-\pi^+\pi^0)}{{\rm BR}(B^-\to D^{*0}\pi^+\pi^-\pi^-\pi^0)}=1.02\pm0.27.\tag{68}$$

For the four pion final state our proof of the factorization theorem does not work, since for many events one or more of the pions will be slow. We therefore would expect less universality in branching ratios involving more than one light meson. (For these decays a different type of factorization involving large N_c works well for the q^2 spectrum [8].)

The result in Eq. (62) also leads to predictions for the ratios of color-suppressed decay amplitudes to final states containing different light mesons $M^0=\pi^0,\rho^0$. We find

$$R_0^{\rho/\pi}\equiv\frac{|A(\bar{B}^0\to D^0\rho^0)|}{|A(\bar{B}^0\to D^0\pi^0)|}=\frac{f_\rho}{f_\pi}\frac{\langle x^{-1}\rangle_\rho}{\langle x^{-1}\rangle_\pi}=1.40\pm0.77,\tag{69}$$

where we used $f_{\pi^\pm}=130.7\pm0.4$ MeV and $f_{\rho^\pm}=210\pm10$ MeV, and inserted the result in Eq. (66) for the moments. This can be compared with the experimental result $(R_{\rho/\pi})^{\rm exp}=1.02\pm0.21$. The large uncertainty in the ratio of moments in Eq. (66) dominates the error in Eq. (69). With the QCD sum-rule result in Eq. (67) we find $R_{\rho/\pi}=1.64\pm0.35$, a result whose central value is farther from the experimental data, but still consistent with it.

In contrast to the first class of predictions, the predictions for the ratios in Eqs. (64), (66), and (69) and the prediction for the universality of ϕ can receive corrections from neglected $[\alpha_s(\mu_0)^2/\pi]$ terms in J. The dominant theoretical corrections to this extraction are expected to come again from these perturbative corrections to J or from power corrections, which we estimate may be at the $\sim30\%$ level. A future study of the perturbative corrections is possible within the framework of our factorization theorem and SCET. If future data indicate large deviations from the predictions in our second class then this points to significant perturbative corrections to the jet function J. However, it would not indicate anything about our first class of predictions which are independent of the functional form of J.

The result in Eq. (62) and the data on $B\to D\pi$ and $B\to D^*\pi$ decays can be used to extract values of the moment parameters $|s_{\rm eff}|$ and strong phase ϕ. We present in Fig. 6 the constraints on the parameter $s_{\rm eff}$ in the complex plane, obtained from $D\pi$ (light shaded region) and $D^*\pi$ data (darker shaded area). We used in this determination $\mu_0=E_\pi=2.31$ GeV, and leading order running which gives $\alpha_s(\mu_0)=0.25$, $C_1(\mu=\mu_0)=1.15$, and $C_2(\mu=\mu_0)=-0.32$. The good agreement between the $D\pi$ and $D^*\pi$ 1σ regions marks a quantitative success of our factorization relation in Eq. (55). Averaging over the $D\pi$ and $D^*\pi$ results, we find the following values for the soft parameters at $\mu=\mu_0$:

$$|s_{\rm eff}|=(428\pm48\pm100\ {\rm MeV})\left(\frac{0.26}{C_L(\mu_0)\alpha_s(\mu_0)}\right)\left(\frac{3.2}{\langle x^{-1}\rangle_\pi}\right),$$

$$\phi=44.0°\pm6.5°.\tag{70}$$

In this determination the inverse moment of the pion wave function was taken from the best fit to the $\gamma^*\gamma\to\pi^0$ data [39], $\langle x^{-1}\rangle_\pi=3.2\pm0.4$. For $|s_{\rm eff}|$ the first error is experimental, while the second is our estimate of the theoretical uncertainty in the extraction from varying μ_0 from $E_\pi/2$ to $2E_\pi$. At the order we are working the extraction of the phase ϕ is independent of the scale, since the prefactor $\alpha_s(\mu_0)\langle x^{-1}\rangle_\pi$ drops out. The result in Eq. (70) agrees well with the dimensional analysis estimates $s_{\rm eff}\sim s^{(0,8)}\sim\Lambda_{\rm QCD}$.

Since ϕ is nonperturbative its value is unconstrained, and a large value of this phase is allowed.

The recent $\bar{B}^0 \to D^0 \rho^0$ data from Belle allow us to extract $|s_{\rm eff}|$ and ϕ in a manner independent of the above determination. Keeping only experimental errors we find

$$|s_{\rm eff}| = (259 \pm 124 \ \ {\rm MeV}) \left(\frac{0.26}{C_L(\mu_0)\alpha_s(\mu_0)} \right) \left(\frac{3.5}{\langle x^{-1} \rangle_\rho} \right),$$

$$\phi = 17^\circ \pm 70^\circ. \tag{71}$$

The results agree with Eq. (70) within 1σ, but currently have errors that are too large to significantly test the factorization prediction of equality on the 20–30 % level of the parameters extracted from $D\rho$ and $D\pi$.

The $\bar{B}^0 \to D_s^{(*)+} K^-$ channels proceed exclusively through the W-exchange graph and have been the object of recent theoretical work [41]. For the result analogous to Eq. (62) we would have $[M = K, K^*]$

$$A(\bar{B}^0 \to D_s^{(*)} M) = \sqrt{2} N_0^M C_L^{(0)} \frac{16\pi\alpha_s(\mu_0)}{9} s_{\rm eff}^E(\mu_0) \langle x_s^{-1} \rangle_M + A_{\rm long}^{D_s^{(*)} M}. \tag{72}$$

Both the $\bar{B}^0 \to D_s^{(*)+} K^-$ modes and the Cabibbo suppressed decays $\bar{B} \to D^{(*)} \bar{K}^{(*)}$ receive this additional contribution from $A_{\rm long}^{D^{(*)} M}$. This makes the factorization theorem less predictive, and so we do not attempt an analysis of ratios $R_c^{D^{(*)} K^{(*)}}$, $R_0^{M/M'}$, or the universal phases ϕ_E and ϕ_C that are analogous to ϕ in Eq. (61).

On the experimental side both Babar and Belle Collaborations [42] recently observed the $\bar{B}^0 \to D_s^+ K^-$ decay, and set an upper limit on the branching ratio of $\bar{B}^0 \to D_s^{*+} K^-$,

$$B(\bar{B}^0 \to D_s^+ K^-) = \begin{cases} [3.2 \pm 1.0({\rm stat}) \pm 1.0({\rm sys})] \times 10^{-5} & ({\rm Babar}) \\ [4.6^{+1.2}_{-1.1}({\rm stat}) \pm 1.3({\rm sys})] \times 10^{-5} & ({\rm Belle}), \end{cases}$$

$$B(\bar{B}^0 \to D_s^{*+} K^-) \le 2.5 \times 10^{-5} (90\% \ \ {\rm C.L.}) \quad ({\rm Babar}). \tag{73}$$

The branching fraction for $\bar{B}^0 \to D_s^+ K^-$ is an order of magnitude smaller than that for $\bar{B}^0 \to D^0 \pi^0$. This indicates that the W-exchange amplitude $E^{D_s K^-}$ is suppressed relative to $(C-E)^{D\pi}$ and $(V_{ud}/\sqrt{2} V_{us}) C^{D^0 \bar{K}^0}$. In SCET the SU(3) breaking between $\phi_\pi(x)$ and $\phi_K(x)$ is generated by masses in the collinear quark Lagrangian [43]. This causes an asymmetry in the light-cone kaon wave function. This SU(3) violation can be expected to be at most a canonical $\sim$20–30 % effect, which would not account for the observed suppression.

However, there is one important source of potentially larger SU(3) breaking from an enhancement in moments of the light-cone kaon wave function which appear in the short-distance amplitude. This may account for the observed suppression. Basically strange quark mass effects imply a larger SU(3) violation for inverse moments than expected for ϕ_π versus ϕ_K alone, and implies that $\langle x_s^{-1} \rangle_K < \langle x_d^{-1} \rangle_K$. Using the result from QCD sum rules the ratio of moments [40] is $\langle x_d^{-1} \rangle_K / \langle x_s^{-1} \rangle_K \sim 1.4$. Furthermore, we anticipate a similar large effect from the moments that appear in the soft matrix elements which again differ by factors of $(k_d^+)^{-1}$ versus $(k_s^+)^{-1}$, and appear in a way that suppresses $D_s K^-$. The combination of these two suppression factors might accommodate the observed factor of 3 suppression in the $D_s K^-$ amplitudes.[11] The long-distance amplitude also involves two inverse momentum fractions through $\bar{J}^{(0,8)}$ in Eq. (26), although admittedly much less is known about the nonperturbative functions $\Psi_M^{(0,8)}$ and $\Phi_{L,R}^{(0,8)}$. Thus, we find that the suppression of $E^{D_s K^-}$ may not imply much about the relative size of $C^{D\pi}$ and $E^{D\pi}$. Finally, we note that the suppression mechanism for $s\bar{s}$ creation that we have identified is particular to problems involving large energies where light-cone wave functions arise.

Further information on the relative size of the short- and long-distance contributions to the kaon factorization theorem is clearly desirable. In Sec. IV it was noted that in type-II decays transverse K^*'s are produced only by the long-distance contribution at this order in $\Lambda_{\rm QCD}/Q$. Therefore, measuring the polarization of the K^* in both the $\bar{B}^0 \to D_s^* K^{*-}$ and $\bar{B}^0 \to D^{*0} K^{*0}$ decays can give us a direct handle on whether there might be additional dynamical suppression of either the long- or short-distance contributions, or whether they are of similar in size as one might expect *a priori* from the power counting.

VII. SUMMARY AND CONCLUSIONS

We presented in this paper the first model-independent analysis of color suppressed $\bar{B}^0 \to D^{0(*)} M^0$ decays, in the limit of an energetic light meson M^0. The soft-collinear effective theory (SCET) was used to prove a factorization theorem for these decay amplitudes at leading order in $\Lambda_{\rm QCD}/Q$, where $Q = \{m_b, m_c, E_M\}$. Compared with decays into a charged pion these decays are suppressed by a factor $\Lambda_{\rm QCD}/Q$. Therefore, in the effective theory they are produced exclusively by subleading operators.[12]

[11] In general this argument gives a dynamic explanation for the suppression of $s\bar{s}$ popping at large energies which could be tested elsewhere. The production of an $s\bar{s}$ pair which end up in different strange hadrons is likely to be accompanied by a suppression from inverse momentum fractions that arise from the gluon propagator that produced these quarks. This enhances the SU(3) violation in a well-defined direction so that less $s\bar{s}$ pairs are produced. A factor of 3 suppression of $s\bar{s}$ popping is implemented in JETSET [44].

[12] In type-I decays, other subleading operators can compete with the time-ordered products we have identified at the same order in Λ/Q. This makes a complete analysis of power corrections to type-I decays more complicated than our analysis of type-II decays.

We have identified the complete set of subleading operators which contribute to $\bar{B}^0 \to D^{0(*)} M^0$ decays with $M = \pi, \rho, K, K^*$, as well as for the decays $\bar{B}^0 \to D_s^{(*)} K^{(*)-}$. After hard-soft-collinear factorization, their matrix elements are given by (i) a short-distance contribution involving a jet function convoluted with nonperturbative soft distribution functions, and the nonperturbative light-cone meson wave function, and (ii) a long-distance contribution involving another jet function and additional $x_\perp$ dependent nonperturbative functions for the soft B,D, and collinear M. The long-distance contributions were shown to vanish for $M = \pi, \rho$ at lowest order in $\alpha_s(Q)/\pi$.

The factorization formula is given in Eqs. (35) and (37). It may seem surprising that the type-II decays factor into a pion light-cone wave function and a $B \to D^{(*)}$ soft distribution function rather than being like the naive a_2 factorization in Eq. (2). Our results indicate that factorization for type-II decays is similar to factorization for type-I decays (albeit with new nonperturbative soft functions and additional long-distance contributions for kaons). To derive Eq. (35), QCD was first matched onto $\mathrm{SCET_I}$ at the scale $\mu^2 = Q^2$. In $\mathrm{SCET_I}$ it is still possible for gluons to redistribute the quarks. This intermediate theory provides a mechanism for connecting the soft spectator quark in B to a quark in the pion and for connecting the energetic quark produced by the four-quark operator with the soft spectator in D (see Fig. 2). This process is achieved by the power suppressed time-ordered products given in Eq. (21). $\mathrm{SCET_I}$ is then matched onto $\mathrm{SCET_{II}}$ at a scale $\mu_0^2 = E_M \Lambda$. In $\mathrm{SCET_{II}}$ the collinear quarks and gluons are nonperturbative and bind together to make the light meson M. This second stage of matching introduces a new coefficient function (jet functions) as in Eq. (24). The jet function J contains the information about the $\mathrm{SCET_I}$ graphs that move the spectator quarks into the pion. The physics at various scales is neatly encoded in Eq. (35). The Wilson coefficient $C(z)$ from matching QCD onto $\mathrm{SCET_I}$ depends on physics at the scale Q^2, the jet functions $J, \bar{J}$ from matching $\mathrm{SCET_I}$ onto $\mathrm{SCET_{II}}$ depends on $Q\Lambda$ physics, which is where quark redistribution occurs, and finally the soft distribution functions S, Φ and the pion light-cone wave function ϕ_M, Ψ_M depend on nonperturbative physics at Λ^2 which is where the binding of hadrons occur.

The soft functions S are complex and encode information about strong rescattering phases. This information is introduced through Wilson lines along the light meson direction of motion, which exchange soft gluons with the final state meson $D^{(*)}$. They provide a new mechanism which generates nonperturbative strong phases. In the literature other mechanisms which generate perturbative strong phases have been proposed. In particular in Refs. [7,45] a method for identifying perturbative strong phases with an expansion in $\alpha_s(Q^2)$ was developed. In Refs. [38,46] it was pointed out that strong phases can also be generated perturbatively at the intermediate scale $\alpha_s(E_M \Lambda)$. In the language of our factorization theorem in Eq. (35) these phases roughly correspond to imaginary parts in the hard coefficients $C_{L,R}^{(0,8)}$ and jet functions J, respectively. These phases exist, but for the $B \to D\pi$ channels they only show up at next-to-leading order in the $\alpha_s(m_b)$ or $\alpha_s(\mu_0)$ expansion. (In type I $B \to D^{(*)}\pi$ decays the hard strong phase is very small, $\sim 2°$ [7].) In contrast, our new source of strong phases is entirely nonperturbative in origin and can produce unconstrained phases. For the case of $B \to D^{(*)} M$ these phases show up in the power suppressed class-II amplitudes.

The factorization theorem proven in this paper leads to predictions which were tested against existing experimental data on color suppressed decays. We derived two model-independent relations, which related (1) the $\bar{B}^0 \to D^0 M^0$ and $\bar{B}^0 \to D^{*0} M^0$ decay branching fractions and (2) the $\bar{B} \to DM$ and $\bar{B} \to D^* M$ strong phases.

Here $M = \pi, \rho, K, K_\parallel^*$, and these relations are true to all orders in the strong coupling at the collinear scale. The same predictions are also obtained for $\bar{B}^0 \to D_s^{(*)} K^-$ and $\bar{B}^0 \to D_s^{(*)} K_\parallel^{*-}$. The good numerical agreement observed between the strong phases and branching fractions in the $D\pi$ and $D^*\pi$ channels gives strong backing to our results. This prediction can be tested further since the equality of the strong phases for the ρ, K, and $K_\parallel^*$ channels have not yet been tested experimentally.

Additional predictions followed from the factorization theorem by using a perturbative expansion for the jet function, including ($M = \pi, \rho$):

(1) the ratios $|R_c| = |A(\bar{B}^0 \to D^{(*)+} M^-)/A(B^- \to D^{(*)0} \times M^-)|$ to subleading order,

(2) the ratios $|R_0^{\rho/\pi}| = |A(\bar{B}^0 \to D^{(*)0} \rho^0)/A(\bar{B}^0 \to D^{(*)0} \pi^0)|$ to subleading order

(3) universal parameters $\{|s_{\rm eff}|, \phi\}$ which appear for both $D^{(*)}\pi$ and $D^{(*)}\rho$, and

(4) a mechanism for enhanced SU(3) violation in $s\bar{s}$ production for the short-distance amplitude which might explain the suppression of the $\bar{B}^0 \to D_s^{(*)} K^-$ rates relative to $\bar{B}^0 \to D^0 \pi^0$.

For $|R_c|$ taking different values of M with the same isospin the power corrections only differ by the moments $\langle x^{-1} \rangle_M$, giving an explanation for the observed quasiuniversality of these ratios. The isospin triangles for these M's are predicted to involve a universal angle ϕ. The ratio of neutral modes $|R_0^{\rho/\pi}|$ are determined by inverse moments of the light-cone wave functions and decay constants. Finally extractions of the nonperturbative soft moment parameter $s_{\rm eff}$ agrees with the $\sim \Lambda_{\rm QCD}$ size estimated by dimensional analysis.

In the case of $\bar{B}^0 \to D_s^{(*)} K^{(*)-}$ an additional suppression mechanism was identified, which arises from enhanced SU(3) violation due to the asymmetry of nonperturbative distributions involving strange versus down quarks. The inverse moments that appear in the factorization theorem enhances this difference, and can lead to a dynamic suppression of $s\bar{s}$ popping. Further information on the size of the short- and long-distance amplitudes would help in clarifying this observation.

A more detailed experimental study of the channels in Tables I and II is crucial to further test the accuracy of the factorization theorem and improve our understanding of the structure of power corrections. Work on extending these results to decays to isosinglet mesons is in progress. It should be evident that B_s decays could also be considered although we have not done so here.

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APPENDIX A: LONG-DISTANCE CONTRIBUTIONS FOR π AND ρ

The factorization theorem derived in Sec. III for the color suppressed $B^0 \to D^0 M^0$ amplitude contains both short- and long-distance contributions. In this appendix we show that, working at lowest order in the Wilson coefficients at the hard scale Q, the long-distance amplitude vanishes for the case of an isotriplet light mesons $M=\pi,\rho$.

We start by recalling the factorized form of the long-distance amplitude, which is given by SCET$_{\rm II}$ time-ordered products $\overline{T}^{(0,8)}_{L,R}$

$$A^{D^{(*)}M}_{\rm long}=\int_0^1 dz\int dk^+ d\omega\int d^2x_\perp[C_L^{(i)}(z)\,\overline{J}^{(i)}(\omega k^+)\times\Phi_L^{(i)}(k^+,x_\perp,\varepsilon^*_{D*})\Psi_M^{(i)}(z,\omega,x_\perp,\varepsilon^*_M)\pm C_R^{(i)}(z)\,\overline{J}^{(i)}(\omega k^+)\,\Phi_R^{(i)}(k^+,x_\perp,\varepsilon^*_{D*})\times\Psi_M^{(i)}(z,\omega,x_\perp,\varepsilon^*_M)]. \tag{A1}$$

The functions $\Psi_M^{(i)}$ and $\Phi_{L,R}^{(i)}$ are SCET$_{\rm II}$ matrix elements of collinear and soft fields, respectively, and their precise definitions are given in Eqs. (38). The jet functions $\overline{J}^{(i)}(\omega k^+)$ appear in the definition of the subleading soft-collinear Lagrangian $\mathcal{L}^{(1)}_{\xi\xi qq}$ and their lowest order expressions are given in Eq. (26).

In the following we derive a few general properties of the functions $\Psi_M^{(i)}$ and $\overline{J}^{(0,8)}$ following from isospin, charge conjugation, parity, and time reversal. The collinear function $\Psi_M^{(i)}(z,\omega,x_\perp,\varepsilon^*_M)$ is defined as the matrix element

$$\langle M^0(\varepsilon)|[(\overline{\xi}_n^{(d)}W)_{\tau_1}\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(u)})_{\tau_2}](0_\perp)\times[(\overline{\xi}_n^{(u)}W)_\omega\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(d)})_\omega](x_\perp)|0\rangle. \tag{A2}$$

We will prove that $\Psi_{M=\pi,\rho}$ is even under $\omega\to-\omega$ and $z\to1-z$. As motivation consider the first bilinear in Eq. (A2), which creates a $d\bar{u}$ collinear quark pair. The second bilinear in Eq. (A2) must act at some point along the collinear quark lines: it either takes a $d\to u$ (for $\omega>0$) or takes a $\bar{u}\to\bar{d}$ (for $\omega<0$). Examination of lowest order graphs contributing to Ψ_M shows that these two types of contributions always appear in pairs, such that the projection of Ψ_M onto an isotriplet state is even under $\omega\to-\omega$. This suggests the existence of a symmetry argument, valid to all orders in perturbation theory.

We will prove that $\Psi_M^{(0,8)}$ is even, as a consequence of G parity. This is defined as usual by $G=C\exp(-i\pi I_2)$ where C is charge conjugation and I_2 is the isospin generator, and is a symmetry of the collinear Lagrangian in the limit $m_{u,d}\ll\Lambda_{\rm QCD}$. Its action on the collinear operators in Eq. (A2) can be worked out from that of its components C and I_2 (cf. Ref. [28]) and is given by

$$G(\overline{\xi}_n^{(d)}W)_{\tau_1}\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(u)})_{\tau_2}G^\dagger=(\overline{\xi}_n^{(d)}W)_{-\tau_2}\bar{n}\!\!\!/P_R(W^\dagger\xi_n^{(u)})_{-\tau_1},$$

$$G(\overline{\xi}_n^{(u)}W)_\omega\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(d)})_\omega G^\dagger=(\overline{\xi}_n^{(u)}W)_{-\omega}\bar{n}\!\!\!/P_R(W^\dagger\xi_n^{(d)})_{-\omega}. \tag{A3}$$

Taking into account the G parity of the states, Eq. (A2) is equal to

$$\pm\langle M^0(\varepsilon)|[(\overline{\xi}_n^{(d)}W)_{-\tau_2}\bar{n}\!\!\!/P_R(W^\dagger\xi_n^{(u)})_{-\tau_1}](0_\perp)\times[(\overline{\xi}_n^{(u)}W)_{-\omega}\bar{n}\!\!\!/P_R(W^\dagger\xi_n^{(d)})_{-\omega}](x_\perp)|0\rangle, \tag{A4}$$

where $\pm$ refer to the ρ^0 and π^0, respectively. Next, we apply parity in the matrix element followed by switching our basis vectors $n\leftrightarrow\bar{n}$. Acting on Eq. (A4) this gives

$$\langle M^0(\varepsilon_P^*)|[(\overline{\xi}_n^{(d)}W)_{-\tau_2}\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(u)})_{-\tau_1}](0_\perp)\times[(\overline{\xi}_n^{(u)}W)_{-\omega}\bar{n}\!\!\!/P_L(W^\dagger\xi_n^{(d)})_{-\omega}](-x_\perp)|0\rangle, \tag{A5}$$

where the overall sign is now the same for $M=\rho,\pi$. Now since $\Psi_M^{(0,8)}$ is a scalar function the only allowed perpendicular dot products are $(-x_\perp)^2=x_\perp^2$ and $-x_\perp\cdot\varepsilon_P^*=x_\perp\cdot\varepsilon^*$. Finally we note that the change in $\tau_{1,2}$ from Eqs. (A2) to (A5) is equivalent $z\to1-z$. Thus the invariance of SCET$_{\rm II}$ under G parity and regular parity has allowed us to prove that

$$\Psi^{(i)}_{\pi,\rho}(z,\omega,x_\perp,\varepsilon^*)=\Psi^{(i)}_{\pi,\rho}(1-z,-\omega,x_\perp,\varepsilon^*). \tag{A6}$$

Next we prove that $\overline{J}^{(0,8)}(\omega k^+)$ is odd under $\omega\to-\omega$. By reparametrization invariance type-III [47] only the product ωk^+ will appear. Consider applying time reversal plus the interchange $(n\leftrightarrow\bar{n})$ to the SCET$_{\rm II}$ Lagrangian. Since this Lagrangian does not have coefficients that encode decays to highly virtual offshell states, it should be invariant under this transformation. Acting on Eq. (25) this implies that $\overline{J}^{(0,8)}$ must be real,

$$[\overline{J}^{(0,8)}(\omega k^+)]^*=\overline{J}^{(0,8)}(\omega k^+). \tag{A7}$$

At tree level this implies that we should drop $i\epsilon$ in the collinear gluon propagator in matching onto this operator. This was done in arriving at the odd functions $J^{(0,8)}\propto 1/(\omega k^+)$ in Eq. (26). The imaginary part would give a $\delta(\omega k^+)$ and corresponds to cases where the SCET$_{\rm I}$ T product is reproduced by a purely collinear SCET$_{\rm II}$ T product ($k^+=0$), or a purely soft SCET$_{\rm II}$ T product ($\omega=0$). Thus dropping $i\epsilon$ also saves us from double counting.

Now consider what functions can be generated by computing loop corrections to $\overline{J^{(0,8)}}$. By dimensional analysis $\overline{J^{(0,8)}}$ must be proportional to $1/(\omega k^+)$ times a dimensionless function of $\omega k^+/\mu^2$. Since at any order in perturbation theory the matching calculation will involve only massless quarks we can only generate logarithms. Therefore, we must study functions of the form

$$\frac{1}{\omega k^+}\ln^n\!\left(\frac{\omega k^+\pm i\epsilon}{\mu^2}\right). \tag{A8}$$

To demand that only the real part of these functions match onto $\overline{J^{(0,8)}}$ we average them with their conjugates. It is straightforward to check that only terms odd in $\omega\to-\omega$ survive. Thus, all the terms that can correct the form of $\overline{J^{(0,8)}}$ at higher orders in α_s are odd under $\omega\to-\omega$.

Now in Eq. (A1) the integration over ω is from $-\infty$ to ∞, while z varies from 0 to 1. Consider the change of variable $\omega\to-\omega$ and $z\to 1-z$. If $C^{(i)}_{L,R}(z)=C^{(i)}_{L,R}(1-z)$, then under this interchange one of the functions in the integrand is odd ($\overline{J}$) and the other two are even ($C^{(i)}_{L,R}$ and $\Psi^{(i)}_{\pi,\rho}$), so the integral would vanish.

Now if $C^{(i)}_{L,R}(z)$ are kept only to leading order, then they are independent of z and thus unchanged under $z\to 1-z$. So at this order in the $\alpha_s(Q)/\pi$ expansion of $C^{(i)}_{L,R}(z)$ we find $A^{D^{(*)}M}_{\rm long}=0$. This completes the proof of the assertion about the vanishing of the long-distance contributions for $M=\pi,\rho$.

APPENDIX B: HELICITY SYMMETRY AND JET FUNCTIONS

In this appendix we discuss the general structure of the jet functions $J^{(0,8)}(z,x,k_j^+)$ in Eq. (35), which are generated by matching SCET$_{\rm I}$ and SCET$_{\rm II}$ at any order in $\alpha_s(\mu_0)$. In Figs. 2(a,b) this means adding additional collinear gluons which generate loops by attaching to the collinear lines already present (as well as vacuum polarization-type collinear quark, gluon, and ghost loops). Additional collinear loops should also be added to Figs. 2(c,d,e), and the difference at lowest order in λ gives J. Throughout this appendix we continue to drop isosinglet combinations of $\bar\xi_n\cdots\xi_n$. These will also have additional contributions from topologies where the outgoing collinear quarks are replaced by outgoing gluons (through $B_\perp^\mu$ operators).

The leading order collinear Lagrangian has a U(1) helicity spin symmetry for the quarks, see the second reference in [25]. It is defined by a generator h_n that has the quark spin projection along the n direction, which is different from usual definition of helicity as the projection of the spin along its momentum. Unlike QCD, the collinear fields in SCET only allow quarks and antiquarks that move in the n direction. For h_n we have

$$h_n=\tfrac14\epsilon_\perp^{\mu\nu}\sigma_{\mu\nu},\quad h_n^2=1,\quad [h_n,\not n]=[h_n,\bar{\not n}]=0,\quad \{h_n,\gamma_\perp^\sigma\}=0. \tag{B1}$$

After making a field redefinition [26] to decouple ultrasoft gluons the leading order collinear quark Lagrangian is

$$\mathcal{L}^{(0)}_{\xi\xi}=\bar\xi_{n,p'}\left\{in\cdot D_c+i\not D_c^\perp\frac{1}{i\bar n\cdot D_c}i\not D_c^\perp\right\}\frac{\bar{\not n}}{2}\xi_{n,p}, \tag{B2}$$

where iD_c^μ contains only collinear $A^\mu_{n,q}$ gluons. $\mathcal{L}^{(0)}_{\xi\xi}$ is invariant under the transformation $\xi_n\to\exp(i\theta h_n)\xi_n$, $\bar\xi_n\to\bar\xi_n\exp(-i\theta h_n)$. This means that any number of leading order collinear quark interactions preserve the quark helicity h_n. The collinear gluon interactions take $u_n(\uparrow)\to u_n(\uparrow)$, $u_n(\downarrow)\to u_n(\downarrow)$, $v_n(\uparrow)\to v_n(\uparrow)$, $v_n(\downarrow)\to v_n(\downarrow)$, and can also produce or annihilate the quark-antiquark combinations $u_n(\uparrow)v_n(\downarrow)$ or $u_n(\downarrow)v_n(\uparrow)$ (the arrows refer to the helicity of the antiparticles themselves rather than their spinors). For this reason we refer to $\mathcal{L}^{(0)}_{\xi\xi}$ as a $\Delta h_n=0$ operator.

The leading order SCET$_{\rm I}$ operators in Eq. (18) are also unchanged by the h_n transformation and therefore does not change collinear quark helicity. In contrast the operators $\mathcal{L}^{(1)}_{\xi q}$ do generate or annihilate a collinear quark giving $\Delta h_n=\pm 1/2$. However, at tree level we showed in Sec. III that the two graphs in Figs. 2(a,b) match onto an overall $\Delta h_n=0$ operator in SCET$_{\rm II}$ as given in Eqs. (28). Since at higher orders $\mathcal{L}^{(0)}_{\xi\xi}$ will not cause a change in the helicity, they also match onto these same operators, so the structure $\not n\gamma_\perp^\nu$ will not occur. At tree level only the structure $\not n P_L\otimes\bar{\not n}P_L$ appeared in Eq. (28). To rule out the appearance of P_R beyond tree level we note that the weak operator projects onto left-handed collinear fermions, and for the jet function the conservation of helicity in $\mathcal{L}^{(0)}_{\xi\xi}$ implies a conservation of chirality. This leaves us with the desired result.

It is perhaps illustrative to see this more explicitly by looking at the spin structure of the loop graphs. We begin by noting that the spin and color structure in $\bar h^{(c)}_{v'}\cdots h^{(b)}_v$ is unaffected by this second stage of matching. Adding additional collinear attachments only can affect the spin and color structure generated in putting the collinear quark fields and light ultrasoft quark fields together.

Consider how additional gluon attachments effect the spin structures that appear in Figs. 2(a,b). The leading order collinear quark Lagrangian is $\mathcal{L}^{(0)}_{\xi\xi}$ in Eq. (B2). Each attachment of a collinear gluon to a collinear quark lines in the figures generates a $\bar{\not n}/2$ from the vertex and a $\not n/2$ from the quark propagator. These combine to a projector which can be eliminated by commuting them to the right or left to act on the collinear quark spinors, via $(\not n\bar{\not n})/4\ \xi_n=\xi_n$. Therefore, at most we have additional pairs of $\gamma_\perp$'s that appear between the light quark spinors. The aim is to show that just like the tree level calculation in Eq. (42) the resulting operators have spin structure $(\bar{d}\bar{n}\!\!\!/ P_L u)(\bar{\xi}_n \bar{n}\!\!\!/ P_L \xi_n)$.

For the contraction of $T_j^{(0,8)}$ which gives the C topology the spin structure is

$$\left[\overline{u_n^{(d)}}\gamma_\perp^{\mu_1}\gamma_\perp^{\mu_2}\cdots\gamma_\perp^{\mu_{2k-1}}\gamma_\perp^{\mu_{2k}}\left(\frac{\bar{n}\!\!\!/}{2}P_L\right)\gamma_\perp^{\nu_1}\cdots\gamma_\perp^{\nu_{2\ell}}\left(\frac{n\!\!\!/}{2}\gamma_\perp^\alpha\right)u_s^{(u)}\right]$$
$$\times[\overline{u_s^{(d)}}(\gamma_\perp^\beta)\gamma_\perp^{\lambda_1}\cdots\gamma_\perp^{\lambda_{2j}}u_n^{(d)}]$$
$$=[\overline{u_n^{(d)}}\gamma_\perp^{\mu_1}\cdots\gamma_\perp^{\mu_{2k'}}\gamma_\perp^\alpha P_L u_s^{(u)}]$$
$$\times[\overline{u_s^{(d)}}\gamma_\perp^\beta\gamma_\perp^{\lambda_1}\cdots\gamma_\perp^{\lambda_{2j}}u_n^{(d)}]. \tag{B3}$$

In the first line $[(\bar{n}\!\!\!/ /2)P_L]$ comes from $\mathcal{Q}_{L,R}^{(0,8)}$, the $\gamma_\perp^\alpha$ and $\gamma_\perp^\beta$ are terms generated by the $\mathcal{L}_{\xi q}^{(1)}$ insertions, and the $n\!\!\!/ /2$ is from the extra collinear quark propagator. In the second line the P_L projector was moved next to $u_s^{(u)}$ without a change of sign (for anticommuting γ_5), and the remaining $\bar{n}\!\!\!/$ and $n\!\!\!/$ were then moved next to the $\overline{u_n^{(d)}}$ and canceled. The remaining free $\perp$ indices in the second line are contracted with each other in some manner. Fierzing the set of γ matrices in Eq. (B3) by inserting $1\otimes 1$ next to the collinear spinors gives

$$[\overline{u_n^{(d)}}\Gamma_1 u_n^{(d)}][\overline{u_s^{(d)}}\gamma_\perp^\beta\gamma_\perp^{\lambda_1}\cdots\gamma_\perp^{\lambda_{2j}}\Gamma_1'\gamma_\perp^{\mu_1}\cdots\gamma_\perp^{\mu_{2k'}}\gamma_\perp^\alpha P_L u_s^{(u)}], \tag{B4}$$

where

$$\Gamma_1\otimes\Gamma_1' = \frac{\bar{n}\!\!\!/}{2}\otimes n\!\!\!/ - \frac{\bar{n}\!\!\!/\gamma_5}{2}\otimes n\!\!\!/\gamma_5 - \frac{\bar{n}\!\!\!/\gamma_\perp^\nu}{2}\otimes n\!\!\!/\gamma_\nu^\perp$$
$$\to \frac{\bar{n}\!\!\!/}{2}(1-\gamma_5)\otimes n\!\!\!/ - \frac{\bar{n}\!\!\!/\gamma_\perp^\nu}{2}\otimes n\!\!\!/\gamma_\nu^\perp. \tag{B5}$$

In the second line of Eq. (B5) we have used the fact that γ_5 in the bracket with soft quark spinors can be eliminated by moving it next to P_L. To eliminate the $n\!\!\!/\gamma_\perp^\nu$ Dirac structure we note that between the soft spinors in Eq. (B4) there are an odd number of $\gamma_\perp$'s to the left and right of $n\!\!\!/\gamma_\nu^\perp$, and so at least one set of indices are contracted between the sets $\{\beta,\lambda_1,\ldots,\lambda_{2j}\}$ and $\{\mu_1,\ldots,\mu_{2k'},\alpha\}$. The identity $\{\gamma_\perp^\sigma,\gamma_\perp^\tau\}=2g_\perp^{\sigma\tau}$ can be used to move these matrices so that they sandwich $\gamma_\nu^\perp$, and this gives the product $\gamma_\perp^\mu\gamma_\nu^\perp\gamma_\mu^\perp=0$. After these manipulations only the spin structure $(\bar{d}\bar{n}\!\!\!/ P_L u)(\bar{\xi}_n\bar{n}\!\!\!/ P_L\xi_n)$ remains. A similar argument can be applied to the E topology with the same result.

In several places in the above argument we made use of Dirac algebra that is particular to four dimensions (anticommuting γ_5 and setting $\gamma_\perp^\mu\gamma_\nu^\perp\gamma_\mu^\perp=0$). If the $\gamma_\perp$'s are taken in full dimensional regulation, then it is not *a priori* clear if the manipulations survive regulation. However, the original helicity symmetry argument shows that as long as the theory can be regulated in a way that preserves this symmetry this will indeed be the case.

APPENDIX C: PROPERTIES OF SOFT DISTRIBUTION FUNCTIONS

In this appendix we derive some useful properties of the soft functions $S^{(0,8)}$. In particular we show that these functions are complex. The imaginary parts have a direct interpretation as nonperturbative contributions to final state rescattering between the $D^{(*)}$ and final energetic meson as discussed in Sec. III.

To be definite we consider the function $S_L^{(0)}$ and suppress the index L. The manipulations for the remaining soft functions $S_R^{(0)}$ and $S_{L,R}^{(8)}$ are identical. The definition in Eq. (34) is

$$\langle D^0(v')|(\overline{h_{v'}^{(c)}}S)n\!\!\!/ P_L(S^\dagger h_v^{(b)})(\bar{d}S)_{k_1^+}n\!\!\!/ P_L(S^\dagger u)_{k_2^+}|\bar{B}^0(v)\rangle$$
$$=S^{(0)}, \tag{C1}$$

where the Wilson lines are defined as

$$W=\left[\sum_{\text{perms}}\exp\left(-\frac{g}{\bar{\mathcal{P}}}\bar{n}\cdot A_{n,q}(x)\right)\right],$$

$$S=\left[\sum_{\text{perms}}\exp\left(-g\frac{1}{n\cdot\mathcal{P}}n\cdot A_{s,q}\right)\right]. \tag{C2}$$

In general $S^{(0)}$ is a dimensionless function of $v\cdot v'$, $n\cdot v$, $n\cdot v'$, $n\cdot k_1$, $n\cdot k_2$, $\Lambda_{\rm QCD}$, and μ. Since $(S^\dagger q)_{k_2^+}=\delta(k_2^+ - n\cdot\mathcal{P})(S^\dagger q)$ the LHS is invariant under a type-III reparametrization transformation [47] $(n\to e^\alpha n,\ \bar{n}\to e^{-\alpha}\bar{n})$. Therefore the RHS can only be a function of w, $t=n\cdot v/n\cdot v'$, $z=n\cdot k_1/n\cdot k_2$, $K/\mu=[n\cdot k_1\, n\cdot k_2/(n\cdot v\, n\cdot v'\mu^2)]^{1/2}$, and $\Lambda_{\rm QCD}/\mu$.

Rather than studying the matrix element in Eq. (C1) directly it is useful to instead consider

$$\langle H_i(v')|(\overline{h_{v'}}S)n\!\!\!/ P_L(S^\dagger h_v)(\bar{q}S)_{k_1^+}n\!\!\!/ P_L\tau^a(S^\dagger q)_{k_2^+}|H_j(v)\rangle$$
$$=S^{(0)}\left(t,z,v\cdot v',\frac{K}{\mu},\frac{\Lambda_{\rm QCD}}{\mu}\right)(\tau^a)_{ij}, \tag{C3}$$

where h_v are doublet fields under heavy quark flavor symmetry, and q and $|H_{i=1,2}(v)\rangle$ are isospin doublets of (u,d). The last three variables in Eq. (C3) will not play a crucial role, so we will suppress this dependence. Taking the complex conjugate of Eq. (C3) gives

$$\langle H_j(v)|(\overline{h_v}S)n\!\!\!/ P_L(S^\dagger h_{v'})(\bar{q}S)_{k_2^+}n\!\!\!/ P_L\tau^a(S^\dagger q)_{k_1^+}|H_i(v')\rangle$$
$$=[S^{(0)}(t,z)]^*(\tau^a)_{ji}=S^{(0)}\left(\frac{1}{t},\frac{1}{z}\right)(\tau^a)_{ji}. \tag{C4}$$

The dependence on w and K is unchanged since they are even under the interchange $v\leftrightarrow v'$, $n\cdot k_1\leftrightarrow n\cdot k_2$. Next, decompose the functions $S^{(0)}$ in terms of even and odd functions under $t\to 1/t$, $z\to 1/z$:

$$S^{(0)} = S_E^{(0)} + S_O^{(0)}, \tag{C5}$$

where $S_{E,O}^{(0)} = [S^{(0)}(t,z) \pm S^{(0)}(1/t,1/z)]/2$. Now Eq. (C4) implies that

$$[S_E^{(0)}(t,z)]^* = S_E^{(0)}(t,z), \quad [S_O^{(0)}(t,z)]^* = -S_O^{(0)}(t,z) \tag{C6}$$

so $S_E^{(0)}$ is real and $S_O^{(0)}$ is imaginary. An identical argument for $S^{(8)}$ implies that it too is a complex function.

For the above analysis it is important to note that $n \cdot v' = m_B/m_D$ is not 1 in the heavy quark limit where we have new spin and flavor symmetries. These symmetries arise from taking $m_B \gg \Lambda_{\mathrm{QCD}}$ and $m_D \gg \Lambda_{\mathrm{QCD}}$, not from having $m_B = m_D$.

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