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Study of $B \to D^{**}\pi$ decays

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We investigate the production of the novel P-wave mesons D_0^* and $D_1'(D_1)$, identified as $J^P = 0^+$ and 1^+, in heavy B meson decays, respectively. With the heavy quark limit, we give our modeling wave functions for the scalar meson D_0^*. Based on the assumptions of color transparency and the factorization theorem, we estimate the branching ratios of $B \to D_0^* \pi$ decays in terms of the obtained wave functions. Some remarks on $D_1^{(\prime)}$ production are also presented.

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I. INTRODUCTION

It is doubtless that the quark model provides a successful method to describe hadron spectroscopy. For instance, based on SU(3) flavor symmetry, a quark and antiquark can comprise octet and singlet states, called a nonet together, such as the well known pseudoscalar mesons of the pion, kaon, and eta with $J^{PC} = 0^{-+}$. However, if we apply the same concept to the light scalar mesons described by 0^{++}, such as the nonet composed of isoscalars $\sigma(600)$ and $f_0(980)$, isovector $a_0(980)$ and isodoublet κ, there are some puzzles: (a) Why are $a_0(980)$ and $f_0(980)$ degenerate in mass? (b) Why are the widths of σ and κ broader than those of $a_0(980)$ and $f_0(980)$ [1]? It is probable that these scalar states consist of four quarks rather than two quarks [2]. Moreover, the possibilities of $K\bar{K}$ molecular states, gluonium states, and scalar glueballs are also proposed. It is clear that the conclusion is still uncertain.

Now, the mysterious event happens not only in the light scalar meson system, but also in the heavy $c\bar{s}$ one. Recently, the BABAR Collaboration has observed one narrow state, denoted by $D_{sJ}^*(2317)$, from a $D_s^+\pi^0$ mass distribution [3]. Furthermore, the same state has been confirmed by CLEO and a new state $D_{sJ}^*(2463)$ is also observed in the $D_s^{*+}\pi^0$ final state [4]. Finally, BELLE verifies the observations [5]. By the data analysis, $D_{sJ}^*(2317)$ and $D_{sJ}^*(2463)$ are identified as parity-even states with $\mathbf{0}^+$ and $\mathbf{1}^+$, respectively. From the observations, the interesting problem is that the states of $D_{sJ}^*(2317)$ and $D_{sJ}^*(2463)$ cannot match with theoretical predictions [6], i.e., the masses (widths) are too low (narrow). To explain the discrepancy, either the theoretical models have to be modified [7] or the observed states are the new composed states. To satisfy the latter, many interesting solutions have been suggested recently in Refs. [8–14].

In fact, before the BABAR's observation, CLEO [15] and BELLE [16] measured the similar states in the $c\bar{q}$ ($q = u,d$) system in heavy B meson decays. With a two-quark picture, there are four parity-even (angular momentum $\ell = 1$) states described by $J^P = 0^+$, 1^+, 1^+, and 2^+, respectively. $J = j_q + S_Q$ is the total angular momentum of the corresponding meson and consists of the angular momentum of the light quark, j_q, and the spin of the heavy quark, S_Q, where $j_q = S_q + \ell$ is combined by the spin and orbital angular momenta of the light quark. In the literature, they are usually labeled by D_0^*, D_1', D_1, and D_2^*, respectively. We also use D^{**} to denote all of them. The first two belong to $j_q = 1/2$, while the last two $j_q = 3/2$. In the heavy quark limit, it is known that $D_{0(1)}^{*(\prime)}$ and $D_{1(2)}^{(*)}$ decay only via S and D waves, respectively. Therefore one expects that the widths of the former are much broader than those of the latter, which is consistent with the observations of CLEO and BELLE [15,16]. Even the BELLE's updated data [17] also show the same phenomenon. We now summarize the results of CLEO and BELLE as follows: in CLEO, the masses (widths) of P-wave states are given by $m_{D_1}(\Gamma_{D_1}) = 2422.0 \pm 2.1$ $(18.9^{+4.6}_{-3.5})$ and $m_{D_2^*}(\Gamma_{D_2^*}) = 2458.9 \pm 2.0$ (23 ± 5) MeV, and the measured branching ratios (BRs) of B decays are given as

$$BR(B^- \to D_1^0 \pi^-) \times BR(D_1^0 \to D^{*+}\pi^-) = (7.8 \pm 1.9) \times 10^{-4},$$

$$BR(B^- \to D_2^{*0} \pi^-) \times BR(D_2^{*0} \to D^{*+}\pi^-) = (4.2 \pm 1.7) \times 10^{-4}. \quad (1)$$

In BELLE, the four states are all measured as $m_{D_0^*}(\Gamma_{D_0^*}) = 2308 \pm 17 \pm 15 \pm 28$ $(276 \pm 21 \pm 18 \pm 60)$, $m_{D_1'}(\Gamma_{D_1'}) = 2427.0 \pm 26 \pm 20 \pm 15$ $(384^{+107}_{-75} \pm 24 \pm 70)$, $m_{D_1}(\Gamma_{D_1}) = 2421.4 \pm 1.5 \pm 0.4 \pm 0.8$ $(23.7 \pm 2.7 \pm 0.2 \pm 4.0)$, and $m_{D_2^*}(\Gamma_{D_2^*}) = 2461.6 \pm 2.1 \pm 0.5 \pm 3.3$ $(45.6 \pm 4.4 \pm 6.5 \pm 1.6)$ MeV, and the BRs with possible decaying chain being

$$BR(B^- \to D_0^{*0} \pi^-) \times BR(D_0^{*0} \to D^{+}\pi^-) = (6.1 \pm 0.6 \pm 0.9 \pm 1.6) \times 10^{-4},$$

$$BR(B^- \to D_1'^{0} \pi^-) \times BR(D_1'^{0} \to D^{*+}\pi^-) = (5.0 \pm 0.4 \pm 1.0 \pm 0.4) \times 10^{-4},$$

$$BR(B^- \to D_1^{0} \pi^-) \times BR(D_1^{0} \to D^{*+}\pi^-) = (6.8 \pm 0.7 \pm 1.3 \pm 0.3) \times 10^{-4}, \quad (2)$$

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$$BR(B^-\to D_2^{*0}\pi^-)\times BR(D_2^{*0}\to D^+\pi^-)$$
$$=(3.4\pm0.3\pm0.6\pm0.4)\times10^{-4},$$

$$BR(B^-\to D_2^{*0}\pi^-)\times BR(D_2^{*0}\to D^{*+}\pi^-)$$
$$=(1.8\pm0.3\pm0.3\pm0.2)\times10^{-4},$$

respectively.

Since the masses of D_{sJ}^* are just below the $D^{(*)}K$ threshold and the corresponding widths are around few 10 KeV [11], both parity-even mesons could only decay through isospin violating channels to $D\pi$ and $D^*\pi$. For this reason, it becomes the main problem how to explain the low masses and narrow widths for D_{sJ}^* states. Unlike D_{sJ}^* cases, however, there are no any suppressions on their decays to $D\pi$ or $D^*\pi$ although the measured masses of D^{**} are slightly different from the predictions of theoretical models. It is believed that the properties of D^{**} could be described by current theoretical models with some improvements. If so, based on the concept of the normal quark model, we could further understand the nature of D^{**} in B decays.

To handle the hadronic effects for $B\to D^{**}\pi$ decays, we use the factorization formalism, called the perturbative QCD (PQCD) approach, which is based on factorization theorem and the transition matrix element, described by the convolution of hadron wave functions and the hard kernel [18,19]. The wave functions in principle can be extracted by experimental data or determined by QCD sum rules or lattice calculations. The hard kernel is related to the hard gluon exchange and high energetic fermion propagator, which are all calculable perturbatively. At the limit of heavy quark, in order to guarantee that the color transparency mechanism [20] is satisfied, i.e., no soft gluon exchange between the final states, we need the hierarchy of $m_B \gg m_{D^{**}} \gg \bar{\Lambda}$ with $\bar{\Lambda} \sim m_B - m_b \sim m_{D^{**}} - m_c$ [21]. It has been shown by Ref. [22] that with the same QCD approach, the calculated results on $B\to D\pi$ decays are consistent with the current observations [23]. Consequently, one expects that PQCD could be also applied to the P-wave meson production in B decaying processes. By the study, we should know more properties related to P-wave mesons.

The paper is organized as follows. We investigate the characteristic of D^{**} and model their amplitude distributions in Sec. II. In terms of the PQCD approach, we derive the factorization formulas for each $B\to D_0^*\pi$ decays in Sec. III. We present our results in Sec. IV. Finally, we give a summary in Sec. V.

II. DECAY CONSTANTS AND WAVE FUNCTIONS OF D^{**}

In order to study the production of the scalar meson D^{**} in B decays, we immediately have to face two questions. The first one is how to write down the hadronic structures and model the wave functions of D^{**}, and the second is what the values of decay constants of D^{**} are. In PQCD, since the wave functions belong to nonperturbative objects and are universal, we can directly apply the wave functions of B and π mesons, which have been discussed in other B-meson decays, such as $B\to\pi\ell\nu$, $B\to\pi\pi$, etc. Therefore the hadronic structures of B and π mesons can be described by [24,25]

$$\langle 0|\bar{b}(0)_j d(z)_l|B,p_1\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{-ixp_1\cdot z} \times\{[\not p_1 + m_B]_{lj}\gamma_5\Phi_B(x)\},$$

$$\langle 0|\bar{u}(0)_j d(z)_l|\pi,p_3\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{-ixp_3\cdot z} \times[[\not p_3]_{lj}\Phi_\pi(x) + m_\pi^0[I]_{lj}\Phi_\pi^p(x) + m_\pi^0[\not n_- \not n_+ - I]_{lj}\Phi_\pi^\sigma(x)], \quad (3)$$

where $n_-=(0,1,\vec{0}_T)$, $n_+=(1,0,\vec{0}_T)$, and x is the momentum fraction of the light parton inside the corresponding meson. Φ_π and $\Phi_\pi^{p(\sigma)}$ are the twist-2 and twist-3 pion wave functions, related to the distribution amplitude of nonlocal operator $\bar{u}\gamma_5\gamma_\mu d(z)$ and associated with $\bar{u}\gamma_5 d(z)$ [$\bar{u}\gamma_5\sigma_{\mu\nu}d(z)$], respectively. We note that m_π^0 is the so-called chiral symmetry breaking parameter and is equivalent to $m_\pi^2/(m_u+m_d)$.

To determine the structures and distribution amplitudes of D^{**}, we need to know their properties. For simplicity, we only concentrate the discussion on the scalar meson of D_0^*. The similar analysis can be applied to other charmed P-wave mesons. As usual, the decay constant of D_0^* is defined as

$$\langle 0|\bar{d}c|D_0^*,p_2\rangle = m_{D_0^*}\tilde{f}_{D_0^*}. \quad (4)$$

By using the equation of motion, we obtain another identity

$$\langle 0|\bar{d}\gamma_\mu c|D_0^*,p_2\rangle = f_{D_0^*}p_{2\mu}, \quad (5)$$

with $f_{D_0^*}=\tilde{f}_{D_0^*}(m_c-m_d)/m_{D_0^*}$, in which $m_{c(d)}$ are the current quark mass of $c(d)$ quark. From the above equation, we see that if the considering case is a light scalar meson, the corresponding transition matrix element will become small. This is the reason why the decay constant of the light scalar meson for the vector current is small. In order to satisfy the conditions of Eqs. (4) and (5), the hadronic structure of D_0^* is adopted to be

$$\langle D_0^*,p_2|\bar{d}(0)_j c(z)_l|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixp_2\cdot z}\{[\not p_2]_{lj}\Phi_{D1}(x) + m_{D_0^*}[I]_{lj}\Phi_{D2}(x)\}, \quad (6)$$

with the normalizations

$$\int_0^1 \Phi_{D1}(x) = \frac{f_{D_0^*}}{2\sqrt{2N_c}}, \quad \int_0^1 \Phi_{D2}(x) = \frac{\tilde{f}_{D_0^*}}{2\sqrt{2N_c}}.$$

The value of the decay constant $\tilde{f}_{D_0^*}$ is the crucial part for concerning whether D_0^* production is interesting or not. To estimate the magnitude of $\tilde{f}_{D_0^*}$, we need help with the scalar meson $K_0^*(1420)$, for which the decay constant has already been estimated in Ref. [26]. As mentioned earlier, the scalar meson generally satisfies the identity

$$(m_{q_1}-m_{q_2})\langle 0|\bar{q}_2 q_1|S\rangle = f_S m_S^2,$$

where m_{q_i}, m_S, and f_S are the current quark mass, the S-meson mass, and its decay constant of vector current, respectively. If we assume $\langle 0|\bar{d}s|K_0^*(1420)\rangle \approx \langle 0|\bar{d}c|D_0^*\rangle$, from the above equation, we can obtain $f_{D_0^*} = f_{K_0^*}\cdot m_{K_0^*}^2/m_{D_0^*}^2\cdot(m_c-m_d)/(m_s-m_d)$. With the values of $f_{K_0^*}\sim 34$ MeV [26], $m_c=1.5$ GeV, $m_s=150$, and $m_d=8.7$ MeV, we get $f_{D_0^*}\approx 130$ MeV. This value is close to the result in Ref. [27], calculated by the relativistic quark model. Finally, from Eq. (5) we have $\tilde{f}_{D_0^*}=m_{D_0^*}/(m_c-m_d)\cdot f_{D_0^*}\approx 200$ MeV. It is known that K_0^* is composed of a two-quark state. Thus it is interesting to have the similar decay constants between the scalar D_0^* and pseudoscalar D_s.

To obtain the shapes of D_0^* wave functions qualitatively, we need to employ the concept of the heavy quark limit. According to Eq. (6), we see that $\Phi_{D1}(x)$ is the distribution amplitude of the nonlocal operator $\bar{d}\gamma_\mu c(z)$ while $\Phi_{D2}(x)$ is associated with $\bar{d}c(z)$. By the equation of motion, we straightforwardly find that the difference between $\Phi_{D1}(x)$ and $\Phi_{D2}(x)$ is of the order of $\bar{\Lambda}/m_{D_0^*}\sim(m_{D_0^*}-m_c)/m_{D_0^*}$. Hence, if we set $m_{D_0^*}\sim m_c$, we can get the information of $\Phi_{D1}(x)\sim\Phi_{D2}(x)$. Furthermore, in order to satisfy the identities of decay constants defined by Eqs. (4) and (5), the simplest forms for both wave functions can be modeled by $\Phi_{Di}\propto x(1-x)+a_i x(1-x)(1-2x)$ in which a_i are free parameters. Since the second term is antisymmetric while x is replaced by $1-x$, we can easily conclude that this term will not change the normalization of the wave function. Therefore we could use it to control the shapes of the wave function. It is worth mentioning that since we consider D_0^* to be a P-wave state, the size of D_0^* is believed to be bigger than that of the particle in the S-wave state. In order to avoid that D_0^* becomes oversize such that the mechanism of color transparency is breakdown, like the b-dependence on the wave function of the B meson, in which b is the conjugate variable of the parton transverse momentum, we also introduce the intrinsic b-dependence on D_0^*. To satisfy Eqs. (4) and (5), the final simplest shapes of the wave functions are expressed as

$$\Phi_{D1}(x,b)=\frac{\tilde{f}_{D_0^*}}{2\sqrt{2N_c}}\left\{6x(1-x)\left[\frac{m_c-m_d}{m_{D_0^*}}+a_{D_0^*}(1-2x)\right]\right\}\times\exp\left[-\frac{\omega_{D_0^*}^2 b^2}{2}\right],$$

$$\Phi_{D2}(x,b)=\frac{\tilde{f}_{D_0^*}}{2\sqrt{2N_c}}\{6x(1-x)[1+b_{D_0^*}(1-2x)]\}\times\exp\left[-\frac{\omega_{D_0^*}^2 b^2}{2}\right], \tag{7}$$

where $\omega_{D_0^*}$, $a_{D_0^*}$, and $b_{D_0^*}$ are the unknown parameters. Although $b_{D_0^*}$ is a free parameter, it can be chosen such that the D_0^* meson wave function has the maximum at $x\approx(m_{D_0^*}-m_c)/m_{D_0^*}\sim 0.35$ for $m_c=1.5$ GeV. As to the value of $a_{D_0^*}$, we refer to the case of $K_0^*(1410)$ [28]. By assuming that $a_{D_0^*}m_{D_0^*}/(m_c-m_d)\sim a_{K_0^*}m_{K_0^*}/(m_s-m_d)\approx 75/f_{K_0^*}$, the order of magnitude of $a_{D_0^*}$ is estimated to be around 1.2.

III. FACTORIZATION FORMULAS

Since the considered decays $B\to D^{**}\pi$ correspond to the $b\to c\bar{u}d$ transition, we describe the effective Hamiltonian as

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}V_c[C_1(\mu)\bar{d}_\alpha u_\beta \bar{c}_\beta b_\alpha + C_2(\mu)\bar{d}_\alpha u_\alpha \bar{c}_\beta b_\beta], \tag{8}$$

where $\bar{q}_\alpha q_\beta=\bar{q}_\alpha\gamma_\mu(1-\gamma_5)q_\beta$, $\alpha(\beta)$ are the color indices, $V_c=V_{ud}^* V_{cb}$ is the product of the CKM matrix elements [30], and $C_{1,2}(\mu)$ are the Wilson coefficients (WCs) [31]. With the light-cone coordinate, the momenta of various mesons and the light valence quarks inside the corresponding mesons are assigned as: $p_1=m_B/\sqrt{2}(1,1,\vec{0}_T)$, $k_1=m_B/\sqrt{2}(x_1,0,\vec{k}_{1T})$; $p_2=m_B/\sqrt{2}(1,r_2^2,\vec{0}_T)$, $k_2=m_B/\sqrt{2}(x_2,0,\vec{k}_{2T})$; $p_3=m_B/\sqrt{2}(0,1-r_2^2,\vec{0}_T)$, $k_3=m_B/\sqrt{2}[0,(1-r_2^2)x_3,\vec{k}_{3T}]$, with $r_2=m_{D_{dJ}^*}/m_B$. As usual, we use

$$\Gamma=\frac{G^2 P_c m_B^2}{16\pi}|V_c|^2|\mathcal{A}|^2 \tag{9}$$

to describe the decay rates of $B\to D^{**}\pi$, in which $P_c\equiv|p_{2z}|=|p_{3z}|\approx m_B(1-r_2^2)/2$ is the momentum of the outgoing meson, $\mathcal{A}$ is the decay amplitude and its value depends on QCD approaches. Since the hadronic structures of the tensor meson have not been derived yet and so far they are not definite, we study the problem elsewhere. Although D_1' and D_1 are the vector mesons and carry the spin degrees of freedom, only longitudinal polarization has the contribution since one of the final states is a pseudoscalar. Therefore the deriving formulas for $B\to D_0^*\pi$ are also proper to the final states with one vector and one pseudoscalar meson. In this paper, we only concentrate on the production of D_0^*.

In terms of the effective interactions, we see that different decaying processes involve different topologies. To be more clear, in the following we analyze each of the $B\to D_0^*\pi$ decays separately.

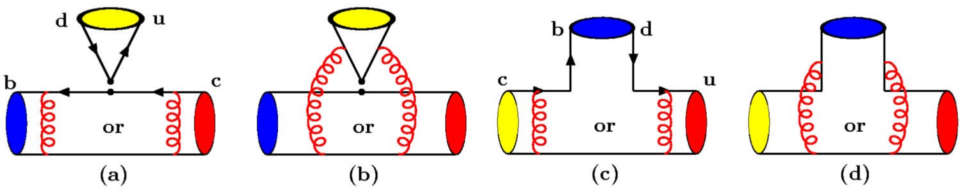

FIG. 1. The topologies (a)[(c)] factorizable emission [annihilation] and (b), (d) nonfactorizable effects for the decays $B_d \to D_0^{*-}\pi^+$.

A. $B_d\to D_0^{*-}\pi^+$ decay

There are two topologies in this decay, emission and annihilation diagrams. The former is color-allowed but the latter belongs to color-suppressed. The corresponding flavor diagrams are illustrated by Fig. 1. Hence the decay amplitude of $B_d\to D_0^{*-}\pi^+$ can be expressed by

$$\mathcal{A}(B_d\to D_0^{*-}\pi^+) = f_\pi F_{BD_0^*} + M_{BD_0^*} + f_B F_a + M_a , \tag{10}$$

where f_π and f_B are the decay constants of π and B mesons and the related contributions are the factorizable emission and annihilation topologies, respectively. The remains denote nonfactorizable contributions. With factorization theorem and hadronic structures of Eqs. (3) and (6), the hard amplitudes $\{F\}$ and $\{M\}$ are formulated as

$$\begin{aligned} F_{BD_0^*} = {} & 8\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \Phi_B(x_1,b_1) \\ & \times\{\Phi_{D1}(x_2,b_2)\mathcal{E}^1_{D_0^*}(t_e^{(1)}) h_e(x_1,x_2,b_1,b_2) \\ & +[r_c\Phi_{D1}(x_2)+2r_2\Phi_{D2}(x_2)] \\ & \times\mathcal{E}^2_{D_0^*}(t_e^{(2)}) h_e(x_2,x_1,b_2,b_1)\}, \end{aligned} \tag{11}$$

$$\begin{aligned} M_{BD_0^*} = {} & 16\pi C_F m_B^2 \sqrt{2N_c} \int_0^1 dx_1 dx_2 dx_3 \\ & \times \int_0^\infty b_1 db_1 b_3 db_3 \Phi_B(x_1,b_1)\Phi_\pi(x_3)\Phi_{D1}(x_2,b_1) \\ & \times\{-(x_2+x_3)\mathcal{E}'^1_{D_0^*}(t_d^{(1)}) h_d^{(1)}(\{x\},b_1,b_3) \\ & +(1-x_3)\mathcal{E}'^2_{D_0^*}(t_d^{(2)}) h_d^{(2)}(\{x\},b_1,b_3)\}, \end{aligned} \tag{12}$$

$$\begin{aligned} F_a = {} & 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 db_3 \Phi_{D1}(x_2,b_2)\Phi_\pi(\zeta) \\ & \times\{-x_3\mathcal{E}_a^1(t_a^{(1)}) h_a(x_2,x_3\eta_3,b_2,b_3) \\ & +x_2\mathcal{E}_a^2(t_a^{(2)}) h_a(x_3,x_2\eta_3,b_3,b_2)\}, \end{aligned} \tag{13}$$

$$\begin{aligned} M_a = {} & 16\pi C_F m_B^2 \sqrt{2N_c} \int_0^1 d[x] \int_0^\infty [b] d[b] \\ & \times\Phi_B(x_1,b_1)\Phi_{D1}(x_2,b_2)\Phi_\pi(\zeta) \\ & \times\{x_3\mathcal{E}'^1_a(t_f^{(1)}) h_f^{(1)}(\{x\},b_1,b_2) \\ & -x_2\mathcal{E}'^2_a(t_f^{(2)}) h_f^{(2)}(\{x\},b_1,b_2)\}. \end{aligned} \tag{14}$$

The hard functions $h_{e(d,a,f)}$, related to the propagators of exchange hard gluon and internal quark, are described by

$$\begin{aligned} & h_e(x_1,x_2,b_1,b_2) \\ & \quad = S_t(x_2) K_0(\sqrt{x_1x_2} m_B b_1) \\ & \qquad \times[\theta(b_1-b_2)K_0(\sqrt{x_2}m_B b_1) I_0(\sqrt{x_2}m_B b_2) \\ & \qquad +\theta(b_2-b_1)K_0(\sqrt{x_2}m_B b_2) I_0(\sqrt{x_2}m_B b_1)], \end{aligned}$$

$$\begin{aligned} & h_d^{(j)}(x_1,x_2,x_3,b_1,b_2) \\ & \quad = [\theta(b_1-b_2)K_0(\sqrt{x_1x_2}m_B b_1) I_0(\sqrt{x_1x_2}m_B b_2) \\ & \qquad +\theta(b_2-b_1)K_0(\sqrt{x_1x_2}m_B b_2) I_0(\sqrt{x_1x_2}m_B b_1)] \\ & \qquad \times\begin{pmatrix} K_0(D_j m_B b_2) & \text{for } D_j^2\geqslant 0 \\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|D_j^2|}m_B b_2) & \text{for } D_j^2\leqslant 0 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} & h_a(x_2,x_3,b_2,b_3) \\ & \quad = S_t(x_3)\left(i\frac{\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_2x_3}m_B b_2) \\ & \qquad \times[\theta(b_2-b_3)H_0^{(1)}(\sqrt{x_3}m_B b_2) J_0(\sqrt{x_3}m_B b_3) \\ & \qquad +\theta(b_3-b_2)H_0^{(1)}(\sqrt{x_3}m_B b_3) J_0(\sqrt{x_3}m_B b_2)], \end{aligned}$$

$$\begin{aligned} h_f^{(j)}(\{x\},b_1,b_2) = {} & i\frac{\pi}{2}[\theta(b_1-b_2)H_0^{(1)}(\sqrt{x_2x_3\eta_3}m_B b_1) \\ & \times J_0(\sqrt{x_2x_3\eta_3}m_B b_2) \\ & +\theta(b_2-b_1)H_0^{(1)}(\sqrt{x_2x_3\eta_3}m_B b_2) \\ & \times J_0(\sqrt{x_2x_3\eta_3}m_B b_1)] \\ & \times\begin{pmatrix} K_0(F_j m_B b_1) & \text{for } F_j^2\geqslant 0 \\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|F_j^2|}m_B b_1) & \text{for } F_j^2\leqslant 0 \end{pmatrix}, \end{aligned}$$

with $D_1^2 = x_1x_2 - x_2x_3\eta_3$, $D_2^2 = x_1x_2 - x_2(1-x_3)\eta_3$, $F_1^2 = (x_1-x_2)x_3\eta_3$, $F_2^2 = x_1+x_2+(1-x_1-x_2)x_3\eta_3$, $\eta_3 = (1-r_2^2)$, and $r_c = m_c/m_B$. The threshold resummation effect is expressed to be $S_t(x) = 2^{1+2c}\cdot\Gamma(3/2+c)[x(1-x)]^c/[\sqrt{\pi}\Gamma(1+c)]$, with $c\approx 0.35$ [32]. The evolution factors $\mathcal{E}^i_{D_0^*}(\mathcal{E}'^i_{D_0^*})$ and $\mathcal{E}^i_a(\mathcal{E}'^i_a)$ are defined by

$$\mathcal{E}^i_{D_0^*}(t_e^{(i)}) = \left(C_2(t_e^{(i)}) + \frac{C_1(t_e^{(i)})}{N_c}\right)\alpha_s(t_e^{(i)})\exp[-S_B - S_{D_0^*}],$$

$$\mathcal{E}'^{i}_{D_0^*}(t_d^{(i)})=\frac{C_1(t_d^{(i)})}{N_c}\alpha_s(t_d^{(i)})\exp[-S_B-S_{D_0^*}-S_\pi]_{b_2=b_1},$$

$$\mathcal{E}^{i}_{a}(t_a^{(i)})=\left(C_1(t_a^{(i)})+\frac{C_2(t_a^{(i)})}{N_c}\right)\alpha_s(t_a^{(i)})\times\exp[-S_{D_0^*}-S_\pi]_{b_3=b_2},$$

$$\mathcal{E}'^{i}_{a}(t_f^{(i)})=\frac{C_2(t_f^{(i)})}{N_c}\alpha_s(t_f^{(i)})\exp[-S_B-S_{D_0^*}-S_\pi]_{b_3=b_2},$$

where the exponents S_M ($M=B,D_0^*,\pi$) are the Sudakov factors. From the above equations, we see clearly that the emission contributions are color-allowed and dictated by effective coupling of C_2+C_1/N_c, while the annihilation contributions are color-suppressed and governed by C_1+C_2/N_c. $t^{(i)}_{e,d,a,f}$ denote the hard scales of the involving diagrams which are expected to be of $\mathcal{O}(\sqrt{\bar{\Lambda} m_B^2})\sim 1.6$ GeV in average and the criteria to determine them are adopted to be

$$\begin{aligned}
t_e^{(1)}&=\max(\sqrt{x_2}m_B,1/b_1,1/b_2),\\
t_e^{(2)}&=\max(\sqrt{x_1}m_B,1/b_1,1/b_2),\\
t_d^{(j)}&=\max(\sqrt{x_1x_2}m_B,\sqrt{D_j^2}m_B,1/b_1,1/b_3),\\
t_a^{(1)}&=\max(\sqrt{x_3}m_B,1/b_2,1/b_3),\\
t_a^{(2)}&=\max(\sqrt{x_2}m_B,1/b_2,1/b_3),\\
t_f^{(j)}&=\max(\sqrt{x_2x_3\eta_3}m_B,\sqrt{F_j^2}m_B,1/b_1,1/b_2).
\end{aligned}\tag{15}$$

Since we deal with the hadronic effects of the B decay by considering six-quarks simultaneously, at lowest order in strong interaction, besides the renormalization group (RG) running from m_W to m_B scales in the μ-scale dependence of WCs, we still need to consider the running from m_B scale to the hard scale $t^{(i)}_{e,d,a,f}$ which indeed dictates the scale of the B meson decay. Hence, in our consideration, the hard scales for WCs are determined by Eq. (15) rather than at the m_B or $m_B/2$ scale. In the formulations of Eqs. (11)–(14) we have dropped the terms related to r_2^2 (r_c and r_2^2) for the right-handed (left-handed) gluon exchange of Fig. 1. Compared to leading power, which is not suppressed by $1/m_B$, they all belong to higher power effects.

B. $B_d\to \bar{D}_0^{*0}\pi^0$ decay

In this decay, the involving annihilation contributions are the same as the decay $B_d\to D_0^{*-}\pi^+$ but the emission topologies become color-suppressed, illustrated by Fig. 2. Due to the neutral pion meson being described by $(\bar{u}u-\bar{d}d)/\sqrt{2}$, the sign of emission topologies is opposite that of annihilation topologies. Therefore the decay amplitude is written as

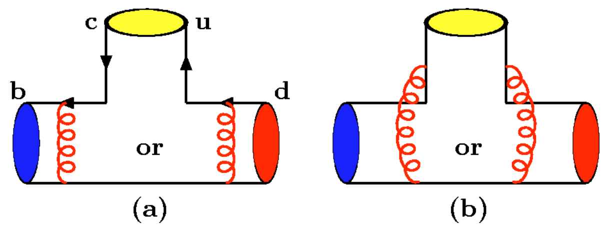

FIG. 2. The topologies (a) factorizable emission and (b) nonfactorizable effects for the decays $B_d\to \bar{D}_0^{*0}\pi^0$.

$$\mathcal{A}(B_d\to \bar{D}_0^{*0}\pi^0)=\frac{1}{\sqrt{2}}[-f_{D_0^*}F_{B\pi}-M_{B\pi}+f_BF_a+M_a].\tag{16}$$

With the same approach and power counting for the $B_d\to D_0^{*-}\pi^+$ decay, the relevant hard amplitudes $F_{B\pi}(M_{B\pi})$ can be derived as

$$\begin{aligned}
F_{B\pi}=&8\pi C_F m_B^2\int_0^1 dx_1dx_3\int_0^\infty b_1db_1b_3db_3\Phi_B(x_1,b_1)\\
&\times\{[(1+x_3)\Phi_\pi(x_3)+r_\pi(1-2x_3)[\Phi_\pi^p(x_3)\\
&+\Phi_\pi^\sigma(x_3)]]\mathcal{E}_\pi^3(t_e^{(3)})h_e(x_1,x_3\eta_3,b_1,b_3)\\
&+2r_\pi\Phi_\pi^p(x_3)\mathcal{E}_\pi^4(t_e^{(4)})h_e(x_3,x_1\eta_3,b_3,b_1)\},
\end{aligned}\tag{17}$$

$$\begin{aligned}
M_{B\pi}=&16\pi C_F m_B^2\sqrt{2N_c}\int_0^1 dx_1dx_2dx_3\\
&\times\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)\Phi_{D1}(x_2,b_2)\\
&\times\{[-(x_2+x_3)\Phi_\pi(x_3)+r_\pi x_3[\Phi_\pi^p(x_3)+\Phi_\pi^\sigma(x_3)]]\\
&\times\mathcal{E}'^3_\pi(t_d^{(3)})h_d^{(3)}(x_1,x_3\eta_3,x_2,b_1,b_2)\\
&+[(1-x_2)\Phi_\pi(x_3)-r_\pi x_3[\Phi_\pi^p(x_3)+\Phi_\pi^\sigma(x_3)]]\\
&\times\mathcal{E}'^4_\pi(t_d^{(4)})h_d^{(4)}(x_1,x_3\eta_3,x_2,b_1,b_2)\}.
\end{aligned}\tag{18}$$

The evolution factors $\mathcal{E}^i_\pi(\mathcal{E}'^i_\pi)$ are defined by

$$\mathcal{E}^i_\pi(t_e^{(i)})=\left(C_1(t_e^{(i)})+\frac{C_2(t_e^{(i)})}{N_c}\right)\alpha_s(t_e^{(i)})\exp[-S_B-S_\pi],$$

$$\mathcal{E}'^i_\pi(t_d^{(i)})=\frac{C_2(t_d^{(i)})}{N_c}\alpha_s(t_d^{(i)})\exp[-S_B-S_{D_0^*}-S_\pi]_{b_3=b_1}.$$

From the above equations, due to the appearance of C_1+C_2/N_c, we know that $B_d\to \bar{D}_0^{*0}\pi^0$ is a color-suppressed process. We note that although nonfactorizable effects are also color-suppressed, since C_2/N_c could be larger than C_1+C_2/N_c, the nonfactorizable effects play an important role in this kind of color-suppressed processes. In fact, the same thing also happens in the decay $B_d\to \bar{D}^0\pi^0$ with $\bar{D}^0$ being a charmed pseudoscalar [22]. The hard scales are determined by

$$t_e^{(3)}=\max(\sqrt{x_3\eta_3}m_B,1/b_1,1/b_3),$$

$$t_e^{(4)}=\max(\sqrt{x_1\eta_3}m_B,1/b_1,1/b_3),$$

$$t_d^{(3)}=\max(\sqrt{x_1x_3\eta_3}m_B,\sqrt{D_3^2}m_B,1/b_1,1/b_2),$$

$$t_d^{(4)}=\max(\sqrt{x_1x_3\eta_3}m_B,\sqrt{D_4^2}m_B,1/b_1,1/b_2),$$

with $D_3^2=(x_1-x_2)x_3\eta_3$ and $D_4^2=(x_1+x_2)r_2^2-(1-x_1-x_2)x_3\eta_3$.

C. $B^+\to\bar{D}_0^{*0}\pi^+$ decay

In this decay, there are no annihilation contributions and new topologies involved. The corresponding flavor diagrams are the same as Figs. 1(a), 1(b), and 2. Hence we can immediately write the decay amplitude as

$$\mathcal{A}(B^+\to\bar{D}_0^{*0}\pi^+)=f_\pi F_{BD_0^*}+M_{BD_0^*}+f_{D_0^*}F_{B\pi}+M_{B\pi}. \tag{19}$$

The hard amplitudes $\{F\}$ and $\{M\}$ are the same as Eqs. (11), (12), (17), and (18).

IV. NUMERICAL ANALYSIS

In our calculations, we adopt the B-meson wave function Φ_B to be

$$\Phi_B(x,b)=N_Bx^2(1-x)^2\exp\left[-\frac{1}{2}\left(\frac{xm_B}{\omega_B}\right)^2-\frac{\omega_B^2b^2}{2}\right], \tag{20}$$

where N_B can be determined by the normalization of the wave function at $b=0$ and ω_B is the shape parameter. Since the π-meson wave functions have been derived in the framework of QCD sum rules, we display them up to twist-3 directly by [25]

$$\Phi_\pi(x)=\frac{3f_\pi}{\sqrt{2N_c}}x(1-x)[1+0.44C_2^{3/2}(2x-1)+0.25C_4^{3/2}(2x-1)],$$

$$\Phi_\pi^p(x)=\frac{f_\pi}{2\sqrt{2N_c}}[1+0.43C_2^{1/2}(2x-1)+0.09C_4^{1/2}(2x-1)],$$

$$\Phi_\pi^\sigma(x)=\frac{f_\pi}{2\sqrt{2N_c}}(1-2x)[1+0.55(10x^2-10x+1)],$$

TABLE I. The values of the $B\to D_0^*$ form factor with $b_{D_0^*}=0.5$ and some variances in $a_{D_0^*}$ and $\omega_{D_0^*}$.

$\omega_{D_0^*}$	$a_{D_0^*}=0.7$	$a_{D_0^*}=0.9$	$a_{D_0^*}=1.1$
0.5	0.29	0.30	0.31
0.6	0.24	0.25	0.26
0.7	0.21	0.22	0.23

with the Gegenbauer polynomials,

$$C_2^{1/2}(\xi)=\frac{1}{2}(3\xi^2-1),\quad C_4^{1/2}(\xi)=\frac{1}{8}(35\xi^4-30\xi^2+3),$$

$$C_2^{3/2}(\xi)=\frac{3}{2}(5\xi^2-1),\quad C_4^{3/2}(\xi)=\frac{15}{8}(21\xi^4-14\xi^2+1).$$

After the wave functions of the π meson are determined, the unknown ω_B can be fixed by decays such as $B\to\pi\pi$. Consequently, the remaining uncertain parameters are the wave functions of the D_0^* meson.

To obtain the numerical results, the values of theoretical inputs are chosen as $\omega_B=0.4$, $f_B=0.19$, $f_\pi=0.13$, $\tilde{f}_{D_0^*}=0.20$, $m_B=5.28$, $m_{D_0^*}=2.29$, and $m_\pi^0=1.4$ GeV. With these values, we get the form factor $F^{B\to\pi}(0)=0.3$. In addition, the values of the $B\to D_0^*$ form factor with some variances in $a_{D_0^*}$ and $\omega_{D_0^*}$ are also shown in Table I. It is interesting that the form factor of $B\to D_0^*$ decay is much smaller than that of $B\to D$ decay, which is calculated to be around 0.57 [22]. We also find that our results are a little bit larger than those calculated by the ISGW2 model [29]. According to Wolfenstein's parametrization [33], we take $A=0.82$ and $\lambda=0.22$ for the CKM matrix element $V_{cb}=A\lambda^2$. Hence in terms of our deriving formulas and by fixing $a_{D_0^*}=0.9$, $b_{D_0^*}=0.5$, and $\omega_{D_0^*}=0.6$, the magnitudes of the hard amplitudes are shown in Table II. From the table, we can see clearly that except $M_{BD_0^*}$, the nonfactorizable effects of the color-suppressed process are comparable to factorizable contributions; even in annihilation topologies, the contributions of the former are much larger than those of the latter. By fixing $b_{D_0^*}=0.5$ and $\omega_{D_0^*}=0.6$ GeV and taking some different values of $a_{D_0^*}$, the decay BRs of $B\to D_0^*\pi$ are displayed in Tables III and IV. We also show the BRs by fixing $a_{D_0^*}=0.9$ and $b_{D_0^*}=0.5$ and some variant values of $\omega_{D_0^*}$. From both tables, we know that with proper values of parameters, the calculated BR of $B^+\to D_0^{*0}\pi^+$ is consistent with the BELLE's observation. It is worth noting that the predicted BR of $B_d\to\bar{D}_0^{*0}\pi^0$ is one order of magnitude smaller than others. The phenomenon can be understood by noticing that,

TABLE II. The values of hard amplitudes (in units of 10^{-3}) by fixing $a_{D_0^*}=0.9$, $b_{D_0^*}=0.5$, and $\omega_{D_0^*}=0.6$.

$f_\pi F_{BD_0^*}$	$M_{BD_0^*}$	f_BF_a	M_a	$f_{D_0^*}F_{B\pi}$	$M_{B\pi}$
36.4	$10^{-2}(1.0-i3.2)$	$-0.06-i0.08$	$-1.85-i3.14$	-7.11	$7.56-i10.95$

TABLE III. The BRs (in units of 10^{-4}) by fixing $b_{D_0^*}=0.5$, $\omega_{D_0^*}=0.6$ GeV, and various values of $a_{D_0^*}$.

a_{D_s}	$B^+\to\overline{D}_0^{*0}\pi^+$	$B_d\to D_0^{*-}\pi^+$	$B_d\to\overline{D}_0^{*0}\pi^0$
1.1	9.75	8.25	0.17
0.9	9.34	7.68	0.19
0.7	8.98	7.13	0.21

TABLE IV. The BRs (in units of 10^{-4}) by fixing $a_{D_0^*}=0.9$ and $b_{D_0^*}=0.5$ and various values of $\omega_{D_0^*}$.

$\omega_{D_0^*}$	$B^+\to\overline{D}_0^{*0}\pi^+$	$B_d\to D_0^{*-}\pi^+$	$B_d\to\overline{D}_0^{*0}\pi^0$
0.5	13.79	10.7	0.26
0.6	9.34	7.68	0.19
0.7	6.28	5.55	0.15

as shown in Table II, the value of $F_{B\pi}$ is very close and opposite in sign to the real part of $M_{B\pi}$ such that there is a strong cancellation in Eq. (16). As a result, we get the small BR in the decay $B_d\to\overline{D}_0^{*0}\pi^0$. That is, the annihilation effects are significant in $B\to\overline{D}_0^{*0}\pi^0$.

As stated before, although we only study the decays $B\to D_0^*\pi$, we still can estimate the BRs of $B\to D_1^{(\prime)}\pi$. Since only the longitudinal polarization has the contributions, except the decay constants, we expect that the involving wave functions of $D_1^{(\prime)}$ should be similar to D_0^*. By neglecting the difference in phase space, the BRs of $B\to D_1^{(\prime)}\pi$ could be estimated by $BR(B\to D_1^{(\prime)}\pi)/BR(B\to D_0^*\pi)\sim(\tilde{f}_{D_1^{(\prime)}}/\tilde{f}_{D_0^*})^2$. If $\tilde{f}_{D_1^{(\prime)}}\approx\tilde{f}_{D_0^*}$, the BRs for producing axial vector mesons $D_1^{(\prime)}$ are close to that for the scalar D_0^*. The tendency is consistent with BELLE's observations, shown in Eq. (2).

V. SUMMARY

We have studied the properties of P-wave mesons in B decays in terms of D_0^*. By taking the concept of the heavy quark limit, we have obtained some information on the shapes of D_0^* wave functions. According to the wave function of $K_0^*(1420)$, we can determine the proper value for the parameter $a_{D_0^*}$ in $\Phi_{D1}(x)$. By the physical argument, the unknown parameter $b_{D_0^*}$ can be chosen so that the maximum of $\Phi_{D2}(x)$ locates at $x\sim0.35$. We have found that with a suitable value of $\omega_{D_0^*}$, our result on $BR(B^+\to\overline{D}_0^{*0}\pi^+)$ can fit BELLE's measurements. Hence the calculated BRs for $B_d\to D_0^{*-}\pi^+$ and $B_d\to\overline{D}_0^{*0}\pi^0$ decays can be viewed as our predictions. Finally, if we regard that the longitudinal wave functions of $D_1^{(\prime)}$ are the same as $D_0^{(*)}$ and assume that $\tilde{f}_{D_1^{(\prime)}}\approx\tilde{f}_{D_0^*}$, we expect that the differences of BRs among them are not significant. The more accurate predictions rely on more definite values of decay constants as well as other unknown parameters.

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