# $\Lambda_b$ lifetime puzzle in heavy-quark expansion

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Lifetime differences of heavy hadrons can be consistently computed in heavy-quark expansion. The leading effects appear through spectator interactions at order $1/m_b^3$. We compute a well-defined subset of $1/m_b^4$ corrections to the lifetime ratio of a $\Lambda_b$ baryon and $B_d$ meson. We find that these corrections are large and should be taken into account in the systematic analysis of heavy hadron lifetimes. We claim that they could shift the ratio $\tau_{\Lambda_b}/\tau_{B_d}$ by as much as $-4.5\%$, significantly reducing the discrepancy between the theoretical predictions and experimental observations.

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Barring possible duality violations, one of the most unambiguous predictions of the heavy-quark effective theory is a prediction of the ratios of lifetimes of heavy mesons. In that respect, the low experimental value of the ratio $\tau(\Lambda_b)/\tau(B_d)$ appears quite puzzling. While the lifetime ratios of heavy mesons appear to be consistent with the experimentally observed ones [1],

$$\left.\frac{\tau(B_u)}{\tau(B_d)}\right|_{ex}=1.074\pm0.014,\quad \left.\frac{\tau(B_u)}{\tau(B_d)}\right|_{th}=1.07\pm0.03,$$

$$\left.\frac{\tau(B_s)}{\tau(B_d)}\right|_{ex}=0.948\pm0.038,\quad \left.\frac{\tau(B_s)}{\tau(B_d)}\right|_{th}=1.00\pm0.02, \tag{1}$$

the latest experimental observations suggest that

$$\tau(\Lambda_b)/\tau(B_d)|_{ex}=0.798\pm0.052, \tag{2}$$

which differs rather significantly from the theoretical predictions of $0.90\pm0.05$ [2–5]. It is therefore worthwhile to look for other effects that affect the baryon lifetime while preserving the lifetime ratios of mesons.

Inclusive decay rates can be computed in heavy-quark expansion. The most convenient way of doing so is to employ the optical theorem to relate the decay width to the imaginary part of the matrix element of the forward scattering amplitude:

$$\Gamma(H_b\to X)=\frac{1}{2M_{H_b}}\langle H_b|\mathcal{T}|H_b\rangle,$$

$$\mathcal{T}=\mathrm{Im}\, i\int d^4x T\{H_{\rm eff}(x)H_{\rm eff}(0)\}. \tag{3}$$

Here $H_{\rm eff}$ represents an effective $\Delta B=1$ Hamiltonian at the scale $\mu=m_b$,

$$H_{\rm eff}=\frac{4G_F}{\sqrt{2}}V_{cb}\sum\,[c_1Q_1^{u'd'}+c_2Q_2^{u'd'}]+\text{H.c.}, \tag{4}$$

where $d'$ and $u'$ are quark flavor eigenstates, $c_i$ are the Wilson coefficients, and the four-quark operators $Q_1$ and $Q_2$ are given by

$$Q_1^{u'd'}=\overline{d'_L}\gamma_\mu u'_L\overline{c_L}\gamma^\mu b_L,\quad Q_2^{u'd'}=\overline{c_L}\gamma_\mu u'_L\overline{d'_L}\gamma^\mu b_L. \tag{5}$$

In the heavy-quark limit, the energy release is large and therefore an operator product expansion (OPE) can be constructed for Eq. (3), which results in a series of local operators of increasing dimension suppressed by powers of $1/m_b$. In other words, the calculation of $\mathcal{T}$ in the expression for the rate in Eq. (3) is equivalent to computing matching coefficients of the effective $\Delta B=0$ Hamiltonian at the scale $\mu=m_b$, with the subsequent computation of its matrix elements.

At the leading order in the heavy-quark expansion all heavy hadrons have the same lifetime. The situation changes at higher orders. At order $1/m_b^2$ the difference between meson and baryon lifetimes appears due to the difference in their structure. The ratio of lifetimes of $\Lambda_b$ and $B_d$ is

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}=1+\frac{1}{2m_b^2}[\mu_\pi^2(\Lambda_b)-\mu_\pi^2(B_d)]$$
$$+\frac{C_G}{m_b^2}[\mu_G^2(\Lambda_b)-\mu_G^2(B_d)]+\mathcal{O}(1/m_b^3), \tag{6}$$

with $C_G\approx1.2$ [2,3]. $\mu_\pi^2$ and $\mu_G^2$ represent kinetic-energy and chromomagnetic interaction corrections [2]. At this order in heavy-quark expansion, the difference is mainly driven by the fact that light quarks in $\Lambda_b$ appear in a $J^P=0^+$ quantum state, diminishing any correlations of spins between the heavy-quark and light cloud. This implies that $\mu_G^2(\Lambda_b)=0$. With matrix elements of kinetic-energy operators canceling each other to a large degree, this difference amounts to at most $1-2\%$, which is not sufficient to explain the observed pattern of lifetimes.

The main contribution comes from the dimension-six operators that enter at the $1/m_b^3$ level. An important subclass of these operators involves four-quark operators, whose contribution is additionally enhanced due to the phase-space factor $16\pi^2$. These effects are commonly called weak annihilation (WA), weak scattering (WS), and Pauli interference (PI). They introduce differences in lifetimes of all heavy mesons and baryons [2,3,6,7]. Their contribution to the lifetime ratios are governed by matrix elements of $\Delta B=0$ four-fermion operators

$$\mathcal{T}_{\rm spec}=\mathcal{T}^{u}_{\rm spec}+\mathcal{T}^{d'}_{\rm spec}+\mathcal{T}^{s'}_{\rm spec}, \tag{7}$$

where $\mathcal{T}_i$ contributing to Eq. (3) are

$$\mathcal{T}^{u}_{\rm spec}=\frac{G_F^2 m_b^2|V_{bc}|^2(1-z)^2}{2\pi}\{(c_1^2+c_2^2)O_1^u+2c_1c_2\tilde{O}_1^u+\delta^u_{1/m}+\delta^u_{1/m^2}\}, \tag{8}$$

$$\mathcal{T}^{d'}_{\rm spec}=-\frac{G_F^2 m_b^2|V_{bc}|^2(1-z)^2}{4\pi}\times\left\{c_1^2\left[(1+z)O_1^{d'}+\frac{2}{3}(1+2z)O_2^{d'}\right]+(N_c c_2^2+2c_1c_2)\left[(1+z)\tilde{O}_1^{d'}+\frac{2}{3}(1+2z)\tilde{O}_2^{d'}\right]+\delta^{d'}_{1/m}+\delta^{d'}_{1/m^2}\right\}, \tag{9}$$

$$\mathcal{T}^{s'}_{\rm spec}=-\frac{G_F^2 m_b^2|V_{bc}|^2\sqrt{1-4z}}{4\pi}\left\{c_1^2\left[O_1^{s'}+\frac{2}{3}(1+2z)O_2^{s'}\right]+(N_c c_2^2+2c_1c_2)\left[\tilde{O}_1^{s'}+\frac{2}{3}(1+2z)\tilde{O}_2^{s'}\right]+\delta^{s'}_{1/m}+\delta^{s'}_{1/m^2}\right\}. \tag{10}$$

$O_i^q$ are the four-fermion operators

$$O_1^q=\overline{b}_i\gamma^\mu(1-\gamma_5)b_i\overline{q}_j\gamma_\mu(1-\gamma_5)q_j,$$
$$O_2^q=\overline{b}_i\gamma^\mu\gamma_5 b_i\overline{q}_j\gamma_\mu(1-\gamma_5)q_j, \tag{11}$$

with $z=m_c^2/m_b^2$. The $\tilde{O}_i^q$ denote the color-rearranged operators that follow from the expressions for $O_i^q$ by interchanging the color indexes of the $b_i$ and $q_j$ Dirac spinors. Our choice of basis operators differs from the one made in [3] due to the relative simplicity of the higher order operators in our basis. $\delta^{q'}_{1/m}$ and $\delta^{q'}_{1/m^2}$ represent power-suppressed corrections to the spectator contributions. These corrections are clearly parametrically more important than the charm-quark mass effects normally taken into account [3], as the latter are suppressed by $z$ or *two powers* of $1/m_b$. This subset of the full set of $1/m_b^4$ corrections also retains the $16\pi^2$ "phase-space" enhancement enjoyed by the leading effects of WS and PI. However, they need not to interfere destructively in their contribution to the $\Lambda_b$ lifetime as do WS and PI [4]. Conspiring, they can produce a sizable shift in the ratio of the $\Lambda_b$ and $B$-meson lifetimes. We shall argue below that this is indeed the case.

Most of the recent progress in understanding lifetimes has been concentrated on computing the next-to-leading order (NLO) QCD corrections to Wilson coefficients of the operators in Eq. (8) [5,8] or calculating matrix elements of these operators in quark models or on the lattice. The simplest parametrization of these matrix elements is inspired by a naive factorization ansatz and represents the matrix elements of the four-fermion operators as products of two matrix elements of current operators separated by a vacuum state. This procedure introduces four new scale-dependent parameters $B_i(\mu)$ and $\epsilon_i(\mu)$ [3] which in our basis parametrize the matrix elements of the operators in Eq. (11) as

$$\langle B_q|O_1^q|B_q\rangle=f_{B_q}^2 m_{B_q}^2\left(2\epsilon_1+\frac{B_1}{N_c}\right),$$
$$\langle B_q|\tilde{O}_1^q|B_q\rangle=f_{B_q}^2 m_{B_q}^2 B_1,$$
$$\langle B_q|O_2^q|B_q\rangle=-f_{B_q}^2 m_{B_q}^2\left[\frac{m_{B_q}^2}{m_b^2}\left(2\epsilon_2+\frac{B_2}{N_c}\right)+\frac{1}{2}\left(2\epsilon_1+\frac{B_1}{N_c}\right)\right],$$
$$\langle B_q|\tilde{O}_2^q|B_q\rangle=-f_{B_q}^2 m_{B_q}^2\left[\frac{m_{B_q}^2}{m_b^2}B_2+\frac{1}{2}B_1\right]. \tag{12}$$

Similar expressions are available for baryons where they are motivated by the valence quark model

$$\langle\Lambda_b|O_1^q|\Lambda_b\rangle=-\tilde{B}\langle\Lambda_b|\tilde{O}_1^q|\Lambda_b\rangle=\frac{\tilde{B}}{6}f_{B_q}^2 m_{B_q}m_{\Lambda_b}r,$$
$$\langle\Lambda_b|O_2^q|\Lambda_b\rangle=-\tilde{B}\langle\Lambda_b|\tilde{O}_2^q|\Lambda_b\rangle=\frac{\tilde{B}}{6}f_{B_q}^2 m_{B_q}m_{\Lambda_b}\delta, \tag{13}$$

where $r=|\psi_{bq}^{\Lambda_b}(0)|^2/|\psi_{bq}^{B_q}(0)|^2$ is the ratio of the wave functions at the origin of the $\Lambda_b$ and $B_q$ mesons, and $\tilde{B}=1$ in the valence-quark model. Estimates of $r$ vary from 0.1 to 1.8 and can potentially be larger [3]. Note that $\delta=\mathcal{O}(1/m_b)$, which follows from the heavy-quark spin symmetry. The above parameters can be computed in QCD sum rules, quark models,

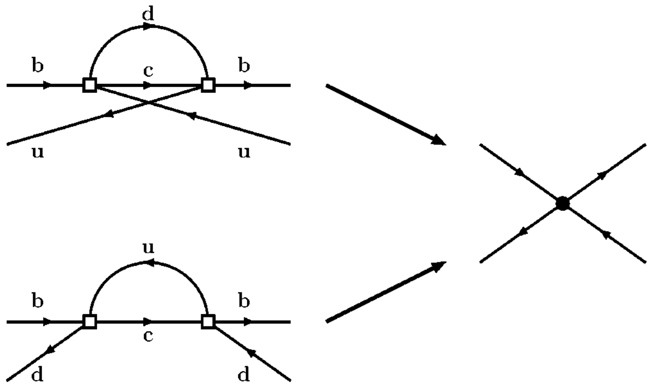


FIG. 1. Higher order $1/m_b$ and $1/m_b^2$ corrections to spectator contributions (derivative insertions).

or on the lattice. Naively, one expects that in the large-$N_c$ limit $B_1 \sim B_2 = \mathcal{O}(1)$, $\epsilon_1 \sim \epsilon_2 = \mathcal{O}(1/N_c)$. Yet, the contributions of the "octet" parameters $\epsilon_i$ are important due to the large Wilson coefficient that accompanies them and the (accidental) cancellation that suppresses the Wilson coefficient accompanying the $B_i$ parameters. A compilation of various estimates of these parameters can be found in [9]. One can parametrize the meson-baryon lifetime ratio as

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.98 - (d_1 + d_2\tilde{B})r - (d_3\epsilon_1 + d_4\epsilon_2) - (d_5B_1 + d_6B_2) + \delta_{1/m}, \tag{14}$$

where the scale-dependent parameters $[d_i(m_b) = \{0.012, 0.021, 0.173, -0.195, \mathcal{O}(10^{-3}), \mathcal{O}(10^{-3})\}$ at LO [3] and $\{0.023, 0.028, 0.16, -0.16, 0.08, -0.08\}$ at NLO [5]] are defined in [3]. While tempting, it is clearly difficult to reduce the $\tau(\Lambda_b)/\tau(B_d)$ lifetime ratio by inflating $\epsilon_i$ without disturbing the meson lifetime ratios. Thus, at least at the $1/m_b^3$ level, the problem can be ameliorated by conjecturing that $r \gg 1$, which runs in contrast with otherwise successful quark-model expectations. $\delta_{1/m}$ represents contributions of order $1/m_b^4$ and higher, which we shall address. The impact of $1/m_b^4$ corrections can be naively expected at the level of 20%. However, as we shall see below, kinetic corrections to WS and PI *conspire* in $\Lambda_b$ and, coupled with large Wilson coefficients, produce a sizable effect. We computed the higher order corrections to Eq. (7) in the heavy-quark expansion, denoted below as $\delta^q_{1/m}$ and $\delta^q_{1/m^2}$, by expanding the forward scattering amplitude in Eq. (3) in the light-quark momentum and matching the result onto the operators containing derivative insertions (see Fig. 1). The result is

$$\delta^u_{1/m} = -2(c_1^2 + c_2^2)R_1^u - 4c_1c_2\tilde{R}_1^u,$$

$$\delta^{d',s'}_{1/m} = \frac{2}{3}c_1^2[R_1^{d',s'} + R_2^{d',s'} - R_3^{d',s'}] + \frac{2}{3}(N_c c_2^2 + 2c_1c_2)[\tilde{R}_1^{d',s'} + \tilde{R}_2^{d',s'} - \tilde{R}_3^{d',s'}]. \tag{15}$$

The operators $R_i$ are defined as

$$R_1^q = \frac{1}{m_b^2}\overline{b_i}\gamma^\mu(1-\gamma_5)\vec{D}^\alpha b_i \overline{q_j}\gamma_\mu(1-\gamma_5)\vec{D}_\alpha q_j,$$

$$R_2^q = \frac{1}{m_b^2}\overline{b_i}\gamma^\mu(1-\gamma_5)\vec{D}^\nu b_i \overline{q_j}\gamma_\nu(1-\gamma_5)\vec{D}_\mu q_j, \tag{16}$$

$$R_3^q = \frac{m_q}{m_b}\overline{b_i}(1-\gamma_5)b_i\overline{q_j}(1-\gamma_5)q_j.$$

Here $\tilde{R}_i^q$ denote the color-rearranged operators that follow from the expressions for $R_i^q$ by interchanging the color indexes of $b_i$ and $q_j$ Dirac spinors. We dropped all the contributions suppressed by light and charm-quark masses, except for $R_3^q$. Since the above result contains "full" QCD $b$-fields, no immediate power counting for these operators is available. The power counting becomes manifest at the level of the matrix elements. We shall present the most general parametrization of these matrix elements elsewhere. Neglecting nonfactorizable contributions, the meson matrix elements are

$$\langle B_q|R_1^q|B_q\rangle = \langle B_q|\tilde{R}_1^q|B_q\rangle/N_c = \frac{\beta_1}{2N_c}f_{B_q}^2 m_{B_q}^2\left[\frac{m_{B_q}^2}{m_b^2} - 1\right],$$

$$\langle B_q|R_{2,3}^q|B_q\rangle = -\frac{\beta_{2,3}}{4N_c}f_{B_q}^2(m_{B_q}^2 - m_b^2), \tag{17}$$

$$\langle B_q|\tilde{R}_{2,3}^q|B_q\rangle = -\frac{\beta_{2,3}}{4}f_{B_q}^2(m_{B_q}^2 - m_b^2),$$

where parameters $\beta_i = 1$ in the factorization approximation [10], which we shall employ hereafter. Similarly, we used the quark-diquark model to *guide* our parametrizations of baryon matrix elements

$$\langle \Lambda_b|R_1^q|\Lambda_b\rangle = -\langle \Lambda_b|\tilde{R}_1^q|\Lambda_b\rangle = -\frac{\tilde{\beta}_1}{24}f_{B_q}^2 m_{B_q} m_{\Lambda_b}\left[\frac{m_{\Lambda_b}^2}{m_b^2} - 1\right],$$

$$\langle \Lambda_b|R_2^q|\Lambda_b\rangle = -\langle \Lambda_b|\tilde{R}_2^q|\Lambda_b\rangle = -\frac{\tilde{\beta}_2}{48m_b^2}f_{B_q}^2\frac{m_{B_q}}{m_{\Lambda_b}}(m_{\Lambda_b}^4 - m_b^4), \tag{18}$$

$$\langle \Lambda_b|R_3^q|\Lambda_b\rangle = -\langle \Lambda_b|\tilde{R}_3^q|\Lambda_b\rangle = -\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\langle \Lambda_b|R_2^q|\Lambda_b\rangle,$$

where $\widetilde{\beta}_i = r$ in the approximation where the color of the quark fields in the operators matches the color of the quarks inside the baryon, which is an analogue of the factorization approximation for baryons. Inserting Eqs. (17) and (18) into (15) gives an estimate of our correction. Numerically, it constitutes 40–60 % of the leading spectator contribution if the leading logarithmic approximation is employed for WS and PI, depending on the chosen renormalization scale (we varied the scale from $m_b/2$ to $2m_b$). Employing the full NLO result for WS and PI [5] we observe that the effect of $1/m^4$ corrections reduces to 36–45 % of the leading spectator contribution. While such a sizable effect is surprising, the main source of such a large correction can be readily identified, at least in factorization. While the individual $1/m_b$ corrections to WS and PI are of order 20%, as expected from the naive power counting, they contribute to the $\Lambda_b$ lifetime with the same sign, instead of destructively interfering WS and PI [4]. This conspiracy of several small $\sim 20\%$ effects produces a sizable shift in the ratio of the $\Lambda_b$ and $B$-meson lifetimes, which can be as large as $-4.5\%$. We expect this effect to persist with more rigorous computations of matrix elements as well.

We checked that higher order $1/m_b^5$ contributions are under control and as large as one would expect based on naive power counting, i.e. of the order of a few percent. These higher order contributions arise from graphs with more derivative insertions and interactions with background gluon fields. Discarding light and charm-quark masses we obtain

$$\delta^u_{1/m^2} = 0, \quad \delta^{d'(s')}_{1/m^2} = \frac{c_1^2}{m_b^2}[P_1^{d'(s')} - P_2^{d'(s')}], \qquad (19)$$

where the only nonzero contribution comes from the gluonic operators depicted in Fig. 2

$$P_1^q = \overline{b}_i \gamma^\mu (1-\gamma_5) \widetilde{G}_{\mu\nu} b_k \overline{d}_j \gamma^\nu (1-\gamma_5) d_j,$$
$$P_2^q = \overline{b}_i \gamma^\mu (1-\gamma_5) b_i \overline{d}_j \gamma^\nu \widetilde{G}_{\mu\nu} (1-\gamma_5) d_k, \qquad (20)$$

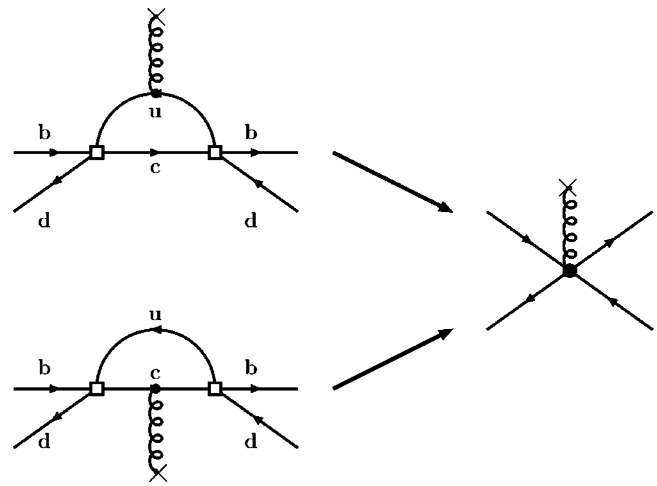


FIG. 2. Higher order $1/m_b^2$ corrections to spectator contributions (background gluon interactions).

where $\widetilde{G}_{\mu\nu} = t^a_{ik} \widetilde{G}^a_{\mu\nu}$. It is easy to see that the naive power counting for the matrix elements of the operators in Eq. (20) implies that $1/m_b^2$ contribution to the spectator effects is of the order of a few percent.

In conclusion, we computed a well-defined subset of $1/m_b^4$ corrections to the lifetime ratio of $\Lambda_b$ and $B_d$. While this subset does not dominate the full $1/m_b^4$ correction in any limit, it receives the same phase-space enhancement factor as the leading spectator effect. We found this correction to be large, of order 40–60 % of the leading spectator effect at LO [2–4] and 36–45 % at NLO [5], reducing the lifetime ratio by as much as 4.5% in addition to the $\mathcal{O}(10\%)$ effect reported earlier. This significantly reduces the discrepancy between the theoretical predictions and experimental observations, making them compatible within error bars.

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