Factorization fits and the unitarity triangle in charmless two-body B decays

N. de Groot
Rutherford Appleton Laboratory, Chilton Didcot OX11 0QX, United Kingdom

W. N. Cottingham
H. H. Wills Physics Laboratory, Bristol University, Tyndall Avenue, Bristol BS8 1TL, United Kingdom

I. B. Whittingham
School of Mathematical and Physical Sciences, James Cook University, Townsville, 4811, Australia

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We present fits to charmless hadronic B decay data from the BaBar, Belle and CLEO experiments using two theoretical models: (i) the QCD factorization model of Beneke *et al.* and (ii) QCD factorization complemented with the so-called *charming penguin* contributions of Ciuchini *et al.* When we include the data from pseudoscalar-vector decays the results favor the incorporation of the *charming penguin* terms. We also present fit results for the unitarity triangle parameters and the CP-violating asymmetries.

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I. INTRODUCTION

A wealth of experimental data on hadronic charmless B decays has become available from the BaBar and Belle experiments. These studies of the numerous B decay channels are designed to test the Cabibbo-Kobayashi-Maskawa (CKM) explanation of CP violation in the standard model as represented by the unitarity triangle condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ on the CKM mixing parameters. Such tests have been facilitated by the recent significant progress in the theoretical understanding of hadronic decay amplitudes based upon QCD factorization which allows the amplitudes to be expressed in terms of numerous soft QCD parameters, such as meson decay constants and transition form factors, and a set of calculable coefficients. In this paper we present an analysis of recent data, based upon QCD factorization. We also investigate the potential contribution to the decay amplitudes of b quark annihilation and so-called charming penguin contributions. The data that we attempt to fit include decays to pseudoscalar (π and K) and vector (ρ, ω, K^* and ϕ) mesons. This is an extension of an earlier study [1] that was based upon simplified formulas derived from the heavy quark limit of QCD factorization.

The starting point for the calculation of all B meson decay amplitudes is the effective Hamiltonian

$$\mathcal{H}_{\rm eff}(\mu) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1(\mu) Q_1^p + C_2(\mu) Q_2^p + \sum_{i=3,\ldots,10} C_i(\mu) Q_i \right] + \text{other terms} \tag{1}$$

where $\lambda_p = V_{pq}^* V_{pb}$ is a product of CKM matrix elements, $q = d,s$ and the local $\Delta B = 1$ four-quark operators are

$$Q_1^p = (\overline{q}_\alpha p_\alpha)_{V-A} (\overline{p}_\beta b_\beta)_{V-A},$$

$$Q_2^p = (\overline{q}_\alpha p_\beta)_{V-A} (\overline{p}_\beta b_\alpha)_{V-A},$$

$$Q_{3,5} = (\overline{q}_\beta b_\beta)_{V-A} \sum_{q'} (\overline{q}'_\alpha q'_\alpha)_{V\mp A},$$

$$Q_{4,6} = (\overline{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\overline{q}'_\alpha q'_\beta)_{V\mp A},$$

$$Q_{7,9} = (\overline{q}_\beta b_\beta)_{V-A} \sum_{q'} (\overline{q}'_\alpha q'_\alpha)_{V\pm A},$$

$$Q_{8,10} = (\overline{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\overline{q}'_\alpha q'_\beta)_{V\pm A}, \tag{2}$$

where $q' \in \{u,d,s,c\}$, α and β are color indices, $e_{q'} = 2/3(-1/3)$ for $u(d)$-type quarks and we use the notation, for example,

$$Q_5 = (\overline{q}_\beta b_\beta)_{V-A} \sum_{q'} (\overline{q}'_\alpha q'_\alpha)_{V+A} = [\overline{q}_\beta \gamma_\mu (1-\gamma_5) b_\beta] \sum_{q'} [\overline{q}'_\alpha \gamma^\mu (1+\gamma_5) q'_\alpha]. \tag{3}$$

$Q_{1,2}$ are the current-current tree operators, $Q_{3,\ldots,6}$ are QCD penguin operators and $Q_{7,\ldots,10}$ are electroweak penguin operators. The "other terms" indicated in Eq. (1) include the electromagnetic and chromomagnetic dipole transition operators. In the standard model the contributions from the electroweak penguin operators and magnetic dipole operators are generally small. The exception is the electroweak penguin operator coefficient C_9 which is larger than the QCD penguin coefficients C_3 and C_5.

It has been shown by Beneke *et al.* [2] that, in the heavy quark limit, the hadronic matrix elements of the four-quark operators in the amplitudes for nonleptonic B decays into two light mesons $M_{1,2}$ have the form

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$$\langle M_1 M_2|Q_i|B\rangle = \langle M_1|J_{1\mu}|B\rangle\langle M_2|J_2^{\mu}|0\rangle \times\left[1+\sum_n r_n\alpha_s^n + O(\Lambda_{\rm QCD}/m_b)\right] \quad (4)$$

and can be calculated from first principles, including nonfactorizable strong interaction corrections. QCD factorization extends naive factorization by separating matrix elements into short distance contributions at scale $O(1/m_b)$ that are perturbatively calculable and long distance contributions $O(1/\Lambda_{\rm QCD})$ that are parametrized.

B meson decay can also be initiated by b quark annihilation with its partner. Although the annihilation contributions to the decay amplitude are formally of $O(\Lambda_{\rm QCD}/m_b)$ and power suppressed, they violate QCD factorization because of end point divergences. However these weak annihilation contributions can be included into the decay amplitudes by treating the end point divergences as phenomenological parameters. Analyses based upon QCD factorization with inclusion of weak annihilation have been undertaken for $B\to PP$ [2,3] and $B\to PV$ [4]. Although general agreement with experiment was found, some branching ratios for PV decays were only marginally consistent and all predictions were plagued by the large uncertainties associated with poorly determined parameters within the theory. A recent global analysis of PP and PV decays [5] found that QCD factorization plus weak annihilation could fit many decay channels but yielded results too low for the $B\to\pi K^*$ decays. More recently, Aleksan *et al.* [6] have undertaken a global analysis of PV decays and have concluded that QCD factorization cannot fit the experimental data when decay channels involving K^* mesons are included, and suggest that this failure is due to some larger than expected nonperturbative contribution. Motivated by the concept of so-called charming penguin contributions, that is nonperturbative $O(\Lambda_{\rm QCD}/m_b)$ corrections from enhanced c-loop penguin contributions, first introduced by Ciuchini *et al.* [12], they introduce additional long-distance contributions to the decay amplitudes and include the two complex parameters from these additional amplitudes in their global fit. They obtain a slightly better fit but their best-fit parameters are at the limits of the allowed domain. A recent study [7], limited to $B\to\rho\pi$ decays, has used QCD factorization to place bounds on the $F_1^{B\to\pi}$ form factor. A detailed study of QCD factorization applied to PP and PV decays of B mesons has just been completed by Beneke and Neubert [8]. Predicted branching ratios and CP asymmetries for a large number of PP and PV channels are given for default values of input parameters and detailed estimates of the theoretical uncertainties in these predictions determined for various scenarios of input parameters. Beneke and Neubert find that there is a scenario for which there is general global agreement between the results of QCD factorization and measurement except for $\overline{B}^0\to\pi^0\overline{K}^0$ and the group of $B\to\pi K^*$ decays.

In this paper we undertake a global analysis of 18 PP, PV and VV channels using two theoretical models, (i) QCD factorization with inclusion of weak annihilation and (ii) this model supplemented with charming penguin contributions. Our study is similar in spirit to that of Aleksan *et al.* [6] for PV decays but we extend the global fit to include PP and some VV decays. This paper is organized as follows. In Sec. II we review the structure of the decay amplitude within QCD factorization and discuss the various parameters that occur in this amplitude. Inclusion of weak annihilation and charming penguin contributions is discussed in Sec. III and Sec. IV, respectively. The method and results of our best fit for our two models to current experimental data is presented in Sec. V, and Sec. VI contains our discussion and conclusions.

II. DECAY AMPLITUDE IN QCD FACTORIZATION

In QCD factorization, the amplitude for B decay into two light hadrons (mesons) $M_{1,2}$ has the form, neglecting weak annihilation processes,

$$\langle M_1 M_2|\mathcal{H}_{\rm eff}|B\rangle = \frac{G_F}{\sqrt{2}}\Bigg\{\sum_{i=1,2}\lambda_u a_i^u[T_i(M_1,M_2)+T_i(M_2,M_1)] + \sum_{p=u,c}\sum_{i=3,\dots,6,9}\lambda_p a_i^p[T_i(M_1,M_2)+T_i(M_2,M_1)]\Bigg\} \quad (5)$$

where

$$T_1(M_1,M_2)=\langle M_1|\overline{u}\gamma^{\mu}(1-\gamma_5)b|B\rangle\langle M_2|\overline{q}\gamma_{\mu}(1-\gamma_5)u|0\rangle,$$

$$T_2(M_1,M_2)=\langle M_1|\overline{q}\gamma^{\mu}(1-\gamma_5)b|B\rangle\langle M_2|\overline{u}\gamma_{\mu}(1-\gamma_5)u|0\rangle,$$

$$T_3(M_1,M_2)=\langle M_1|\overline{q}\gamma^{\mu}(1-\gamma_5)b|B\rangle \times\langle M_2|\overline{q'}\gamma_{\mu}(1-\gamma_5)q'|0\rangle,$$

$$T_4(M_1,M_2)=\langle M_1|\overline{q'}\gamma^{\mu}(1-\gamma_5)b|B\rangle \times\langle M_2|\overline{q}\gamma_{\mu}(1-\gamma_5)q'|0\rangle,$$

$$T_5(M_1,M_2)=\langle M_1|\overline{q}\gamma^{\mu}(1-\gamma_5)b|B\rangle \times\langle M_2|\overline{q'}\gamma_{\mu}(1+\gamma_5)q'|0\rangle,$$

$$T_6(M_1,M_2)=-2\langle M_1|\overline{q'}(1-\gamma_5)b|B\rangle\langle M_2|\overline{q}(1+\gamma_5)q'|0\rangle,$$

$$T_9(M_1,M_2)=\langle M_1|\overline{q}\gamma^{\mu}(1-\gamma_5)b|B\rangle \times\langle M_2|e_{q'}\overline{q'}\gamma_{\mu}(1-\gamma_5)q'|0\rangle. \quad (6)$$

The two-quark matrix elements are the well determined electroweak decay constants f_π, f_K, f_ρ, etc. and the B transition form factors F_π, F_K, A_ρ, etc. In principle transition form factors are independently measurable in B semileptonic decays but to date they are only loosely constrained by measurements and model estimations.

TABLE I. Leading order Wilson coefficients C_i in the NDR scheme calculated at the scales μ and $\mu_h=\sqrt{\Lambda_h\mu}$, where $\Lambda_h=0.5$ GeV, for $\mu=m_b$ and $\mu=m_b/2$. The input parameters are $\Lambda_{\rm QCD}^{\overline{\rm MS}(5)}=0.225$ GeV, $m_t(m_t)=167.0$ GeV, $m_b(m_b)=4.2$ GeV, $M_W=80.42$ GeV, $\alpha=1/129$, and $\sin^2\theta_W=0.23$.

Scale (GeV)	4.20	2.10	1.45	1.02
C_1	1.1174	1.1848	1.2392	1.3120
C_2	−0.2678	−0.3873	−0.4755	−0.5862
C_3	0.0121	0.0185	0.0235	0.0299
C_4	−0.0274	−0.0383	−0.0459	−0.0551
C_5	0.0080	0.0105	0.0120	0.0136
C_6	−0.0341	−0.0526	−0.0677	−0.0883
C_7/α	−0.0140	−0.0282	−0.0314	−0.0303
C_8/α	0.0288	0.0432	0.0555	0.0734
C_9/α	−1.2913	−1.3785	−1.4408	−1.5190
C_{10}/α	0.2888	0.4213	0.5194	0.6424

The coefficients a_i have to be calculated from the Wilson coefficients [2]. We have calculated the Wilson coefficients at several scales μ of $O(m_b)$ using the next-to-leading-order (NLO) renormalization group equations

$$C_i(\mu)=U_{ij}(\mu,M_W)C_j(M_W). \tag{7}$$

We follow the Beneke *et al.* [2] prescription of (i) dropping terms of $O(\alpha_s^2)$, $O(\alpha^2)$ and $O(\alpha_s\alpha)$ in Eq. (7), (ii) neglecting the effect of the electromagnetic penguin contributions $C_{7,\ldots,10}(M_W)$ on the evolution of the QCD penguin coefficients $C_{1,\ldots,6}$, and (iii) in $C_j(M_W)$, splitting the $O(\alpha)$ electroweak penguin terms into those enhanced by large m_t or $1/\sin^2\theta_W$, which are treated as leading order (LO), and treating the remainder, together with the $O(\alpha_s)$ terms, as NLO. Our calculated values are shown in Tables I and II and are very similar to those of Beneke *et al.* [2] and Du *et al.* [3,4].

To lowest order in the strong coupling constant α_s, the a_i coefficients are the same as in naive factorization, that is

TABLE II. Next to leading order Wilson coefficients C_i in the NDR scheme calculated at the scales μ and $\mu_h=\sqrt{\Lambda_h\mu}$, where $\Lambda_h=0.5$ GeV, for $\mu=m_b$ and $\mu=m_b/2$. The input parameters are $\Lambda_{\rm QCD}^{\overline{\rm MS}(5)}=0.225$ GeV, $m_t(m_t)=167.0$ GeV, $m_b(m_b)=4.2$ GeV, $M_W=80.42$ GeV, $\alpha=1/129$, and $\sin^2\theta_W=0.23$.

Scale (GeV)	4.20	2.10	1.45	1.02
C_1	1.0813	1.1374	1.1820	1.2405
C_2	−0.1903	−0.2948	−0.3700	−0.4619
C_3	0.0137	0.0212	0.0274	0.0358
C_4	−0.0357	−0.0506	−0.0618	−0.0762
C_5	0.0087	0.0102	0.0105	0.0096
C_6	−0.0419	−0.0653	−0.0854	−0.1146
C_7/α	−0.0026	−0.0139	−0.0147	−0.0100
C_8/α	0.0618	0.0986	0.1317	0.1813
C_9/α	−1.2423	−1.3181	−1.3526	−1.4149
C_{10}/α	0.2283	0.3388	0.4015	0.4991

$$a_i^{\rm LO}=C_i+C_{i'}/N_c \tag{8}$$

where $i'=i-(-1)^i$ and $N_c=3$ is the number of quark colors. The higher order corrections include lowest order gluon exchange between the quarks in the basic tree amplitudes which are calculated and folded into the light cone distribution functions $\Phi_M(x,\mu)$ of the participating mesons (see, for example [9]). This results in the a_i coefficients having the form

$$a_i(M_1M_2)=a_{i,I}(M_2)+a_{i,II}(M_1M_2) \tag{9}$$

where M_1 is the recoil meson containing the spectator (anti) quark and M_2 is the emitted meson. The complex quantities $a_{i,I}$ describe the formation of M_2, including nonfactorizable corrections from hard gluon exchange or light quark loops in penguin contributions. They do not involve the hard gluon exchanges with the spectator quark, these are described by the (possibly) complex quantities $a_{i,II}$.

In the correction terms the leading-twist light cone distribution functions for both pseudoscalar and vector mesons are expanded in the first few terms of a Gegenbauer expansion

$$\Phi_M(x,\mu)=6x(1-x)\left[1+\sum_n \alpha_n^M C_n^{3/2}(2x-1)\right]. \tag{10}$$

The asymptotic limit $\Phi_M(x,\mu)=6x(1-x)$ is valid for the mass scale $\mu\to\infty$. The parameters α_n^M are anticipated to be small but they are not well established. To economize in the number of fitting parameters in the initial fits to data presented here they are taken to be zero. With this simplification all light mesons included in our analysis have the same spatial wave function and all coefficients $a_{i,I}$ except $a_{6,I}$ are the same for all decays. Formulas for the evaluation of the $a_{i,I}$ can be found in [2,4,8]. Although Beneke and Neubert [8] obtain a different expression for $a_{6,I}(M_2=V)$ to that of Du *et al.* [4], this is not important here as $a_6(PV)$ does not occur in the decay amplitudes for $M_2=V$ since $\langle V|(\overline{q}q)_{S\pm P}|0\rangle=0$. The $O(\alpha_s)$ corrections to $a_i^{\rm LO}$ include contributions from one-loop vertex corrections $V_M(\mu,\alpha_n^M)$ to all $a_{i,I}$ and from penguin corrections $P_M(\mu,\alpha_n^M,s_p)$, involving the quark mass ratio $s_p=(m_p/m_b)^2$, to $a_{4,I}$ and $a_{6,I}$. We neglect the small electroweak penguin corrections to P_M. Typical parton off-shellness in the loop diagrams contributing to V_M and P_M is $O(m_b)$, suggesting that the scale $\mu\propto m_b$ should be used in evaluating $a_{i,I}$. The results of our calculations of $a_{i,I}$ coefficients without light cone corrections are given in Table III. Results including the light cone corrections $\alpha_{1,2}^M$ taken from [10] are given in Table IV. The coefficients most affected are $a_{2,I}$ and $a_{4,I}$.

The coefficients $a_{i,II}$ are not universal even when light cone corrections are neglected. They contain not only the low energy parameters (decay constants and form factors) from the lowest order calculations, in common with naive factorization, but low energy contributions to the folding integrals involving another nonperturbative complex parameter X_H which is only loosely constrained by model estimations. To discuss the form of the $a_{i,II}$, we first note that, from Eq.

TABLE III. Factorization $a_{i,I}(M_2)$ coefficients evaluated at the scale μ using the expressions of [2,8]. The light cone corrections $\alpha_n^{M_2}$ are set equal to zero, making all coefficients except $a_{6,I}$ universal.

μ(GeV)	4.20	2.10
$a_{1,I}^u$	$1.0572+0.0200i$	$1.0791+0.0369i$
$a_{2,I}^u$	$0.0060-0.0836i$	$-0.0377-0.1130i$
$a_{3,I}$	$0.0058+0.0021i$	$0.0083+0.0037i$
$a_{4,I}^u$	$-0.0312-0.0161i$	$-0.0338-0.0205i$
$a_{4,I}^c$	$-0.0369-0.0068i$	$-0.0415-0.0079i$
$a_{5,I}$	$-0.0070-0.0026i$	$-0.0106-0.0050i$
$a_{6,I}^u(P)$	$-0.0433-0.0152i$	$-0.0586-0.0188i$
$a_{6,I}^c(P)$	$-0.0465-0.0056i$	$-0.0630-0.0056i$
$a_{6,I}^u(V)$	$-0.0075-0.0007i$	$-0.0094-0.0013i$
$a_{6,I}^c(V)$	$0.0009-0.0115i$	$0.0019+0.0152i$
$a_{9,I}$	$-0.0094-0.0002i$	$-0.0097-0.0003i$

(1), the contributions of the a_i coefficients to the decay amplitude for $B\rightarrow M_1M_2$ are of the form

$$m_B^2\frac{G_F}{\sqrt{2}}\lambda_p[g_{1,2}^i f_{M_1}F_{M_2}+g_{2,1}^i f_{M_2}F_{M_1}]a_i, \tag{11}$$

where $g_{1,2}^i$ and $g_{2,1}^i$ are products of Clebsch-Gordan coefficients tabulated in, for example, [1,11,21]. Using the formulas of [2], we can write, with $N_c=3$,

$$f_{M_2}F_{M_1}a_{i,II}=\frac{4\pi}{9}\epsilon_i C_{i'}\alpha_s\beta_i \tag{12}$$

where $\epsilon_i=+1(i=1,\ldots,4,9)$, $\epsilon_5=-1$, $\epsilon_6=0$, and

$$\beta_i=\frac{f_B f_{M_2} f_{M_1}}{m_B\lambda_B}[3(1+\epsilon_i\alpha_1^{M_2}+\alpha_2^{M_2})(1+\alpha_1^{M_1}+\alpha_2^{M_1}) + r_\chi^{M_1}(1-\epsilon_i\alpha_1^{M_2}+\alpha_2^{M_2})X_H^{M_1}]. \tag{13}$$

The chiral factors $r_\chi^{M_1}$ are zero for M_1 a vector meson and are

$$r_\chi^\pi=\frac{2m_\pi^2}{m_b(m_u+m_d)},\quad r_\chi^K=\frac{2m_K^2}{m_b(m_u+m_s)} \tag{14}$$

for the pseudoscalar mesons. It should be noted that these $a_{i,II}$ contributions to the decay amplitudes are independent of the B transition form factors. However, they do involve the poorly determined parameter f_B/λ_B where f_B is the B leptonic decay constant and $\lambda_B\approx 0.6$ GeV is related to the B light cone distribution function. The parameter $X_H^{M_1}$ is the contribution of a logarithmic end-point divergence in the integration over the M_1 light cone distribution function

$$X_H^{M_1}=\int_0^1\frac{dx}{1-x}. \tag{15}$$

These functions take no account of the internal quark transverse momenta which, if included, would make the integrals finite but not calculable within perturbative QCD. $X_H^{M_1}$ is parametrized as

$$X_H^{M_1}=\ln\left(\frac{m_B}{\Lambda_{\rm QCD}}\right)+\rho_H e^{i\phi_H} \tag{16}$$

where ρ_H is not expected to be larger than 3. We take $\ln(m_B/\Lambda_{\rm QCD})=3.03$. The energies involved in the calculation of $a_{i,II}$ imply that the appropriate scale is not that of the scale μ used in calculating the $a_{i,I}$ but $\mu_h=\sqrt{\Lambda_h\mu}$ where [2] $\Lambda_h=0.5$ GeV. We use this with $\mu=m_b/2$ in our fitting so that $\alpha_s f_B/(m_B\lambda_B)=0.0209$. We note that substantial light cone corrections $\alpha_{1,2}^M$ can significantly enhance the $a_{i,II}$ coefficients.

III. ANNIHILATION CONTRIBUTIONS

Because of the heavy b quark mass it is expected that perturbative QCD calculations will give a reliable estimate of the annihilation contribution to the decay amplitude. In these calculations the basic perturbative quark amplitudes are again folded into the participating meson light cone distribution functions. Apart from the low energy regions of the folding integrals the only low energy parameters that appear in the lowest order calculations are the participating meson

TABLE IV. Factorization $a_{i,I}(M_2)$ coefficients evaluated at the scale $\mu=2.10$ GeV using the expressions of [2,8] and Lü and Yang [10] values for the light cone corrections $\alpha_{1,2}^{M_2}$.

	π	K	ρ,ω	K^*
$a_{1,I}^u$	$1.0809+0.0369i$	$1.0762+0.0432i$	$1.0798+0.0369i$	$1.0753+0.0439i$
$a_{2,I}^u$	$-0.0432-0.1130i$	$-0.0290-0.1322i$	$-0.0400-0.1130i$	$-0.0261-0.1344i$
$a_{3,I}$	$0.0085+0.0037i$	$0.0080+0.0043i$	$0.0084+0.0037i$	$0.0079+0.0043i$
$a_{4,I}^u$	$-0.0306-0.0206i$	$-0.0313-0.0210i$	$-0.0325-0.0206i$	$-0.0321-0.0210i$
$a_{4,I}^c$	$-0.0343-0.0072i$	$-0.0364-0.0061i$	$-0.0386-0.0076i$	$-0.0383-0.0061i$
$a_{5,I}$	$-0.0108-0.0050i$	$-0.0102-0.0059i$	$-0.0107-0.0050i$	$-0.0101-0.0060i$
$a_{6,I}^u$	$-0.0586-0.0188i$	$-0.0586-0.0188i$	$-0.0097-0.0012i$	$-0.0102-0.0013i$
$a_{6,I}^c$	$-0.0630-0.0058i$	$-0.0630-0.0058i$	$0.0022+0.0147i$	$0.0032+0.0152i$
$a_{9,I}$	$-0.0097-0.0003i$	$-0.0097-0.0004i$	$-0.0097-0.0003i$	$-0.0097-0.0004i$

electroweak decay constants f_B, f_π, f_ρ, etc. Again the low energy contributions to the integral introduce another nonperturbative complex parameter X_A, which is only loosely constrained by the model estimations. Detailed formulas are to be found in [2,4]. In this paper we follow the more extensive calculations of Du *et al.* [4] but express the annihilation contribution to the decay amplitude in the form

$$\begin{aligned}\langle M_1M_2|\mathcal{H}_{\rm eff}^{\rm ann}|B\rangle = \frac{G_F}{\sqrt{2}}B_{M_1M_2}\{\lambda_u(d_1C_1+d_2C_2)A_1^i \\ -\lambda_t[d_3(C_3A_1^i+C_5A_3^i) \\ +(d_4C_4+d_6C_6)A_1^i+d_5(C_5+N_cC_6)A_3^f]\}\end{aligned} \tag{17}$$

where

$$B_{M_1M_2}=\frac{C_F}{N_c^2}f_Bf_{M_1}f_{M_2} \tag{18}$$

and $C_F=(N_c^2-1)/2N_c$. The quantities $A_{1,3}^{i,f}(M_1,M_2)$, where the superscript $i(f)$ denotes gluon emission from initial (final) state quarks, result from folding the quark amplitudes into the meson distribution functions. If the asymptotic form of Eq. (10) is used then [4]

$$\begin{aligned}A_1^i(P_1,P_2)&=\pi\alpha_s\left[18\left(X_A-4+\frac{\pi^2}{3}\right)+2r_\chi^{P_1}r_\chi^{P_2}X_A^2\right],\\ A_3^i(P_1,P_2)&=6\pi\alpha_s(r_\chi^{P_1}-r_\chi^{P_2})\left(X_A^2-2X_A+\frac{\pi^2}{3}\right),\\ A_3^f(P_1,P_2)&=6\pi\alpha_s(r_\chi^{P_1}+r_\chi^{P_2})(2X_A^2-X_A),\\ A_1^i(P,V)&=\pi\alpha_s\left[18\left(X_A-4+\frac{\pi^2}{3}\right)\right],\\ A_3^i(P,V)&=\pi\alpha_s r_\chi^P[2\pi^2+6(X_A^2-2X_A)],\\ A_3^f(P,V)&=6\pi\alpha_s r_\chi^P(2X_A^2-X_A),\\ A_1^i(V_1,V_2)&=A_1^i(P,V),\\ A_3^i(V_1,V_2)&=A_3^f(V_1,V_2)=0.\end{aligned} \tag{19}$$

The coefficients $d_i(M_1,M_2)$ are Clebsch-Gordan type factors and are given in Table V for the particle sign conventions of [1,3,4,11]. Note that, in using Eq. (17) with Table V, there is no need to distinguish between PV and VP decays.

TABLE V. Annihilation coefficients $d_i(M_1,M_2)$ for $B\to M_1M_2$. The VV channels refer to zero helicity states only.

M_1M_2	d_1	d_2	d_3	d_4	d_5	d_6
$\pi^+\pi^-$	-1	0	-1	-2	-1	-2
$\pi^0\pi^0$	-1	0	-1	-2	-1	-2
$\pi^0\pi^-$	0	0	0	0	0	0
$\rho^+\pi^-$	1	0	1	2	-1	-2
$\rho^-\pi^+$	1	0	1	2	1	-2
$\rho^0\pi^0$	1	0	1	2	0	-2
$\rho^0\pi^-$	0	0	0	0	$-\sqrt{2}$	0
$\rho^-\pi^0$	0	0	0	0	$\sqrt{2}$	0
$\omega\pi^-$	0	$\sqrt{2}$	$\sqrt{2}$	0	0	0
$\rho^0\rho^0$	1	0	1	2	-1	2
$\rho^0\rho^-$	0	0	0	0	0	0
ωK^-	0	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
$\omega\bar{K}^0$	0	0	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
π^0K^-	0	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
$\pi^-\bar{K}^0$	0	-1	-1	0	-1	0
$\pi^-\bar{K}^{*0}$	0	1	1	0	1	0
$\pi^0\bar{K}^0$	0	0	$1/\sqrt{2}$	0	$1/\sqrt{2}$	0
ρ^0K^-	0	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
ρ^0K^{*-}	0	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	0
π^+K^-	0	0	-1	0	-1	0
π^+K^{*-}	0	0	1	0	1	0
ρ^+K^-	0	0	1	0	-1	0
ϕK^-	0	1	1	0	1	0
ϕK^{*-}	0	1	1	0	-1	0
$\phi\bar{K}^0$	0	0	1	0	1	0
$\phi\bar{K}^{*0}$	0	0	1	0	-1	0

IV. CHARMING PENGUIN CONTRIBUTIONS

In our attempts to fit the data set within the scheme outlined above we found that the PP branching ratios could be accommodated with acceptable values of the CKM parameters and transition form factors. However, data on the πK^* channels is consistently too large to be accounted for. The problem is that the πK branching ratios are only marginally larger than the πK^* ratios. In the QCD factorization scheme described above the penguin operators Q_4 and Q_6 contribute coherently and almost equally and dominate the πK decay amplitudes whereas Q_6 is missing from the amplitude for πK^* decay. This results in the predicted ratio of the πK and πK^* branching fractions being too small. Perhaps, staying within the QCD factorization scheme, this failing can be removed by taking radically different light cone distribution functions for the K and K^* mesons. We investigate the possibility of significant additional contributions from so-called *charming penguin contributions* [12].

The largest term in the effective Hamiltonian that produces a strange quark comes from

$$\begin{aligned}\mathcal{H}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{cb}[C_1(\overline{c}_\beta b_\beta)_{V-A}(\overline{s}_\alpha c_\alpha)_{V-A} \\ +C_2(\overline{s}_\alpha b_\alpha)_{V-A}(\overline{c}_\beta c_\beta)_{V-A}].\end{aligned} \tag{20}$$

The charming penguin contributions originate from these terms when the c and $\overline{c}$ quarks annihilate. In the general description of two body B decays as given by Buras and Silvestrini [13], charming penguin contributions have the topologies CP_1 and DP_2 of connected and disconnected penguin contributions respectively. Ciuchini *et al.* [12] consider the contribution of these terms to $\pi\pi$ and πK decays in the $SU(3)$ limit. In their notation, and including the small contribution from the u quark loop, this results in a contribution to the decay amplitudes which they express as

$$A_{\pi K} = -m_B^2[V_{tb}V_{ts}^*\overline{P_1} + V_{ub}V_{us}^*\overline{P_1^{GIM}}]g,$$
$$A_{\pi\pi} = -m_B^2[V_{tb}V_{td}^*\overline{P_1} + V_{ub}V_{ud}^*\overline{P_1^{GIM}}]g, \tag{21}$$

where $\overline{P_1}$ and $\overline{P_1^{GIM}}$ are two complex numbers that are independent of the particular channel, $\pi\pi$ or πK. The only channel dependence is through the Clebsch-Gordan factor g which is the same as the Clebsch-Gordan factor in the a_4 contribution from QCD factorization. Ciuchini *et al.* suggest that all chirally suppressed terms should be dropped and replaced with this term.

We take the charming penguin contribution to be from the penguin topology but to be in addition to the QCD factorization of the penguin contribution. However, to retain the notation of [12], we express $\overline{P_1}$ and $\overline{P_1^{GIM}}$ as

$$\overline{P_1} = \frac{G_F}{\sqrt{2}}(f_\pi F_\pi)|\overline{D}|e^{i\phi},$$

$$\overline{P_1^{GIM}} = \frac{G_F}{\sqrt{2}}(f_\pi F_\pi)|\overline{D^{GIM}}|e^{i\phi_{GIM}}. \tag{22}$$

With the factor $(f_\pi F_\pi)$, taken here to be 0.042 GeV, the dimensionless parameters $|\overline{D}|$ and $|\overline{D^{GIM}}|$ must be less than unity as the charming penguin contributions are expected to be small $O(\Lambda_{\rm QCD}/m_B)$ corrections.

This simple model must be extended to include vector mesons. To this end we note that (i) in PV decays the vector meson must have zero helicity and that, for example, the s and $\overline{q}$ quarks forming the decay meson M can be expected to have zero spin projection in their direction of motion irrespective of whether they form a pseudoscalar or vector meson and (ii) that, when folded into the same light cone distribution functions, the amplitudes would be the same. This most simple model extends the $SU(3)$ symmetry to $SU(6)$. With this *albeit* simple extension, the charming penguin contribution to a particular amplitude is obtained from the factorization contribution by reference to the $a_{4,I}$ term. For example, the decay amplitude for $\overline{B}^0 \to \pi^+\rho^-$ contains a term

$$-m_B^2\frac{G_F}{\sqrt{2}}f_\rho F_\pi[V_{ub}V_{ud}^*(a_{4,I}^c - a_{4,I}^u) + V_{tb}V_{td}^*a_{4,I}^c]. \tag{23}$$

The charming penguin contribution is obtained by replacing this with

$$-m_B^2[V_{ub}V_{ud}^*\overline{P_1^{GIM}} + V_{tb}V_{td}^*\overline{P_1}]. \tag{24}$$

TABLE VI. Measured branching ratios Br(exp), experimental error σ, best fit theoretical branching ratios Br(BBNS) and Br(CP) for the Beneke *et al.* model (BBNS) and charming penguin model (CP), respectively, and the contribution to χ^2 for various B decay channels. All branching ratios are in units of 10^{-6}. For the VV channels the predictions are for longitudinal polarization states only as these decays are expected to be dominant.

Decay	Br(exp)	σ	Br(BBNS)	χ^2	Br(CP)	χ^2
$\pi^+\pi^-$	4.8	0.5	5.0	0.2	4.9	0.0
$\pi^0\pi^-$	5.6	0.9	4.0	2.0	5.9	0.0
$\rho^+\pi^-$	25.4	4.2	24.4	0.1	23.3	0.2
ρ^+K^-	16	5	7.3	2.7	11.1	0.7
$\rho^0\pi^-$	9.4	2.0	9.5	0.0	9.6	0.0
$\omega\pi^-$	6.4	1.3	7.4	0.5	6.4	0.0
π^0K^-	12.9	1.2	13.0	0.0	12.9	0.0
$\pi^-\overline{K}^0$	18.2	1.7	20.9	2.1	19.5	0.5
$\pi^-\overline{K}^{*0}$	12.4	2.6	4.4	7.7	9.1	0.8
ωK^-	3.1	1.0	3.4	0.1	5.0	2.2
ϕK^-	8.8	1.1	8.4	0.1	8.4	0.1
ϕK^{*-}	5.4	2.4	8.9	1.0	9.0	1.6
π^+K^-	18.5	1.0	18.7	0.1	19.0	0.2
π^+K^{*-}	16	6	4.1	3.8	9.5	1.0
$\pi^0\overline{K}^0$	10.4	1.4	7.4	3.9	7.1	4.6
$\omega\overline{K}^0$	6.5	1.7	2.4	5.2	4.4	1.2
$\phi\overline{K}^0$	8.4	1.6	7.8	0.2	7.8	0.2
$\phi\overline{K}^{*0}$	7.2	1.7	8.3	0.3	8.3	0.2

In the charming penguin amplitude, the s quark is produced from a left-handed field and can be expected to have predominantly negative helicity. The $q\overline{q}$ pair emanates from either right-handed or left-handed fields and, with zero helicity for the produced $(s\overline{q})$ meson, we expect that the left-handed contribution will dominate to form an a_4 type term.

TABLE VII. Value of the parameters used with their variation.

Parameter	Central value	Variation
f_π	0.1307	± 0.00046
f_K	0.1598	± 0.00184
f_ρ	0.216	± 0.005
f_ω	0.194	± 0.004
f_{K^*}	0.216	± 0.010
f_ϕ	0.233	± 0.004
λ	0.2205	± 0.0010
$\tau_{B^0}/\tau_{B^\pm}$	1.081	± 0.015
μ	$0.5m_b$	m_b
$\|\rho_H\|$	2.0	± 1.0
$\arg(\rho_H)$	4.7	1.6
$\|X_A\|$	1.85	± 1.0
$\arg(X_A)$	2.86	3.7
$r_\chi^{\pi,K}$	1.0	± 0.2 (CP only)
$A_{\rm Wolf}$	0.82	± 0.05 (CP only)
α_n^M	0	Table IV (Lü and Yang)

TABLE VIII. Best fit values and one standard deviation errors for the Beneke *et al.* (BBNS) and charming penguin (CP) models. $F_{\pi,K}$ and $A_{\rho,\omega,K*}$ are the transition form factors $F^{B\to h}(0)$ for P and V mesons, respectively, and $r_\chi^{\pi,K}$ are the chiral enhancement factors which are nominally power suppressed but are $O(1)$ in practice.

Model	F_π	F_K	A_ρ	A_ω	A_{K*}
BBNS	0.244 ± 0.038	0.369 ± 0.031	0.344 ± 0.098	0.300 ± 0.094	0.321 ± 0.136
CP	0.291 ± 0.022	0.349 ± 0.077	0.320 ± 0.065	0.298 ± 0.072	0.309 ± 0.117
	r_χ^π	r_χ^K	A	$\bar\rho$	$\bar\eta$
BBNS	1.09 ± 0.24	1.24 ± 0.17	0.813 ± 0.045	0.068 ± 0.071	0.383 ± 0.090
CP	1.0	1.0	0.82	-0.044 ± 0.112	0.397 ± 0.050
	$\lvert D\rvert$	Arg(D)	D^{GIM}	Arg(D^{GIM})	
CP	0.068 ± 0.007	1.32 ± 0.10	0.32 ± 0.14	1.00 ± 0.27	

The right-handed term will be suppressed by a factor $\Lambda_{\mathrm{QCD}}/m_B$. We appreciate that the expectation of considerable suppression is false for the corresponding factorization term a_6Q_6 for which the suppression factor r_χ^K is of order unity. a_6Q_6 is a product of matrix elements of local operators containing $\langle M|(\bar{s}q)|0\rangle$ which is zero for M being a vector meson. We suspect that the chiral enhancement of scalar meson production in factorization penguin contributions is a property of factorization of the local product rather than a general feature of all charming penguin contributions.

V. FITTING METHOD

We have attempted to fit the theoretical expressions for branching ratios with the available data as averaged by the Heavy Flavor Averaging Group [15]. Measured branching ratios for 18 channels are shown in Table VI. We take the measured branching ratios to be the mean of the B and $\bar{B}$ decays. For the two vector-vector channels ϕK^* we multiply the measured branching ratios by the longitudinal polarization as measured by BaBar [14]. *CP* asymmetries are not included in the fit. The measurements are not always consistent between the experiments and the errors are large. We therefore prefer to compare the measured results with the predictions from the fit. To economize in the number of soft QCD parameters we have not included decay channels involving η and η' mesons. These amplitudes involve the mixing angle between the $(u\bar{u}+d\bar{d})$ and $s\bar{s}$ combinations. Also, in principle, there is mixing with $c\bar{c}$ which, though small, could make a significant contribution to decay modes through the enhanced quark decay modes $b\to cq\bar{c}$.

For convenience we assign to each channel (M_1,M_2) a number α. The statistical and systematic errors have been combined [15] into a single error σ_α. The systematic errors are in general small and we ignore any correlations. We then form a χ^2 function

$$\chi^2(P_i)=\sum_\alpha [|Br_\alpha(P_i)-Br_\alpha(\mathrm{exp})|/\sigma_\alpha]^2 + \text{additional constraints.} \qquad (25)$$

$Br_\alpha(P_i)$ are the theoretical branching ratios expressed in terms of ten parameters P_i which we take to be the three Wolfenstein CKM parameters $\{A,\rho,\eta\}$ and seven soft QCD parameters $\{r_\chi^\pi,r_\chi^K,F_\pi,F_K,A_\rho,A_\omega,A_{K*}\}$. For the fit to the charming penguin model we introduce four more parameters: the modulus and phase of $\bar{P}_1$ and $\bar{P}_1^{GIM}$. In this case we fix A to the world average of 0.82 and keep $r_\chi^{\pi,K}$ fixed at 1.0. The well established decay parameters $\{f_\pi,f_K,f_\rho,f_\omega,f_\phi,f_{K*}\}$ are held at their mean values and the Wolfenstein CKM parameter λ is set to 0.2205. The results are not very sensitive to the divergence parameters and we held them fixed at $\rho_H=2e^{4.7i}$ and $X_A=1.85e^{2.86i}$, values suggested by some preliminary investigations. Additional terms were included in the χ^2 to take into account experimental and theoretical constraints from outside the data on B decay branching ratios. We search for a minimum of χ^2 as a function of the P_i using the MINUIT [16] program.

Next to the experimental error on the measurement we have to consider the error from our assumptions on the QCD

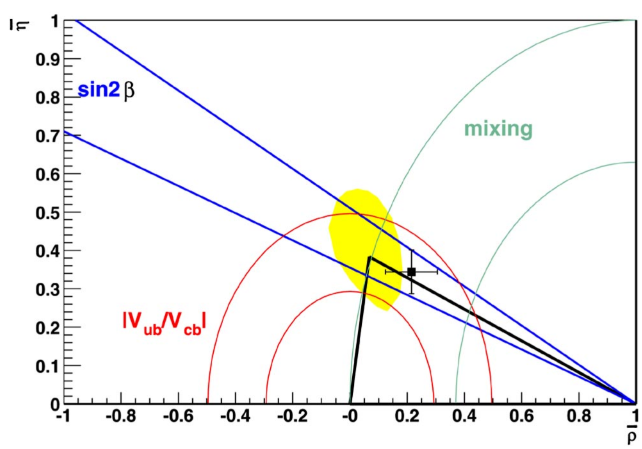

FIG. 1. Result for the unitarity triangle fit in the BBNS model. The shaded area shows the 3σ allowed region for the apex of the unitarity triangle. The data point shows the fit result from the unitarity triangle fit from other measurements taken from *CKMfitter* [17].

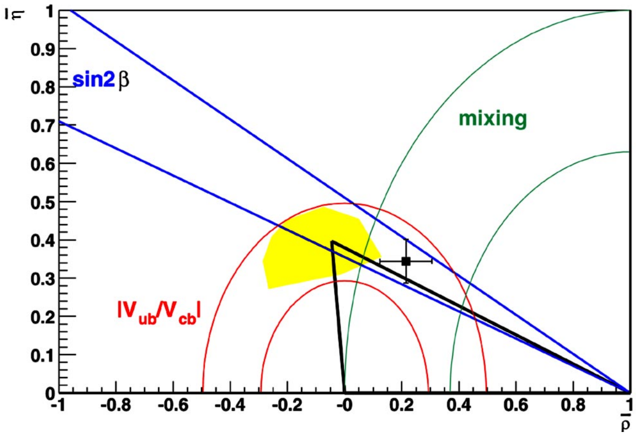

FIG. 2. Result for the unitarity triangle fit in the charming penguin model. The shaded area shows the 3σ allowed region for the apex of the unitarity triangle. The data point shows the fit result from the unitarity triangle fit from other measurements taken from *CKMfitter* [17].

parameters that we do not fit for. Table VII shows their central value and an estimate of the allowed variation. First we performed the fit using experimental errors only. With the best value we calculated a set of reference branching fractions. We then varied the parameters according to Table VII while keeping the value of the CKM parameters ρ and η fixed. For each parameter this leads to a difference for each branching ratio. For every branching ratio we sum the difference in quadrature and consider this to be the model uncertainty. We add this uncertainty in quadrature to the experimental error and repeat the fit.

The theoretical branching ratios and the contributions of the individual channels to χ^2 based upon these best fit values are given in Table VI. The best fit parameter values are shown in Table VIII together with our estimates of the errors. These errors are of course highly correlated. Plots of the error matrix ellipse for the Wolfenstein parameters $\overline{\rho}$ and $\overline{\eta}$ are shown in Figs. 1 and 2. The CKM angles are $\alpha=(78\pm9)°$, $\beta=(22\pm2)°$, $\gamma=(80\pm7)°$ for the BBNS model and $\alpha=(63\pm7)°$, $\beta=(21\pm2)°$, $\gamma=(96\pm6)°$ for the CP model. These can be compared to the world averages of [17] $\alpha=(96\pm13)°$, $\beta=(23.3\pm1.5)°$, $\gamma=(61\pm12)°$. All errors are one standard deviation. For both models the results in Table VIII for the values of the various form factors lie within the spread of theoretical estimates. The χ^2/dof is 34.5/17 for the BBNS fit and 14.5/13 for the charming penguin contributions.

VI. DISCUSSION AND CONCLUSIONS

The first conclusion is that the factorization approach works quite well. Most of the branching ratios in Table VI are predicted correctly by both models. The fitted parameters in Table VIII look reasonable for both fits. The χ^2 for the charming penguin fit is significantly better, due mainly to the poor fit of the BBNS model for the decay modes involving the K^* meson. Also, the experimental value for ωK^0 is not easily accommodated within the BBNS model. Both models have a problem in fitting the $\pi^0 K^0$ mode. Figures 1 and 2 show the position of the apex of the unitarity triangle for both fits. The results are consistent with each other for the angle γ but give a larger value than that suggested by the CKM Fitting Group. The angle β agrees well for both fits.

Our theoretical best-fit values for those *CP* asymmetries that have been measured are shown in Table IX. The direct *CP*-violating parameter $A_{\rm CP}$ for the decay channel M_1M_2 is

$$A_{\rm CP}=\frac{\Gamma(\overline{B}\to\overline{M}_1\overline{M}_2)-\Gamma(B\to M_1M_2)}{\Gamma(\overline{B}\to\overline{M}_1\overline{M}_2)+\Gamma(B\to M_1M_2)} \tag{26}$$

where $\overline{B}=b\overline{u}$ or $b\overline{d}$. The definitions of the other *CP*-violating parameters can be found in [19]. Regarding these asymmetries it is too early to reach a conclusion. In many cases the different experiments disagree, in others the errors are so large that a meaningful discrimination is not possible. The theoretical asymmetries are very sensitive to the parameters and to the different models and, with improved

TABLE IX. Measurements and theoretical best fit values for *CP* asymmetries. Only statistical errors are shown.

A_{CP}	BaBar	Belle	BBNS	CP
$K^+\pi^-$	-0.102 ± 0.05	-0.07 ± 0.06	0.00	-0.08
$K^+\pi^0$	-0.09 ± 0.09	0.23 ± 0.11	0.06	-0.02
$K^0\pi^+$	-0.17 ± 0.10	0.07 ± 0.09	0.01	0.11
$\pi^+\pi^0$	-0.03 ± 0.18	-0.14 ± 0.24	0.00	0.00
$\rho^+\pi^-$	-0.22 ± 0.08		-0.03	-0.03
ρ^+K^-	0.28 ± 0.17		0.10	-0.40
$C_{\pi\pi}$	-0.30 ± 0.25	-0.77 ± 0.27	0.01	-0.23
$S_{\pi\pi}$	0.02 ± 0.34	-1.23 ± 0.41	-0.20	0.25
$C_{\rho\pi}$	0.36 ± 0.15		0.03	-0.31
$S_{\rho\pi}$	0.19 ± 0.24		0.38	0.65
$\Delta C_{\rho\pi}$	0.28 ± 0.19		0.08	0.31
$\Delta S_{\rho\pi}$	0.15 ± 0.25		0.02	0.11
$C_{\phi K_s^0}$	-0.80 ± 0.38	0.56 ± 0.41	-0.01	-0.19
$S_{\phi K_s^0}$	-0.18 ± 0.51	-0.73 ± 0.64	0.73	0.62

TABLE X. Predicted branching ratios (in units of 10^{-6}) and *CP* asymmetries for some channels not included in the fit. For the *VV* channels the predictions are for longitudinal polarization states only as these decays are expected to be dominant. The predictions are for the central values of the model fits with 1σ errors estimated by sampling the parameter error matrix.

	Mean Branching Ratio		A_{CP}	
Decay	BBNS	CP	BBNS	CP
$\pi^0\pi^0$	0.5 ± 0.1	1.1 ± 0.2	0.59 ± 0.08	-0.63 ± 0.15
$\rho^0\rho^0$	0.5 ± 0.1	1.2 ± 0.3	0.69 ± 0.16	-0.66 ± 0.07
$\pi^0\rho^-$	8.4 ± 0.7	11.9 ± 1.4	0.04 ± 0.01	0.06 ± 0.01
$\rho^0\rho^-$	19.0 ± 3.1	17.8 ± 3.3	0.00 ± 0.00	0.00 ± 0.00
ρ^0K^+	2.0 ± 0.6	4.6 ± 0.5	0.00 ± 0.09	-0.01 ± 0.64
ρ^0K^{*+}	4.6 ± 1.3	5.2 ± 0.7	0.33 ± 0.06	-0.35 ± 0.06

statistics, could become the final test of factorization. In Table X we show our predictions for branching ratios and *CP* asymmetry A_{CP} for some channels not included in the fit [22]. We include some *VV* channels that are currently under investigation. For these channels we have assumed that the decays are to longitudinally polarized vector mesons as it is expected [1,20] that decays to the other polarization states will be suppressed by at least a factor of $(m_V/m_B)^2$.

In their most recent work Beneke and Neubert [8] give formulas for the weak annihilation functions $A_{1,3}^{i,f}(P,V)$ that, in addition to the terms given in Eq. (19), include terms containing a parameter r_χ^V. This parameter r_χ^V has a similar origin to r_χ^P and, like r_χ^P, is suppressed by a power of $\Lambda_{\mathrm{QCD}}/m_b$ but, unlike r_χ^P, is not chirally enhanced. We only became aware of [8] after completion of this study. To check for the effects of these terms we have modified our program to include the r_χ^V contribution to annihilation and have reminimized χ^2. For the charming penguin model we found the best fit occurred with $X_A = 1.09\exp(2.79i)$ and that there were very small changes in the best fit parameters of Table VIII and the results of Tables VI, IX and X. For the BBNS model we found similar results but the overall fit was better in that the χ^2 was reduced from 34.5 to 27.5.

Finally, the results presented here are slightly different from preliminary numbers presented earlier [18], for three reasons. The Heavy Flavour Averaging Group has updated some of the results and included a new branching fraction $(\rho^\pm K^\mp)$. We have also corrected the vector-vector channels $K^*\phi$ for the effect of polarization. Finally we now include the systematic uncertainties in the fit.

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[21] It should be noted that these tables conform to the sign conventions of Ali and Greub [11] and differ from the isospin convention of Beneke *et al.* [2].

[22] Experimental results for two of these decay modes are now available from the BaBar and Belle experiments. They find [15] $B^0 \to \pi^0\pi^0 = (1.9 \pm 0.5) \times 10^{-6}$ and $B^- \to \rho^-\pi^0 = (11.0 \pm 2.7) \times 10^{-6}$. The former is on the high side of our predictions, particularly for the BBNS model; the latter is in good agreement with the prediction from the *CP* model but less so with that of the BBNS model.