Worldvolume supersymmetries for branes in plane waves

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We study the worldvolume supersymmetries of $M2$ branes in the maximally supersymmetric plane wave background of M theory. For certain embeddings the standard probe analysis indicates that the worldvolume theory has less than 16 supersymmetries. We show that at the quadratic level the worldvolume theory admits additional linearly realized supersymmetries, and that the spectra of the branes are organized into multiplets of these symmetries. We find, however, that these supersymmetries are not respected by worldvolume interactions. Our analysis was motivated by recent work showing that D branes in the maximally supersymmetric plane wave background of type IIB string theory admit supersymmetries beyond those of the probe analysis. The construction of the additional supercharges in this case was specific to a string worldsheet that is a strip and the present results suggest that string interactions do not preserve these symmetries.

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I. INTRODUCTION

One of the most elementary questions that one can ask about a supersymmetric theory is "what are the supersymmetric states of the theory." In superstring theory a class of supersymmetric states is represented by D branes. Recent work on branes in the maximally supersymmetric plane wave background type IIB string theory shows that this issue is more subtle than the corresponding analysis in flat spacetimes. A tree-level open string analysis of boundary conditions and spectra in [1,2] revealed that certain branes have more supersymmetries than the probe analysis gives [3]. This implies that either the standard probe analysis needs to be amended or that the extra supersymmetries are not respected by string interactions. One of the aims of the present work is to settle this issue.

Recall that in string perturbation theory D branes specify boundary conditions for open strings. In the Green-Schwarz formalism the spacetime supersymmetries preserved by the D-brane manifest themselves as global symmetries of the worldsheet action. Some of the global symmetries of the open worldsheet directly descend from corresponding symmetries of the closed string. These symmetries are exactly the ones found by the probe analysis. It was found in [1,2], however, that in certain cases the worldsheet action admits additional supersymmetries and that the spectrum of the theory is organized with respect to the extra supersymmetries as well.

The branes in the type IIB plane wave can be divided into two different classes: the D_- and D_+ branes. This classification originates from the specific form of the (worldsheet) fermionic boundary conditions, but one can also understand it from the features of the spectrum. In D_- branes, the mass parameter μ of the plane wave lifts some of the degeneracy of states present in the flat space limit. In particular, the lowest lying states which in the flat space limit form a $d=10$ vector multiplet (i.e., a multiplet with $8+8$ degrees of freedom with the same lightcone energy P^-) now split as $1+4+6+4+1$ (see Table 1 of [2]). On the other hand, the lowest lying states for D_+ branes are as degenerate as in flat space (see Table 3 of [2]).

The branes under investigation are located at a constant transverse position and wrap specific directions. According to the probe analysis, D_+ branes always break all kinematical supersymmetries.[1] D_- branes preserve eight kinematical supercharges along with an additional eight dynamical supercharges only when the brane is located at the origin [3]. In the string theory analysis [1], however, one finds eight alternative supercharges for the D_+ branes and for the D_- branes located away from the origin. A clue for the existence of the extra supersymmetries was that the spectrum of the brane exhibits more symmetries than the probe analysis suggests. In particular, the spectrum of D_- branes at and away from the origin is identical up to an overall additive (positive) constant in the lightcone energy [2].

Notice that the existence of extra *local* Noether currents corresponding to the new supersymmetries is not an automatic consequence of the degeneracy of the spectrum. In-

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[1]We call kinematical supersymmetries the supercharges that square to the lightcone momentum, and dynamical the ones that square to the lightcone Hamiltonian (plus other charges).

deed, the string states of the D_+ branes also appear to be organized in multiplets of ''dynamical supercharges.'' However, these charges are associated with *nonlocal* currents [2].

The extra eight supersymmetries for D_+ branes are completely new symmetries, unrelated to the closed string symmetries. They satisfy the standard lightcone superalgebra, i.e., they square to the lightcone momentum. The extra eight supersymmetries for D_- branes are a combination of the corresponding closed string supersymmetries and new transformation rules. The new supercharges when evaluated on-shell are identical to the corresponding supercharges of the brane at the origin [2]. It follows that the corresponding superalgebras are also identical (up to certain c-number shifts). In particular, the new supercharges square to the lightcone Hamiltonian plus rotational charges plus a c-number that has the interpretation of the energy of an open string in a harmonic oscillator potential with ends at the constant position of the brane. This c-number contribution is also the on-shell value of an additional worldsheet charge [see Eqs. (4.31) and (4.32) of [1]].

The construction of the extra supercharges crucially used the fact that the gauge fixed worldsheet action is quadratic in the fields and that the worldsheet is a strip. The former is a special property of strings propagating in the type IIB plane wave background. The latter indicates that the extension of the extra symmetries to higher genus surfaces is not immediate, and that string interactions may invalidate them.

Consistency requires that the string theory and probe analysis yield the same results. Recall that the worldvolume theory of D branes captures the low-lying open string excitations and their (low-energy) interactions. In particular, the spectrum of small fluctuations around the D-brane embedding should coincide with the zero slope limit of the open string spectrum. As mentioned, the open string spectrum is more supersymmetric than the probe analysis implies. It follows that the quadratic part of the D-brane action should exhibit additional supersymmetries. If string interactions respect the extra symmetries then the worldvolume interactions should also respect them. Conversely, if we show that the worldvolume interactions do not preserve the extra symmetries then this shows that string interactions do not respect the extra symmetries.

Recall that the brane worldvolume theories are by construction invariant under target space supersymmetry and they also possess a local kappa symmetry invariance. Upon gauge fixing the kappa symmetry, the target space supersymmetry turns into worldvolume supersymmetry. The plane wave backgrounds we discuss in this paper admit 32 supercharges. This means that the worldvolume theory is by construction invariant under 32 fermionic symmetries. However, at most 16 of them are linearly realized, i.e., they are of the schematic form (the exact expressions are given in the main text)

$$\delta X^A \sim \overline{\theta}\Gamma^A \epsilon + \cdots, \quad \delta\theta \sim \Gamma^{\mu A}\partial_\mu X^A \epsilon + \mu \Gamma^A X^A \epsilon + \cdots, \tag{1.1}$$

where ϵ is the supersymmetry parameter and the dots indicate additional terms which are at least quadratic in the fields. The remaining transformations have an inhomogeneous term

$$\delta\theta = \beta + \cdots, \tag{1.2}$$

where β is field independent, and the associated θ can be identified with the Goldstone fermion associated with the breaking of supersymmetry.

The probe analysis by construction counts the number of linearly realized supersymmetries that arise from a combination of target space supersymmetries with kappa symmetry. This does not exclude, however, the possibility that there are extra nongeneric symmetries when the brane is in particular backgrounds. The string theory analysis in [1,2] can be viewed as an example of such phenomenon: because of special properties of the background (i.e., the worldsheet action is quadratic in fields) the worldsheet theory exhibits more symmetries than in generic situations.

The extra linearly realized supersymmetries can be completely new symmetries, unrelated to the supersymmetries associated with target space supersymmetries. Alternatively, if the brane in special backgrounds exhibits a new gauge invariance that allows one to gauge away the Goldstone fermion then the corresponding symmetry would be linearly realized. Both mechanisms are suggested by the string theory computation in [1]: the former is the analogue of the new kinematical supersymmetries in D_+ branes and the latter is the analogue of the restoration of dynamical supersymmetries for D_- branes using worldsheet symmetries. We will see that both mechanisms are realized, albeit only at the quadratic approximation of the worldvolume theories.

The purpose of this paper is to analyze the issue of worldvolume supersymmetries in detail. Instead of working with the worldvolume theory of the type IIB D branes, however, we will analyze the same issues for $M2$ branes for which the worldvolume theory is much simpler (since there are no gauge fields). The probe analysis for this case has been worked out in [4] (see also [5]). The results for supersymmetric $M2$ embeddings are directly analogous to the results in [3]. Recall that in the maximally supersymmetric plane wave of M theory the transverse to the lightcone coordinates split as $3+6$. The $M2$ branes that wrap the lightcone directions and one of the three coordinates preserve 16 supercharges when located at the origin of transverse space but only eight when located away from it. These are the analogues of D_- branes and we will refer to them as M_-2 branes. $M2$ branes that wrap the lightcone coordinates and one of the six coordinates preserve no supersymmetry and are the analogue of D_+ branes. They will be referred to as M_+2 branes. We will see here that their fluctuation spectra are similar to those of the corresponding D_- and D_+ branes.

Of course in this case there is no computation corresponding to the string theory analysis in [1,2] so strictly speaking there is no *a priori* reason for expecting the worldvolume theory to exhibit extra supersymmetries. Since the probe computations for M and D branes are similar, we expect extra supersymmetries at the quadratic level of fluctuations of the $M2$ probe action, and our calculations confirm this.

Conversely our results in Secs. IV and V indicate that the extra supersymmetries fail at the interacting level, and we expect the same to be true for D branes.

This paper is organized as follows. We review the properties of the plane wave background in Sec. II and discuss the computation of the gauge fixed supermembrane action in Sec. III. In Sec. IV we consider the spectrum and worldvolume supersymmetries of M_-2 branes while Sec. V addresses the same issues for M_+2 branes. In the discussion section, Sec. VI, we review our results and comment on the implications for D branes in type IIB plane waves and for the stability of the branes.

II. THE PLANE WAVE BACKGROUND

The maximally supersymmetric plane wave background of 11-dimensional supergravity is [6]

$$\begin{aligned} ds^2 &= -2dx^+dx^- + \sum_{A=1}^{9}(dx^A)^2 \\ &\quad - \frac{\mu^2}{36}\left(4\sum_{i=1}^{3}(x^i)^2 + \sum_{a=4}^{9}(x^a)^2\right)(dx^+)^2, \\ F_{+123} &= -\mu \rightarrow C_{ijk} = -\mu x^+ \epsilon_{ijk}, \end{aligned} \tag{2.1}$$

where a specific gauge choice for the 3-form gauge gauge field C is made.

For coset spaces such as the plane wave exact expressions for the supervielbein were computed in [7] (see also [8])

$$\begin{aligned} E &= D\theta + \sum_{n=1}^{16}\frac{1}{(2n+1)!}\mathcal{M}^n D\theta, \\ E^r &= e^r + \bar{\theta}\Gamma^r D\theta + 2\sum_{n=1}^{15}\frac{1}{(2n+2)!}\bar{\theta}\Gamma^r\mathcal{M}^n D\theta. \end{aligned} \tag{2.2}$$

Here $(r,\bar{a})$ are tangent space vector and spinor indices, respectively, and (m,α) are the corresponding curved indices. In these expressions[2]

$$\begin{aligned} D\theta &= d\theta + e^r T_r^{stuv}\theta F_{stuv} + \tfrac{1}{4}\omega^{rs}\Gamma_{rs}\theta, \\ T_r^{stuv} &= \frac{1}{2!3!4!}(\Gamma_r^{stuv} - 8\delta_r^{[s}\Gamma^{tuv]}), \\ \mathcal{M} &= 2(Trstuv\,\theta)Fstuv(\theta\Gamma r) - 1288(\Gamma rs\,\theta) \end{aligned} \tag{2.3}$$

The bosonic vielbein and the spin connection are

$$\begin{aligned} e_-^- = e_+^+ = 1, \quad e_B^A = \delta_B^A, \quad e_+^- = -\tfrac{1}{2}G_{++}, \\ \omega_+^{-A} = -\tfrac{1}{2}\partial_A G_{++} \end{aligned} \tag{2.4}$$

(G_{mn} denotes the spacetime metric). The Killing spinors are

$$\begin{aligned} \epsilon &= \left(1 + \frac{\mu}{12}(x^a\Gamma^a - 2x^i\Gamma^i)\Gamma^{+123}\right)\exp\left(\frac{\mu}{12}x^+\Gamma^{+-123}\right) \\ &\quad \times \exp\left(-\frac{\mu}{6}x^+\Gamma^{123}\right)\epsilon_0, \end{aligned} \tag{2.5}$$

where ϵ_0 is a constant spinor.

III. SUPERMEMBRANE ACTION

The supermembrane action [10] is

$$S = -\int d^3\xi\sqrt{-\det g_{\mu\nu}} + \int B, \tag{3.1}$$

where the induced worldvolume supermetric is $g_{\mu\nu} = \Pi_\mu^r\Pi_\nu^s\eta_{rs}$ and $\Pi_\mu^r = \partial_\mu Z^M E_M^r$. Here $Z^M = (X^m, \theta^\alpha)$ are the coordinates of the target superspace and ξ^μ are the worldvolume coordinates. The explicit expression for B in the coset background is [7]

$$B = \frac{1}{6}e^r\wedge e^s\wedge e^t C_{rst} - \int_0^1 dt\,\bar{\theta}\Gamma_{rs}E(t)\wedge E^r(t)\wedge E^s(t), \tag{3.2}$$

where t is auxiliary and $E(t), E^r(t)$ are obtained from the supervielbeins by the shift $\theta \rightarrow t\theta$.

This action is invariant under the kappa symmetry transformations [10]

$$\delta Z^M E_M^r = 0, \quad \delta Z^M E_M^a = [(1-\Gamma)\kappa]^a, \tag{3.3}$$

where

$$\Gamma = \frac{1}{6}\frac{\epsilon^{\mu\nu\rho}}{\sqrt{-g}}\Pi_\mu^r\Pi_\nu^s\Pi_\rho^t\Gamma_{rst}, \tag{3.4}$$

which satisfies $\Gamma^2 = 1$ and $\mathrm{Tr}(\Gamma) = 0$. The action is also invariant under superspace diffeomorphisms $\delta Z^M = -K^M(Z)$ which act as

$$\begin{aligned} \delta E_M^A &= K^N\partial_N E_M^A + \partial_M K^N E_N^A, \\ \delta B_{MNP} &= K^Q\partial_Q B_{MNP} + 3\partial_{[M}K^Q B_{|Q|NP]}, \end{aligned} \tag{3.5}$$

where $K^M(Z)$ is a Killing supervector, along with worldvolume diffeomorphisms

$$\delta Z^M = \eta^\mu\partial_\mu Z^M, \quad \delta g_{\mu\nu} = \eta^\rho\partial_\rho g_{\mu\nu} + 2\partial_{(\mu}\eta^\rho g_{\nu)\rho}. \tag{3.6}$$

The action also admits other symmetries, such as tensor gauge transformations, but these will not play a role here. To leading order the kappa symmetry and supersymmetry transformations of the supermembrane action are

[2]Note that we will use here the usual Wess-Zumino gauge for the plane wave superspace, namely $\theta^{\bar{a}} = \delta_\alpha^{\bar{a}}\theta^\alpha$. An alternative choice would be the Killing spinor adapted gauge [9] $\theta^{\bar{a}} = K_\alpha^{\bar{a}}\theta^\alpha$ where the Killing spinors of the curved target space are $\epsilon^{\bar{a}} = K_\alpha^{\bar{a}}\epsilon_0^\alpha$ with constant ϵ_0^α. In contrast to the AdS backgrounds considered in [9] this choice does not appear to lead to substantial simplifications in the supervielbeins or the worldvolume action.

$$\delta_\kappa\theta=(1-\Gamma)\kappa, \quad \delta_\kappa X^m=-\bar{\theta}\Gamma^m\delta_\kappa\theta, \tag{3.7}$$

$$\delta_\epsilon\theta=\epsilon, \quad \delta_\epsilon X^m=-\bar{\epsilon}\Gamma^m\theta, \tag{3.8}$$

where ϵ are the Killing spinors of the target space (2.5). Implicit in Eqs. (3.3) and (3.5) are corrections to these expressions which are higher order in θ and can be neglected in what follows.

A. Embeddings

Membrane embeddings are given by solutions of the bosonic field equations of Eq. (3.1),

$$\frac{1}{\sqrt{-\gamma}}\partial_\mu(\sqrt{-\gamma}\gamma^{\mu\nu}\partial_\nu X^m)+\gamma^{\mu\nu}\partial_\mu X^n\partial_\nu X^p\Gamma^m_{np} =\frac{1}{3!}\epsilon^{\lambda\mu\nu}F^m_{\lambda\mu\nu}, \tag{3.9}$$

where $\gamma_{\mu\nu}=\partial_\mu X^m\partial_\nu X^n G_{mn}$ is the induced worldvolume metric and Γ^m_{np} is the Christoffel symbol of the plane wave metric.

The solutions of interest here have been discussed in [4], so our discussion will be brief. These solutions are

$$M_-2: \quad X^\mu=\xi^\mu, \ \ \mu=\{+,-,1\}, \quad X^{A'}=x_0^{A'}, \ \ A'=\{i',a\},$$

$$M_+2: \quad X^\mu=\xi^\mu, \ \ \mu=\{+,-,4\}, \quad X^{A'}=x_0^{A'}, \ \ A'=\{i,a'\}, \tag{3.10}$$

where in each case A' runs over directions transverse to the brane, $i'=2,3$ and $a'=5,\ldots,9$.

The supersymmetries of branes at the origin vs branes displaced along the parabolic "potential" in the transverse directions are one of the main concerns of this paper. Although the plane wave space-time is homogeneous, rigid translations in transverse directions are not isometries. So branes at $x_0^{A'}=0$ and $x_0^{A'}\neq 0$ are not related by symmetry and are physically distinct.

The condition for unbroken supersymmetry is [11]

$$0=\delta\theta=\epsilon(X)+(1-\gamma^*)\kappa(X), \tag{3.11}$$

where γ^* is Γ evaluated at the embedding and ϵ is the Killing spinor of the background evaluated at the embedding. Clearly one can choose $\kappa(X)$ to cancel the effect of the $(1-\gamma^*)\epsilon(X)$ projection of the Killing spinor, so the condition reduces to

$$\gamma^*\epsilon=-\epsilon. \tag{3.12}$$

For M_-2 branes, $\gamma^*=\Gamma_{+-1}$, and for M_+2 branes $\gamma^*=\Gamma_{+-4}$. Using the decomposition of the Killing spinors (2.5) into eigenspinors of γ^* given in Eqs. (4.23) and (5.6) one can easily solve Eq. (3.12). The situation on supersymmetries of the embeddings in Eq. (3.10) may be summarized as follows:

$$M_-2: \quad x_0^{A'}=0, \ \Gamma_{+-1}\epsilon_0=-\epsilon_0, \quad 16 \text{ supercharges}, \tag{3.13}$$

$$M_-2: \quad x_0^{A'}\neq 0 \quad \text{above and } \Gamma^+\epsilon_0=0, \quad 8 \text{ supercharges}, \tag{3.14}$$

$$M_+2: \quad \text{no preserved supercharges}. \tag{3.15}$$

It is guaranteed that supersymmetries of the embedding are preserved by the kinetic and interaction Lagrangians of fluctuations (in both X^m and θ) about the static configurations in Eq. (3.10). By construction the embedding SUSY's satisfy $\delta\theta=0$ when $\theta=0$. They are therefore realized linearly on fluctuations. The principal question investigated below is whether there are new linear fermionic symmetries of the fluctuations about displaced M_-2 branes and M_+2 branes which effectively increase the number of supercharges beyond those counted in Eqs. (3.13)–(3.15).

B. Gauge fixing

The next step of our investigation requires gauge fixing of both worldvolume diffeomorphisms and κ symmetry. We encountered some initially puzzling issues of compatibility of gauge-fixing conditions with the specific embeddings (3.10). These issues were not known to the previous investigators we consulted, so we will describe them in some detail.

It was shown in [12,13] that the plane wave membrane action is quadratic in fermions in lightcone gauge, just as it is in flat space [10]. Here lightcone gauge consists of the conditions

$$\Gamma^+\theta=0, \quad X^+=p^+\tau, \quad g_{\tau p}=0, \quad g_{\tau\tau}=-\det g_{pq}, \tag{3.16}$$

where (τ,σ^p) with $p=1,2$ are the worldvolume coordinates. Note that these conditions do not entirely fix the worldvolume diffeomorphisms; the group of area preserving diffeomorphisms remain.

Given the simplicity of the action in lightcone gauge, this gauge appears at first sight to be the natural choice for us. However, the embeddings in which we are interested are degenerate in this gauge. To prove this consider a bosonic embedding and gauge fix $X^+=p^+\tau$. Then the induced worldvolume metric is

$$\gamma_{\tau\tau}=-2p^+\partial_\tau X^-+(\partial_\tau X^A)^2+(p^+)^2G_{++}, \tag{3.17}$$

$$\gamma_{\tau p}=-p^+\partial_p X^-+\partial_\tau X^A\partial_p X^A, \quad \gamma_{pq}=\partial_p X^A\partial_q X^A.$$

Now impose the next condition from Eq. (3.16), namely $\gamma_{\tau p}=0$; this condition will determine X^- from the remaining scalars X^A (see [14] for details). The final condition in Eq. (3.16) is needed to remove the square root $\sqrt{-\det g_{\mu\nu}}$ and give a polynomial action. However, even before imposing this, one finds

$$\det(g_{\mu\nu})=g_{\tau\tau}\det(g_{pq}). \tag{3.18}$$

Since the configurations in Eq. (3.10) describe branes extended in $(X^+,X^-,X^{\tilde{A}})$, there is a single worldvolume direction $X^{\tilde{A}}$ transverse to the lightcone. The induced brane metrics are thus degenerate for our embeddings, i.e., $\det(-g_{\mu\nu})=0$ and are thus inadmissible in lightcone gauge.

It may seem surprising that these embeddings are degenerate in this gauge, given that they are clearly not degenerate in the static gauge $X^\pm=\xi^\pm$ and $X^{\tilde{A}}=\xi^{\tilde{A}}$, which is necessarily related to the lightcone gauge by a worldvolume diffeomorphism (3.6). However, the Jacobian of the transformation from $(\xi^\pm,\xi^{\tilde{A}})$ to (τ,σ^p) is zero. This follows from imposing the conditions

$$\xi^+=p^+\tau, \quad 0=-p^+\partial_p\xi^- +\partial_\tau\xi^{\tilde{A}}\partial_p\xi^{\tilde{A}}, \tag{3.19}$$

which enforce $X^+=p^+\tau$ and $\gamma_{\tau p}=0$, respectively.

Thus we must give up lightcone gauge for the bosons in favor of the static gauge which is immediate for our embeddings. However, we might consider a ''hybrid'' gauge in which the bosonic static gauge conditions are combined with lightcone gauge for the fermions $(\Gamma^+\theta=0)$. The matrix $\mathcal{M}$ vanishes in this gauge [12] so the supervielbeins in Eq. (2.2) are quadratic in the fermions. The square root $\sqrt{-\det g_{\mu\nu}}$ will still contain terms up to order θ^{16}, but the Lagrangian is still much simpler than for other fermionic gauges.

However, this hybrid gauge is also singular in the neighborhoods of our embeddings. To prove that a given fermionic gauge is admissible one needs to show that there always exists a kappa symmetry transformation to bring any theta into this gauge. In the case at hand this requires that

$$\Gamma^+[\theta+(1-\Gamma)\kappa]=0 \tag{3.20}$$

admits solutions for κ which remove all 16 (arbitrary) components of θ^+, where $\theta^+=-\frac{1}{2}\Gamma^-\Gamma^+\theta$.

Now let $\Gamma=\gamma^*+\delta\Gamma$ where γ^* is Γ evaluated on the classical embedding and $\delta\Gamma$ contains field fluctuations; then $\gamma^{*2}=1$ and $(1\pm\gamma^*)$ are projectors of rank 16. For the embeddings (3.10), γ^* is Γ_{+-1} or Γ_{+-4}; thus $[\gamma^*,\Gamma^+]=0$. These two facts immediately imply

$$(1+\gamma^*)\Gamma^+(\theta-\delta\Gamma\kappa)=0. \tag{3.21}$$

This means that the kappa transformations needed to remove the eight components of theta satisfying $\gamma^*\theta^+=\theta^+$ are nonperturbative in that $\kappa\sim(\delta\Gamma)^{-1}\theta^+$. The conclusion is that the fermion lightcone gauge is singular at the embeddings of interest.

The singularity of the hybrid gauge near the embedding is also manifest on gauge fixing within the functional integral. When one tries to introduce ghosts for the hybrid gauge fixing one finds as usual that one needs an infinite number of ghosts for ghosts. Leaving this well-known problem aside, one also finds that the leading term in the ghost action is cubic in the fields for 24 out of the 32 ghost components. Thus the ghost action does not admit a traditional perturbative formulation, which is directly connected to the observation above that the compensating kappa transformation is nonperturbative.

Given these problems, we choose to work instead with the physical gauge, namely $\gamma^*\theta=\theta$, which can manifestly always be reached in the neighborhood of the embedding. Given the complexity of the supervielbeins we will work only to quadratic order in the fermions. Fortunately this turns out to be sufficient to investigate the question of enhanced supersymmetry for displaced M_-2 branes and M_+2 branes.

C. Action to quadratic order in fermions

It is straightforward to compute the supermembrane theory on the plane wave supergeometry specified by Eq. (2.2) to quadratic order in the fermions. From Eqs. (2.2) and (2.3) we obtain

$$\Pi^r_\mu=\partial_\mu X^n e^r_n+\overline{\theta}\Gamma^r\tilde{D}_\mu\theta+O(\theta^4), \tag{3.22}$$

where

$$\tilde{D}_\mu\theta=\partial_\mu\theta+\partial_\mu X^m(e^r_m T_r+\tfrac{1}{4}\omega^{rs}_m\Gamma_{rs})\theta \tag{3.23}$$

and $T_r=T^{stuv}_r F_{stuv}$. From this expression we obtain $g_{\mu\nu}$ to quadratic order in the fermions

$$g_{\mu\nu}=\gamma_{\mu\nu}+2\overline{\theta}\partial_{(\mu}X^n\tilde{\Gamma}_n\tilde{D}_{\nu)}\theta+O(\theta^4), \tag{3.24}$$

where $\gamma_{\mu\nu}=\partial_\mu X^m\partial_\nu X^n G_{mn}$ is the induced worldvolume metric and $\tilde{\Gamma}_n=e^r_n\Gamma_r$ are curved gamma matrices. Using these results we obtain

$$\begin{aligned} S=&-\int d^3\xi\sqrt{-\det\gamma_{\mu\nu}}(1+\gamma^{\mu\nu}\partial_\mu X^n\overline{\theta}\tilde{\Gamma}_n\tilde{D}_\nu\theta+O(\theta^4))\\ &+\int d^3\xi\,\epsilon^{\lambda\mu\nu}\Big(\frac{1}{6}C_{lmn}\partial_\lambda X^l\partial_\mu X^m\partial_\nu X^n\\ &-\frac{1}{2}\overline{\theta}\tilde{\Gamma}_{mn}\tilde{D}_\lambda\theta\partial_\mu X^m\partial_\nu X^n+O(\theta^4)\Big). \end{aligned} \tag{3.25}$$

IV. M_-2 BRANES

We now discuss the case of M_-2 branes along $(+,-,1)$. The physical gauge corresponds to

$$X^\mu=\xi^\mu, \quad \Gamma_{+-1}\theta=\theta, \tag{4.1}$$

where $\mu=(+,-,1)$. We are interested in both the brane at the origin and the brane away from the origin. Recall that the worldvolume scalars parametrize the transverse position of the brane. To obtain the action for the brane at the origin we expand around $X^{A'}_{cl}=\theta=0$, whereas for the brane localized at $x^{A'}_0$ we instead expand around $X^{A'}_{cl}=x^{A'}_0$, $\theta=0$ where $A'=(i',a)$ runs over all transverse directions.

Clearly the action for the brane at the origin is given by Eq. (3.25) and to obtain the action for the brane away from the origin we simply have to shift $X^{A'}$ by $x^{A'}_0$. It will be useful to introduce a double grading to count the order in fluctuations and in the constant position x_0. The action and variation are then split into terms of definite order and we

will denote them as S_q^p and δ_q^p, where the superscript p denotes the order of fluctuating fields and the subscript q denotes the order of the constant positions. For instance, the quadratic part of the action for the brane at the origin will be denoted by S_0^2, etc.

To explicitly evaluate the action we need to know $\tilde{D}_\mu\theta$ and $h^{\mu\nu}\equiv\sqrt{-\det\gamma_{\mu\nu}}\gamma^{\mu\nu}$:

$$\tilde{D}_\mu\theta=\left[\partial_\mu+\partial_\mu B+\left(\frac{\mu}{12}(\Gamma_{23}+2\Gamma_{123})-\frac{1}{4}\partial_A G_{++}\Gamma_{-A}\right)\delta_{\mu+}+\frac{\mu}{6}\Gamma_{-23}\delta_{\mu 1}\right]\theta,$$

$$h^{++}=-\tilde{\gamma}_{--},\quad h^{+1}=\tilde{\gamma}_{-1},\quad h^{-1}=\tilde{\gamma}_{+1}+G_{++}\tilde{\gamma}_{-1},\tag{4.2}$$

$$h^{--}=-G_{++}-\tfrac{1}{2}G_{++}\tilde{\gamma}_{11}-\tilde{\gamma}_{++}-G_{++}\tilde{\gamma}_{+-}-\tfrac{1}{2}G_{++}^2\tilde{\gamma}_{--},$$

$$h^{+-}=-1-\tfrac{1}{2}\tilde{\gamma}_{11}-\tfrac{1}{2}G_{++}\tilde{\gamma}_{--},$$

$$h^{11}=1-\tfrac{1}{2}G_{++}\tilde{\gamma}_{--}-\tfrac{1}{2}\tilde{\gamma}_{11}-\tilde{\gamma}_{+-},$$

where $B=\mu/6\,(X^2\Gamma_{-3}-X^3\Gamma_{-2}-\frac{1}{2}X^a\Gamma_{-23a})$, $\tilde{\gamma}_{\mu\nu}=\partial_\mu X^{A'}\partial_\nu X^{A'}$, and we have only kept terms quadratic in fluctuations.

A. Quadratic action

The action to quadratic order in the fields is

$$S_0^2=-\int d^3\xi\left[1+\frac{1}{2}\gamma_{(0)}^{\mu\nu}\partial_\mu X^{A'}\partial_\nu X^{A'}+2\overline{\theta}\tilde{\Gamma}^\mu D_\mu\theta+\mu X^+(\partial_+X^2\partial_-X^3-\partial_+X^3\partial_-X^2)\right].\tag{4.3}$$

where $\gamma_{(0)\mu\nu}$ and $\tilde{\Gamma}^\mu$ are the fluctuation independent part of the induced metric $\gamma_{\mu\nu}$ and $\gamma^{\mu\nu}\partial_\nu X^n\tilde{\Gamma}_n$, respectively. Note that we take $\epsilon^{+-1}=1$. Notice that the fermion kinetic term receives a contribution both from the Dirac and the Wess-Zumino part of the action. The Dirac operator appearing in Eq. (4.3) can be written as

$$\tilde{\Gamma}^\mu D_\mu=(\Gamma^-\partial_-+\Gamma^+\partial_++\Gamma^1\partial_1+\tfrac{1}{2}G_{++}\Gamma^+\partial_-+\tfrac{1}{2}\mu\Gamma^{+123}).\tag{4.4}$$

The last term couples the eight physical worldvolume spinors, i.e., the SO(8) part is not diagonal.

The action (4.3) describes fluctuations of a brane located at the origin in the transverse directions; here $G_{++}=-\mu^2/9\,(x^1)^2$. To describe fluctuations of a brane embedded at constant nonzero transverse position one needs to expand instead about $X^{A'}=x_0^{A'}$. The resulting action to quadratic order in the fluctuations is

$$S_{\text{shifted}}=S_0^2+S_2^2,$$

$$S_2^2=-\frac{1}{2}\mathcal{A}\int d^3\xi[(\partial_-X^{A'})^2-2\overline{\theta}\Gamma^+\partial_-\theta],\tag{4.5}$$

$$\mathcal{A}=\frac{\mu^2}{36}(4(x_0^{i'})^2+(x_0^a)^2),$$

where $i'=2,3$.

Note that Eq. (4.5) can be obtained from Eq. (4.3) provided that in the latter G_{++} is kept exact. Although terms $(x^{A'})^2(\partial X^{B'})^2$ are clearly subleading (quartic) and do not contribute at quadratic order for the brane at the origin they do contribute to the quadratic term $(x_0^{A'})^2(\partial X^{B'})^2$ appearing in the action for the shifted brane.

B. Interactions

We next compute a subset of the interaction terms. For the brane at the origin there are no cubic interaction terms; the leading order interactions are quartic. The terms quartic in bosonic fluctuations can be obtained straightforwardly by expanding the $\sqrt{-\gamma}$. To obtain the terms quartic in fermion fluctuations one would need to extend the results of Sec. III C. This is a somewhat tedious computation (which could, however, be done using the results of Secs. II and III). Fortunately, it is sufficient for our purpose (as we explain below) to consider only quartic terms that are quadratic in both bosonic and fermionic fluctuations. The Dirac term contributions to these are

$$\begin{aligned}S_{0(D)}^4=-\int d^3\xi(&\overline{\gamma}_{--}\overline{\theta}\Gamma^-\partial_+\theta-\overline{\gamma}_{+-}\overline{\theta}\Gamma^1\partial_1\partial\theta\\&+(\tilde{\gamma}_{+1}+G_{++}\tilde{\gamma}_{-1})\overline{\theta}\Gamma^1\partial_-\theta\\&+\tilde{\gamma}_{-1}\overline{\theta}(-\Gamma^-\partial_1+\Gamma^1\partial_+)\theta\\&+\frac{1}{2}\tilde{\gamma}_{11}\overline{\theta}(\Gamma^-\partial_--\Gamma^1\partial_1)\theta\\&+\frac{1}{4}\mu(\tilde{\gamma}_{--}\overline{\theta}\Gamma^{-23}\theta+\tilde{\gamma}_{-1}\overline{\theta}\Gamma^{123}\theta)\\&-\frac{1}{2}G_{++}\tilde{\gamma}_{--}\overline{\theta}(\Gamma^1\partial_1-\Gamma^-\partial_-)\theta\\&+\frac{1}{4}\partial_1G_{++}\tilde{\gamma}_{--}\overline{\theta}\theta+\cdots).\end{aligned}\tag{4.6}$$

The relevant Wess-Zumino term contributions are

$$S^4_{0(WZ)} = -\int d^3\xi \Big(\frac{1}{2} e^{\mu\nu\lambda} \overline{\theta}\Gamma^{A'B'} \partial_\mu \theta \partial_\nu X^{A'} \partial_\lambda X^{B'} - \frac{1}{12}\mu \partial_- X^a \partial_1 X^b \overline{\theta}\Gamma^{23ab}\theta + \frac{1}{12}\mu[(\partial_1 X^a \partial_- X^2 - \partial_- X^a \partial_1 X^2)\overline{\theta}\Gamma^{13a}\theta + 5(\partial_1 X^2 \partial_- X^3)\overline{\theta}\theta - 2 \leftrightarrow 3] + \cdots \Big), \tag{4.7}$$

where the ellipses in both contributions denote terms containing $(\overline{\theta}\Gamma^+ \cdots \theta)$ which are irrelevant in what follows (the ellipses do not contain Γ^-).

For the brane at nonzero transverse position there are cubic interaction terms. The relevant terms are those which are linear in bosonic and quadratic in fermionic fluctuations:

$$S^3_1 = \int d^3\xi \Big(-\mathcal{B}[(\partial_- X^{A'})^2 - 2\overline{\theta}\Gamma^+ \partial_- \theta] + \frac{1}{2}\mathcal{B}_{B'}(\partial_- X^{A'})\overline{\theta}\Gamma^{+A'B'}\theta \Big),$$
$$\mathcal{B} = \frac{\mu^2}{36}(4x_0^{i'} X^{i'} + x_0^a X^a), \tag{4.8}$$
$$\mathcal{B}_{B'} = \frac{\mu^2}{18}(4x_0^{i'}\delta_{i'B'} + x_0^a \delta_{aB'}).$$

C. Fluctuation spectrum

We now use the quadratic actions (4.3) and (4.5) to work out the fluctuation spectra of the branes. The bosonic equations of motion following from Eqs. (4.3) and (4.5) are

$$\Box X^a = 0, \quad \Box \phi = -i\mu\partial_- \phi, \tag{4.9}$$

where

$$\Box = (-2\partial_+\partial_- - G_{++}\partial_-^2 + \partial_1^2), \tag{4.10}$$

and $\phi = (X^2 + iX^3)$ is a complex scalar. Note that the scalars X^2 and X^3 are coupled by the Wess-Zumino term so one has to diagonalize their equations of motion. Here $G_{++} = -[\mu^2/9(x^1)^2 + \mathcal{A}]$ where $\mathcal{A}$ is given in Eq. (4.5) and is zero for a brane at the origin.

The Dirac equation is

$$0 = \tilde{\Gamma}^\mu D_\mu \theta = (\Gamma^-\partial_- + \Gamma^+\partial_+ + \Gamma^1\partial_1 + \tfrac{1}{2}G_{++}\Gamma^+\partial_- - \tfrac{1}{4}\mu\Gamma^{+23})\theta. \tag{4.11}$$

Iterating we get

$$0 = \tilde{\Gamma}^\nu D_\nu \tilde{\Gamma}^\mu D_\mu \theta = \Box\theta + \tfrac{1}{2}(\mu\Gamma^{23} + \partial_1 G_{++}\Gamma^+)\partial_-\theta. \tag{4.12}$$

Recall that the 16 θ satisfy $\Gamma_{+-1}\theta = \theta$.[3] Let us further decompose the fermions into eigenspinors of Γ^1,

$$\Gamma^1\theta^\pm = \pm\theta^\pm. \tag{4.13}$$

Multiplying Eq. (4.12) by $\Gamma^-\Gamma^+$ yields

$$\Box\theta^- = -\tfrac{1}{2}\mu\Gamma^{23}\partial_-\theta^-, \tag{4.14}$$

where we used the relations $\Gamma^+\theta^+ = 0$ and $\Gamma^-\theta^- = 0$. Since Γ^1 commutes with Γ^{23} we can further decompose θ^- into eigenspinors of Γ^{23},

$$\Gamma^{23}\theta^-_\pm = \pm i\theta^-_\pm. \tag{4.15}$$

We thus obtain

$$\Box\theta^-_\pm = \mp\tfrac{1}{2}i\mu\partial_-\theta^-_\pm \tag{4.16}$$

which is of the same form as the scalar field equation (4.9). It remains to discuss θ^+ components of θ. Multiplying the fermion field equation (4.11) by Γ^+ yields

$$\partial_-\theta^+ = -\tfrac{1}{2}\Gamma^+\partial_1\theta^-. \tag{4.17}$$

Provided $p^+ \neq 0$ this equation determines θ^+ from θ^-. Thus there are eight independent fermion modes.

So for both bosonic and fermionic fluctuations we need to solve equations of the form

$$\Box\varphi = ic\partial_-\varphi, \tag{4.18}$$

for various values of c. Decomposing into Fourier modes along the lightcone, $\varphi = \exp(ip^+x^- + ip^-x^+)\varphi(x^1)$, this becomes

$$[2p^+p^- - \tfrac{1}{9}(p^+)^2\mu^2(x^1)^2 + \partial_1^2 - \Delta]\varphi(x^1) = 0, \tag{4.19}$$

with

$$\Delta = (p^+)^2\frac{\mu^2}{36}\Big[4\sum_{i'}(x_0^{i'})^2 + \sum_a (x_0^a)^2\Big] - cp^+ \equiv 2p^+\Delta H - cp^+, \tag{4.20}$$

where the $x_0^{i'}$ and x_0^a are the constant transverse positions about which the brane is fluctuating. Recall that the eigenfunctions of the harmonic oscillator satisfy

$$(\partial_1^2 + [1 + 2n - \tfrac{1}{9}(p^+)^2\mu^2(x^1)^2])H_n(x^1) = 0. \tag{4.21}$$

The Gaussian part of the Hermite function behaves as $\exp[-\tfrac{1}{6}\mu p^+(x^1)^2]$ and decays exponentially. Notice that we take $p^+ > 0$. Thus the p^- eigenvalue is determined as

[3]Notice that in later sections we use the notation $\Gamma_{+-1}\theta^\pm = \pm\theta^\pm$ and in this notation the 16 θ are the θ^- ones. In order to avoid clumsy notation such as $\theta^{-,\pm}_\pm$ we suppress this superscript below.

$$p^- = (1+2n)\frac{\mu}{6} + \Delta H - \frac{1}{2}c. \tag{4.22}$$

The spectra of fluctuations are characterized by their (p^-,n) eigenvalues for a given p^+. From Eq. (4.22) one finds that the lowest p^- eigenvalues for given p^+ (i.e., those for which $n=0$) for the fluctuations are, respectively,

ϕ		$+\frac{1}{2}\mu$
θ_+^-		$+\frac{1}{4}\mu$
X^a	$\Delta H + \frac{1}{6}\mu$	$+0$
θ_-^-		$-\frac{1}{4}\mu$
$\bar{\phi}$		$-\frac{1}{2}\mu$

Furthermore raising n by one unit increases the p^- eigenvalue by $\mu/3$. This analysis shows that the transverse position enters the spectrum only as a universal shift ΔH in the p^- eigenvalue for all bosonic and fermionic fluctuations. Thus the brane away from the origin is as supersymmetric as the brane at the origin, just as in the corresponding computation of D-brane spectra in the maximally supersymmetric type IIB plane wave in [2].

D. Worldvolume supersymmetry

We now discuss in detail the worldvolume supersymmetries of these branes. The emergence of worldvolume supersymmetry from spacetime supersymmetry on gauge fixing kappa symmetry was first discussed in detail in the context of the four-dimensional supermembrane in [15]. The discussion here follows closely that of [16]: we determine which combined kappa and supersymmetry transformations leave the gauge fixed action invariant.

Let us split both κ and ϵ into eigenspinors of Γ_{+-1}, defining $\Gamma_{+-1}\lambda^\pm = \pm\lambda^\pm$ for any spinor λ. The appropriate splitting of the Killing spinors is

$$\epsilon = \epsilon^- + \epsilon^+,$$

$$\epsilon^- = \left(1 + \frac{\mu}{6}\Gamma^{+23}x^1\right)e^{(\mu/12)x^+\Gamma^{23} - (\mu/6)x^+\Gamma^{123}}\epsilon_0^- - \frac{\mu}{12}(x^a\Gamma^a - 2x^{i'}\Gamma^{i'})\Gamma^{+23}e^{(\mu/12)x^+\Gamma^{23}}\epsilon_0^+, \tag{4.23}$$

$$\epsilon^+ = \left(1 + \frac{\mu}{6}\Gamma^{+23}x^1\right)e^{-(\mu/12)x^+\Gamma^{23} - (\mu/6)x^+\Gamma^{123}}\epsilon_0^+ + \frac{\mu}{12}(x^a\Gamma^a - 2x^{i'}\Gamma^{i'})\Gamma^{+23}e^{-(\mu/12)x^+\Gamma^{23}}\epsilon_0^-,$$

where $i'=2,3$.

As discussed, we fix kappa symmetry by setting

$$\theta^- = 0. \tag{4.24}$$

Notice that the kappa symmetry transformations (3.7) have their own gauge invariance: taking $\kappa \to \kappa + (1+\Gamma)\kappa'$, for a local spinor $\kappa'(x)$, leaves the transformation rules invariant. We gauge fix this invariance by setting $\kappa^+ = 0$.

Demanding that a combined kappa symmetry and supersymmetry transformation preserves the gauge $\theta^- = 0$ requires that we choose the parameters such that

$$\kappa^- = -\tfrac{1}{2}\epsilon^-. \tag{4.25}$$

The worldvolume supersymmetry transformations are now given by

$$\delta\theta = \epsilon + (1-\Gamma)\left(-\tfrac{1}{2}\epsilon^-\right),$$
$$\delta X^{A'} = -\bar{\epsilon}\Gamma^{A'}\theta - \bar{\theta}\Gamma^{A'}(1-\Gamma)\left(-\tfrac{1}{2}\epsilon^-\right). \tag{4.26}$$

In what follows we will only be concerned with the transformation rules up to terms linear in the fluctuations. The reason is that we are only interested in whether the transformation rules contain an inhomogeneous term (which means the supersymmetry is nonlinearly realized) or not. Notice that the symmetry rules (3.7) themselves receive higher order corrections in θ, as noted at the end of Sec. III.

The field dependent Γ expanded to leading order about the embedding is

$$\Gamma = \Gamma_{+-1} + \Gamma_{+-A'}\partial_1 X^{A'} + \Gamma_{-1A'}\partial_+ X^{A'} + \Gamma_{1+A'}\partial_- X^{A'} + \tfrac{1}{2}G_{++}\Gamma_{-1A'}\partial_- X^{A'}. \tag{4.27}$$

This Γ contains all the terms needed to obtain the transformation rules to linear order in fluctuations. The resulting combined transformations are then

$$\delta\theta = \tfrac{1}{2}\tilde{\Gamma}^\mu\Gamma^{A'}\partial_\mu X^{A'}\epsilon^- + \epsilon^+,$$
$$\delta X^{A'} = 2\bar{\theta}\Gamma^{A'}\epsilon^-, \tag{4.28}$$

where $\tilde{\Gamma}^\mu = \gamma^{\mu\nu}\partial_\nu X^n e_n^r \Gamma_r$ (but only the fluctuation independent part contributes) and ϵ^+ and ϵ^- are given in Eq. (4.23). Note that these combined transformations do not preserve the static gauge (since $\delta X^\mu = \bar{\theta}\tilde{\Gamma}^\mu\epsilon^+$ is nonzero) and thus a compensating diffeomorphism (3.6) with parameter $\eta^\mu = -\bar{\theta}\tilde{\Gamma}^\mu\epsilon^+$ is needed to maintain the gauge. The latter implies further terms in the combined transformations (4.28) which are, however, always at least quadratic in the fluctuating fields and can be neglected below.

1. Symmetries to quadratic order

Let us now consider the supersymmetries of the quadratic actions, for branes located both at and away from the origin. In the former case the action is given in Eq. (4.3) and in the latter case in Eq. (4.5).

The explicit form of the symmetries follows from substituting the Killing spinors into Eq. (4.28) and keeping terms to the appropriate order. This gives

$$\delta_0^1\theta=\frac{1}{2}\tilde{\Gamma}^{\mu A'}\partial_\mu X^{A'}\left(1+\frac{\mu}{6}\Gamma^{+23}x^1\right)\times e^{(\mu/12)x^+\Gamma^{23}-(\mu/6)x^+\Gamma^{123}}\epsilon_0^- +\left(1+\frac{\mu}{6}\Gamma^{+23}x^1\right)\times e^{-(\mu/12)x^+\Gamma^{23}-(\mu/6)x^+\Gamma^{123}}\epsilon_0^+ +\frac{\mu}{12}(X^a\Gamma^a-2X^{i'}\Gamma^{i'})\Gamma^{+23}e^{-(\mu/12)x^+\Gamma^{23}}\epsilon_0^-, \tag{4.29}$$

$$\delta_0^1X^{A'}=2\overline{\theta}\Gamma^{A'}\left(1+\frac{\mu}{6}\Gamma^{+23}x^1\right)e^{(\mu/12)x^+\Gamma^{23}-(\mu/6)x^+\Gamma^{123}}\epsilon_0^-,$$

where the subscript p in the variation δ_p^q denotes the order of field fluctuations and the superscript q denotes the order of the constant positions. Of these transformations the ϵ_0^+ transformations are clearly inhomogeneous and nonlinearly realized (see second line). The ϵ_0^- transformations are, however, linearly realized; these correspond to the 16 worldvolume supersymmetries of the brane at the origin found in the probe analysis.

Now consider expanding about constant transverse positions; the action is given in Eq. (4.5). The supersymmetry transformations follow from shifting the fields $X^{A'}$ in the previous expressions. To show this explicitly notice that the action at the origin S^0 is of the schematic form

$$S^0=\int\sum_n X^nF_n(\theta,dX), \tag{4.30}$$

where we suppress space-time indices and F_n are expressions that depend on θ and on derivatives of $X^{A'}$ but not on undifferentiated $X^{A'}$. In other words we make explicit in Eq. (4.30) the dependence on undifferentiated $X^{A'}$. Similarly the supersymmetry rules are of the form

$$\delta X=\sum_n X^nG_n(\theta,dX,\epsilon),\quad \delta\theta=\sum_n X^nH_n(\theta,dX,\epsilon) \tag{4.31}$$

for appropriate G_n and H_n (ϵ is the supersymmetry parameter). The explicit form of the lowest order F_n, G_n, and H_n can be read off from Eqs. (4.3) and (4.29) but we will not need them. We now shift the brane away from the origin by setting $X=x_0+Y$, where for clarity we call the fluctuating part Y,

$$S_{\text{shift}}=\int\sum_n (x_0+Y)^nF_n(\theta,dY). \tag{4.32}$$

It is simple to show that invariance of Eq. (4.30) under Eq. (4.31) implies that Eq. (4.32) is invariant under

$$\delta Y=\sum_n (x_0+Y)^nG_n(\theta,dY,\epsilon),$$

$$\delta\theta=\sum_n (x_0+Y)^nH_n(\theta,dY,\epsilon). \tag{4.33}$$

The issue is whether the new transformation rules contain inhomogeneous pieces or not. If they do then the corresponding supersymmetries will be nonlinearly realized.

Let us now return to our specific case. Notice that to obtain all terms that are at most linear in fluctuations for the brane away from the origin we need to know certain terms that are higher order in fluctuations for the SUSY variation for the brane at the origin. The only quantities that contain undifferentiated $X^{A'}$ are G_{++} and spin-connection $\omega_+^{-A'}$, so one has to keep track of the dependence on them.

The transformation rules that depend on ϵ_0^+ acquire new contributions linear in fluctuation but they still contain the inhomogeneous term given in Eq. (4.29). These supersymmetries are nonlinearly realized and they will not be discussed further. The supersymmetries that depend on ϵ_0^- are given by

$$\delta_{\text{shifted}}=\delta_0^1+\delta_1^0+\delta_2^1, \tag{4.34}$$

$$\delta_1^0\theta=\frac{\mu}{12}(x_0^a\Gamma^a-2x_0^{i'}\Gamma^{i'})\Gamma^{+23}e^{-(\mu/12)x^+\Gamma^{23}}\epsilon_0^-, \tag{4.35}$$

$$\delta_2^1\theta=-\tfrac{1}{4}\mathcal{A}\Gamma^{+A'}\partial_-X^{A'}e^{-(\mu/12)x^+\Gamma^{23}}\epsilon_0^-, \tag{4.36}$$

where $\mathcal{A}$ is given in Eq. (4.5). The $\delta_2^1\theta$ term originates from the term in $\delta_0^1\theta$ containing $\tilde{\Gamma}_+=\Gamma_+-\frac{1}{2}G_{++}\Gamma_-$. The supersymmetry transformation of $X^{A'}$ remains unchanged. The ϵ_0^- transformations now contain an inhomogeneous piece for spinors such that $\Gamma^+\epsilon_0^-\neq 0$. Thus only eight worldvolume supersymmetries seem to be linearly realized, in agreement with the probe analysis.

The analysis so far explicitly confirms general expectations: the linearly realized supersymmetries are exactly the ones predicted by Eq. (3.12). We now show, however, that for the brane away from the origin and to quadratic approximation in fluctuations the worldvolume theory admits eight additional linearly realized supersymmetries. As discussed above, the invariance of the action at the origin implies a corresponding invariant for the brane away from the origin,

$$(\delta_0^1+\delta_1^0+\delta_2^1)(S_0^2+S_2^2)=0. \tag{4.37}$$

This leads to a number of relations obtained by collecting terms that contain the same number of fields and are of the same order in x_0:

$$\delta_0^1S_0^2=0, \tag{4.38}$$

$$\delta_1^0S_0^2=0, \tag{4.39}$$

$$\delta_0^1S_2^2+\delta_2^1S_0^2=0, \tag{4.40}$$

$$\delta_1^0S_2^2=0. \tag{4.41}$$

That these relations should hold follows from the general argument given earlier but we have also explicitly verified them. Equation (4.38) is the supersymmetry invariance of the brane at the origin. What is important is Eq. (4.39) which says that the action for the brane at the origin is symmetric by itself under the inhomogeneous symmetry transformation δ_1^0. This follows from the fact that the action for the brane shifted away from the origin does not contain a term linear in x_0. Furthermore, this symmetry extends to a symmetry of the brane away from the origin, see Eq. (4.41). For this to be true it is crucial that the shifted rules do not contain δ_3. The invariance of the shifted action under δ_0^1 implies that

$$\delta_{\rm hom} = \delta_0^1 + \delta_2^1 \tag{4.42}$$

is also a symmetry of the shifted action by itself [as can also be verified using Eqs. (4.38) and (4.40)]. This is, however, a homogeneous transformation generated by all 16 ϵ_0^-, so the quadratic approximation of the worldvolume theory for brane away from origin admits 16 linear supersymmetries.

The invariance of the action under δ_1^0 is a special case of the "semilocal" invariance

$$\delta\theta = \Gamma^+ \chi(x^+), \tag{4.43}$$

where $\chi(x^+)$ is an arbitrary spinor that depends only on x^+. One may check that the quadratic part of the action is invariant under this transformation. There is also a similar bosonic "semilocal" invariance

$$\delta X^{A'} = f^{A'}(x^+), \tag{4.44}$$

where $f^{A'}(x^+)$ are arbitrary functions of x^+. This additional "gauge" invariance allows one to gauge away the inhomogeneous term leading to an additional set of eight linear realized supersymmetries.

These considerations explain why the spectrum of the shifted brane is as supersymmetric as that of the brane at the origin. Furthermore, one can understand the shifted values of p^- for the fluctuations of the former as follows. The structure of the superalgebra implies that the anticommutation of $\delta_{\rm hom}$ with itself should generate (amongst other terms related to the rotation charges) a transformation corresponding to P^-. Since $\{\delta_0^1, \delta_2^1\} \sim \mathcal{A}$ and $\{\delta_2^1, \delta_2^1\} = 0$ the p^- values for the brane away from the origin are clearly shifted by a term proportional to $\mathcal{A}$ as we found.

2. Interactions

We now turn to the question of whether the extra supersymmetries are respected by interactions. By construction the full action is invariant under the symmetries generated by ϵ_0^-. A sufficient and necessary condition for there to be an extension of the homogeneous symmetry (4.42) to the full theory is hence that the inhomogeneous symmetry δ_1^0 extends to a symmetry of the interacting theory. In other words, there should be a deformation of δ_1^0 (possibly containing fluctuating fields) which leaves the action of the interacting theory invariant.

The action for the interacting theory of the shifted brane is

$$S = S_0^2 + S_2^2 + S_0^4 + S_1^3 + \cdots, \tag{4.45}$$

where (relevant parts of) S_0^4 and S_1^3 are given in Eqs. (4.6), (4.7), and (4.8). We have seen in the previous section that $\delta_1^0 S_0^2 = 0 = \delta_1^0 S_2^2$. Furthermore, it is also true by inspection that

$$\delta_1^0 S_1^3 = 0. \tag{4.46}$$

Let us now discuss S_0^4. A direct computation shows that $\delta_1^0 S_0^4$ is not zero. In fact the appropriate extension of Eq. (4.37) implies the relation

$$(\delta_1^0 S_0^4 + \delta_0^1 S_1^3 + \delta_1^2 S_0^2) = 0, \tag{4.47}$$

where the explicit form of δ_1^2 (not needed here) follows from extending the previous arguments to higher order. One can verify explicitly that each term in the above is nonvanishing.

The issue is, however, whether there is an appropriate deformation of δ_1^0,

$$\delta = \delta_1^0 + \sum_{p,q} \tilde{\delta}_p^q, \tag{4.48}$$

for appropriate p and q, that leaves the interacting action invariant. Notice that $\delta_1^0 S_0^4$ contains three fluctuating fields and is linear in the constant positions. This means that terms of the same order as $\delta_1^0 S_0^4$ are only produced by $\tilde{\delta}_0^1 S_1^3$ and $\tilde{\delta}_1^2 S_0^2$ [just as in Eq. (4.47)]. In order not to upset the lowest order invariance (i.e., the invariance of S_0^2) the new transformation should be at least quadratic in fluctuations. The only possible deformation $\tilde{\delta}_0^1$ which leaves invariant the lowest order action is δ_0^1 itself, but such a deformation leads us back to the original supersymmetry variations.

We are thus led to look for a variation $\tilde{\delta}_1^2$ such that

$$\delta_1^0 S_0^4 + \tilde{\delta}_1^2 S_0^2 = 0 \Leftrightarrow \delta_1^0 S_0^4 = -\frac{\delta S_0^2}{\delta Z^M} \tilde{\delta}_1^2 Z^M \Leftrightarrow \delta_1^0 S_0^4 \approx 0, \tag{4.49}$$

where $\approx$ means equality when the lowest order equations hold. We thus obtain that a necessary and sufficient condition for δ_1^0 to be extendible to a symmetry of the leading interactions is that $\delta_1^0 S_0^4$ vanishes weakly.

Notice that δ_1^0 does not mix terms with a different number of fermions. This means that the variation of terms in S_0^4 that are quadratic in θ should vanish separately from the variation of terms quartic in θ. Clearly the terms containing $(\bar{\theta}\Gamma^+ \cdots \theta)$, where the ellipses do not contain Γ^-, are trivially invariant under δ_1^0 [because $(\Gamma^+)^2 = 0$]. Thus we only need to examine the remaining terms; this is the reason why only these terms were listed in Sec. IV B. Now notice that the structure of the bosonic fluctuation terms in the Dirac and Wess-Zumino part of S_0^4, Eqs. (4.6) and (4.7) respectively, is different: the latter is antisymmetric under the exchange of two bosons while the former is symmetric. This means that the two sets of terms cannot mix with each other under the δ_1^0 variation, except possibly through the use of lowest order

field equations. The latter, however, are diagonal for X^a ($a=4,\ldots,9$). It follows that a necessary condition for the extension of δ_1^0 (and thus of the extra supersymmetries) into a symmetry of the leading interactions is that the δ_1^0 variation of the terms in Eq. (4.6) that do not depend on X^2 and X^3 vanishes weakly.

Explicit computation yields

$$\delta_1^0 S_D = -\int d^3\xi (2\tilde{\chi}'\Gamma^+(\partial_-^2 X^a\Gamma^- + \partial_1\partial_- X^a\Gamma^1)\theta X^a + \frac{1}{2}\partial_-(X^aX^a)\tilde{\chi}'\Gamma^+(\Gamma^-\partial_- + \Gamma^1\partial_1)\theta - \partial_- X^a\Box X^a\tilde{\chi}\Gamma^+(\Gamma^- + \Gamma^1)\theta), \tag{4.50}$$

where χ is defined by $\delta_1^0\theta = \Gamma^+\chi$, $\delta_1^0\theta$ is given in Eq. (4.35), and $\chi' = (\partial_+ + \mu/4\Gamma^{23})\chi$. The last line is proportional to the lowest order field equations. To check whether the term in the first line is a total derivative up to lowest order field equations we use the lemma that a term is a total derivative of a local field if and only if its Euler-Lagrange derivative[4] with respect to all fields vanishes (for a proof see, for instance, Sec. 4.4 of [17]). One finds by inspection that the Euler-Lagrange derivative with respect to θ is nonvanishing (even when the free field equations hold). We therefore conclude that the variation $\delta_1^0 S_0^4$ does not vanish weakly and thus that the extra supersymmetries are not respected by interactions.

V. M_+2 BRANES

We consider in this section an M_+2 brane oriented along $(+,-,4)$ and compute the action to quadratic order in fermions in the physical gauge $\Gamma_{+-4}\theta = \theta$. One can use the general expression given in Sec. II, along with explicit expressions

$$\tilde{D}_\mu\theta = \left[\partial_\mu + \partial_\mu B + \left(\frac{\mu}{12}(\Gamma^{-+123} + 2\Gamma^{123}) - \frac{1}{4}\partial_A G_{++}\Gamma_{-A}\right)\delta_{\mu+} - \frac{\mu}{12}\Gamma_4\Gamma^{+123}\delta_{\mu 4}\right]\theta, \tag{5.1}$$

where $a'=5,\ldots,9$ and $B=\mu/6(X^i\Gamma_i\Gamma^{+123} - \frac{1}{2}X^{a'}\Gamma^{+123a'})$. Substituting these expressions into the action one finds the following action to quadratic order:

$$S^2 = -\int d^3\xi\left(1 + \frac{1}{2}\gamma_{(0)}^{\mu\nu}\partial_\mu X^{A'}\partial_\nu X^{A'} + 2\overline{\theta}\tilde{\Gamma}^\mu\partial_\mu\theta\right), \tag{5.2}$$

where A' labels the eight transverse scalars and $\gamma_{(0)\mu\nu}$ and $\tilde{\Gamma}^\mu$ are as given for the M_-2 brane. Note that there is no coupling of the fluctuations to the background Ramond-Ramond (RR) flux to quadratic order. This leads to a degeneracy in the spectrum; the equations of motion are

$$\Box X^{A'} = 0, \quad \tilde{\Gamma}^\mu\partial_\mu\theta = 0, \tag{5.3}$$

and thus following the previous analysis one finds that all eight physical bosons and fermions have a ground state energy of $\Delta H + \frac{1}{6}\mu$. As previously advertised this spectrum is akin to that of the D_+ branes considered in [1,2].

The target space supersymmetries are realized to this order on the brane as

$$\delta\theta = \tfrac{1}{2}\tilde{\Gamma}^\mu\Gamma^{A'}\partial_\mu X^{A'}\epsilon^- + \epsilon^+, \quad \delta X^{A'} = 2\overline{\theta}\Gamma^{A'}\epsilon^-, \tag{5.4}$$

where the relevant splitting is now $\Gamma_{+-4}\epsilon^\pm = \pm\epsilon^\pm$. Decomposing the Killing spinors one finds

$$\begin{aligned}\epsilon^+ = &\left(\cos\left(\frac{\mu}{6}x^+\right)\cos\left(\frac{\mu}{12}x^+\right) - \Gamma^4\sin\left(\frac{\mu}{6}x^+\right)\sin\left(\frac{\mu}{12}x^+\right)\right.\\ &\left. - \frac{\mu}{12}\Gamma^+\left[(x^{a'}\Gamma^{a'} - 2x^i\Gamma^i)\Gamma^{123}\cos\left(\frac{\mu}{12}x^+\right) + x^4\sin\left(\frac{\mu}{12}x^+\right)\right]\right)\epsilon_0^+\\ &+\left(\Gamma^{123}\left[-\sin\left(\frac{\mu}{6}x^+\right)\cos\left(\frac{\mu}{12}x^+\right) + \Gamma^4\cos\left(\frac{\mu}{6}x^+\right)\sin\left(\frac{\mu}{12}x^+\right)\right]\right.\\ &\left. - \frac{\mu}{12}\Gamma^+\left[(x^{a'}\Gamma^{a'} - 2x^i\Gamma^i)\sin\left(\frac{\mu}{12}x^+\right) - x^4\Gamma^{123}\cos\left(\frac{\mu}{12}x^+\right)\right]\right)\epsilon_0^-\end{aligned} \tag{5.5}$$

and

[4]The Euler-Lagrange derivative of a local function f is defined as $\delta f/\delta\phi = \partial f/\partial\phi - \partial_\mu[\partial f/\partial(\partial_\mu\phi)] + \cdots$.

$$\epsilon^- = \Bigg(-\Gamma^{123}\left[\sin\left(\frac{\mu}{6}x^+\right)\cos\left(\frac{\mu}{12}x^+\right)+\Gamma^4\cos\left(\frac{\mu}{6}x^+\right)\sin\left(\frac{\mu}{12}x^+\right)\right]$$
$$+\frac{\mu}{12}\Gamma^+\left[(x^{a'}\Gamma^{a'}-2x^i\Gamma^i)\sin\left(\frac{\mu}{12}x^+\right)-x^4\Gamma^{123}\cos\left(\frac{\mu}{12}x^+\right)\right]\Bigg)\epsilon_0^+$$
$$+\Bigg(\cos\left(\frac{\mu}{6}x^+\right)\cos\left(\frac{\mu}{12}x^+\right)+\Gamma^4\sin\left(\frac{\mu}{6}x^+\right)\sin\left(\frac{\mu}{12}x^+\right)$$
$$-\frac{\mu}{12}\Gamma^+\left[(x^{a'}\Gamma^{a'}-2x^i\Gamma^i)\Gamma^{123}\cos\left(\frac{\mu}{12}x^+\right)+x^4\sin\left(\frac{\mu}{12}x^+\right)\right]\Bigg)\epsilon_0^- . \tag{5.6}$$

The splitting indicates that all 32 target space supersymmetries are realized nonlinearly regardless of the brane location; this follows from the inhomogeneous first terms in the first and third lines of ϵ^+ and ϵ^-.

Unlike the case studied in the previous section, we do not find in this case extra symmetries that can be used to remove the inhomogeneous parts of the transformations. Furthermore, the spectrum suggests that the supersymmetries of the M_+2 branes should not be directly related to the Killing spinors. This follows from the fact that the symmetries of the spectrum manifestly commute with the lightcone Hamiltonian, according to the degeneracy found above, while the target space supercharges do not commute with the lightcone Hamiltonian [18,19]. Also in the corresponding string analysis in [1,2] the restored kinematical symmetries were not directly related to the target space kinematical symmetries.

These considerations lead us to the following homogeneous symmetries of the action

$$\delta^0\theta=\frac{1}{2}\tilde{\Gamma}^\mu\Gamma^{A'}\partial_\mu X^{A'}\epsilon_0^- , \quad \delta^0 X^{A'}=2\overline{\theta}\Gamma^{A'}\epsilon_0^- , \tag{5.7}$$

where the constant parameter ϵ_0^- satisfies $\Gamma^+\epsilon_0^-=0$. This gives eight linearly realized symmetries irrespective of the brane location. Note also that these symmetries are directly analogous to the restored kinematical symmetries of the D_+ branes discussed in [1,2].

To determine whether interactions respect these symmetries one needs to compute the action to next order. In this case there are cubic interaction terms given by

$$S^3=\frac{1}{2}\mu\int d^3\xi(\partial_- X^{a'}\overline{\theta}\Gamma^{a'123}\theta-\partial_- X^i\overline{\theta}\Gamma_i\Gamma^{4123}\theta$$
$$-\partial_4 X^i\overline{\theta}\Gamma_i\Gamma^{+123}\theta)-\mu\int d^3\xi x^+\epsilon^{\mu\nu\rho}(\partial_\mu X^1\partial_\nu X^2\partial_\rho X^3). \tag{5.8}$$

Now suppose that the symmetry (5.7) can be extended to the full action. This implies that there is a transformation $\delta=\delta^0+\delta^1+\cdots$ such that

$$\delta^0 S^3+\delta^1 S^2=0, \tag{5.9}$$

which can only be satisfied for some δ^1 (to be determined) provided that $\delta^0 S^3\approx 0$ up to terms which vanish on-shell with respect to the lowest order field equations.

However, $\delta^0 S^3$ contains the term

$$\delta^0 S^3=-\frac{1}{2}\mu\int d^3\xi[\overline{\theta}\Gamma^{4a'b'123}\epsilon_0^-(\partial_4 X^{b'}\partial_- X^{a'})+\cdots]. \tag{5.10}$$

This is the only term in the variation which contains both $X^{a'}$ and $X^{b'}$ (with $b'\neq a'$) and thus cannot cancel against other terms. It cannot, however, be written either as a total derivative or as a term proportional to the lowest order field equations, which can be most easily seen using the Euler-Lagrange test of the previous section, and thus it violates the symmetry.

VI. DISCUSSION

We have studied in this paper the worldvolume supersymmetries for $M2$ branes in the maximally supersymmetric plane wave background of M theory. By construction, the $M2$ worldvolume theory has as many supersymmetries as the background. However, only a subset of them are linearly realized. We explicitly constructed all worldvolume supersymmetries using standard methods and found results in agreement with the probe analysis, i.e., the number of linearly realized supersymmetries matched the probe results.

We showed, however, that the quadratic approximation to the worldvolume theory admits additional linearly realized supersymmetries. In the case of M_-2 branes localized away from the origin this came about by the use of an extra "semilocal" invariance that the quadratic action possesses. This "gauge" invariance allows one to gauge away the inhomogeneous term from certain supersymmetry rules, thus providing eight additional linearly realized supersymmetries. In the case of M_+2 branes the additional supersymmetries were completely new symmetries, unrelated to target space supersymmetries. In both cases, we explicitly computed the spectrum of small fluctuations and showed that it is organized into multiplets of the additional supersymmetries. In both cases we showed that the new supersymmetries are not respected by the worldvolume interactions.

This investigation was motivated by the analysis in [1,2,3]

of D branes on the maximally supersymmetric plane wave background of type IIB string theory. The probe analysis [3] leads to results analogous to those for $M2$ branes. Moreover, as in the cases studied here the open string spectrum appears to be organized by more supercharges than given by the probe analysis. Corresponding additional exceptional supercharges were constructed in [1,2] as additional Noether charges. The additional fermionic symmetries depend crucially on special properties of the plane wave background and the fact that the string worldsheet was considered to be a strip.

The results in [1,2,3] imply that the D-brane worldvolume theory should admit more supersymmetries, albeit only at the quadratic level if the interactions break the extra symmetries. In this paper we found that the $M2$ branes do exhibit extra supersymmetries but only at the quadratic order. The mechanism of supersymmetry enhancement mimics the corresponding string theory construction. Our results strongly suggest that the extra symmetries in the case of D branes are not respected by string interactions and one could verify this by repeating the computation described here for the worldvolume theory of D branes.

An interesting question is whether the branes studied here as well as the corresponding D branes in the type IIB plane wave are stable. Consider the case of M/D_- branes localized away from the origin. These branes have the same mass density as the corresponding branes at the origin, i.e., the worldvolume $\sqrt{-\det\gamma}$ is independent of the position of the brane, and there is no classical force that acts on them. However, their excitations have an excess of energy that depends on the position of the brane. This suggests that the brane will emit the extra energy and recoil to the origin. In the case of D branes a possible decay channel is via emission of closed strings.[5] Since the position of the D brane is arbitrary one may consider the brane localized very far from the origin so that the excess energy is very large and semiclassical methods may be applicable.

It would be interesting to understand under which conditions the quadratic part of the worldvolume theory admits more supersymmetries and the spectrum is more supersymmetric than one would naively expect. It seems likely that there is corresponding supersymmetry enhancement in other pp-wave backgrounds, given the similar generic form of the Killing spinors.

Since interactions do not respect the extra supersymmetries, the masses of the states are expected to be split by quantum corrections. As masses are mapped to conformal dimension in the gravity/gauge theory correspondence this would translate into a prediction for anomalous dimensions.

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APPENDIX: CONVENTIONS

The gamma matrices satisfy $\{\Gamma^r,\Gamma^s\}=2\eta_{rs}$ where η_{rs} is the tangent space metric. They can be taken to be real in the Majorana representation. Gamma matrices with multiple indices denote antisymmetrized products with unit strength. The Dirac conjugate is defined by $\bar{\psi}=i\psi^t\Gamma^0$ for a generic spinor ψ. Here Γ^0 is the charge conjugation matrix which satisfies $(\Gamma^0)^t=-\Gamma^0$. An important property of gamma matrices in $D=11$ is that the matrix $\Gamma^0\Gamma_{\alpha_1\cdots\alpha_p}$ is symmetric for $p=1,2,5$ and antisymmetric for $p=0,3,4$ (the cases $p>5$ are related by duality to these).

A useful explicit basis for gamma matrices in terms of $\mathrm{SO}(2,1)\times\mathrm{SO}(9)$ matrices is

$$\Gamma^{\pm}=\gamma^{\pm}\otimes\gamma,\quad \Gamma^1=\gamma^1\otimes\gamma,\quad \Gamma^{A'}=1_2\otimes\gamma^{A'},\qquad \text{(A1)}$$

with $\gamma^1=\sigma^3$, $\sqrt{2}\gamma^{\pm}=(i\sigma^2\pm\sigma^1)$, and

$$\gamma^{A'}=\begin{pmatrix}0 & \sigma^{A'}\\ (\sigma^{A'})^t & 0\end{pmatrix},\quad \gamma=\begin{pmatrix}1_8 & 0\\ 0 & -1_8\end{pmatrix}.\qquad \text{(A2)}$$

Here the $\gamma^{A'}$ form a real representation of SO(8). With these choices $\Gamma_{+-1}=1_2\otimes\gamma$. Note that $\sqrt{2}\Gamma^{\pm}=(\Gamma^0\pm\Gamma^{10})$.

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