Black hole entropy associated with the supersymmetric sigma model

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By means of an identity that equates the elliptic genus partition function of a supersymmetric sigma model on the N-fold symmetric product S^NX of X ($S^NX=X^N/S_N$, where S_N is the symmetric group of N elements) to the partition function of a second-quantized string theory, we derive the asymptotic expansion of the partition function as well as the asymptotic for the degeneracy of spectrum in string theory. The asymptotic expansion for the state counting reproduces the logarithmic correction to the black hole entropy.

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I. INTRODUCTION

In the correspondence between a black hole and a highly excited string the black hole horizon is governed by some conformal operator algebra on a two-dimensional surface. This provides a string representation of black hole quantum states. Conversely, it may be possible to give a black hole interpretation of strings [1]. Obtaining the black hole entropy by counting the number of excited strings states (statistical interpretation of the black hole entropy) has been subsequently presented in several papers [2–12]. A comparison of the asymptotic state density of (twisted) p-branes and mass level state density of black holes has also been established in Refs. [2,13–16]. In this work, we calculate the black hole entropy for a supersymmetric sigma model. In the remainder of this section, we set the relevant mathematical method used in this paper. Then, in Secs. II and III, we derive the asymptotic state density and black hole entropy, respectively. We end up with some concluding remarks in Sec. IV.

Mathematical notation

We start by considering a supersymmetric sigma model on the N-fold symmetric product S^NX of a Kähler manifold X, which is the orbifold space $S^NX=X^N/S_N$. Here S_N is the symmetric group of N elements. The Hilbert space of an orbifold field theory can be decomposed into twisted sectors $\mathcal{H}_\gamma$, which are labeled by the conjugacy classes $\{\gamma\}$ of the orbifold group S_N [17–19]. For a given twisted sector one can keep the states invariant under the centralizer subgroup Γ_γ related to the element γ. Let $\mathcal{H}_\gamma^{\Gamma_\gamma}$ be an invariant subspace associated with Γ_γ; the total orbifold Hilbert space takes the form $\mathcal{H}(S^NX)=\oplus_{\{\gamma\}}\mathcal{H}_\gamma^{\Gamma_\gamma}$. Taking into account the group S_N one can compute the conjugacy classes $\{\gamma\}$ by using a set of partitions $\{N_n\}$ of N: namely, $\Sigma_n nN_n=N$, where N_n is the multiplicity of the cyclic permutation (n) of n elements in the decomposition of $\gamma:\{\gamma\}=\Sigma_{j=1}^s(j)^{N_j}$. For this conjugacy class the centralizer subgroup of a permutation γ is $\Gamma_\gamma=S_{N_1}\otimes_{j=2}^s(S_{N_j}\rtimes Z_j^{N_j})$ [19], where each subfactor S_{N_n} and Z_n permutes the N_n cycles (n) and acts within one cycle (n) correspondingly. Following the lines of Ref. [19] we may decompose each twisted sector $\mathcal{H}_\gamma^{\Gamma_\gamma}$ into a product over the subfactors (n) of N_n-fold symmetric tensor products, $\mathcal{H}_\gamma^{\Gamma_\gamma}=\otimes_{n>0}S^{N_n}\mathcal{H}_{(n)}^{Z_n}$, where $S^N\mathcal{H}\equiv(\otimes^N\mathcal{H})^{S_N}$.

Let $\chi(;q,y)$ be the partition function for every (sub) Hilbert space of a supersymmetric sigma model. It has been shown [20–25] that the partition function coincides with the elliptic genus. If $\chi(\mathcal{H}_{(n)}^{Z_n};q,y)$ admits the extension $\chi(\mathcal{H};q,y)=\Sigma_{m\geqslant 0,\ell}C(nm,\ell)q^my^\ell$, the following result holds (see Refs. [19,26]):

$$\sum_{N\geqslant 0}p^N\chi(S^N\mathcal{H}_{(n)}^{\mathbf{Z}_n};q,y)=\prod_{m\geqslant 0,\ell}(1-pq^my^\ell)^{-C(nm,\ell)}, \quad (1)$$

$$W(p;q,y)=\sum_{N\geqslant 0}p^N\chi(S^NX;q,y)$$
$$=\prod_{n>0,m\geqslant 0,\ell}(1-p^nq^my^\ell)^{-C(nm,\ell)}, \quad (2)$$

where $p=\mathbf{e}[\rho]$, $q=\mathbf{e}[\tau]$, $y=\mathbf{e}[z]$, and $\mathbf{e}[x]\equiv\exp[2\pi ix]$. Here ρ and τ determine the complexified Kähler form and complex structure modulos of $\mathbf{T}^2$, respectively, and z parametrizes the $U(1)$ bundle on $\mathbf{T}^2$. The Narain duality group $SO(3,2,\mathbf{Z})$ is isomorphic to the Siegel modular group $Sp(4,\mathbf{Z})$ and it is convenient to combine the parameters ρ,τ and a Wilson line module z into a 2×2 matrix belonging to the Siegel upper half-plane of genus 2,

$$\Xi=\begin{pmatrix}\rho & z\\ z & \tau\end{pmatrix},$$

with $\Im\rho>0, \Im\tau>0, \det\Im\Xi>0$. The group $Sp(4,\mathbf{Z})\cong SO(3,2,\mathbf{Z})$ acts on the matrix Ξ by fractional linear transformations: namely $\Xi\to(A\Xi+B)(C\Xi+D)^{-1}$.

Finally we go into some facts related to the orbifoldized elliptic genus of $N=2$ superconformal field theory. The contribution of the untwisted sector to the orbifoldized elliptic genus is the function

$$\chi(X;\tau,z)\equiv\phi(\tau,z)\equiv {}_0\underset{0}{\square}(\tau,z),$$

whereas

$$\phi\left(\frac{a\tau+b}{c\tau+d},\frac{z}{c\tau+d}\right)= {}_0\underset{0}{\square}(\tau,z)\mathbf{e}\left[\frac{rcz^2}{c\tau+d}\right],$$

$$\begin{pmatrix} a & b\\ c & d\end{pmatrix}\in SL(2,\mathbf{Z}), \tag{3}$$

$r=d/2$. The contribution of the twisted μ sector projected by ν is [25]

$${}_\nu\underset{\mu}{\square}(\tau,z)=\phi(\tau,z+\mu\tau+\nu)\mathbf{e}\left[\frac{d}{2}(\mu\nu+\mu^2\tau+2\mu z)\right],$$

$$\mu,\nu\in\mathbf{Z}. \tag{4}$$

The orbifoldized elliptic genus can be defined by

$$\phi(\tau,z)_{\rm orb}\overset{def}{=}\frac{1}{h}\sum_{\mu,\nu=0}^{h-1}(-1)^{P(\mu+\nu+\mu\nu)}\,{}_\nu\underset{\mu}{\square}(\tau,z), \tag{5}$$

where P,h are some integers.

II. ASYMPTOTIC DENSITY OF STATES

If $y=\mathbf{e}[z]=1$, then the elliptic genus degenerates to the Euler number or Witten index [27,28]. For the symmetric product this gives the identity

$$W(p)=\sum_{N\geqslant 0}p^N\chi(S^NX)=\prod_{n>0}(1-p^n)^{-\chi(X)}. \tag{6}$$

Thus this character is almost a modular form of weight $-\chi(X)/2$. Equation (6) is similar to the denominator formula of a (generalized) Kac-Moody algebra [29,30]. A denominator formula can be written as follows:

$$\sum_{\sigma\in\mathcal{W}}[\mathrm{sgn}(\sigma)]e^{\sigma(v)}=e^{v}\prod_{r>0}(1-e^{r})^{\mathrm{mult}(r)}, \tag{7}$$

where v is the Weyl vector, the sum on the left hand side is over all elements of the Weyl group $\mathcal{W}$, the product on the right side runs over all positive roots (one has the usual notation of root spaces, positive roots, simple roots, and Weyl group, associated with Kac-Moody algebra), and each term is weighted by the root multiplicity mult(r). For the $su(2)$ level, for example, an affine Lie algebra (7) is just the Jacobi triple-product identity. For generalized Kac-Moody algebras there is the following denominator formula:

$$\sum_{\sigma\in\mathcal{W}}[\mathrm{sgn}(\sigma)]\sigma\left(e^{v}\sum_{r}\varepsilon(r)e^{r}\right)=e^{v}\prod_{r>0}(1-e^{r})^{\mathrm{mult}(r)}, \tag{8}$$

where the correction factor on the left hand side involves $\varepsilon(r)$ which is $(-1)^n$ if r is the sum of n distinct pairwise orthogonal imaginary roots and zero otherwise.

The logarithm of the partition function $W(p;q,y)$ is the one-loop free energy $F(p;q,y)$ for a string on $\mathbf{T}^2\times X$:

$$\begin{aligned}F(p;q,y)&=\log W(p;q,y)\\ &=-\sum_{n>0,m,\ell}C(nm,\ell)\log(1-p^nq^my^\ell) \qquad (9)\\ &=\sum_{n>0,m,\ell,k>0}\frac{1}{k}C(nm,\ell)p^{kn}q^{km}y^{k\ell}\\ &=\sum_{N>0}p^N\sum_{kn=N}\frac{1}{k}\sum_{m,\ell}C(nm,\ell)q^{km}y^{k\ell}. \qquad (10)\end{aligned}$$

The free energy can be written as a sum of Hecke operators T_N [31] acting on the elliptic genus of X [19,29,32]: $F(p;q,y)=\Sigma_{N>0}p^NT_N\chi(X;q,y)$.

The goal now is to calculate an asymptotic expansion of the elliptic genus $\chi(S^NX;q,y)$. The degeneracies for the sigma model are given by the Laurent inversion formula

$$\chi(S^NX;q,y)=\frac{1}{2\pi i}\oint\frac{W(p,q,y)}{p^{N+1}}dp, \tag{11}$$

where the contour integral is taken on a small circle around the origin. Let the Dirichlet series

$$\mathcal{D}(s;\tau,z)=\sum_{(n,m,\ell)>0}\sum_{k=1}^{\infty}\frac{\mathbf{e}[\tau mk+z\ell k]C(nm,\ell)}{n^sk^{s+1}} \tag{12}$$

converge for $0<\Re s<\alpha$. We assume that series (12) can be analytically continued in the region $\Re s\geqslant -C_0$ $(0<C_0<1)$ where it is analytic excepting a pole of order 1 at $s=0$ and $s=\alpha$, with residue Res$[\mathcal{D}(0;\tau,z)]$ and Res$[\mathcal{D}(\alpha;\tau,z)]$, respectively. Besides, let $\mathcal{D}(s;\tau,z)=\mathcal{O}(|\Im s|^{C_1})$ uniformly in $\Re s\geqslant -C_0$ as $|\Im s|\to\infty$, where C_1 is a fixed positive real number. The Mellin-Barnes representation of the function $F(t;\tau,z)$ has the form

$$\mathcal{M}[F](t;\tau,z)=\frac{1}{2\pi i}\int_{\Re s=1+\alpha}t^{-s}\Gamma(s)\mathcal{D}(s;\tau,z)ds. \tag{13}$$

The integrand in Eq. (13) has a first-order pole at $s=\alpha$ and a second-order pole at $s=0$. Shifting the vertical contour from $\Re s=1+\alpha$ to $\Re s=-C_0$ (this procedure is permissible) and making use of the residues theorem one obtains

$$F(t;\tau,z)=t^{-\alpha}\Gamma(\alpha)\mathrm{Res}[\mathcal{D}(\alpha;\tau,z)]+\lim_{s\to 0}\frac{d}{ds}[s\mathcal{D}(s;\tau,z)]$$
$$-(\gamma+\log t)\mathrm{Res}[\mathcal{D}(0;\tau,z)]$$
$$+\frac{1}{2\pi i}\int_{\Re s=-C_0}t^{-s}\Gamma(s)\mathcal{D}(s;\tau,z)ds, \quad (14)$$

where $t\equiv 2\pi(\Im\rho-i\Re\rho)$. The absolute value of the integral in Eq. (14) can be estimated to behave as $\mathcal{O}((2\pi\Im\rho)^{C_0})$. We are ready now to state the main result.

In the half-plane $\Re t>0$ there exists an asymptotic expansion for $W(t;\tau,z)$ uniformly in $|\Re\rho|$ for $|\Im\rho|\to 0$, $|\arg(2\pi i\rho)|\leqslant\pi/4$, $|\Re\rho|\leqslant 1/2$ and given by

$$W(t;\tau,z)=\mathbf{e}\left[\frac{1}{2\pi i}\left\{\mathrm{Res}[\mathcal{D}(\alpha;\tau,z)]\Gamma(\alpha)t^{-\alpha}\right.\right.$$
$$-\mathrm{Res}[\mathcal{D}(0;\tau,z)]\log t-\gamma\,\mathrm{Res}[\mathcal{D}(0;\tau,z)]$$
$$\left.\left.+\lim_{s\to 0}\frac{d}{ds}[s\mathcal{D}(s;\tau,z)]+\mathcal{O}(|2\pi\Im\tau|^{C_0})\right\}\right]. \quad (15)$$

The asymptotic expansion at $N\to\infty$ for the elliptic genus (see also Refs. [7,33] is given by the formula

$$\chi(S^NX;\tau,z)_{N\to\infty}=\mathcal{C}(\alpha;\tau,z)N^{\{2\,\mathrm{Res}[\mathcal{D}(0;\tau,z)]-2-\alpha\}/[2(1+\alpha)]}$$
$$\times\mathbf{e}\left[\frac{1+\alpha}{2\pi i\alpha}\{\mathrm{Res}[\mathcal{D}(\alpha;\tau,z)]\right.$$
$$\left.\times\Gamma(1+\alpha)\}^{1/(1+\alpha)}N^{\alpha/(1+\alpha)}\right]$$
$$\times[1+\mathcal{O}(N^{-k})], \quad (16)$$

$$\mathcal{C}(\alpha;\tau,z)=\{\mathrm{Res}[\mathcal{D}(\alpha;\tau,z)]$$
$$\times\Gamma(1+\alpha)\}^{\{1-2\,\mathrm{Res}[\mathcal{D}(0;q,y)]\}/(2+2\alpha)}$$
$$\times\mathbf{e}\left[\frac{1}{2\pi i}\left(\lim_{s\to 0}\frac{d}{ds}[s\mathcal{D}(0;\tau,z)]\right.\right.$$
$$\left.\left.-\gamma\,\mathrm{Res}[\mathcal{D}(0;\tau,z)]\right)\right][2\pi(1+\alpha)]^{1/2}, \quad (17)$$

where $k<\alpha/(1+\alpha)$ is a positive constant. In the above formulas the complete form of the prefactor $\mathcal{C}(\alpha;\tau,z)$ appears. The results (16), (17) have a universal character for all elliptic genera associated with Calabi-Yau manifolds.

III. BLACK HOLE ENTROPY

In the context of string dynamics the asymptotic state density gives a precise computation of the free energy and entropy of a black hole. The corresponding black hole entropy $\mathcal{S}(N)$ takes the form

$$\mathcal{S}(N)=\log\chi(S^NX;\tau,z)\simeq\mathcal{S}_0+\mathcal{A}(\alpha)\log(\mathcal{S}_0)+(\mathrm{const}), \quad (18)$$

$$\mathcal{A}(\alpha)=(2\alpha)^{-1}\{2\,\mathrm{Res}[\mathcal{D}(0;\tau,z)]-2-\alpha\}. \quad (19)$$

The leading term in Eq. (18) is $\mathcal{S}_0=\mathcal{B}(\alpha)N^{\delta(\alpha)}$, where

$$\mathcal{B}(\alpha)=\frac{1}{\delta(\alpha)}\{\mathrm{Res}[\mathcal{D}(\alpha;\tau,z)]\Gamma(1+\alpha)\}^{\delta(\alpha)/\alpha},$$

$$\delta(\alpha)=\frac{\alpha}{1+\alpha}, \quad (20)$$

while $\mathcal{A}(\alpha)$ is the coefficient of the logarithmic correction to the entropy.

The asymptotic state density at level $N(N\gg 1)$ for fundamental p-branes compactified on manifold with topology $\mathbf{T}^p\times\mathbf{R}^{d-p}$ can be calculate within the semiclassical quantization scheme (see for details Refs. [2,33]). The coefficient $\mathcal{A}(p)$ in this case takes the form

$$\mathcal{A}(p)=(2p)^{-1}[Z_p(0)-2-p], \quad (21)$$

where $Z_p(s)$ is the p-dimensional Epstein zeta function. Since $Z_p(s=0)=-1$, we have $\mathcal{A}(p)=-(d+1)/(2p)$. In string theory, in the case of zero modes, the dependence on embedding spacetime can be eliminate [12]. In fact, the coefficient logarithmic correction $\mathcal{A}(p)$ becomes $-3/2$, which agrees with the results obtained in the spin network formalism. The coefficient of the logarithmic correction to the supersymmetric string entropy, $\mathcal{A}(\alpha)$, depends on the complex dimension d of a Kähler manifold X.

Using the transformation properties (4) in Eqs. (16), (17) one can obtain the asymptotic expansion for the orbifoldized state density. Thus starting with the expansion of the state density of the untwisted sector we can compute the asymptotics of the state density of the twisted sector.

IV. CONCLUDING REMARKS

Our results can be used in the context of the brane method's calculation of the ground-state degeneracy of systems with quantum numbers of certain Bogomol'nyi-Prasad-Sommerfield (BPS) extreme black holes [34–36,4]. We note here the BPS black hole in toroidally compactified ($M=\mathbf{T}^5\times X^5$) type II string theory. One can construct a brane configuration such that the corresponding supergravity solutions describe five-dimensional black holes. Five-branes and one-brane are wrapped on $\mathbf{T}^5$ and the system is given by the Kaluza-Klein momentum N in one of the directions. Thus black holes in these theories can carry both an electric charge Q_F and an axion charge Q_H. The brane picture gives the entropy in terms of partition function $W(t)$ for a gas of Q_FQ_H species of massless quanta: $W(t)=\prod_{\mathbf{n}\in\mathbb{Z}^m/\{\mathbf{0}\}}\{1-\exp[-t\omega_{\mathbf{n}}(\mathbf{a},\mathbf{g})]\}^{-(\dim M-m-1)}$, where $t=y+2\pi ix$, $\Re t>0$, $\omega_{\mathbf{n}}(\mathbf{a},\mathbf{g})=[\Sigma_j a_j(n_j+g_j)^2]^{1/2}$, and g_j and a_j are some real numbers. For unitary conformal theories of fixed central charge c, Eq. (16) represents the degeneracy of the state $\chi(N)$ with momentum N and for $N\to\infty$ one has [14]

$$\log\chi(N)\simeq 2\sqrt{\Lambda\zeta_R(2)cN}-\frac{\Lambda c+3}{4}\log(N), \qquad (22)$$

where $\Lambda=(\dim M-m-1)/4$ and $\zeta_R(s)$ is the Riemann zeta function. The entropy takes the form

$$\mathcal{S}(N)=\log\chi(N)\simeq\mathcal{S}_0+\mathcal{A}\log(\mathcal{S}_0), \qquad (23)$$

where for $\Lambda=1$ we have

$$\mathcal{S}_0=2\pi\sqrt{cN/6}, \quad \mathcal{A}=-\frac{c+3}{2}. \qquad (24)$$

Following Ref. [36], we can put $c=3Q_F^2+6$, $N=Q_H$, and get the growth of the elliptic genus (or the degeneracy of BPS solitons) for $N=Q_H\gg 1$. However, this result is incorrect when the black hole becomes massive enough for its Schwarzschild radius to exceed any microscopic scale such as the compactification radii [4,35]. Such models, stemming from string theory, would therefore be incompatible; in view of the present result, this might be presented as a useful constraint for the underlying microscopic field theory.

Finally, note that for a Calabi-Yau space the χ_y genus [37] is a weak Jacobi form of weight 0 and index $d/2$ and it transforms as $\chi_y(T_X)=(-1)^{r-d}y^r\chi_{y^{-1}}(T_X)$. This relation can also be derived from the Serre duality $H^j(X;\wedge^s T_X)\cong H^{d-j}(X;\wedge^{r-s}T_X)$. For $q=0$ the elliptic genus reduces to a weighted sum over the Hodge numbers: namely, $\chi(X;0,y)=\Sigma_{j,k}(-1)^{j+k}y^{j-d/2}h^{j,k}(X)$. For the trivial line bundle the symmetric product (6) can be associated with the simple partition function of a second-quantized string theory.

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