An inflation model with large variations in the spectral index

Bo Feng,[1] Mingzhe Li,[1] Ren-Jie Zhang,[2,3] and Xinmin Zhang[1]
[1]*Institute of High Energy Physics, Chinese Academy of Science, P.O. Box 918-4, Beijing 100039, China*
[2]*School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540, USA*
[3]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China*
(Received 4 March 2003; published 21 November 2003)

Recent fits of cosmological parameters by a Wilkinson Microwave Anisotropy Probe (WMAP) measurement favor a primordial scalar spectrum with a varying index. This result, if it stands, could severely constrain inflation model building. Most extant slow-roll inflation models allow for only a tiny amount of scale variations in the spectrum. We propose in this paper an extra-dimensional inflation model which is natural theoretically and can generate the required variations of the spectral index as implied by the WMAP for suitable choices of parameters.

DOI: 10.1103/PhysRevD.68.103511 PACS number(s): 98.80.Cq, 11.10.Kk, 98.70.Vc

The recently released Wilkinson Microwave Anisotropy Probe (WMAP) data [1] have been used to fit the cosmological parameters and confront the predictions of inflationary scenarios, respectively, in Refs. [2] and [3]. It is found that the data (with no other significant priors) can be best fitted by a standard cold dark matter model with a cosmological constant (ΛCDM) seeded by an almost scale-invariant, adiabatic, Gaussian primordial fluctuation; predictions inferred from the model also agree with other cosmological measurements with high accuracies [4,5]. It is noted that there might be possible discrepancies between predictions and observations on the largest and smallest scales. The problem at the smallest scales is improved when combined with data from finer-scale CMB experiments (ACBAR and CBI) and structure formation measurements (2dFGRS and Lyman α forest), for which a varying scalar primordial spectrum is favored by the best fit. At the pivot scale $k_0 = 0.05$ Mpc^{-1}, the best-fit values for the scalar power spectrum are $n_s = 0.93 \pm 0.03$ and $dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$ [2]; this means that at a 2σ level the spectrum runs from blue to red as the comoving wave number k increases. As noted, suppression of the power at the smallest scale might offer an interesting solution to the problem of the standard ΛCDM model at small scales [6].

The intriguing result of a varying spectral index needs to be closely evaluated with improved statistics and with future data. If it stands, it could severely constrain inflation model building. Most extant inflationary models allow for only a tiny amount of scale variations. In this paper we propose a single-field inflation model which is natural theoretically and can generate the required variations of the spectral index as implied by the WMAP.

For the single-field slow-roll inflationary models, one usually defines the slow-roll parameters [7]

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta \equiv M_P^2 \frac{V''}{V}, \quad \xi \equiv M_P^4 \frac{V' V'''}{V^2}, \qquad (1)$$

where primes represent derivatives with respect to the inflaton field ϕ and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The slow-roll approximation requires ϵ, $|\eta|$, $|\xi| \ll 1$ in the inflationary epoch.

The primordial curvature (scalar) and tensor perturbation power spectra are given by [7]

$$\mathcal{P}_{\mathcal{R}} \approx \left.\frac{V}{24\pi^2 M_P^4 \epsilon}\right|_{k=aH}, \quad \mathcal{P}_h \approx \left.\frac{2V}{3\pi^2 M_P^4}\right|_{k=aH}, \qquad (2)$$

evaluated when a particular mode crosses the horizon. The tensor to scalar ratio $r \equiv \mathcal{P}_h/\mathcal{P}_{\mathcal{R}} \approx 16\epsilon$ is generally small in the slow-roll inflation. The spectral indices and their running are the slopes and curvatures of the power spectra. In terms of the slow-roll parameters, for scalar perturbations, they are

$$n_s - 1 \approx -6\epsilon + 2\eta, \quad \frac{dn_s}{d\ln k} \approx 16\epsilon\eta - 24\epsilon^2 - 2\xi. \qquad (3)$$

In most inflationary models, $\epsilon, |\eta| \sim M_P^2/(\Delta\phi)^2$, $|\xi| \sim M_P^4/(\Delta\phi)^4$, and the number of e-foldings, $\mathcal{N} \sim (\Delta\phi)^2/M_P^2$, where $\Delta\phi$ is the displacement of the homogeneous field ϕ, so there is a hierarchy in the slow-roll parameters ϵ, $|\eta| \sim \mathcal{N}^{-1}$, $|\xi| \sim \mathcal{N}^{-2}$, and the variations of the spectral indices, $dn_s/d\ln k$, are negligible.

To achieve a running spectrum in the order favored by the WMAP, one has to consider inflation models with more exotic potentials [8], such as the running mass model [9] and models with an oscillating primordial spectrum [10]. These models can generate significant spectral runnings, which could be as large as $n_s - 1$.

The Achilles heel of these inflationary models (and of the inflation paradigm in general) lies, arguably, in the difficulty of obtaining a sufficiently flat and stable (against radiative corrections) inflaton potential from the perspective of particle physics. Symmetry principles must be invoked. There are only two known symmetries which can protect the flatness of a scalar potential: supersymmetry and the shift symmetry for a pseudo Nambu-Goldstone boson (PNGB). However, as shown in Ref. [11] and recently reemphasized in Refs. [12] and [13], supersymmetry alone cannot protect the flatness of the inflaton potential, since it is explicitly broken during inflation, and the gravitational effects generically give a Hubble-scale mass correction to the inflaton.

Shift symmetry was first realized in the natural inflation model [14]. Still there are some difficulties in this model. The flatness condition, in the simplest scenario with a single PNGB, requires the scale of spontaneous symmetry breaking and the values of the inflaton during the slow roll above the Planck scale, taking the model outside the regime of validity of an effective field theory description. Moreover, it is expected that the gravity-induced higher-dimensional operators are not suppressed.

These issues have been recently reexamined in the context of extra dimensions (called extra-natural inflation in Ref. [12]). Consider a five-dimensional Abelian gauge field model with the fifth dimension compactified on a circle of radius R; we identify the inflaton field θ with the gauge-invariant Wilson loop of the extra component A_5 propagating in the bulk,

$$\theta = g_5 \oint dx^5 A_5, \tag{4}$$

where g_5 is the five-dimensional gauge coupling constant. At energies below $1/R$, θ is a four-dimensional field with an effective Lagrangian

$$\mathcal{L} = \frac{1}{2g_4^2(2\pi R)^2}(\partial\theta)^2 - V(\theta), \tag{5}$$

with $g_4^2 = g_5^2/2\pi R$ the four-dimensional effective gauge coupling constant. The nonlocal potential $V(\theta)$ is generated in the presence of particles charged under Abelian symmetry [15].

For bulk fields with bare masses M_a and charges q_a the potential takes the form [16]

$$V(\theta) = \frac{1}{128\pi^6 R^4}\mathrm{Tr}[V(r_a^F,\theta) - V(r_a^B,\theta)], \tag{6}$$

where the trace is over the number of degrees of freedom, and the superscripts F and B stand for fermions and bosons, respectively. Here

$$V(r_a,\theta) = x_a^2 \mathrm{Li}_3(r_a e^{-x_a}) + 3x_a \mathrm{Li}_4(r_a e^{-x_a}) + 3\mathrm{Li}_5(r_a e^{-x_a}) + \mathrm{H.c.}, \tag{7}$$

with

$$r_a = e^{iq_a\theta}, \quad x_a = 2\pi R M_a, \tag{8}$$

and the polylogarithm function $\mathrm{Li}_k(z)$ is

$$\mathrm{Li}_k(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^k}. \tag{9}$$

For the massless particles $(x_a = 0)$ considered in Ref. [12] the potential is

$$V(\theta) = -\frac{3}{64\pi^6 R^4}\sum_I (-)^{F_I}\sum_{n=1}^{\infty}\frac{\cos(nq\theta)}{n^5}, \tag{10}$$

where $F_I = 0$ and 1 stand for massless bosonic and fermionic fields, respectively.

Neglecting the higher-power terms in Eq. (10), one obtains the same form of the potential as that of the natural inflation model. The effective decay constant of the spontaneously broken Abelian symmetry is

$$f_{\mathrm{eff}} = \frac{1}{2\pi g_4 R}, \tag{11}$$

which can be naturally greater than M_P for a sufficiently small coupling constant g_4 [12]. Moreover, as a result of the extra-dimension nature, gravity-induced higher-dimensional operators are generally exponentially suppressed. This solves the forementioned problems of the four-dimensional natural inflation model.[1] The extra-natural model predicts a red-tilted scalar spectrum with negligible spectral runnings, the same as that of natural inflation.

The model we propose in this paper includes one massless and one massive field[2] coupled to A_5, i.e., $M_1 = 0$, $M_2 \gtrsim 1/R$; the corresponding potential for θ is

$$V(\theta) = -\frac{3}{64\pi^6 R^4}\sum_{n=1}^{\infty}\frac{1}{n^3}\left[(-)^{F_1}\frac{\cos(nq_1\theta)}{n^2} + (-)^{F_2}e^{-nx_2}\left(\frac{x_2^2}{3} + \frac{x_2}{n} + \frac{1}{n^2}\right)\cos(nq_2\theta)\right]. \tag{12}$$

Neglecting the higher-power terms in the sum and defining a canonical field $\phi = f_{\mathrm{eff}}\theta$, the effective Lagrangian of our model becomes[3]

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V_0\left[1 - \cos\left(\frac{q_1\phi}{f_{\mathrm{eff}}}\right) - \sigma\cos\left(\frac{q_2\phi}{f_{\mathrm{eff}}}\right)\right], \tag{13}$$

where

$$\sigma = (-)^{F_2+1}e^{-x_2}\left(\frac{x_2^2}{3} + x_2 + 1\right), \quad V_0 = \frac{3}{64\pi^6 R^4}. \tag{14}$$

Note that in our calculations we have added a σ-dependent term to the potential, Eq. (13), to make it vanish at the minimum. For $\sigma = 0$, this potential coincides with that of the natural inflation model.

The slow-roll parameters of Eq. (1) are

[1] If the natural inflation model includes a large Z in the kinetic term [17], redefining the field gives rise to an effective decay constant $f_{eff} = \sqrt{Z}f$, which (as well as the inflaton field itself) can also be larger than M_p; however, a question remained is how to get a large Z naturally [18].

[2] Massive particles have also been considered in Ref. [19], in the context of extra-dimensional quintessence models.

[3] For a specific presentation, we set $F_1 = 1$. The $F_1 = 0$ case is equivalent since it only corresponds to a coordinate shift in the potential.

$$\epsilon = \frac{\mu^2}{2}\frac{(\sin\tilde{\theta}+\sigma\kappa\sin\kappa\tilde{\theta})^2}{[1-\cos\tilde{\theta}-\sigma\cos\kappa\tilde{\theta}]^2}, \tag{15}$$

$$\eta = \frac{\mu^2(\cos\tilde{\theta}+\sigma\kappa^2\cos\kappa\tilde{\theta})}{1-\cos\tilde{\theta}-\sigma\cos\kappa\tilde{\theta}}, \tag{16}$$

$$\xi = -\frac{\mu^4(\sin\tilde{\theta}+\sigma\kappa\sin\kappa\tilde{\theta})(\sin\tilde{\theta}+\sigma\kappa^3\sin\kappa\tilde{\theta})}{[1-\cos\tilde{\theta}-\sigma\cos\kappa\tilde{\theta}]^2}, \tag{17}$$

where we have defined $\tilde{\theta}\equiv q_1\theta$ and

$$\mu \equiv q_1 M_P/f_{\rm eff}, \quad \kappa\equiv q_2/q_1. \tag{18}$$

To see analytically the effects of the σ term, we consider the following choice of parameters: $\kappa \gg 1$, $\sigma \ll 1$, $|\sigma|\kappa \ll 1$, and $|\sigma|\kappa^2 \sim \mathcal{O}(1)$. When cosmological scales begin to cross the horizon in the inflationary epoch, $\tilde{\theta}\sim\pi$, the slow-roll parameters are approximately

$$\epsilon \sim \mu^2(\pi-\tilde{\theta})^2, \tag{19}$$

$$\eta \sim \mu^2(-1+\sigma\kappa^2\cos\kappa\tilde{\theta}), \tag{20}$$

$$\xi \sim -\mu^4\sigma\kappa^3(\pi-\tilde{\theta})\sin\kappa\tilde{\theta}. \tag{21}$$

Hence $\epsilon \ll |\eta|$. The spectral index is determined by the η parameter, $n_s - 1 \simeq 2\eta$, and the tensor fluctuation is negligible.

From the above equations, one generically would have $|\xi| \sim \eta^2 \sim 10^{-4}$, corresponding to a negligible spectral variation. However, the large $\sigma\kappa^3$ term in Eq. (21) can enhance ξ substantially (to the size as large as η) and lead to a scale varying spectrum. Furthermore, for appropriate choices of σ and κ, the scalar power spectrum can run from blue to red as the comoving scale k increases. Such a tilted spectrum is favored by the current data of WMAP [2].

We show this behavior of power spectrum running in Fig. 1 for the following choices of parameters: (a) $\sigma = -3.8\times10^{-4}$, $\mu = 1/2$, and $\kappa = 100$ (solid lines) and (b) $\sigma = 0.012$, $\mu = 1/3$, and $\kappa = 15$ (dashed lines). To ensure the accuracy of our results, we have retained the higher-power terms (up to $n=6$ for the massless particle) in the numerical calculations. We have also set the number of e-foldings, $\mathcal{N}(k_*) = 50$, at a reference comoving scale k_*; the WMAP analyses used pivot scales $k_0 = 0.002$ and 0.05 Mpc^{-1} [2,3]. (This scale arbitrariness is the inherent theoretical uncertainty in our analysis; it can be resolved if the detail reheating history is known [20].) Normalizing the spectral index to the WMAP central value $n_s = 1.1$ at $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ [3], we find $dn_s/d\ln k \approx -0.041$ and ≈ -0.021 for cases (a) and (b), respectively; they are in good agreement with the current WMAP fits [2,3]. Such a running feature of the spectral index does not exist in the usual natural or extra-natural inflation models.

In Fig. 2 we delineate the parameter space that gives the required amount of spectral runnings ($-0.084 < dn_s/d\ln k < -0.027$) [3] in our model. We have normalized the spectral index $n_s = 1.13$ at the same pivot scale k_0. We show allowed regions in the κ-σ plane, for $\mu = 1/3$. The correlation between κ and σ can be understood because one needs a certain amount of cancellations between the two terms in Eq. (20) (to achieve a spectral running from blue to red).

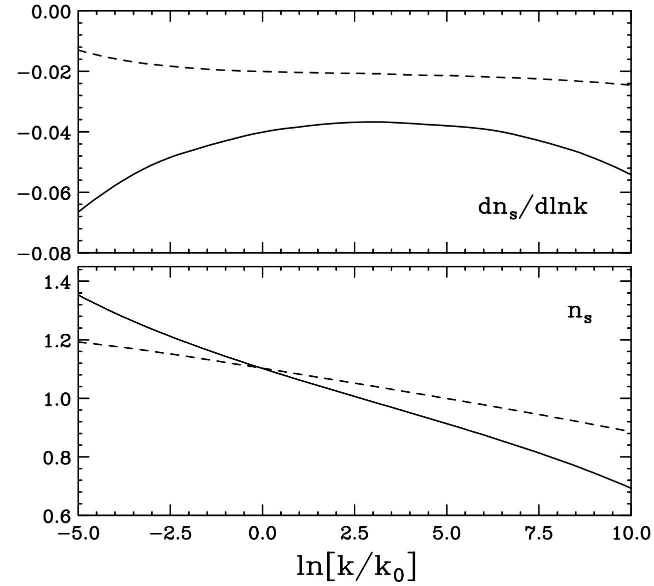

FIG. 1. Spectral indices n_s and their runnings $dn_s/d\ln k$ for $\sigma = -3.8\times10^{-4}$, $\mu = 1/2$, $\kappa = 100$ (solid lines) and for $\sigma = 0.012$, $\mu = 1/3$, $\kappa = 15$ (dashed lines). $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ is the pivot scale of WMAP.

In our models the size of the fifth dimension is determined to be of the order $R \sim 10/M_P$ from the COBE normalization, $\mathcal{P}_{\mathcal{R}}^{1/2} \sim 10^{-5}$. For our choices of parameters, $\mu \sim 0.1$, this implies the four- and five-dimensional gauge coupling constants $g_4 \lesssim 10^{-3}/q_1$, $g_5^2 \lesssim 10^{-4}/q_1^2 M_P$.

These parameter choices could be accommodated in models of fundamental theories. For example, string theory pre-

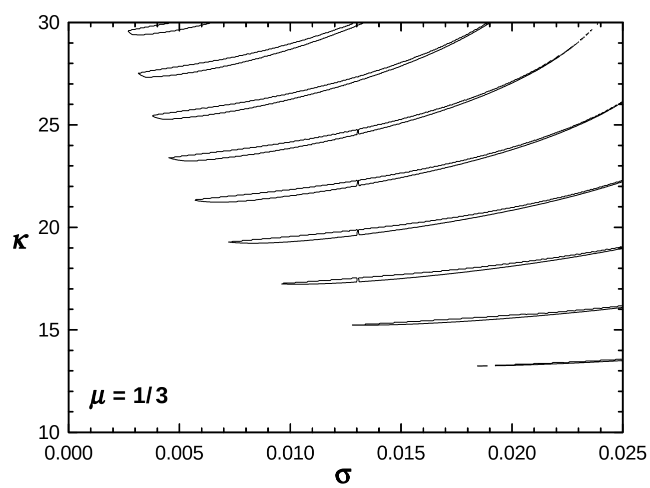

FIG. 2. Regions in the κ-σ plane which give variations of the spectral index in the range $-0.084 < dn_s/d\ln k < -0.027$, for $\mu = 1/3$. The spectral indices have been normalized to $n_s = 1.13$ at the pivot scale k_0.

dicts a host of exotic particles with fractional charges; one can obtain large charge ratios of the massive to massless particles ($\kappa \sim 10$–100) as required in our models. The massive particle has a mass of the order $M_2 \sim \mathcal{O}(1) R^{-1}$ and falls in the range of massive string states. It remains to be seen whether this kind of model can arise naturally from specific string models.

The small gauge coupling constant [$g_4 \lesssim 10^{-3}$ when $q_1 \sim \mathcal{O}(1)$] might seem worrisome, since string theory compactification generally requires $g_4 M_P R \gtrsim 1$ [12]. This problem can be improved by introducing many Abelian gauge fields in higher dimensions as in the assisted inflation model [21]. For a system with N Wilson loops θ_i, the Lagrangian is given by

$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{2 g_4^2 (2\pi R)^2} (\partial \theta_i)^2 - \sum_{i=1}^{N} V(\theta_i). \tag{22}$$

This system has a solution where all the θ_i fields are equal; consequently, it can be described by a single-field model through the redefinitions $\tilde{f}_{\rm eff} = \sqrt{N}/2\pi g_4 R$, $\tilde{V} = N V_i$. The COBE normalization gives $R N^{-1/4} \sim 10 M_P^{-1}$, which implies $g_4 \lesssim 10^{-3} N^{1/4}$. The above requirement can be naturally satisfied if $N \gtrsim 100$.

Before concluding, we comment on the possible discrepancy between predictions and COBE [22] and WMAP observations on the largest scales [2,23]. As argued in Ref. [24], the need of a large running is due to the suppressed multipoles at $l = 2,3,4$. However, as shown in Ref. [2] the possibility of finding a lower value of the quadrupole in the presence of a constant running of the spectral index is no more than 0.9% for a spacial flat ΛCDM cosmology. In our model, the running index $dn_s/d\ln k$ is not a constant, but we have checked that it does not improve the fit significantly. With many Abelian gauge fields in the higher dimensions, we will show now that this discrepancy can be alleviated. For simplicity of our discussion, consider two Wilson lines θ_1 and θ_2; the Lagrangian for such a system is given by

$$\mathcal{L} = \frac{1}{2}(\partial \phi_1)^2 + \frac{1}{2}(\partial \phi_2)^2 - V_0 \left[1 - \cos\left(\frac{q_1 \phi_1}{f_{\rm eff}} \right) \right] - V_0 \left[1 - \cos\left(\frac{q_2 \phi_2}{f_{\rm eff}} \right) \right]. \tag{23}$$

Assuming an adiabatic process with a certain choice of parameters, e.g., $q_1 = 5 q_2$, inflation will be first driven by the heavy field ϕ_1 and then by ϕ_2, with no interruption in between. The "double natural inflation" can generate a primordial spectrum with a feature at the largest scales favored by current data. In Fig. 3 we give an example with $q_1 = 5$ and $q_2 = 1$. The details of our calculations are similar to Ref. [25]: we numerically calculate the primordial scalar and tensor spectra and fit the resulting CMB TT and TE spectra using a WMAP likelihood code [26] with the CMBFAST program [27]. We compare our result with the WMAP team's best-fit power-law ΛCDM (PΛCDM) and running spectral index ΛCDM (RΛCDM) models. The CMB quadrupole as shown in Fig. 3 is better suppressed in this model.

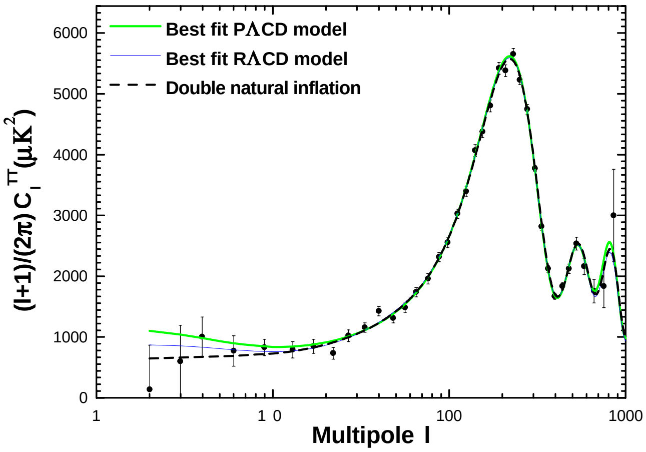

FIG. 3. Suppression of CMB quadrupole in the double natural inflation model. The CMB data with error bars are taken from WMAP collaborations [2].

In summary, the WMAP result of a varying spectral index, if it stands, could be used as a discriminator for inflationary models; most extant models allow for only a tiny amount of scale variations in the spectral index and could face a severe challenge. We have studied in this paper the possibility of building inflation models with large running spectral indices and specifically proposed a higher-dimensional model which can generate $dn/d\ln k \sim -\mathcal{O}(10^{-2})$, favored by the WMAP analyses.

This work was supported in part by the National Natural Science Foundation of China and by the Ministry of Science and Technology of China under Grant No. NKBRSF G19990754, and in part by NSF Grant No. NSF-0070928.

[1] C.L. Bennett *et al.*, Astrophys. J. Suppl. **148**, 1 (2003); also see the WMAP homepage http://map.gsfc.nasa.gov.

[2] D.N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).

[3] H.V. Peiris *et al.*, Astrophys. J. Suppl. **148**, 213 (2003).

[4] A. Melchiorri and C.J. Ödman, Phys. Rev. D **67**, 081302(R) (2003).

[5] V. Barger, H.-S. Lee, and D. Marfatia, Phys. Lett. B **565**, 33 (2003).

[6] B. Moore, Nature (London) **370**, 629 (1994); R.A. Flores and J.A. Primack, Astrophys. J. Lett. **427**, L1 (1994); W.J.G. De Blok and S.S. McCaugh, Mon. Not. R. Astron. Soc. **290**, 533 (1997); D.N. Spergel and P.J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000); M. Kamionkowski and A.R. Liddle, *ibid.* **84**, 4525 (2000); A.R. Zentner and J.S. Bullock, Phys. Rev. D **66**, 043003 (2002); W. Lin, D. Huang, X. Zhang, and R. Brandenberger, Phys. Rev. Lett. **86**, 954 (2001).

[7] See, e.g., A.R. Liddle and D.H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cam-

bridge, England, 2000).
[8] S. Dodelson and E.D. Stewart, Phys. Rev. D **65**, 101301(R) (2002).
[9] E.D. Stewart, Phys. Lett. B **391**, 34 (1997); Phys. Rev. D **56**, 2019 (1997).
[10] X. Wang *et al.*, astro-ph/0209242.
[11] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart, and D. Wands, Phys. Rev. D **49**, 6410 (1994).
[12] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall, Phys. Rev. Lett. **90**, 221302 (2003); J. Cosmol. Astropart. Phys. **07**, 003 (2003).
[13] D.E. Kaplan and N. Weiner, hep-ph/0302014; M. Fairbairn, L.L. Honorez, and M.H.G. Tytgat, Phys. Rev. D **67**, 101302(R) (2003).
[14] K. Freese, J.A. Frieman, and A.V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); F. Adams, J.R. Bond, K. Freese, J.A. Frieman, and A.V. Olinto, Phys. Rev. D **47**, 426 (1993).
[15] Y. Hosotani, Phys. Lett. **126B**, 309 (1983); **129B**, 193 (1983); Ann. Phys. (N.Y.) **190**, 233 (1989).
[16] A. Delgado, A. Pomarol, and M. Quiros, Phys. Rev. D **60**, 095008 (1999).
[17] S. Dimopoulos and S. Thomas, hep-th/0307004.
[18] B. Feng, M. Li, and X. Zhang (in preparation).
[19] L. Pilo, D.A.J. Rayner, and A. Riotto, Phys. Rev. D **68**, 043503 (2003).
[20] B. Feng, X. Gong, and X. Wang, astro-ph/0301111.
[21] A.R. Liddle, A. Mazumdar, and F.E. Schunck, Phys. Rev. D **58**, 061301 (1998).
[22] C.L. Bennett *et al.*, Astrophys. J. Lett. **464**, L1 (1996).
[23] S.L. Bridle, A.M. Lewis, J. Weller, and G. Efstathiou, Mon. Not. R. Astron. Sco. **342**, L72 (2003).
[24] A. Oliveira-Costa, M. Tegmark, M. Zaldarriaga, and A. Hamilton, astro-ph/0307282.
[25] B. Feng and X. Zhang, Phys. Lett. B **570**, 145 (2003).
[26] L. Verde *et al.*, Astrophys. J. Suppl. **148**, 195 (2003).
[27] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996).