Big bang nucleosynthesis and Λ_{QCD}

James P. Kneller* and Gail C. McLaughlin†
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA
(Received 3 June 2003; revised manuscript received 7 July 2003; published 19 November 2003)

Big bang nucleosynthesis (BBN) has increasingly become the tool of choice for investigating the permitted variation of fundamental constants during the earliest epochs of the Universe. Here we present a BBN calculation that has been modified to permit changes in the QCD scale, Λ_{QCD}. The primary effects of changing the QCD scale upon BBN are through the deuteron binding energy B_{D} and the neutron-proton mass difference δm_{np}, which both play crucial roles in determining the primordial abundances. In this paper we show how a simplified BBN calculation allows us to restrict the nuclear data we need to just B_{D} and δm_{np} yet still gives useful results so that any variation in Λ_{QCD} may be constrained via the corresponding shifts in B_{D} and δm_{np} by using the current estimates of the primordial deuterium abundance and helium mass fraction. The simplification predicts the helium-4 and deuterium abundances to within 1% and 50%, respectively, when compared with the results of a standard BBN code. But Λ_{QCD} also affects much of the remaining required nuclear input so this method introduces a systematic error into the calculation and we find a degeneracy between B_{D} and δm_{np}. We show how increased understanding of the relationship of the pion mass and/or B_{D} to other nuclear parameters, such as the binding energy of tritium and the cross section of $\mathrm{T}+\mathrm{D}\rightarrow {}^4\mathrm{He}+n$, would yield constraints upon any change in B_{D} and δm_{np} at the 10% level.

DOI: 10.1103/PhysRevD.68.103508 PACS number(s): 98.80.Ft, 21.10.Dr, 26.35.+c

I. INTRODUCTION

Speculation that fundamental constants may vary with time began as early as the 1930s [1] and, although there is no mechanism for such time variation in the context of the standard model of particle physics, recent observations have created renewed interest in this idea. Webb *et al.* [2] report observations of quasar absorption lines at a redshift of $z=1-2$ that suggest the fine structure constant α may have been smaller at this time. The effect is at the level of one part in 10^5 and further analysis of a new data set by the same group gives similar results [3] although Bahcall, Steinhardt and Schlegel [4], using a different analysis method, find a different limit.

Independently of these observations, however, it is interesting to consider what happens if the fundamental constants were different in earlier times than they are today. There are many suggestions for beyond the standard model theories that could accommodate time variation in the fundamental constants and link the changes in some constants to others, e.g. Langacker [5], though in general studies have typically derived constraints on one constant when all the others are fixed. If these constants were different at an early epoch than they are today, their relative shifts can only be determined by the underlying theory which causes the changes. In the context of grand unified theories (GUTs), all couplings are correlated via the GUT scale parameter, which varies with time, and relative ratios are determined by the renormalization group equations. Langacker *et al.* [5] (see also [6,7]) find that

$$\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}\sim 30\,\frac{\Delta\alpha}{\alpha}. \tag{1}$$

The relationship between the shift in the fine structure constant and the vacuum expectation value of the Higgs boson is very model dependent even in the context of grand unified theories. For some supersymmetric theories it is as large as $\Delta v/v\sim 70\Delta\alpha/\alpha$. While the limits on the variation of the fine structure constant derived from quasar spectra (which were formed $\sim$1 billion years after the big bang) are severe and would seem to limit the amount we could expect in a variation of Λ_{QCD} at that epoch [for example, from Eq. (1)] they by no means rule out the possibility of much larger changes in Λ_{QCD}, α or indeed any other constant at the much earlier phase of BBN unless a model is provided that allows one to calculate how their value at one epoch is related to the value at another.

To constrain the permitted variations one can look to several places to confront theory with data and the strongest constraints upon the time variation of fundamental constants are expected to emerge when the observables come from events in the distant past. Such places include the Oklo nuclear reactor, quasar absorption spectra, the cosmic microwave background and big bang nucleosynthesis (BBN). There have been a number of studies that consider the variation of particular fundamental constants in these scenarios. Data from the cosmic microwave background anisotropy measurements and BBN were used by Yoo and Scherrer [8] for the Higgs vacuum expectation value while the effect of changing the fine structure on big bang nucleosynthesis has been extensively studied by Bergström, Iguri and Rubinstein [9], Avelino *et al.* [10] and Nollett and Lopez [11]. Flambaum and Shuryak [12,13] give constraints upon the quark masses and Λ_{QCD} after using BBN and limits derived from the Oklo nuclear reactor have also been studied [13,14]. In a study that permitted changes in all the gauge and Yukawa couplings by relating them to the evolution of a single scalar field (the "dilaton") Ichikawa and Kawasaki [15] again use

*Electronic address: Jim_Kneller@ncsu.edu
†Electronic address: Gail_McLaughlin@ncsu.edu

BBN to limit the variations of these couplings. Fundamental coupling constraints on BBN were also studied by [16].

In this paper we revisit the study of the influence of the strong coupling on big bang nucleosynthesis yields. BBN occurs in the first few minutes and hours after the big bang and hence we would naively expect the maximal difference from the current values of the fundamental constants if indeed they vary with time. A variation in Λ_{QCD} is much more difficult to implement than changes in most other constants because of the uncertainty of how the difference would impact the nuclear data required to make a prediction. Nevertheless, that is our intention in this paper. In Sec. II we show that of all the inputs to the calculation, it is the deuteron binding energy, B_{D}, and the neutron-proton mass difference, δm_{np} that play the most crucial role in determining the primordial helium and deuterium abundances—the two nuclei with, currently, the two most accurately known primordial abundances with which to compare the calculation. In Sec. III we discuss how the variation in fundamental parameters is related to the deuteron binding energy and the neutron-proton mass difference and discuss how these quantities enter into the important reaction rates. In Sec. IV we describe our calculation of big bang nucleosynthesis using a modified reaction network to reduce its complexity and show explicitly the effects of δm_{np} and B_{D} on abundance yields before deriving constraints on these quantities. In Sec. V we give our conclusions and point to where further understanding of the interactions/structure of nuclei can improve our results.

II. STANDARD BBN

BBN represents the marriage of nuclear physics with cosmology and, in comparison to the majority of nucleosynthesis settings, the calculation is relatively simple since there are no production zones to deal with (the whole Universe participates) and no spatial gradients of any kind that lead to a transport of entropy, momentum or mass. But the continual dilution and cooling of the cosmic fluid certainly does not mean the Universe is in a steady state during these early phases. The expansion is driven by the energy density of the relativistic particles, the nucleons/nuclei represent only a small fraction of the total, and the inexorable decrease in temperature and density means that the nuclear reactions can occur for only a brief period. Despite their cosmological inconsequence the (inferred) abundances of the nuclei at the end of BBN represent the best set of observables that probe sub-horizon scale physics at this early epoch. Like all nucleosynthesis mechanisms BBN is sensitive to three key characteristics of the setting: the duration, the energy available and the interactions between the nuclei. Simplistically the primordial abundances are determined in three (somewhat) distinct phases whose boundaries are determined by just two nuclear parameters: the neutron-proton mass difference δm_{np} and the binding energy of the deuteron B_{D}. In each, the behavior of the nuclei is distinguished by their interactions and, in order to set the stage for later discussions, we shall skip briefly through each pointing out the importance of these two parameters.

A. δm_{np}: Weak equilibrium

Throughout the entire evolution of the baryons in BBN it is the relativistic particles that drive the expansion, provide the thermal bath in which the nuclear reactions take place, and set the time-temperature relationship. At a sufficiently early epoch all particles are in thermal contact through electromagnetic and/or weak interactions and so possess a common temperature T. At a temperature of 10 MeV, a common initial temperature for BBN calculations, the radiation includes photons, electron/positrons and three light neutrinos. The inconsequence of the nucleons/nuclei for cosmology at this epoch is implied by the smallness of the ratio of baryon and photon number densities, denoted by η, which is $\eta = n_B/n_\gamma \sim \mathcal{O}(10^{-10})$. Any chemical potential of the photons is driven to zero from such rapid processes as double Compton scattering, in the absence of any (significant, nonthermal) source of neutrinos the ratio $\xi_i = \mu_i/T_i$ is constant, and the chemical potential of the electrons/positrons is set by the proton density and is therefore extremely small, $\xi_e \sim \eta$ [17]. The energy density of the Universe is thus determined from the common temperature and the chemical potential of each relativistic fluid component so that the expansion rate of the Universe, denoted by the Hubble parameter H, is simply

$$H^2 = \frac{8\pi G_N}{3} \sum_i \rho_i(T_i, \mu_i) \tag{2}$$

and the age of the Universe is related to the temperature through

$$tT^2 = 0.74 \ \ \mathrm{MeV}^2\mathrm{s}, \tag{3}$$

when all neutrino chemical potentials are zero.

The neutrons and protons are also held in chemical equilibrium via such weak interactions as

$$\mathrm{n} + \overline{e} \leftrightarrow \mathrm{p} + \overline{\nu_e}. \tag{4}$$

From the requirement $\mu_{\mathrm{n}} - \mu_e = \mu_{\mathrm{p}} - \mu_{\nu_e}$ the neutron to proton ratio F is

$$F = \frac{n_{\mathrm{n}}}{n_{\mathrm{p}}} = \exp\left[-\frac{\delta m_{np}}{T} + \xi_e - \xi_{\nu_e} \right]. \tag{5}$$

As the Universe cools the plethora of equilibria established above 10 MeV is broken by the increasingly infrequent weak interactions. The simplicity of a single temperature for every fluid constituent falters when the neutrinos "decouple." The ebb of the weak interactions occurs when the interaction time scale becomes longer than the age of the Universe so that the neutrino decoupling temperature represents the point at which the two are equal. Below this temperature a neutrino is, on average, unlikely to ever experience another interaction that would allow energy to be transferred from one fluid component to another. For ν_μ and ν_τ the decoupling temperature is $T \sim 3.3$ MeV, for ν_e it is slightly lower at $T \sim 2$ MeV because ν_e may also interact with the electron/positron fluid via $W^\pm$ exchange whereas the other two flavors do not. Despite the fact that the three neutrinos no longer interact with any other fluid component there is no change in the evolution of the Universe. Since there was no change in the number of degrees of freedom in any of the fluid components during decoupling the three flavors all possess the same temperature T_ν equal to the electromagnetic temperature T_γ. Neutrino decoupling has no direct impact upon the nucleons at this point in their evolution; its importance is resurrected during a later epoch.

Concurrent with neutrino decoupling the nucleons also undergo an equilibrium crisis. The neutron/proton interconversion reaction $\mathrm{n}\leftrightarrow\mathrm{p}$, which actually represents three processes,

$$\mathrm{n}\leftrightarrow\mathrm{p}+e+\overline{\nu_e} \tag{6a}$$

$$\mathrm{n}+\overline{e}\leftrightarrow\mathrm{p}+\overline{\nu_e} \tag{6b}$$

$$\mathrm{n}+\nu_e\leftrightarrow\mathrm{p}+e, \tag{6c}$$

is also governed by the weak interaction and is the only process that can significantly alter the neutron/proton ratio.

At temperatures of ~ 10 MeV the three-body reaction $\mathrm{p}+e+\overline{\nu_e}\rightarrow\mathrm{n}$ and its inverse, neutron decay, both occur at a rate smaller than the Hubble parameter and are therefore incapable of affecting the neutron to proton ratio. In contrast the two-body reactions (6b) and (6c) are sufficiently rapid initially to establish an equilibrium but below $T_\gamma \sim 1.2$ MeV, a temperature we shall denote by T_{np}, the rates become smaller than the Hubble rate and are therefore incapable of maintaining this equilibrium. The neutron to proton ratio is said to "freeze out" though, in truth, even if all complex nuclei were prohibited and neutrons did not decay the ratio would not become a constant until much later [18,19].

The departure from neutron/proton weak equilibrium sets the first milestone in the path to BBN. The boundary between the equilibrium phases prior to ~ 1 MeV and the nonequilibrium phase that follows is determined essentially by the single parameter δm_{np}, and this quantity also determines the freeze-out ratio of neutrons and protons. The freeze-out ratio sets the upper limit to the number of neutrons that can participate in BBN and will therefore limit the primordial abundance of every complex nucleus. As the Universe continues to evolve below ~ 1 MeV the n/p ratio decreases from the freeze-out ratio because of both the fading residual two-body interactions and the emerging importance of neutron decay: the extent to which this limits the number of neutrons that will go on to participate in BBN proper is determined by the second parameter B_{D}.

B. B_{D}: NSE

There are several key events that occur during this second stage of the maturing Universe. The first is to the background fluid when the number/energy density of $e,\overline{e}$ begins to depart from the relativistic value. This occurs at roughly $T_\gamma \sim 800$ keV and does not cease until $T_\gamma \sim 10$ keV when the annihilation rate becomes smaller than the Hubble rate. The electron/positron annihilation deposits energy and entropy only into the photon gas, the neutrinos have decoupled and cannot share in this energy release. The change in the number of electromagnetic degrees of freedom leads to an increase of T_γ relative to T_ν in order to maintain the entropy within the co-moving volume though the increase is never capable of reversing the redshifting due to the expansion. After annihilation the electromagnetic temperature T_γ is related to the neutrino temperature T_ν via the well known $T_\nu^3/T_\gamma^3 = 4/11$ and the time-electromagnetic temperature relationship evolves to become

$$tT_\gamma^2 = 1.32\ \ \mathrm{MeV}^2\mathrm{s} \tag{7}$$

again with the standard assumption of zero neutrino degeneracy. The changing time-temperature relationship is important here because it determines how much neutron decay can occur before nuclei begin to form. Hereafter we drop the subscript for T_γ so that whenever we mention temperature it is always the electromagnetic.

So far we have made no mention of the complex nuclei. The temperatures are so high that their abundances are suppressed relative to the free nucleons but of course the nuclear reactions that form them do occur. In contrast with the $\mathrm{n}\leftrightarrow\mathrm{p}$ interconversion processes the nuclear reactions, such as $\mathrm{n}+\mathrm{p}\leftrightarrow\mathrm{D}+\gamma$, are rapid at T_{np} and so the abundances reach, and maintain, chemical/nuclear statistical equilibrium (NSE). In equilibrium the abundance,[1] $Y_A = n_A/n_B$, of the complex nuclei A is derived from $\mu_A = Z\mu_{\mathrm{p}} + (A-Z)\mu_{\mathrm{n}}$ so inserting the expressions for the Boltzmann number density we find

$$Y_A = \frac{g_A A^{3/2}}{2^A}\left[n_B \left(\frac{2\pi}{m_N T}\right)^{3/2}\right]^{A-1} Y_{\mathrm{p}}^Z Y_{\mathrm{n}}^{A-Z} e^{B_A/T}. \tag{8}$$

From this equation we see that for a temperature of $T \sim 1$ MeV the abundance of deuterons is $Y_{\mathrm{D}} \sim 10^{-12}$. We can rewrite Eq. (8) to illustrate what is happening in this temperature regime, by replacing the thermal factors with the Y_{D} so that

$$Y_A = \frac{g_A A^{3/2}}{2[3\sqrt{2}]^{A-1}} Y_{\mathrm{p}}^{1+Z-A} Y_{\mathrm{n}}^{1-Z} Y_{\mathrm{D}}^{A-1}\ \exp\left(\frac{B_A-(A-1)B_{\mathrm{D}}}{T}\right). \tag{9}$$

This equation makes it much clearer that the abundance of a nucleus with mass $A+1$ relative to a nucleus with mass A is smaller by approximately Y_{D}, indicating just how severe the suppression is when Y_{D} is small.

There are many different reactions in which the nuclei participate. For BBN the most important are [20]

$$\mathrm{n}+\mathrm{p}\leftrightarrow\gamma+\mathrm{D} \tag{10}$$

$$\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{n}+{}^3\mathrm{He} \tag{11a}$$

$$\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{p}+\mathrm{T} \tag{11b}$$

[1]The term "abundance" is also used for the ratio Y_A/Y_H.

$$\mathrm{D+p} \leftrightarrow \gamma + {}^{3}\mathrm{He} \tag{11c}$$

$$\mathrm{T+D} \leftrightarrow \mathrm{n} + {}^{4}\mathrm{He} \tag{12a}$$

$$\mathrm{T} + {}^{4}\mathrm{He} \leftrightarrow \gamma + {}^{7}\mathrm{Li} \tag{12b}$$

$${}^{3}\mathrm{He+n} \leftrightarrow \mathrm{p+T} \tag{13a}$$

$${}^{3}\mathrm{He+D} \leftrightarrow \mathrm{p} + {}^{4}\mathrm{He} \tag{13b}$$

$${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \leftrightarrow \gamma + {}^{7}\mathrm{Be} \tag{13c}$$

$${}^{7}\mathrm{Li+p} \leftrightarrow 2\quad {}^{4}\mathrm{He} \tag{14}$$

$${}^{7}\mathrm{Be+n} \leftrightarrow \mathrm{p} + {}^{7}\mathrm{Li}. \tag{15}$$

In every case the rate of change of abundance of a nucleus j, a member[2] of the set of products $\{P\}$, from any given reaction involving the reactant set $\{R\}$ is

$$\frac{1}{\nu_j}\frac{dY_j}{dt} = \Gamma_{R\to P}\prod_{i\in\{R\}}\frac{Y_i^{\nu_i}}{\nu_i!} \tag{16}$$

where the ν's are the stoichiometric coefficients and $\Gamma_{R\to P}$ is the rate of the reaction per unit abundance of the reactants. In equilibrium, the production and destruction of the nuclei by any one reaction are almost equal with only a small difference between them. This allows us to relate the forward, $\Gamma_{R\to P}$, and back reaction, $\Gamma_{P\to R}$, rate coefficients, i.e.

$$\Gamma_{R\to P}\prod_{i\in\{R\}}\frac{Y_i^{\nu_i}}{\nu_i!} \approx \Gamma_{P\to R}\prod_{j\in\{P}\frac{Y_j^{\nu_j}}{\nu_j!}. \tag{17}$$

If we insert into this expression the Bolztmann distribution of the number densities then we find

$$\Gamma_{P\to R} \propto \Gamma_{R\to P}\ \exp\left(\frac{(M_P-M_R)-(\mu_P-\mu_R)}{T}\right) \tag{18}$$

$$\propto \Gamma_{R\to P}\ \exp\left(\frac{-Q}{T}\right) \tag{19}$$

where M_R and M_P (μ_P and μ_R) are the sum of the reactant or product particle masses (chemical potentials) and Q is the difference. If the reaction involves only nucleons/nuceli and photons then $\mu_P = \mu_R$ and the "Q value" is simply the difference in the total binding energy of reactants and products, but in those cases where the reaction involves particles we have not included in $\{R\}$ and $\{P\}$ the equality is not ensured. An analytic discussion of BBN from the point of view of the reaction rates can be found in [21].

As the Universe cools the reactions are unable to process the nuclei leading to a departure of the abundance of each from the NSE abundance: the heavier nuclei departing earlier than the lighter. The ratio of the NSE abundance and the actual abundance from a standard BBN code is shown in Fig. 1 and was first discussed in Smith, Kawano and Malaney [20]. The figure shows that the ^{4}He abundance departs from NSE when $T \sim 600$ keV while ^{3}He and T depart at $T \sim 200$ keV. Below $T \sim 200$ keV only the abundance of the deuteron is given by its NSE value, the rest are many orders of magnitude smaller. The departure from NSE is not because the rates fall beneath the Hubble rate but rather the departure occurs because the amount that can be provided from the reactions falls short of the amount required to *remain* in equilibrium. For any particular nuclear species the former is simply the sum of all the relevant production and destruction reactions listed in Eqs. (10) through (15) while the latter can be found from Eq. (8) (since deuterons are only beginning to form we ignore the change in Y_{p} and Y_{n} and remember $n_B \propto T^3$),

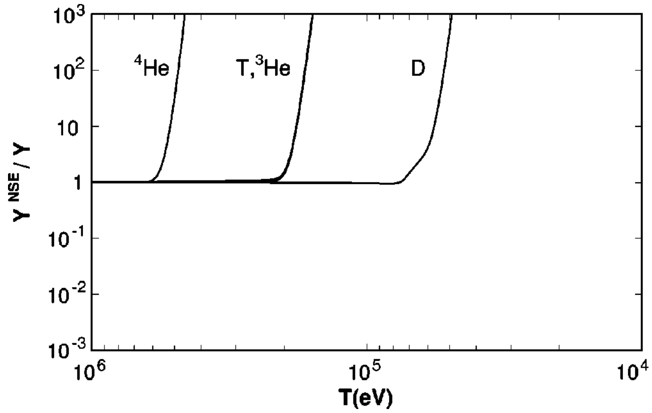

FIG. 1. The ratio Y_i^{NSE}/Y_i, the NSE abundance to the actual abundance of D, T, ^{3}He, and ^{4}He as a function of the photon temperature when the baryon/photon ratio is 5.6×10^{-10}.

$$\frac{dY_A}{dt} = \frac{Y_A}{T}\frac{dT}{dt}\left[\frac{3(A-1)}{2} - \frac{B_A}{T}\right]. \tag{20}$$

The departure from NSE has a major impact upon the reactions. The rate of change of nucleus k in, for example, a two-body reaction $i+j \leftrightarrow k+l$ with $i\neq j,\ k\neq l,$ is

$$\frac{dY_k}{dt} = Y_iY_j\Gamma_{ij\to kl} - Y_kY_l\Gamma_{kl\to ij} \tag{21}$$

$$= Y_iY_j\Gamma_{ij\to kl}\left[1 - \frac{Y_k}{Y_k^{NSE}}\frac{Y_l}{Y_l^{NSE}}\right], \tag{22}$$

with the superscript NSE indicating the equilibrium abundance, is now very lopsided because $Y_k/Y_k^{NSE} \ll 1$. Essentially there is no destruction of k via this reaction, the back reaction has switched off.

By $T\sim200$ keV the only compound nucleus in NSE is the deuteron. Its abundance is rising rapidly and BBN is said to begin when the nuclear reactions finally begin to process the deuterium leading to the final departure from NSE. The increasing significance of such reactions as $\mathrm{D+D} \to \mathrm{T+p}$ halts the rise in the deuterium abundance and Bernstein, Brown, and Feinberg [18] define the temperature at which BBN be-

[2] The set does not include non-nuclear particles.

gins, T_{BBN}, as the point where the deuteron abundance reaches its peak, i.e. $dY_{\mathrm{D}}/dt=0$. Using this definition they find $T_{BBN}\approx B_{\mathrm{D}}/26=86$ keV, or $t_{BBN}=180$ s.

The second stage to BBN, from 1.2 MeV to 86 keV, is characterized by departure from equilibria. At its inception non-equilibrium is limited to the neutron/proton ratio: by its end all other nuclei have fallen out of equilibrium with deuterium the last to succumb. Formation of the heavier nuclei in significant amounts begins to occur towards the end of this stage and indeed the abundance of ^{4}He can already be larger than D by T_{BBN}. What T_{BBN} represents is the point where the (leaky) dam bursts, so to speak. The initiation of the next stage of BBN proper is controlled by the deuteron binding energy B_{D} and, therefore, in conjunction with δm_{np}, the initial conditions for BBN proper are essentially controlled by just these two parameters.

C. BBN proper

From the departure of n/p weak equilibrium at $T\sim1$ MeV to the inception of BBN at $T\sim100$ keV some of the free neutrons have decayed and reduced the pool available to be assimilated. But once the balance has been tipped in favor of complex nuclei the free nucleons are rapidly dragged into and through $A=2$ and neutrons become stabilized. This phase is BBN proper and during it the abundances of the complex nuclei can become very large as the nuclear reactions process them. But by the time the Universe has cooled to the point where the reactions have ceased virtually all the neutrons present at T_{BBN} now reside in helium-4 with a small fraction in the trace abundances of the other nuclei. The trace abundances of the intermediary nuclei D, T and ^{3}He are the ashes of processing from free nucleons to ^{4}He. Because helium-4 essentially acts as the end point of the reactions its abundance is very insensitive to the exact details of the nuclear reactions that lead to its formation. The abundance of ^{4}He instead probes the much earlier epoch of n/p freeze-out and the time delay until BBN commences. In contrast, the final abundance of the intermediary nuclei D, T and ^{3}He are strong functions of the nuclear physics. The efficiency of the processing from free nucleons to ^{4}He depends crucially on the conditions during BBN, the temperature and the interaction cross sections, so their abundances can vary markedly if BBN begins at a higher temperature or the cross sections change.

III. NUCLEAR REACTION RATES AND Λ_{QCD}

As discussed above, the deuteron binding energy and the neutron-proton mass difference are two of the most important parameters that control big bang nucleosynthesis. Each of these depends on the QCD scale, although each in a different way.

Shifts in the QCD scale might not, at first glance, be expected to have significant effects. This is because most nuclear quantities, e.g. nuclear masses, nucleon masses, will shift together with this scale and therefore not be observable. However, there is no reason to expect that all the fundamental constants shift at the same rate, and the relative change may cause observable effects. For example, while the masses of the proton and neutron are mainly determined by the QCD scale, the difference between the neutron and proton masses is dominated by electroweak effects. As a consequence, the ratio $(m_{\mathrm{n}}-m_{\mathrm{p}})/m_{\mathrm{n}}$ may be sensitive to changes in the fundamental couplings. Furthermore, the pion mass will not shift in the same way as the nucleon or nuclear masses, and the pion mass is an important ingredient in setting the deuteron binding energy.

The neutron-proton mass difference is approximately given by [22]

$$\delta m_{np}=M_{\mathrm{n}}-M_{\mathrm{p}}=m_d-m_u-\alpha M_{elm}. \tag{23}$$

The coefficient of α in the electromagnetic contribution, M_{elm}, is determined by strong interactions and therefore proportional to Λ_{QCD}. We use the estimate $\alpha M_{elm}\simeq0.76$ MeV, while the difference in the down and up quark masses is approximately 2 MeV [22]. Other calculations of αM_{elm} yield different results, for example [23]. Shifts in the mass of the up and down quark masses depend on the vacuum expectation value of the Higgs and their respective Yukawa couplings. Specifically,

$$\Delta(\delta m_{np})=-\left(\frac{\Delta\alpha}{\alpha}+\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}\right)\alpha M_{elm}+\left(\frac{\Delta y_d-\Delta y_u}{y_d-y_u}+\frac{\Delta v}{v}\right)\times(m_d-m_u). \tag{24}$$

Here the y's are the Yukawa couplings and v is the vacuum expectation value (VEV) of the Higgs boson. From this equation, it can be seen that the shift in the neutron-proton mass difference is determined by several fundamental parameters, not just Λ_{QCD}.

In principle, we should define a scale which remains fixed, while others vary. This could be a scale such as the GUT scale. In our calculations in the next section however, we only vary Λ_{QCD} so this scale could even be the electron mass. A discussion of scales as well as a preliminary estimate of the deuteron binding energy may be found in [24].

The scale Λ_{QCD} also has great importance in determining the binding energy of the deuteron. The small binding energy prevents a significant abundance of deuterium from building up early in big bang nucleosynthesis. And therefore, as discussed above, the deuteron binding energy controls the nuclear flow to elements with greater mass number. The long-range part of the nuclear force is governed by one-pion exchange and therefore the deuteron binding energy is sensitive to pion properties. The mass of the pion is determined by the Gell-Mann-Oakes-Renner relation [25],

$$f_\pi^2 m_\pi^2=(m_u+m_d)\langle\bar{q}q\rangle. \tag{25}$$

Here f_π is the coupling of the pion to the axial current, and $\langle\bar{q}q\rangle$ is the quark condensate.

Shifts in the pion mass are then calculated in a similar manner to Eq. (24) and determined by the shifts of the Yukawa couplings, v and Λ_{QCD}. Since $f_\pi\propto\Lambda_{QCD}$ and $\langle\bar{q}q\rangle\propto\Lambda_{QCD}^3$,

$$\frac{\Delta m_\pi}{m_\pi} = \frac{1}{2}\left[\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}} + \frac{\Delta v}{v} + \frac{\Delta y_u + \Delta y_d}{y_u + y_d}\right]. \tag{26}$$

Recent studies of the dependence of the nuclear potential on the mass of the pion have shown a strong variation with the mass of the pion [26–28]. According to these authors, changes in the pion mass of a few percent could lead to changes in the deuteron binding energy of a factor of two. Although a relationship between the pion mass and the deuteron binding energy exists [26–28], several parameters in this relationship are uncertain. These include parameters that describe the pion nucleon coupling and the coefficient of a quark mass dependent four nucleon operator. Next we turn to the reaction rates themselves that are most important for determining the deuterium and helium abundances.

The first is the neutron-proton interconversion reaction listed in Eqs. (6a) through (6c). The rates for the reactions must include the temperature chemical potentials of the electrons and neutrinos which also participate in the reactions. In the Born approximation the neutron/proton interconversion rates are

$$\Gamma_{\mathrm{n}\to\mathrm{p}} = \frac{G_F^2(C_V^2+3C_A^2)}{2\pi^3}\int_{m_e}^{\infty} dE_e E_e p_e[(E_e + \delta m_{np})^2 f_{\bar{e}}(E_e)\overline{f_{\bar{\nu}}}(E_e+\delta m_{np}) + (E_e - \delta m_{np})^2 \overline{f_e}(E_e) f_\nu(E_e-\delta m_{np})] \tag{27}$$

$$\Gamma_{\mathrm{p}\to\mathrm{n}} = \frac{G_F^2(C_V^2+3C_A^2)}{2\pi^3}\int_{m_e}^{\infty} dE_e E_e p_e[(E_e + \delta m_{np})^2 \overline{f_{\bar{e}}}(E_e) f_{\bar{\nu}}(E_e+\delta m_{np}) + (E_e - \delta m_{np})^2 f_e(E_e)\overline{f_\nu}(E_e-\delta m_{np})] \tag{28}$$

where $f(E)$ is the Fermi-Dirac distribution, $\overline{f}(E)$ its compliment and the subscripts indicate the chemical potential and temperature to be used in f. Though these two expressions capture the essence of how δm_{np} enters into these rates there are a number of corrections that must be taken into account before the n↔p reaction rates reach sufficient accuracy [29–31] given the accuracy of the data we will eventually use in our constraints.

The next is the reaction which creates deuterium, $\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma$, which is well studied in a low energy effective field theory without pions [32,33]. These authors give an expression for the cross section as a function of the deuteron binding energy, the scattering length in the singlet channel, the phase shift, and so on. In principle each of these parameters should be treated as free; however, the one which has by far the largest leverage on the rate is the binding energy of the deuteron. In our calculation, we use the expression for the cross section given by [33] and integrate over the thermal distribution of the particles in the manner prescribed in Fowler, Caughlan and Zimmerman [34].

The other crucial reaction is the one that destroys most of the deuterium, $\mathrm{D}+\mathrm{D}\to\mathrm{T}+\mathrm{p}$ during BBN proper. Ideally, we would like to use the results of a nuclear effective field theory calculation in order to determine the dependence of this rate on the pion mass, but there are no effective field theory calculations available yet.

IV. NON-STANDARD BBN

Standard BBN is a well studied problem and even though the compatibility of its predictions with the observed abundances remains a contentious issue much of the recent efforts have been towards using it as a test of the state of the Universe at the earliest epochs. Modifying standard BBN to include new effects can be relatively painless: for example, the use of BBN as a probe of the number of light neutrino species or the chemical potentials of the μ and/or τ neutrino flavors is simply a case of modifying the energy density and hence the expansion rate with no direct influence upon the nuclear physics [35–37]. Similarly the expansion rate can be modified by changing the cosmological equations [38,39], introducing extra energy density in the form of ‘‘quintessence’’ [38,40–42] and even quite general evolutions have been explored [43]. The effects of new physics from scenarios such as a non-zero electron neutrino chemical potential [44–51] or neutrino oscillations/mass/decay [52–56], show up primarily in the neutron-proton interconversion reactions. There the known expressions for these reaction rates permit a high degree of confidence that all the consequences of the new physics have been accurately taken into account.

By far the most difficult proposals to implement touch every nuclear reaction. For example, the impact of a varying fine structure constant was examined by Bergström, Iguri and Rubinstein [9], updated by Avelino *et al.* [10] and improved upon by Nollett and Lopez [11], and to examine its effects these authors had to recalculate every relevant rate. In this type of study one is forced to make a number of approximations because we do not have a complete understanding of how a change in a fundamental constant alters low energy nuclear physics parameters, such as the binding energies and cross sections. An alternative approach would be to map the parameter space from a single, or handful of, fundamental constant(s) to a greater number of nuclear physics parameters, and then constrain the nuclear physics parameters while treating them as independent. This way we avoid the uncertainty in how changes in the fundamental constants manifest themselves in nuclear quantities but this comes at the expense of exploring larger regions of parameter space. Despite this trade-off, we shall turn in this direction.

The number of nuclear physics parameters is huge but, as we have been at pains to stress, the most important two are δm_{np} and B_{D} and indeed we can limit our parameter space to just these two quantities. Before we set out our plan for constraining δm_{np} and B_{D} we examine why we can restrict the number of nuclear parameters to just this limited set yet still obtain good predictions for the results of BBN and how these two parameters will affect the predicted primordial abundances.

A. Simplifying BBN

A prediction for the mass fraction of helium-4 is reliably calculated by simply counting the number of neutrons that become stabilized inside nuclei. This approximation for the helium-4 mass fraction emerges from the empirical results of standard BBN discussed above where we showed that all the neutrons that survive BBN reside in helium-4. To determine the number of stabilized neutrons we must first be able to calculate the neutron-proton ratio during the earliest, weak-equilibrium, phase of BBN and then, secondly, follow it through to the inception of BBN proper. The first is relatively easy to implement since we have known expressions to use for the neutron-proton interconversion rates. As we mentioned earlier, there are many corrections to Eqs. (27) and (28) that must be included in order to improve their accuracy. Of these we only explicitly included the radiative electromagnetic corrections and do not introduce the corrections for the finite mass or thermal radiative corrections. Since they are relatively small when compared with the effects we want to consider, we take them into account only by an overall scale factor normalized to the measured neutron lifetime. The second step means that we must be able to follow the neutrons into the first reaction that must occur in processing the nuclei: $\mathrm{n}+\mathrm{p}\leftrightarrow\mathrm{D}+\gamma$. As we mentioned in Sec. III, an analytic expression for the cross section of this reaction is available from Rupak [33] which shows how the deuteron binding energy enters into this important quantity.

As we learned from standard BBN the final abundance of ^{4}He is very insensitive to the exact rate of its formation so if we truncated our reaction network at this step we could still get a good estimate for the helium-4 mass fraction and thus we only need two parameters for the calculation: the neutron-proton mass difference δm_{np} and the deuteron binding energy B_{D}. But with two parameters (three if we include the baryon to photon ratio η) and one prediction a degeneracy is expected to arise between δm_{np} and B_{D} in the abundance of ^{4}He. Therefore, it is worth endeavoring to make a prediction for the abundance of at least one other nucleus. The most obvious candidate is deuterium because its primordial abundance is also a function of these same two parameters. To make a prediction for the amount of unprocessed deuterium we have to follow not only how this nucleus is formed courtesy of the $\mathrm{n}+\mathrm{p}\leftrightarrow\mathrm{D}+\gamma$ reaction but also how much is destroyed. The majority of the destruction of deuterium occurs via the reactions (11a), (11b) and (12a) with (11c) and (13b) following behind in importance [20]. As we saw in Eq. (19) the inverse rate is related to the forward rate by a term that is proportional to $\exp(-Q/T)$ and Q may be expressed in terms of the binding energies. By including these reactions we are adding the tritium and helium-3 binding energies explicitly to the δm_{np} and B_{D} parameter list through this exponential dependence. (In addition, there may be additional dependence of these binding energies in the forward cross sections, although there is no explicit calculation currently available.) But, here another result from standard BBN comes to our rescue: the nuclear abundances of all the nuclei heavier than D during BBN fall far below their nuclear statistical equilibrium values and the inverse reactions essentially switch off long before BBN begins. This allows us to use tritium and helium-3 as the substitutes for helium-4 *if* we can assume that any neutron that ever forms one of these nuclei is prevented from ever participating in another reaction. If we are able to ignore all inverse rates then we are eliminating the explicit dependence upon the binding energies of tritium and helium-3 (although the cross sections will still implicitly be functions of these quantities). Furthermore, if we truncate our reaction network at these two $A=3$ nuclei then we need only be concerned with reactions (11a) and (11b). With the absence of an analytic expression to use, we take the cross sections of these two reactions to be related to the size of the deuteron radius, i.e. $\sigma \sim 1/(m_N B_{\mathrm{D}})$. In terminating the network so early we miss the significant destruction of deuterium from $\mathrm{T}+\mathrm{D}\to\mathrm{n}+{}^4\mathrm{He}$ and the less important ${}^3\mathrm{He}+\mathrm{D}\to\mathrm{p}+{}^4\mathrm{He}$ but nevertheless, we can use the abundance of D from the calculation as an estimate of the primordial abundance even though this approximation is much cruder than the one used for the primordial ^{4}He abundance.

B. Implementation

This simplified BBN scheme is implemented in a code of our own construction. The neutron lifetime at $\Delta\delta m_{np}=0$ was taken to be 885.7 s while the zero-temperature radiative and Coulomb corrections from Dicus *et al.* [29] were included explicitly in the $\mathrm{n}\leftrightarrow\mathrm{p}$ interconversion rates. These are the two most important corrections and, furthermore, the radiative correction is a function of δm_{np} so it will change as we vary this quantity. As mentioned previously, the $\mathrm{n}+\mathrm{p}\leftrightarrow\mathrm{D}+\gamma$ rate was calculated by integrating the Rupak [33] cross section over the thermal distribution. At $\Delta B_{\mathrm{D}}=0$ the rate for $\mathrm{n}+\mathrm{p}\leftrightarrow\mathrm{D}+\gamma$ we obtain from the integration is virtually identical to value from the Smith, Kawano and Malaney [20] expression. For the remaining reactions we rescaled the Smith, Kawano and Malaney [20] rates with B_{D}. The differential equations were solved using the "Rosenbrock" algorithm in Press *et al.* [57]. The numerical error introduced by taking finite time steps have been found to be considerable in standard BBN codes and they are usually removed by adding a correction to the results from some "default" setting (see [58,60] for more details). We also found a similar, time-step error; however, in our case, this simplistic correction factor approach proved inadequate. Empirically we found that the results at fixed $\eta,\Delta B_{\mathrm{D}}$ and $\Delta\delta m_{np}$ varied with the algorithm's "accuracy parameter," ϵ, as $\ln Y\propto\epsilon$ whereas the run time varied as $\propto 1/\epsilon$. Therefore, rather than setting ϵ to a small value and running the code once it proved more efficient to record the results at, for example, five different moderate values of ϵ and extrapolate to $\epsilon=0$ using the empirical relation. By determining the intercept and its variance of the straight line fit to $\ln Y$ versus ϵ for the five "jacknifed" subsets of results the primordial abundance was taken to be the weighted average of these five intercepts. This jacknife methodology reduces the influence of results if they deviate from $\ln Y\propto\epsilon$ since the variance of the intercept will be larger in those cases and hence have less weight in the final determination of the abundance. We examined the validity of this procedure by comparing the result against those with small ϵ with several test runs. In addition, we extensively examined many other aspects of the code that employ numerical algorithms, such as the integrators for the thermodynamic properties of the electron/positrons, by adjusting the accuracy parameters within them or by replacing them with more sophisticated (and computationally expensive) routines but found that they did not effect the results at a level greater that 0.01%.

Of course the convergence of our code to a result does not mean that its predictions are accurate since our approximation to BBN obviously introduces considerable error. Nevertheless, the agreement with the standard (Kawano) BBN calculation results is impressive. The standard BBN code is certainly not error free but it is not unreasonable to assume that its results are more accurate than those made with our approximation. By comparing the results at $\Delta B_{\rm D}=\Delta\delta m_{np}=0$ we find that as a function of the baryon to photon ratio η the helium-4 abundance is systematically too low by $\lesssim 1\%$. The reason lies in the fact mentioned previously: the abundance of ^{4}He surpasses the abundance of D at T_{BBN}, the beginning of BBN proper, but since we have essentially removed this nucleus from the network we overestimate the amount of neutron decay and therefore calculate a lower than expected yield of helium-4. In contrast deuterium is systematically overestimated but this time by $\sim 50\%$. Neglecting $\mathrm{T}+\mathrm{D}\rightarrow \mathrm{n}+{}^4\mathrm{He}$ has introduced insufficient deuterium destruction, which is of the order of the observation uncertainty. These systematic errors must be included when we make the comparison with observed abundances and we will also show how powerful the use of deuterium could be if we had more information about how other nuclear cross sections/binding energies vary with the pion mass.

C. The effect of changing $B_{\rm D}$

The effects of a change in $B_{\rm D}$ occur through both the change in the NSE deuteron abundance and the cross sections that process D to heavier nuclei. As $B_{\rm D}$ increases the deuteron becomes more stable and consequently more difficult to disassociate. Therefore we expect as $B_{\rm D}$ increases BBN will commence at a higher temperature/earlier epoch. Perhaps counterintuitively the increased stability of the deuteron leads to a decrease in the primordial deuterium abundance. The rate coefficients are functions of both the temperature and the powers of the baryon density n_B so if T_{BBN} increases interactions occur more rapidly even if the cross sections have changed. This leads to a more efficient processing of the neutrons to ^{4}He so that the amount of unprocessed, and thus the primordial, deuterium decreases. From a more efficient processing of the nucleons we would expect an increase in the primordial ^{4}He abundance and this is enhanced by the shorter interval between n/p freeze-out and the beginning of BBN and consequently less n decay. These expectations are confirmed in Fig. 2 where we plot the deuterium abundance $Y_{\rm D}$ and helium-4 mass fraction, confusingly denoted also by Y, as functions of the temperature for $\Delta B_{\rm D}/B_{\rm D}=0$ and $\Delta B_{\rm D}/B_{\rm D}=0.3$.

At temperatures above 100 keV when deuterium is in NSE the offset due to the change in its binding energy is clear, as is the shift in the peak deuterium abundance, and hence T_{BBN}. The lower figure shows that the change in the deuterium binding energy leads to an increase in the final helium-4 mass fraction as predicted.

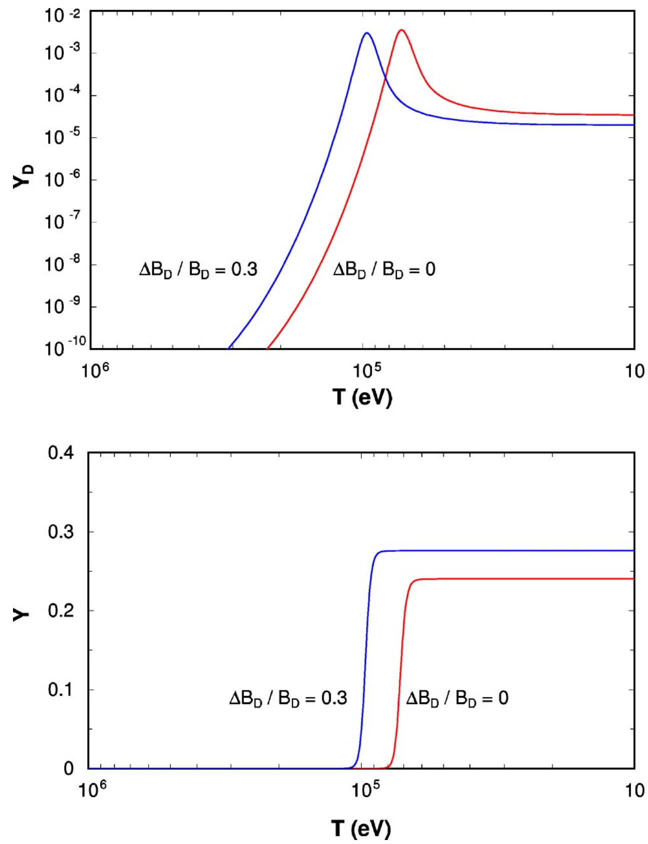

FIG. 2. The deuterium abundance, $Y_{\rm D}$, top panel, and the helium-4 mass fraction, Y, bottom panel, as functions of the photon temperature for $\Delta B_{\rm D}/B_{\rm D}=0$ and $\Delta B_{\rm D}/B_{\rm D}=0.3$ when the baryon/photon ratio is 6.14×10^{-10}.

D. The effect of changing δm_{np}

There are two effects that become apparent when we change the neutron-proton mass difference: an altered neutron lifetime and a change in the n/p freeze-out ratio. Before we add the radiative corrections, the neutron lifetime $\tau_{\rm n}$ is simply

$$\frac{1}{\tau_{\rm n}} \approx \frac{G_F^2(C_V^2+3C_A^2)}{2\pi^3}\int_{m_e}^{\delta m_{np}} dE_e E_e p_e (E_e-\delta m_{np})^2 \quad (29)$$

$$= \delta m_{np}^5 I(m_e/\delta m_{np}). \quad (30)$$

Since function $I(m_e/\delta m_{np})$ varies less rapidly than δm_{np}^5 as δm_{np} increases the lifetime drops considerably. We therefore expect more neutron decay in the period from neutron/proton freeze-out until the inception of BBN and consequently less ^{4}He and less D.

Secondly, the neutron-proton mass difference δm_{np} sets the neutron to proton ratio prior to the cessation of the interconversion reactions. From Eqs. (27) and (28) we can see that increasing δm_{np} will lead to increases in both rates per particle but this is more than overwhelmed by the decrease in the equilibrium neutron abundance $F \propto \exp(-\delta m_{np}/T)$. The increase in δm_{np} therefore also leads to lower primordial D and ^{4}He abundances because there are less neutrons around that can form these nuclei. We can add some quantitativeness to these remarks by following the simplified treatment of neutron/proton freeze-out of Bernstein, Brown and Feinberg [18]. By ignoring n-decay[3] these authors derive an expression for the freeze-out abundance of neutrons $Y_{\rm n}^{\star}$ as

$$Y_{\rm n}^{\star}=\int_0^{\infty}\frac{dq}{2}\,\frac{1}{1+\cosh(q)}\exp\left(-\frac{f(m_e/\delta m_{np})}{\tau\delta m_{np}^2}\left[\left(\frac{4}{q^3}+\frac{3}{q^2}+\frac{1}{q}\right)+\left(\frac{4}{q^3}+\frac{1}{q^2}\right)e^{-q}\right]\right) \tag{31}$$

where the function $f(m_e/\delta m_{np})$ varies slowly at $m_e/\delta m_{np} \approx 1/2$. The change of $Y_{\rm n}^{\star}$ in Eq. (31) is therefore primarily due to the effect of the $\tau\delta m_{np}^2$ factor in the denominator of the exponential. From Eq. (29) we know that the lifetime scales as $\sim 1/\delta m_{np}^5$ so the factor in the exponent is proportional to $1/\delta m_{np}^3$. As δm_{np} increases the exponential term in Eq. (31) fades more rapidly yielding a faster convergence with q and thus confirming our expectation of a lower freeze-out abundance of neutrons.

Finally, as a consequence of the lower freeze-out neutron abundance the NSE deuteron abundance is lowered. In turn the temperature, T_{BBN}, at which BBN proper commences is also lower because the reactions such as D+D→T+p will not begin to process the deuterium until a slightly lower temperature. The Universe is therefore older at the beginning of BBN thus permitting even more neutron decay than would be expected from just the change in neutron lifetime. For ^{4}He this effect has the same sign as the previous two, as δm_{np} increases the amount of ^{4}He decreases. In contrast, a lower T_{BBN} actually leads to an increase in D because there will be less destruction of this nucleus but the increase is insufficient to compensate for the previous two effects.

The effects of a change in δm_{np} are best summarized in Fig. 3 where we plot the deuterium abundance, $Y_{\rm D}$, the (free) neutron/proton ratio, F, and the helium-4 mass fraction, Y, as function of the photon temperature at $\Delta\delta m_{np}/\delta m_{np}=0$ and $\Delta\delta m_{np}/\delta m_{np}=0.3$. At high temperatures, above $\sim$1 MeV, the shift in the neutron-proton mass difference is seen as the difference in the two neutron/proton ratio curves while the change in neutron lifetime appears as the different gradients for these curves close to $T \sim 10^5$. The different amounts of neutrons that survive from T_{np} to T_{BBN} results in the large shift in the helium-4 mass fraction. This is primarily due to the change in the neutron lifetime but is enhanced by a small shift in the onset of BBN proper as witnessed in the small shift in the position of the peak deuterium abundance seen in the top panel.

[3] Although Bernstein, Brown and Feinberg [18] ignore n-decay in deriving their expressions they still require neutrons to decay eventually in order to normalize the interconversion reactions.

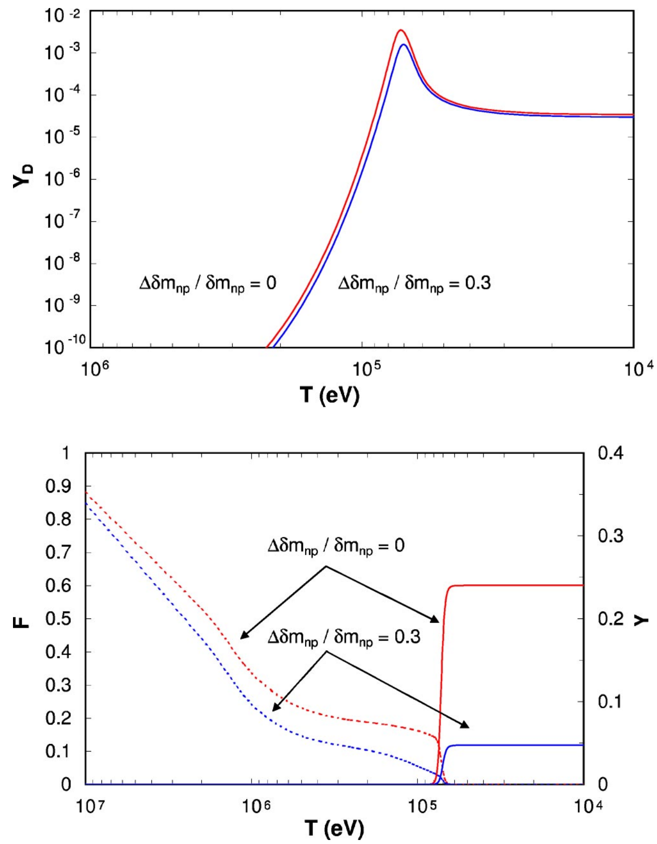

FIG. 3. The deuterium abundance, $Y_{\rm D}$ (top panel), the (free) neutron/proton ratio, F (bottom panel, dashed lines) and the helium-4 mass fraction, Y (bottom panel, solid lines) as functions of the photon temperature when $\Delta\delta m_{np}/\delta m_{np}=0$ and $\Delta\delta m_{np}/\delta m_{np}=0.3$ with a baryon/photon ratio of 6.14×10^{-10}.

E. Constraining δm_{np} And $B_{\rm D}$

We can use the discussion in the preceding two sections to show how a degeneracy between δm_{np} and $B_{\rm D}$ in the prediction of the helium-4 mass fraction occurs and, furthermore, illustrate why using deuterium is/could be the ideal foil. Raising δm_{np} will lower the neutron/proton freeze-out ratio and so, in turn, reduce the helium-4 mass fraction but if we also increase $B_{\rm D}$ then this can be entirely compensated for by an earlier inception of BBN proper, the point where neutrons become stabilized inside nuclei. At the same time the predicted abundance of deuterium is expected to fall as we increase δm_{np} but an increase in $B_{\rm D}$ also leads to less deuterium. Thus, while the prediction for the abundance of deuterium also possesses a degeneracy between the two parameters, it is orthogonal to that of helium-4. Therefore, using both allows us, in principle, to gain strong constraints. This orthogonality is best illustrated by Fig. 4 where the complementarity of the two nuclei is evident.

The figure also shows where the approximations begin to fail: significant deviations begin to occur when $\Delta B_{\rm D}/B_{\rm D} \sim 0.7$, which, using the binding energy for the deuteron of 2.22 MeV, is remarkably early. The explanation lies in the Q value of the reactions we have used. During the phase of NSE we require the binding energies of tritium and helium-3 in order to compute the reverse reactions and for these we

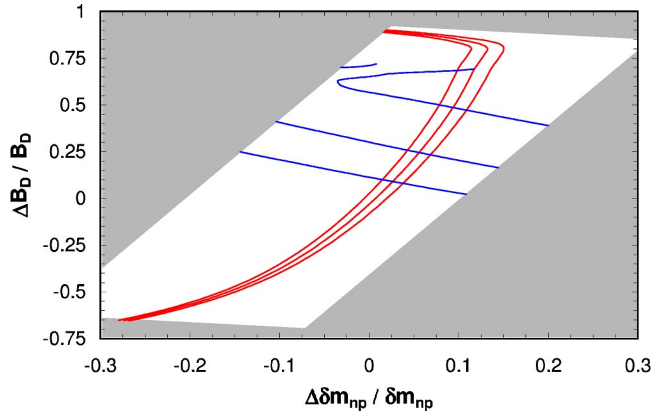

FIG. 4. The iso-abundance contours D/H of deuterium (horizontal) and iso-mass fraction contours of helium-4 (vertical) from our calculation as a function of the fractional change in the neutron-proton mass difference δm_{np} and the deuteron binding energy B_{D} at $\eta=6.14\times10^{-10}$. The parameter space in the shaded region was not explored and the deuterium contours in the upper portion of the unshaded region are not shown as we experienced significant numerical difficulties in that portion as discussed below. From top to bottom the deuterium contours are 4×10^{-5}, 3×10^{-5} and 2×10^{-5} while from left to right the helium-4 contours are 0.25, 0.24 and 0.23.

used the present measured values. For reference the binding energies are $B_{\mathrm{T}}=8.48$ MeV and $B_{^3\mathrm{He}}=7.72$ MeV. The abundances of tritium and helium-3 while they are in NSE are so small that even if this introduces a considerable error the final results will not reflect this fault. The Q-value for the reaction $\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{n}+{}^3\mathrm{He}$ is $Q=B_{^3\mathrm{He}}-2B_{\mathrm{D}}$ while that for $\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{p}+\mathrm{T}$ is $Q=B_{\mathrm{T}}-2B_{\mathrm{D}}$. So as we increase the deuteron binding energy the Q values for both reactions decrease and at a 70% increase the Q value for $\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{n}+{}^3\mathrm{He}$ reaches zero. With such a low Q value the inverse reactions are significant and our use of tritium and helium-3 as neutron sinks is no longer valid. In our numerical calculations, when the Q values of these reactions became small, we saw a very different flow of the nuclei through the reaction network compared to standard BBN. If we persist with the increases in B_{D} and make the Q values negative we enter very dangerous territory since our simple rescaling of the cross sections cannot still apply. Endothermic reactions are very different from exothermic at low energy/tempertaure. This is not to say that the deuteron binding energy cannot be greater than $1.7B_{\mathrm{D}}$ during BBN, it is simply a statement that we cannot reliably predict the abundances in this domain. But a 70% increase in the deuteron binding energy cannot be regarded as a safe upper limit to the permitted variation in B_{D}. At 50% the change in the deuteron binding energy is ~1 MeV. Since we do not know if the change in B_{T} and $B_{^3\mathrm{He}}$ is correlated or anti-correlated with B_{D}, if B_{T} and $B_{^3\mathrm{He}}$ vary with similar magnitude to B_{D}, the Q value of $\mathrm{D}+\mathrm{D}\leftrightarrow\mathrm{n}+{}^3\mathrm{He}$ may already have been forced to zero at a 1 MeV increase in B_{D} and our approximations cannot be used. Therefore we estimate a 50% increase in B_{D} as a limit to the permitted variation of this parameter, in the absence of further information about how the cross sections and binding energies vary with pion mass.

In contrast, it appears we can vary δm_{np} in the region we want to explore without running into similar difficulties. There is a lower limit to the variation we can allow: when δm_{np} drops beneath the electron mass, a decrease in δm_{np} of 60%, neutrons cannot decay so that our rescaling of the reaction rates with the neutron lifetime to correct for the finite mass and thermal radiative corrections cannot be applied. Again, a neutron-proton mass difference smaller than the electron mass during BBN cannot be ruled out, it is just that we cannot calculate what occurs when this happens.

We are now in the position where we can begin to constrain our two parameters δm_{np} and B_{D} by comparing the predicted abundances with observation. The primordial abundance D/H of deuterium is taken to be $\mathrm{D/H}=(2.6\pm0.4)\times10^{-5}$ [59] while we use the Olive, Steigman and Walker [60] value of $Y=0.238\pm0.005$ for the helium mass fraction Y. The exact primordial abundances remain a topic of debate with two, largely incompatible, determinations for the helium mass fraction [61–65] and excessive scatter in the measurements of deuterium [59,66] but these two nuclei still represent the best probes of BBN because the other nuclei that could be used, such as ^{3}He and ^{7}Li, suffer from large uncertainties in the derivation of their primordial values. From comparing the observed abundances of D and ^{4}He and their associated errors with the iso-abundance contours in Fig. 4 it is apparent that helium-4 will be the chief source of constraints on δm_{np} while deuterium will play the same role for B_{D}. The errors for these observations also reflect the level to which we must beat down the systematic errors in order to avoid contaminating our results with large offsets. For helium-4 we have succeeded handsomely since 0.005 represents a 2% error on 0.238 while our systematic was at 1%. For deuterium we have not done so well since the observation has an error of 15% and our systematic was at 50%.

So in addition to the observational errors σ_{D} and σ_Y we must also include into the analysis the systematic errors S_{D} and S_Y that we introduced when we made our approximations. The systematic errors differ from the statistical observational errors in that they are correlated. The covariance matrix, V, is therefore of the form

$$V=\begin{pmatrix}\sigma_{\mathrm{D}}^2+S_{\mathrm{D}}^2 & S_{\mathrm{D}}S_Y\\ S_{\mathrm{D}}S_Y & \sigma_Y^2+S_Y^2\end{pmatrix}. \tag{32}$$

If we knew the exact predictions for Y_{D} and Y at any combination of δm_{np}, B_{D} and η then we could easily calculate S_{D} and S_Y by comparing the exact value with that achieved with our approximations. Unfortunately this is not the case and we can only make this comparison at $\Delta\delta m_{np}/\delta m_{np}=\Delta B_{\mathrm{D}}/B_{\mathrm{D}}=0$. From comparing our results with that from a standard BBN code we can represent the systematic errors as the product of the fractional errors and the predicted values. We take the fractional errors to be the same at arbitrary δm_{np} and B_{D}. With this understanding we can approximate the covariance matrix at all δm_{np} and B_{D} and calculate a likelihood via

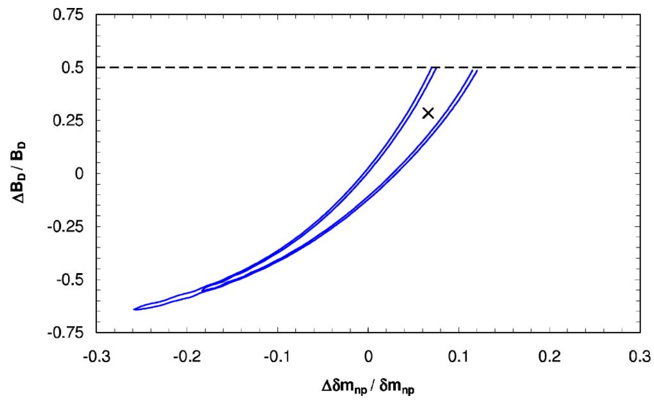

FIG. 5. The 95% and 99% χ^2 contours derived from using a primordial deuterium abundance of D/H$=(2.6\pm0.4)\times10^{-5}$ and a primordial helium-4 mass fraction of $Y=0.238\pm0.005$. The cross, located at $\Delta\delta m_{np}/\delta m_{np}=0.07$ and $\Delta B_{\mathrm{D}}/B_{\mathrm{D}}=0.29$, indicates the position of the best fit point.

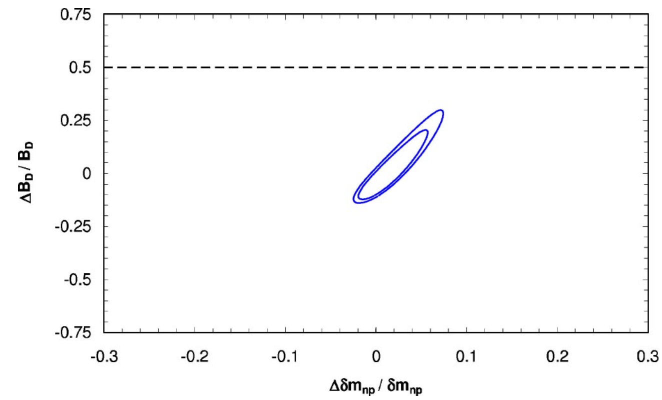

FIG. 6. The 95% and 99% χ^2 contours derived from using a primordial deuterium abundance of D/H$=(2.6\pm0.4)\times10^{-5}$ and a primordial helium-4 mass fraction of $Y=0.238\pm0.005$ after rescaling the deuterium and helium-4 results from our numerical calculation by the systematic errors we identified from comparison with a standard BBN code. This methodology is expected to produce results that are representative of the situation where we do not have to terminate the nuclear reaction network at tritium and helium-3.

$$\mathcal{L}=\frac{1}{2\pi\sqrt{|V|}}\exp\left(\frac{-\delta^T V^{-1}\delta}{2}\right) \tag{33}$$

where the vector δ is $\delta=\{Y_{\mathrm{D}}-\hat{Y}_{\mathrm{D}},Y-\hat{Y}\}$ and the hat indicates the observed values. If we also allow η to vary then we have three adjustable parameters and only two constraints so we choose to fix this quantity at the value of $\eta=6.14\times10^{-10}$ given by the WMAP observations [67]. The CMB, which is essentially ‘‘atomic physics meets cosmology,’’ is not expected to show any dependence upon Λ_{QCD} but other fundamental constants relevant to atomic physics have been constrained by using it [8,10,68,69]. At this fixed value we derive the constraints on δm_{np} and B_{D} shown in Fig. 5. The figure shows that δm_{np} and B_{D} are not as well constrained as we might hope because of the large systematic uncertainty in the prediction for deuterium, but nevertheless the figure clearly indicates that the primordial abundances are only compatible along a narrow band in the δm_{np}-B_{D} plane.

It is worthwhile exercise to show how much stronger the constraints would be if deuterium were not contaminated in this way. To this end we show in Fig. 6 the 95% and 99% χ^2 contours when we rescale the results by systematic error factors (deuterium down by $\sim$50% and helium up by $\sim$1%) and remove the systematic errors from the covariance matrix. In this way we can simulate the situation of a complete understanding of how the cross sections and the binding energies are related and there was no reason to truncate the reaction network. The size of the contours is now determined by the errors in the observational values we used and we can see that the current deuterium abundance, which primarily constrains B_{D}, would permit only a variation of 20% in this parameter at 95% confidence while the neutron-proton mass difference would be constrained to within 4%, again at 95% confidence. BBN can provide meaningful constraints on the extent to which δm_{np} and B_{D} could differ from their current values at the earliest epochs of the Universe if we could determine, with more reliability, how the nuclear data we need for the calculation depend upon either of these two parameters or on the underlying fundamental constants. Finally, we show how we can begin to remove portions of the δm_{np}-B_{D} parameter space by using their relationships with the fundamental constants such as Λ_{QCD}. We would like to use the relationship for the deuteron binding energy as a function of the pion mass derived in Beane and Savage [26,27] and compare it with our constraints. Beane and Savage quote their results in terms of the pion mass, but they are varying the ratio of the quark mass to Λ_{QCD}. They show a range of B_D vs m_π which is effectively a function $B_D=\Lambda_{QCD}f(\sqrt{m_q/\Lambda_{QCD}})$, where the function contains unknown, but constrained coefficients. We use this function to vary the QCD scale and relate it to the neutron-proton mass difference given in Eq. (24), i.e.

$$\frac{\Delta B_D}{B_D}=\frac{\Delta\Lambda_{QCD}}{\Lambda_{QCD}}+\frac{\Delta f}{f}. \tag{34}$$

In Fig. 7 we superimpose their limits on B_{D} upon our results from Fig. 5. Beane and Savage only considered pion masses smaller than its current value so while the two sets of curves seem to overlap in a small area close to $\Delta\delta m_{np}/\delta m_{np}=\Delta B_{\mathrm{D}}/B_{\mathrm{D}}=0$ a much larger overlap might be expected if heavier pions were considered. The figure shows that even with the large systematic uncertainty in deuterium BBN can rule out much of the region of increases in δm_{np} and B_{D} up to $\Delta B_{\mathrm{D}}/B_{\mathrm{D}}=0.5$ because it is incompatible with the B_{D}-δm_{np} relationship from Beane and Savage despite the large uncertainty also found in that calculation. New limits will be found when heavier pions are considered, and also when we have sufficient information to explore $\Delta B_{\mathrm{D}}/B_{\mathrm{D}}>0.5$.

V. CONCLUSIONS

BBN presents a golden opportunity to study possible changes in the fundamental constants of nature, particularly those related to the structure of nuclei and their interactions,

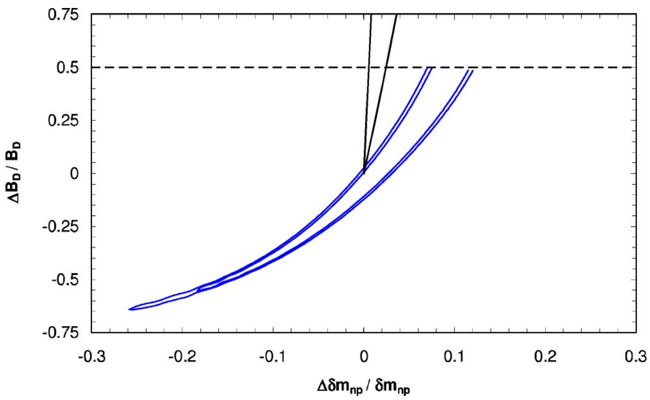

FIG. 7. The 95% and 99% χ^2 contours as in Fig. 5 plus the limits upon B_{D} from [26,27] after changing the variable from the pion mass to the neutron-proton mass difference. Beane and Savage only considered lighter pions and the overlap of the two sets of curves occurs is minimal; a larger overlap would be expected if heavier pions had been used.

at one of the earliest epochs of the Universe. We have examined the impact of variations in one of these constants, Λ_{QCD}, upon the predictions for the primordial deuterium abundance and helium-4 mass fraction. A change in Λ_{QCD} will manifest itself through shifts in the properties of the nuclei such as the neutron-proton mass difference and the deuteron binding energy. We have shown how these two parameters are crucial for determining the predictions for D and ^{4}He and how we can simplify BBN to the extent that it is a function of only these two quantities. While the simplification gives very good predictions for the mass fraction of ^{4}He when compared to a standard BBN code the results for D were offset by 50%. This large systematic error in deuterium swamped the statistical error associated with the observation allowing the degeneracy between δm_{np} and B_{D} in the prediction of helium-4 to show through. By simulating the case when the systematic error in the prediction for deuterium can be removed we found that BBN can limit their variation to the 10% level. In order to make BBN a better probe of the time variation of Λ_{QCD}, we need to know, in particular, the dependence of the ^{3}He and T (and ^{4}He) binding energies on the pion mass and the dependence of all the binding energies in the cross sections. Even without this input, much stronger constraints are obtained when the result of the BBN calculation is combined with the results of the Beane and Savage calculations for the deuteron binding energy. If Λ_{QCD} is the only constant that varies with time, and we only consider decreases in the pion mass, then in order to be compatible with the results from Beane and Savage the variations of B_{D} and δm_{np} are limited to $\Delta\delta m_{np}/\delta m_{np} \leqslant 0.002$ and $\Delta B_{\mathrm{D}}/B_{\mathrm{D}} \leqslant 0.04$.

ACKNOWLEDGMENTS

The authors would like to thank Rebecca Surman, Martin Savage and Silas Beane for useful discussions. This work was supported by the U.S. Department of Energy under grant DE-FG02-02ER41216.

[1] P.A.M. Dirac, Nature (London) **139**, 323 (1937).
[2] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[3] J.K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[4] J.N. Bahcall, C.L. Steinhardt, and D. Schlegel, astro-ph/0301507.
[5] P. Langacker, G. Segre, and M.J. Strassler, Phys. Lett. B **528**, 121 (2002).
[6] X. Calmet and H. Fritzsch, Phys. Lett. B **540**, 173 (2002).
[7] X. Calmet and H. Fritzsch, Eur. Phys. J. C **24**, 639 (2002).
[8] J.J. Yoo and R.J. Scherrer, Phys. Rev. D **67**, 043517 (2003).
[9] L. Bergstrom, S. Iguri, and H. Rubinstein, Phys. Rev. D **60**, 045005 (1999).
[10] P.P. Avelino *et al.*, Phys. Rev. D **64**, 103505 (2001).
[11] K.M. Nollett and R.E. Lopez, Phys. Rev. D **66**, 063507 (2002).
[12] V.V. Flambaum and E.V. Shuryak, Phys. Rev. D **65**, 103503 (2002).
[13] V.V. Flambaum and E.V. Shuryak, Phys. Rev. D **67**, 083507 (2003).
[14] Y. Fujii *et al.*, hep-ph/0205206.
[15] K. Ichikawa and M. Kawasaki, Phys. Rev. D **65**, 123511 (2002).
[16] B.A. Campbell and K.A. Olive, Phys. Lett. B **345**, 429 (1995).
[17] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
[18] J. Bernstein, L.S. Brown, and G. Feinberg, Rev. Mod. Phys. **61**, 25 (1989).
[19] S. Sarkar, Rep. Prog. Phys. **59**, 1493 (1996).
[20] M.S. Smith, L.H. Kawano, and R.A. Malaney, Astrophys. J., Suppl. Ser. **85**, 219 (1993).
[21] R. Esmailzadeh, G.D. Starkman, and S. Dimopoulos, Astrophys. J. **378**, 504 (1991).
[22] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[23] E.M. Henley and G.A. Miller, Nucl. Phys. **A518**, 207 (1990).
[24] T. Dent and M. Fairbairn, Nucl. Phys. **B653**, 256 (2003).
[25] M. Gell-Mann, R.J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[26] S.R. Beane and M.J. Savage, Nucl. Phys. **A713**, 148 (2003).
[27] S.R. Beane and M.J. Savage, Nucl. Phys. **A717**, 91 (2003).
[28] E. Epelbaum, U.G. Meissner, and W. Glockle, Nucl. Phys. **A714**, 535 (2003).
[29] D. Dicus *et al.*, Phys. Rev. D **26**, 2694 (1982).
[30] D.H. Wilkinson, Nucl. Phys. **A377**, 474 (1982).
[31] S. Esposito *et al.*, Nucl. Phys. **B540**, 3 (1999).
[32] J.W. Chen and M.J. Savage, Phys. Rev. C **60**, 065205 (1999).
[33] G. Rupak, Nucl. Phys. **A678**, 405 (2000).
[34] W.A. Fowler, G.R. Caughlan, and B.A. Zimmerman, Annu. Rev. Astron. Astrophys. **5**, 525 (1967).
[35] G. Steigman, D.N. Schramm, and J.R. Gunn, Phys. Lett. **66B**, 202 (1977).

[36] J. Yang, D.N. Schramm, G. Steigman, and R.T. Rood, Astrophys. J. **227**, 697 (1979).
[37] J.P. Kneller, R.J. Scherrer, G. Steigman, and T.P. Walker, Phys. Rev. D **64**, 123506 (2001).
[38] J.P. Kneller and G. Steigman, Phys. Rev. D **67**, 063501 (2003).
[39] X. Chen, R.J. Scherrer, and G. Steigman, Phys. Rev. D **63**, 123504 (2001).
[40] A. Serna and R. Dominguez-Tenreiro, Phys. Rev. D **48**, 1591 (1993).
[41] A. Serna, R. Dominguez-Tenreiro, and G. Yepes, Astrophys. J. **391**, 433 (1992).
[42] R. Bean, S.H. Hansen, and A. Melchiorri, Phys. Rev. D **64**, 103508 (2001).
[43] S.M. Carroll and M. Kaplinghat, Phys. Rev. D **65**, 063507 (2002).
[44] G. Beaudet and A. Yahil, Astrophys. J. **218**, 253 (1977).
[45] R.V. Wagoner, W.A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3 (1967).
[46] H.-S. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
[47] K. Kohri, M. Kawasaki, and K. Sato, Astrophys. J. **490**, 72 (1997).
[48] K.A. Olive *et al.*, Phys. Lett. B **265**, 239 (1991).
[49] R.J. Scherrer, Mon. Not. R. Astron. Soc. **205**, 683 (1983).
[50] A. Yahil and G. Beaudet, Astron. Astrophys. **206**, 415 (1976).
[51] V. Barger *et al.*, Phys. Lett. B **566**, 8 (2003).
[52] R. Foot, Phys. Rev. D **61**, 023516 (2000).
[53] M. Kaplinghat, G. Steigman, and T.P. Walker, Phys. Rev. D **61**, 103507 (2000).
[54] E.W. Kolb and R.J. Scherrer, Phys. Rev. D **25**, 1481 (1982).
[55] X. Shi, D.N. Schramm, and B.D. Fields, Phys. Rev. D **48**, 2563 (1993).
[56] X. Shi, G.M. Fuller, and K. Abazajian, Phys. Rev. D **60**, 063002 (1999).
[57] W.H. Press *et al.*, *Numerical Recipes in C*, 2nd ed. (Cambridge University Press, Cambridge, 1988).
[58] R.E. Lopez and M.S. Turner, Phys. Rev. D **59**, 103502 (1999).
[59] V. Barger *et al.* (unpublished).
[60] K.A. Olive, G. Steigman, and T.P. Walker, Phys. Rep. **333**, 389 (2000).
[61] Y.I. Izotov, T.X. Thuan, and V.A. Lipovetsky, Astrophys. J., Suppl. Ser. **108**, 1 (1997).
[62] K.A. Olive and G. Steigman, Astrophys. J., Suppl. Ser. **97**, 49 (1995).
[63] K.A. Olive, E.D. Skillman, and G. Steigman, Astrophys. J. **483**, 788 (1997).
[64] B.D. Fields and K. Olive, Astrophys. J. **506**, 177 (1998).
[65] Y.I. Izotov and T.X. Thuan, Astrophys. J. **500**, 188 (1998).
[66] D. Kirkman *et al.*, astro-ph/0302006.
[67] D. N. Spergel *et al.* (unpublished).
[68] M. Kaplinghat, R.J. Scherrer, and M.S. Turner, Phys. Rev. D **60**, 023516 (1999).
[69] S. Hannestad, Phys. Rev. D **60**, 023515 (1999).